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A simplified method for bearing capacity analysis of energy piles integrating temperature-dependent model of soil-water characteristic curve

Tuan A. Pham [a],

[a] Heriot-Watt University, School of Energy, Geoscience, Infrastructure and Society, EH14 4AS, Edinburgh, UK

Corresponding email: ta.pham@hw.ac.uk;

Present address: William Arrol Building, Heriot-Watt University, Edinburgh, EH14 4AS, UK

Melis Sutman [a]

[a] Heriot-Watt University, School of Energy, Geoscience, Infrastructure and Society, EH14 4AS, Edinburgh, UK

Email: m.sutman@hw.ac.uk;

Present address: William Arrol Building, Heriot-Watt University, Edinburgh, EH14 4AS, UK

Abstract

The bearing resistance of energy piles in the presence of temperature effect has not been thoroughly investigated, preventing the perfecting of energy pile design methods. Quantifying the relationship between soil suction and the temperature of unsaturated soils therefore becomes an important step in predicting the bearing resistance of energy piles. A new constitutive model based on interfacial energy and thermodynamic theories is therefore presented to predict the effect of temperature on soil suction as well as the soil-water characteristic curve (SWCC) in this paper. The analytical model for the nonisothermal matric suction was developed by combining five different temperature-dependent functions for the surface tension, air-water contact angle, void ratio, and thermal expansion of solid and water density, thereby providing a more complete approach than the one that considers surface tension only. The proposed formulation is expressed under a simplified form which is believed as a useful and convenient tool to apply to a range of possible field situations. The temperature-dependent relationship of soil suction is then used to extend existing isothermal SWCCs to nonisothermal conditions that allow obtaining the SWCC at any temperature. The validity of the proposed model is verified by comparison to several test data sets for five different soils: swelling clay, hard clay, clayey-silty soil, ceramic material, and sand. The satisfactory agreement between predicted and measured curves proved that the proposed model has a good performance in predicting the effect of temperature on the SWCCs of unsaturated soils. The nonisothermal SWCC model was then coupled with the bearing resistance theory to produce a simplified method for analysis of energy piles. The results show that the proposed method successfully predicted pile resistance at various temperatures when compared to experimental data. The pile resistance reduces as the temperature rises for a specific degree of saturation or if the soil is in an undrained condition. However, water evaporation may cause a decrease in water content and an increase in matric suction as the temperature increases. Therefore, as soils dry out, pile resistance may increase with increasing temperature.

Keywords: energy pile; soil-water characteristic curve; unsaturated soil; soil suction; thermodynamics; design method; bearing capacity

1. Introduction

Energy piles are a relatively new and cost-effective method of obtaining underground geothermal energy that has recently gained popularity (Behbehani and McCartney 2022). The design of the energy pile structure needs to be not only safe but also economical. Regarding the latter aspect, less attention has yet been paid to the strengthening role of interparticle capillary forces in unsaturated conditions on bearing resistance enhancement. In fact, energy piles are frequently designed under the assumption of saturated soils (Song and Pei 2022; Elzeiny et al. 2020; Ravera et al. 2020; Faizal et al. 2019; Di Donna et al. 2016). However, in some semiarid to arid areas, groundwater tables are typically found at a depth below the ground surface, leading to unsaturated soil layers along the length of the pile. Furthermore, during the summer operation of energy piles, soil temperature increases due to heat injection into the ground. Temperature increases lead to a decrease in the viscosity and density of the water in soil pores, resulting in the water to flow away from the heat source. The migration of the water leads to the soil around the pile becoming unsaturated, causing the change in friction resistance of the energy piles.

However, because of the air-water interaction phenomena, also known as matric suction, the challenge related to bearing resistance estimation of structures in unsaturated soils becomes more complicated. The term "matric suction" refers to the difference between air pore pressure and water pore pressure in soils, which is determined by the amount of water in the pore structure (Pham and Sutman 2022a). The constitutive relationship between the water amount in the soil and matric suction is called the soil-water characteristics curve (SWCC), or soil-water retention curve (SWRC). Because the shear strength of unsaturated soils is greatly influenced by the matric suction and degree of saturation, variation in SWCC causes changes in the bearing resistance of the structure. Particularly, many experimental results from full-scale field tests, laboratory model tests, and temperature-controlled direct shear tests showed that the SWCC varies significantly with temperature (Pham et al. 2023a; Pham and Sutman 2022b; Kelishadi et al. 2018; She and Sleep 1998; Constantz 1991), which may lead to uncertainty in estimating the bearing resistance of energy pile. Unfortunately, the bearing resistance of energy piles in the presence of temperature effect has not been thoroughly investigated, preventing the perfecting of energy pile design methods. It is therefore important to predict the effect of temperature on SWCC and to be able to quantify changes in the bearing resistance of energy piles.

In recent years, several researchers have contributed to a better understanding of the relationship between SWCC and temperature. A number of non-isothermal models have been proposed by considering only surface tension as a temperature-dependent function (Romero et al. 2001; Salager et al. 2010; Roshani and Sedano 2016; Liu et al. 2020). Several other authors dedicated efforts to taking further into account the effect of temperature on the air-water contact angle (Grant and Salehzadeh 1996; Bachmann et al. 2002; Vahedifard et al. 2018). However, several gaps and limitations among existing models need to be filled and improved. Firstly, it is not sufficient to consider the air-water surface tension as the only temperature-dependent variable and ignore the temperature influence on other parameters. Many experimental results suggest that the dependent relationship between the matric suction and temperature should be considered through various factors such as surface tension, contact angle, void ratio, and pore size (Pham et al. 2023a). Besides, almost all existing models use a linear form to describe the relationship between surface tension and temperature. This existing drawback probably comes from the fact that the temperature-dependent function of surface tension has been established eighty years ago for a low-temperature range. Yet, the measured data proved that the relationship between matric suction and temperature is a nonlinear function, particularly at high temperatures (Vargaftik et al. 1983; Saito et al. 2006). Another limitation is that not enough attention was paid to the effect of the thermal expansion of water and the influence of the thermal volume change of soils on the matric suction when the temperature is increased. It is obvious that when the temperature effect is not addressed thoroughly, the temperature change will continue to remain a great challenge in the field of geotechnical engineering.

The main aim of this paper is to present an analytical model for predicting the effect of temperature on the bearing resistance of energy piles. Firstly, the nonisothermal formulation for the matric suction is presented under a reduced form which is believed to be a useful and comfortable tool to apply to a range of possible field situations. Five temperature-dependent functions accounting for the temperature effects on (i) surface tension, (ii) air-water contact angle, (iii) void ratio, (iv) thermal expansion of solids, and (v) thermal expansion of water phase are incorporated in the proposed model. The temperature-dependent relationship of matric suction is then extended to produce a nonisothermal SWCC equation for unsaturated soils. The validity of the proposed model is verified by comparing it to several experimental data sets. Finally, the nonisothermal SWCC is extended to combine with the bearing resistance theory to produce a new design method for energy piles, which is applicable for piles in both saturated and unsaturated soils. The proposed approach is expected to complement current energy pile design approaches and provide a useful tool for determining the thermomechanical behaviour of energy piles.

2. Temperature-dependent model of matric suction

2.1. Model assumptions

As a starting point, the actual case is simplified by postulating a soil consisting of uniform rigid spheres in regular packing, which provides convenience to apply mathematical laws, allowing precise theoretical deductions to be made. The simplified arrangement was also adopted commonly in many previous studies for both granular soils (Haines 1925; Fisher 1926; Erle et al. 1971; Arya and Paris 1981; Snyder and Miller 1985; Lian et al. 1993; Willett et al. 2000; Soulie et al. 2006; Richefeu et al. 2008; Pham 2020a) and fine-grained soils (Cho and Santamarina 2001; Likos and Lu 2004; Rojas 2008; Zhou et al. 2016; Khorshidi et al. 2017; Pham 2022b). However, it should be noted that spherical soils are solely used for illustrative purposes while all the following formulas are obtained using the volume relationship. Moreover, this assumption is also minimized through the parameter $\boldsymbol{\delta}$ which reflects the effect of actual particle sizes, shapes, and orientations in a natural soil sample. Finally, the proposed model is subject to predict non-isothermal SWCC based on the reference SWCC at room temperature fitted based on the measured data. The effect of soil characteristics is included in the reference SWCC and is thus taken into account in the proposed model. As a result, the proposed model is suitable for both granular and fine-grained soils.

In this study, T_0 is assumed to represent reference temperature while T is defined as any current temperature. The reference temperature is defined as the initial temperature of the soil sample during matric suction measurement in the laboratory and is often the same as room temperature. For the present purpose, variables at reference temperature are given symbols with subscript zero.

2.2. Surface tension phenomenon

Figure 1 illustrates an ideal model with systematic packing of soil grains. In the considered model, the water is confined to annuli of a wedge-shaped cross-section around the points of contact of the spheres. A film membrane that appears between the air phase and the water phase is so-called the meniscus. Figure 2 defines the variables in the idealized model. The neck of fluid connecting the two spheres is considered to have a radius of R_n, and the meridian curve or the curvature of the surface film is considered to have a radius of R_c. According to the geometry of the water-air interface model between two soil particles, the total meniscus curvature (R) corresponding to the specific moisture content is determined by the following expression:

$$\frac{1}{R} = \frac{1}{R_c} - \frac{1}{R_n} \tag{1}$$

The radius R_c and R_n can be expressed as follows, in terms of particle radius (r) and air-water contact angle (α):

$$R_c = r. tan\, \alpha\, . tan(\alpha/2) \tag{2}$$

$$R_n = r. tan\, \alpha\, . (1 - tan(\alpha/2)) \tag{3}$$

It should be noted that the surface tension causes the contractile skin, which behaves as an elastic membrane and is subjected to different pressures on each side (Pham et al. 2023a). The equilibrium state of the curvature membrane is established by air-pore pressure, water-pore pressure, and surface tension, as shown in Figure 3. Vertical pressure equilibrium along the curvature membrane gives:

$$\psi = u_a - u_w = \frac{2\sigma_s}{R} \tag{4}$$

where ψ = matric suction in soils, u_a = air-pore pressure, u_w = water-pore pressure, σ_s= air-water surface tension.

Replacing Eqs. (1) to (3) into Eq. (4), the matric suction can be expressed as a function of air-water contact angle:

$$\psi = \frac{\sigma_s}{r} . \frac{1 - tan(\alpha/2) - 2\, tan^2(\alpha/2)}{tan^2(\alpha/2)} \tag{5}$$

2.3. Determination of wetting coefficient

A quadratic equation representing the relationship between air-water contact angle, matric suction, and particle radius are derived from Eq. (5) as follows:

$$(r\psi + 2\sigma_s).t^2 + \sigma_s.t - \sigma_s = 0 \tag{6}$$

$$t = tan(\alpha/2) \tag{7}$$

The solution of the quadratic equation (6) should be a positive value. Therefore,

$$t = \frac{-\sigma_s + \sqrt{{\sigma_s}^2 + 4\sigma_s(r\psi + 2\sigma_s)}}{2(r\psi + 2\sigma_s)} \tag{8}$$

The wetting coefficient is directly calculated by the following relations:

$$cos\,\alpha = \frac{1-t^2}{1+t^2} \tag{9}$$

Figure 4 illustrates the relationship between the matric suction, air-water contact angle, and particle size. It is noted that the air-water contact angle decreases with increasing matric suction and approaches zero at a very high value of matric suction. This tendency agrees well with the Kevin-Laplace equation that maximum air-water contact angle is obtained at minimum matric suction. It is also interesting to observe that the influence of matric suction on the air-water contact angle curve depends on particle size. As noted, the dependence of air-water contact angle on matric suction becomes more significant with a decrease in particle size.

2.4. Thermodynamic behaviour of unsaturated soils.

In porous media, the mechanical and thermodynamic equilibrium is often derived from the relationship between capillary potential and the free energy of soil moisture. According to Edlefsen and Anderson (1943), the matric suction or capillary pressure is determined by the following relation:

$$\frac{\psi}{g.\rho_w} = \frac{2.(\sigma_{sa} - \sigma_{sw})}{g.\rho_w.r_i} \tag{10}$$

where σ_{sa} = interfacial tension along with the solid-air interface, σ_{sw} = interfacial tension along with the solid-water interface, r_i= mean pore radius, g = gravity acceleration, ρ_w = water density.

However, accurate measurements of the interfacial tensions σ_{sa} and σ_{sw} are quite difficult, and Eq. (10) thus must be recast with parameters that can be obtained experimentally. Unsaturated soils are considered as an interaction system between air-water-solid phases. Three interfacial tensions appear to correspond to three interfaces as shown in Figure 5. It is assumed that three solid-air-water phases intersect at point A, and the derivative of the following relation is obtained:

$$\sigma_s.cos\,\alpha = \sigma_{sa} - \sigma_{sw} \tag{11}$$

On the other hand, with the assumption that the soil volume can be approximated as the total volume of spherical particles while the total pore volume can be approximated as the volume of cylindrical capillary tubes, Pham et al. (2023b) established the relationship between pore radius and particle radius as follows:

$$r_i = r.\sqrt{\frac{4e_0{n_s}^{1-\delta}}{6}} \tag{12}$$

where e_0 = initial void ratio of soils, n_s = number of soil particles, δ = empirical constant to consider the effect of particle shape and size.

Combining Eqs. (10) to (12), the matric suction based on the thermodynamic approach can be obtained:

$$\frac{\psi}{g.\rho_w} = \frac{\sigma_s.\cos\alpha}{g.\rho_w.r.\sqrt{e_0 n_s^{1-\delta}/6}} \tag{13}$$

Equation (13) implies that the matric suction can vary by temperature through variations in surface tension, contact angle, particle radius, void ratio, and water density as follows:

$$\frac{\partial\psi}{\partial T} = \frac{\psi}{\sigma_s}.\frac{\partial\sigma_s}{\partial T} + \frac{\psi}{\cos\alpha}.\frac{\partial\cos\alpha}{\partial T} - \frac{\psi}{r}.\frac{\partial r}{\partial T} - \frac{\psi}{e}.\frac{\partial e}{\partial T} - \frac{\psi}{\rho_w}.\frac{\partial\rho_w}{\partial T} \tag{14}$$

2.5. Temperature-dependent function of matric suction

The matric suction at two different states, namely, the reference temperature (room temperature), and the current temperature are re-written respectively as follows:

$$\frac{\psi_0}{g.\rho_w} = \frac{\sigma_{s0}.\cos\alpha_0}{\rho_{w0}.r_0.\sqrt{e_0{n_{s0}}^{1-\delta}/6}} \tag{15}$$

$$\frac{\psi_T}{g.\rho_w} = \frac{\sigma_{sT}.\cos\alpha_T}{\rho_{wT}.r_T.\sqrt{e_T{n_{sT}}^{1-\delta}/6}} \tag{16}$$

where ψ_0 = matric suction at the reference temperature, ψ_T = matric suction at the current temperature, r_0= particle radius at the reference temperature, r_T = particle radius at the current temperature considering thermal expansion, e_0= initial void ratio at the reference temperature, e_T= void ratio at the current temperature considering thermal volume change, n_{s0} = number of soil particles at the reference temperature, n_{sT} = number of soil particles at the current temperature, ${\sigma_s}_0$= air-water surface tension at the reference temperature, σ_{sT}= air-water surface tension at the current temperature, α_0= air-water contact angle at the reference temperature, and α_T= air-water contact angle at the current temperature, ρ_{w0} = water density at the reference temperature, ρ_{wT} = water density at the current temperature.

Dividing Eq. (16) by Eq. (15), the temperature-dependent function of matric suction is derived:

$$\psi_T = \psi_0 \times f_\sigma \times f_\alpha \times f_r \times f_e \times f_\rho \tag{17}$$

$$f_\sigma = \frac{\sigma_{sT}}{\sigma_{s0}} \tag{18}$$

$$f_\alpha = \frac{\cos \alpha_T}{\cos \alpha_0} \tag{19}$$

$$f_r = \frac{r_0}{r_T} \tag{20}$$

$$f_e = \sqrt{\frac{e_0}{e_T} \cdot \left(\frac{n_{s0}}{n_{sT}}\right)^{1-\delta}} \tag{21}$$

$$f_\rho = \frac{\rho_{w0}}{\rho_{wT}} \tag{22}$$

where f_σ is a surface tension factor that represents the dependence of surface tension on temperature, f_α is a contact angle factor that represents the dependence of air-water contact angle on temperature, f_r is a particle size factor that represents the dependence of particle radius on temperature, f_e is a void ratio factor that represents the dependence of void ratio on temperature, f_ρ is a water density factor that represents the thermal expansion of the water phase.

Equation (17) emphasizes that an increase in temperature causes the concurrent change of surface tension, contact angle, particle radius, void ratio, water density leading to the temperature-dependent definition of matric suction. Another significance of Eq. (17) is allowing to consider the relation between the thermal expansion of particles and void ratio with matric suction.

Dependence of surface tension on temperature

Surface tension is defined as the tensile force per unit length of the air-water interface. Almost all existing models concentrate on considering the relationship between matric suction and temperature through the function of surface tension. Several studies proposed empirical equations to predict the surface tension with temperature (Andelfsen and Anderson 1943; Dorse 1940; Harr et al. 1984; Romero et al. 2001; Saito et al. 2006**).** Because these equations are empirical in nature and were derived from a previously measured data range, two main limitations were identified in these expressions. Firstly, these equations were established based on a low range of temperature between 0°C and 40°C. However, in many geo-environmental applications, the soil may be exposed to temperatures above 40°C. For example, this is truly the case for geothermal structures with temperatures up to 60°C (Amatya et al. 2012), engineered barrier systems with temperatures up to 80°C (Imbert et al. 2005), nuclear waste storage with temperatures up to 100°C (Romero et al. 2001; Delage et al. 2013), and heat storage with temperatures up to 300°C (Zheng et al. 2015). Another drawback is that almost all available equations use a linear form to describe temperature-dependent surface tension. However, experimental data showing surface tension values up to 100°C exists in the literature (Kaye and Laby 1966). Moreover, Vargaftik et al. (1983) presented other sets of data for the surface tension up to 200°C, which presents a nonlinear form.

Therefore, by using the regression analysis technique, the following equation is proposed for the temperature-dependent function of surface tension:

$$\sigma_{sT} = (96.76 - 0.0125 \times T - 0.000238 \times T^2) \times 10^{-3} \qquad \text{(N/m)} \qquad (23)$$

where, T = current absolute temperature in degree Kelvin (K).

The surface tension of the air-water interface at the reference temperature is expressed as follows:

$$\sigma_{s0} = (96.76 - 0.0125 \times T_0 - 0.000238 \times T_0^2) \times 10^{-3} \qquad \text{(N/m)} \qquad (24)$$

where, T_0 = reference absolute temperature in degree Kelvin (K).

Figure 6 shows a comparison between predicted and measured values of surface tension with temperature. It is observed that the proposed equation agrees better with the measured data, compared to the other equations, for the entire range of considered temperature. Other equations agree relatively well with measured data until 40°C but differ significantly as the temperature increases above this range. As mentioned earlier, these equations were established from the limited data range of temperature below 40°C, which can be considered the reason for this error.

Replacing Eqs. (23) and (24) back into Eq. (17), the surface tension factor f_σ is derived as follows:

$$f_\sigma = \frac{96.76 - 0.0125 \times T - 0.000238 \times T^2}{96.76 - 0.0125 \times T_0 - 0.000238 \times T_0^2} \qquad (25)$$

Dependence of contact angle on temperature

Grant and Salehzadeh (1996) assumed that a change in interfacial energy equals a change in interfacial tension. Based on the interfacial energy approach, the following expression is stated:

$$-\Delta h = \sigma_{sT} . \cos \alpha_T - T \frac{d(\sigma_{sT} \cos \alpha_T)}{dT} \qquad (26)$$

where Δh = immersion enthalpy per unit area at temperature T.

Rearranging Eq. (26), the expression for the temperature dependence of air-water contact angle can be obtained:

$$\frac{d \cos\alpha_T}{dT} = \frac{\cos\alpha_T}{T} - \frac{\cos\alpha_T}{\sigma_{sT}} . \frac{d\sigma_{sT}}{dT} + \frac{1}{T} . \frac{\Delta h}{\sigma_{sT}} \qquad (27)$$

The solution of the differential equation (27) allows the determination of the wetting coefficient as well as the air-water contact angle as a function of temperature. However, the immersion enthalpy (Δh) is an empirical parameter that is difficult to measure and has not been adopted widely. Furthermore, the majority of published enthalpies of immersion were for SiO_2, which might not be appropriate for other materials (Grant and Salehzadeh 1996; Bachmann et al. 2002). It is therefore expected to simplify the calculation problem and limit the effect of the immersion enthalpy parameter using the largest boundary condition.

For the sake of simplicity, Eq. (23) can be re-written as follows:

$$\sigma_{sT} = \lambda_1 T^2 + \lambda_2 T + \lambda_3 \tag{28}$$

where $\lambda_1, \lambda_2, \lambda_3$ are constants with $\lambda_1 = -0.000238 \times 10^{-3}$; $\lambda_2 = -0.0125 \times 10^{-3}$; $\lambda_3 = 96.76 \times 10^{-3}$

At the largest immersion enthalpy per unit area, $\Delta h = \Delta h_{max}$, the change in the wetting parameter approaches zero:

$$\frac{d\cos\alpha_T}{dT} = 0 \tag{29}$$

$$\frac{d\sigma_{sT}}{dT} = 2\lambda_1.T + \lambda_2 \tag{30}$$

Substituting Eqs. (29) and (30) back into Eq. (26), and rearranging gives:

$$\Delta h_{max} = \omega.(\lambda_1 T^2 - \lambda_3).cos\alpha_T \tag{31}$$

Replacing Eq. (31) into Eq. (27) and conducting an integration procedure for temperature range from T_0 to T, a solution can be derived as follows:

$$cos\ \alpha_T = \frac{\sigma_{sT}.T_0}{\sigma_{s0}.T - \omega.(\lambda_1 T^2 - \lambda_3).(T - T_0)} cos\ \alpha_0 \tag{32}$$

where, ω is interface energy-related coefficient that takes into account the effects of several complicated aspects in plastic clays. It should be noted that coefficient ω was also added to compensate for the above-mentioned simplified assumption. The coefficient ω can be easily calculated by doing calibration with experimental data, which requires at least two data sets corresponding to two different temperatures. Otherwise, ω can be assumed to be 1 for the ideal assumption.

Using Eq. (32), the relationship between air-water contact angle and temperature is demonstrated in Figure 7. It is interesting to note that the air-water contact angle increases with increasing temperature under a highly nonlinear relationship. Moreover, it is observed that the increase in air-water contact angle with temperature becomes more significant for the lower initial contact angle. Such an increase is estimated to be from 41% at an initial contact angle of 60° to 441% at an initial contact angle of 15° when the temperature increases from 15°C to 200°C.

The wetting coefficient $cos\ \alpha_0$ at the reference, the temperature can be determined using Eq. (9). In this equation, the matric suction ψ_T is replaced by the value of matric suction at the reference temperature ψ_0 which is usually measured from laboratory tests.

Replacing Eq. (32) back into Eq. (19), the air-water contact angle factor f_α is derived:

$$f_\alpha = \frac{(\lambda_1 T^2 + \lambda_2 T + \lambda_3).T_0}{\sigma_{s0}.T - \omega.(\lambda_1 T^2 - \lambda_3).(T - T_0)} \tag{33}$$

Dependence of particle radius on temperature

The matric suction is known as a dependent function of the particle size. The volume of soil particles is considered to expand with increasing temperature, and therefore the particle radius is also changed (Nayak and Preetham 2020). The thermal expansion of a solid particle is considered by the equation below:

$$\frac{\Delta V}{V_{s0}} = \alpha_s . \Delta T \tag{34}$$

where ΔV = volume increment of the solid particle due to temperature, V_{s0}= original volume of the solid particle at a reference temperature, α_s= coefficient of volumetric thermal expansion of solid particle ($\alpha_s \approx 35\ \mu\varepsilon/℃$), ΔT = temperature increment.

Considering the soil particles to have a spherical shape, the relation between particle radius and temperature is as follows:

$$r_T = r_0 \sqrt[3]{1 + \alpha_s . \Delta T} \tag{35}$$

Replacing Eq. (35) into Eq. (20), the particle size factor is derived:

$$f_r = \frac{1}{\sqrt[3]{1+\alpha_s . \Delta T}} \tag{36}$$

Dependence of void ratio on temperature

The physical properties of soils are influenced by the relative proportions of solid to void phases. The overall considered volume of the soil sample (V) is equal to the sum of the solids (V_s) and void (V_v) volumes, that is:

$$V = V_s + V_v \tag{37}$$

The solid and void volumes can fluctuate at any temperature, but the overall considered volume is assigned to remain constant. The void volume change caused by temperature variations can therefore result in various void ratios. Because the void volume changes while the considered volume is the same, the number of particles needs to change to compensate for the change in solid volume (Pham et al. 2023b). As a result, the corresponding number of soil particles for each different void ratio must be different. Moreover, by assuming that the pore volume can be approximated by the assemblage of particles, void volume variation (density change) will require the number of soil particles to change. The relationship between the number of soil particles and the void ratio at reference and current temperatures is expressed as:

$$n_{s0} = \frac{V}{V_{si}^{0}} . \frac{1}{1+e_0} \tag{38}$$

$$n_{sT} = \frac{V}{V_{si}^{T}} . \frac{1}{1+e_T} \tag{39}$$

Where V_{si}^0 = volume of a soil particle at the reference temperature, V_{si}^T = volume of a soil particle at the current temperature.

Dividing side-by-side of Eq. (38) to (39) gives:

$$\frac{n_{s0}}{n_{sT}} = \frac{1+e_T}{1+e_0}.\frac{V_{si}^T}{V_{si}^0} = \frac{1+e_T}{1+e_0}.(1 + \alpha_s.\Delta T) \tag{40}$$

Substitute Eq. (40) back into Eq. (21), and the void ratio factor is found as follows:

$$f_e = \sqrt{\frac{e_0}{e_T}.\left(\frac{1+e_T}{1+e_0}.[1 + \alpha_s.\Delta T]\right)^{1-\delta}} \tag{41}$$

In which,

$$e_T = e_0 \pm \Delta e_T \tag{42}$$

Where Δe_T = void ratio variation due to temperature, sign (+) is used if the soil is contracted while sign (-) is used if the soil is dilative by increasing temperature.

It should be noted that the significance of parameter δ in Eq. (41) is associated with considering the effect of actual particle shapes, sizes, and orientations in a natural soil sample. The value of δ can be determined empirically by calibration for two SWCC tests. Alternatively, Arya and Paris (1981) conducted tests on five different materials and found that $\boldsymbol{\delta}$ is arranged in value from 1.3 to 1.45.

On the other hand, it is generally known that soils tend to undergo a volume change when subjected to temperature change. Demars and Charles (1982) found that the void ratio variation of normally consolidated soils due to temperature fluctuation is directly related to soil plasticity, in which soils of high plasticity are more susceptible to thermal volume changes than soils of low plasticity. Based on the collection of test results, Pham and Sutman (2023) have been proposed an equation that describes the relationship between change in void ratio due to temperature cycle and plasticity index (PI) as follows:

$$\Delta e_T = [10^{-8}(PI)^2 + 3.10^{-6}(PI) + 0.0002](1 + e_0).\Delta T \tag{43}$$

Dependence of water density on temperature

Water density is defined as the mass-to-volume ratio of water. The density of water under isothermal conditions is commonly estimated to be 1000 kg/m^3 for most geotechnical engineering applications. However, due to water volume expansion, the density of water varies with temperature fluctuations. Using the regression analysis technique of the data from Lide (1995), the closed-form equation for the temperature-dependent component of water density is expressed as follows:

$$\rho_{wT} = (0.6582 + 0.002509T - 0.000004606T^2).10^3 \qquad (\text{kg/m}^3) \tag{44}$$

The water density factor can be obtained by substituting Eq. (44) in (22) as follows:

$$f_\rho = \frac{\rho_{wT}}{\rho_{w0}} = \frac{0.6582+0.002509T-0.000004606T^2}{0.6582+0.002509T_0-0.000004606T_0^2} \tag{45}$$

2.6 Combination of thermodynamic and interfacial energy theories

It should be noted that the temperature-dependent relationship of matric suction can be solved fully by the combination of Eqs. (17), (25), (33), (36), (41), and (45). This relationship was established using the interfacial energy approach and thermodynamic approach. Compared to several other models in this field, the proposed model has several advantages as stated in this section. Firstly, the surface tension was considered to be the main factor when analysing how temperature affects suction in the current studies (Romero et al. 2001; Salager et al. 2010; Roshani and Sedano 2016). In contrast, this study developed a new nonisothermal model by taking into account the surface tension, air-water contact angle, thermal void ratio, thermal expansion of particles, and water density when analysing the impact of temperature on suction. Secondly, the existing models treated the influence of temperature on surface tension as a linear function, whereas a nonlinear temperature-dependent function of surface tension is presented in this study. Furthermore, some approaches, like the model developed by Vahedifard et al. (2018), required input parameters with the initial contact angle and reference enthalpy of immersion. However, due to time-consuming and sophisticated physical instruments, as argued by Grant and Salehzadeh (1996), it is quite challenging to ascertain these factors. In contrast, the proposed model offered a logical method for calculating the initial contact angle. By applying the solution presented in this paper, a value of α_0 may now be precisely determined rather than being assumed. In particular, the proposed model also demonstrated that the dependence of contact angle on temperature did not relate to the enthalpy of immersion, making the model significantly easier to apply, which can be considered as the four main advantages.

Figure 8a demonstrates the contribution of five different components to the reduction of matric suction with temperature. It is noted that the air-water contact angle factor shows the most significant contribution while the particle expansion factor shows less contribution to the overall suction reduction factor. Unfortunately, the majority of the current models ignored the influence of temperature on the air-water contact angle, void ratio, particle size, and water density which possibly leads to an insufficient evaluation of the temperature effect on matric suction. Moreover, the analytical results also indicate a highly nonlinear relationship between the overall suction reduction factor and temperature. It is worth noting that the influence of temperature on matric suction reduction becomes less significant for the temperature range higher than 120°C. This is because the contact angle factor reduces slower with increasing temperature over 120°C. For example, when temperature increases from 10°C to 120°C (ΔT = 110°C), the matric suction decreases by 70%, or about 0.64 %/°C. Meanwhile, the matric suction only decreases by 20% when the temperature increases from 120°C to 200°C (ΔT = 80°C), which is approximately 0.25 %/°C.

Figure 8b shows the variation in the void ratio factor with increasing temperature at different plasticity indexes of soils. It is interesting to note that the influence of the void ratio factor is expected to be relatively large with increasing the temperature and plasticity index of soils while it will become less with increasing initial void ratio. The results also show that the void ratio factor changes nonlinearly with temperature, but the nonlinear level depends strongly on the properties of soils. Such when temperature increases from 10°C to 200°C, the void ratio factor is estimated to increase by 25% for soil with a plasticity index of 40, and 90% for soil with a plasticity index of 120.

Figure 9 shows the temperature-dependent relationship of matric suction with variation in pore size for three different cases considering (1) temperature-dependent surface tension only, (2) two temperature-dependent functions of surface tension and contact angle, and (3) all five temperature-dependent functions of surface tension, contact angle, particle expansion, void ratio, and water density. According to the results, the change in matric suction with temperature is less significant for case 1 where the effects of air-water contact angle, void ratio, thermal expansion of particle, and water were discarded. Accordingly, for a pore size radius of 1 mm, the reduction in matric suction is estimated to be 19.6% with increasing temperature from 15°C to 100°C. In cases 2 and 3, where temperature dependence of air-water contact angle, void ratio, and thermal expansion of particle and water is considered, the changes in matric suction with temperature are more significant as compared to case 1. For example, for a pore size radius of 1 mm, the reduction in matric suction is estimated to be 64.5% and 69.9% for cases 2 and 3, respectively, when the temperature is increased from 15°C to 100°C. The findings emphasize that the effects of air-water contact angle, void ratio, and thermal expansion of particle and water on the variation in matric suction with a temperature increase are more significant while this aspect was ignored in the majority of existing non-isothermal models.

3. Validation of the proposed model

To validate the proposed model, two sets of laboratory test data are selected for comparison in this section. The first data set is from Uchaipichat and Khalili (2009) who tested silty soil for a temperature range of 25°C to 60°C. The fundamental properties of the tested soils are as follows: Specific gravity G_s = 2.65, dry density γ_d = 1.53 g/cm^3, saturated water content w = 27.6%, and void ratio e_0 = 0.732. The second data set is from Liu et al. (2020) who tested clayey-silty sand by applying a temperature range between 10°C and 40°C. The fundamental properties of the tested soils are as follows: Specific gravity G_s = 2.67, dry density γ_d = 1.50 g/cm^3, saturated water content w = 28%, and void ratio e_0 = 0.75

Figure 10 shows the comparison between predicted and measured matric suction with different temperatures under constant water content for the data set of Uchaipichat and Khalili (2009). It should be emphasized that the proposed formula requires initial reading of matric suction at room temperature. In these data sets, the initial readings of the matric suctions at the reference temperatures were reported for two soil samples at 100 kPa and 300 kPa. The figure shows that the proposed model agrees well with the measured data in predicting the variation in matric suction with temperature. The largest relative errors between predicted and measured values are 10.1% and 12.9% for matric suctions at reference temperatures of 100 kPa and 300 kPa, respectively. However, the average relative errors between predicted and measured results for the entire range of applied temperatures were 4.7%, which yields an excellent agreement between the proposed model and the measured data.

A comparison between predicted and measured values of matric suction with variation in temperature and saturation degree for the data set of Liu et al. (2020) is presented in Figure 11. This data set is particularly interesting for comparison because the unsaturated soil samples were tested under a temperature range between 10°C and 40°C for three different saturation degrees (*S* = 60%, 65%, and 70%). According to the results, the proposed model matches well with measured data for the different values of temperature and saturation degree. The average relative errors between predicted and measured values are 2%, 3.7%, and 5.3% for the saturation degrees of 60%, 65%, and 70%, respectively. It is concluded that the proposed model has a good performance in predicting the effect of temperature variations on the matric suction of soils.

4. Nonisothermal equation for soil-water characteristic curve

Several well-known isothermal equations of SWCC are available in the literature for unsaturated soils. Among these existing equations, the SWCC equation of Fredlund and Xing (1994) was found to show a high-fitting performance (Leong and Rahardjo 1997; Zapata et al. 2000; Sillers and Fredlund 2001; Pham et al. 2023a). Therefore, the isothermal equation of Fredlund and Xing (1994) is selected to be extended to nonisothermal conditions. The main refinements are (i) the inclusion of correction factors and (ii) the application of the temperature-dependent model of matric suction.

It should be emphasized that the temperature-dependent matric suction function was established with a constant volumetric water content (no evaporation). Because of this, the volumetric water content at arbitrary temperatures is kept constant as that at the reference temperature, while the matric suction changes with temperature, in order to plot the SWCCs at various temperatures. The non-isothermal equation to predict SWCCs at different temperatures is thus proposed as follows:

$$\begin{cases} \theta_T = \theta_0 = \dfrac{C_f . \theta_s}{(ln[2.71828 + (\psi_0/a)^n])^m} \\ \psi_T = \psi_0 \times f_\sigma \times f_\alpha \times f_r \times f_e \times f_\rho \end{cases} \tag{46}$$

$$C_f = 1 - \frac{ln(1+\psi_i/\psi_r)}{ln(1+10^6/\psi_r)} \tag{47}$$

$$a = \psi_i \tag{48}$$

$$m = 3.67\, ln\left(\frac{\theta_s . C_i}{\theta_i}\right) \tag{49}$$

where θ_T = volumetric water content at the current temperature, θ_s = saturated volumetric water content, θ_i = volumetric water content at an inflection point on SWCC, β_w= thermal expansion coefficient of water, C_f = correction factor, ψ_i = matric suction at an inflection point on SWCC, ψ_r = matric suction corresponds to the residual volumetric water content, a and m = calculation parameters, n = fitting parameter which controls the slope of the SWCC.

The proposed model, however, mainly focused on the influence of matric suction without taking into account osmotic suction because matric suction contributes significantly greater to the shear strength of soils compared to osmotic suction. Additionally, the reference SWCC at room temperature must first be known in order to estimate the SWCC at any temperature. To plot the reference SWCC, laboratory tests must be performed at room temperature. Lastly, this study did not take into account the impact of thermally induced vapour diffusion.

5. Experimental validation of nonisothermal SWCC equation

In this section, the validation of the proposed SWCC equation is evaluated by conducting a comparison with published experimental results in the literature. The degree of curve match is considered a key criterion to evaluate the accuracy of the proposed SWCC equation (Pham 2020b; Pham and Dias 2021; Pham et al. 2023). The degree of curve match is defined as the agreement degree between measured and predicted results for a total number of considered points. It is assessed by the normalized sum of squared error (SSE) as follows:

$$SSE = \sum_{i=1}^{N}\left(\frac{\psi_{measured}-\psi_{predicted}}{\psi_{measured}}\right)^2 \tag{50}$$

where $\psi_{measured}$= measured suction corresponding to the volumetric water content of ith data, $\psi_{predicted}$ = predicted suction of ith data, N = total number of data available.

5.1 Summary of experimental data sets for comparison

For validation purposes, four well-documented experimental data sets corresponding to several different soil types were selected in this study. Salager et al. (2010) presented interesting test results for clayey-silty sands, which were compacted in order to obtain a dry density of 1.5 g/cm^3. The data of SWCC was presented under two different temperatures (20°C and 60°C). The second study is the experimental campaign of Romero et al. (2001) applied to swelling clay. Two soil samples with different densities (loose and dense soils) were tested at temperatures of 22°C and 80°C. The third experiment selected for comparison is by Laloui et al. (2013). Tests were performed on hard clay specimens with a high density of 2.06 g/cm^3 under temperatures of 21°C and 80°C. The last one is the experimental outcomes of Roshani and Sedano (2016). They conducted tests on two different soil types, namely, Unimin silica sand and superfine sand. The measured data of SWCC was reported for three different temperatures (4°C, 20°C, and 49°C). A summary of tested soil properties in the selected models is presented in Table 1.

To predict the soil-water characteristic curve at any temperature, the SWCC at the reference temperature is required in advance. The procedure to draw the SWCC at the reference temperature was presented in the previous section. Table 2 shows the fundamental parameters used to draw the SWCC for the examined soils at a reference temperature. Among all fundamental parameters, only the fitting-parameter n is varied to control the shape of SWCC while other remaining parameters can be derived from the measured data or estimation equations.

5.2 Comparison outcomes for swelling clays

Figure 12a shows a comparison between the predicted and measured SWCC of the loose swelling clay sample for two different temperatures. It is observed that the proposed equation generates the SWCC that matched well with measured data in describing the relationship between matric suction and volumetric water content. The normalized sum of squared error (SSE) between predicted and measured values at 20°C and 80°C were only 2.7% and 3%, respectively. It is also worthy to note that almost all measured points are located very close to the predicted SWCC for temperatures of 22°C and 80°C. It can be concluded that the proposed equation produces a good performance in predicting the SWCC with temperature for loose swelling clay.

A comparison between the predicted and measured SWCC of the dense swelling clay is shown in Figure 12b. It is noted that the results obtained from the proposed SWCC equation are in good agreement with the experimental data. The majority of the measured data points locate closely to the predicted SWCC. The normalized sum of squared error between predicted and measured values at 22°C and 80°C were 3.2% and 4.3%, respectively. It is also observed that the SWCC slope of dense soil (Fig. 12b) is significantly larger than the one of loose soil (Fig. 12a). Denser soils generally producing a higher suction than loose soils at the same degree of saturation can be considered as a reason behind this observation.

5.3 Comparison outcomes for hard clays

Figure 13 shows a comparison between the predicted and measured SWCC of the hard clay. It is observed that the predicted SWCC passes through the majority of measured data points, which yields an excellent match between the proposed model and experimental data. At the temperature of 80°C, the measured points distributed close to the predicted SWCC yield the conclusion that the proposed model shows a good performance in predicting matric suction with temperature change. According to the results, the SSE between predicted and measured values at 21°C and 80°C were 3.1% and 3.8%, respectively. It has been deduced from the experimental data that the residual point was reached at a very high value of matric suction which is much larger than the range suggested by Fredlund and Xing (1994). These results may derive from the fact that the tested soils were hard clay with very high density which is different from normally consolidated clayey soils tested previously.

5.4 Comparison outcomes for clayey-silty sand

Figure 14a demonstrates the measured and predicted SWCC of clayey-silty sand. An excellent agreement is observed between measured data and the predicted SWCC for both temperatures. It should be noted that the distribution of measured points is nearly on the predicted curves. It is therefore concluded that the proposed model demonstrated a good performance in predicting the change of SWCC with the temperature. The results indicate that the SSE between predicted and measured values at 20°C and 60°C were 0.89% and 1.1%, respectively. It is obvious that the prediction performance of the proposed model for clayey-silty sand is better as compared to swelling clays. Furthermore, the experimental results also showed that temperature has a significant influence on the matric suction of soils and generally the matric suction decreases with increasing temperature. A good agreement between the proposed model and measured data is also observed for the ceramic material, as shown in Figure 14b. The value of SSE, in this case, was only 2.25% which reveals a good prediction performance of the proposed model.

5.5 Comparison outcomes for sands

Figure 15a compares the measured and predicted SWCC of Unimin silica sand for three different temperatures (4°C, 20°C, and 49°C). At a temperature of 4°C, the proposed best-fit curve matched well with measured data when the majority of measured points are very close to the predicted curve. At temperatures of 20°C and 49°C, it is observed that the majority of measured points matched well with the predicted SWCC curve. The proposed model, therefore, shows a good performance in predicting the effect of temperature on SWCC. According to the results, the SSE between predicted and measured values are 2.7%, 6.3%, and 13.4% at 4°C, 20°C, and 49°C, respectively.

Figure 15b shows a comparison between the measured and predicted SWCC of super fine sand. The results obtained from the proposed equation are in satisfactory agreement with measured data for three different temperatures (4°C, 20°C, and 49°C). It is observed that most of the measured points locate close to the predicted SWCC, which yields a good prediction performance of the proposed model. The comparison results indicate that the SSE values between predicted and measured curves at 4°C, 20°C, and 49°C are 8.4%, 6.6%, and 6.5%, respectively. It has also been found from experimental results that the temperature has a significant effect on SWCC as a considerable difference in the soil suctions at 4°C and 49°C is observed. Furthermore, it is also worthy to note that the residual state is not observed on the SWCC of the super-fine sand while it was readily detected on the SWCC of Unimin silica sand.

5.6 Performance evaluation of proposed model

A summary of the average normalized sum of squared error (SSE) values, which represents the matching degree between predicted and measured curves among seven experimental cases, is presented in Table 3. It is observed that all selected cases demonstrated SSE values under 10% which yields a good agreement between the predicted model and measured data. Another finding obtained is that the predicted results for sands are more sensitive than that for clays. A significantly lower suction range for sands compared to one of the clays can be considered a potential reason behind larger SSE values. It is observed from the comparison outcomes that when the suction range increased, the SSE values often decreased. Figure 16 shows a comparison between the measured and predicted volumetric water contents for 152 test points. It is interesting to note that the majority of comparison points are very close to the linear line, which indicates an excellent performance of the proposed model in predicting the SWCC with temperature variation. It is therefore concluded that the proposed model for nonisothermal SWCC remains an effective method to be considered for unsaturated soils.

The performance of the proposed model is verified further by conducting a comparison with three existing models. The first selected model is proposed by Grant and Salehzadeh (1996) and improved by Vahedifard et al. (2018) later. The second model is one proposed by Salager et al. (2010) and the last one is the model of Roshani and Sedano (2016). Figure 17 shows the comparison between the proposed model and three existing models for 6 different soil types. It can be observed that the three existing models underpredict significantly in almost all cases. It is noted that the model of Grant and Salehzadeh (1996) is relatively sensitive to initial contact angle value and immersion enthalpy which are often determined empirically. However, it is generally difficult, and expensive to conduct tests for measuring these parameters. Meanwhile, the effect of temperature on SWCC is quite small in the models of Salager et al. (2010), and Roshani and Sedano (2016). This can be explained because both these models considered the effect of temperature on SWCC through only surface tension function while all factors were neglected. Consequently, the effect of temperature on SWCC was significantly underpredicted in these models. The results also indicate that the proposed model has a better performance as compared to other existing models.

6. Model application to the resistance analysis of energy piles

Simplified analytical approach

The nonisothermal SWCC model will be extended to apply to the resistance analysis of energy piles in this section. One of the notable features of energy piles is that their temperature differs from that of the surrounding soil, causing water to migrate away from the pile. This behavior may result in unsaturated soil zones along the length of the energy pile, which must be considered in bearing resistance calculations. Due to the presence of groundwater, however, the unsaturated zone does not extend the entire length of the pile. Instead, both saturated and unsaturated zones exist in most circumstances. In this scenario, the pile body can be divided into segments according to the soil layers to compute the bearing resistance of the energy pile. Figure 18 shows a typical schematic with shear resistance distribution at various depths. It is worth noting that the soil zone above the groundwater level with length L_u is unsaturated, whilst the soil zone below the groundwater level with length L_s is saturated. The total length of the pile is $L = L_u + L_s$.

The ultimate bearing capacity of an energy pile at an arbitrary temperature T can be written as follows:

$$(Q_{ult})_T = (Q_f)_T + (Q_b)_T - (W_p)_T = (Q_{fs} + Q_{fu})_T + (Q_b)_T - (W_p)_T \qquad (51)$$

Where Q_{ult} = ultimate bearing capacity; Q_f = total skin resistance; Q_{fs} = skin resistance along saturated zone; Q_{fu} = skin resistance along unsaturated zone; Q_b = end-bearing resistance; W_p = weight of the pile. It is noted that the subscript "T" refers to the corresponding variable at the present temperature T

The skin resistance can be evaluated using the β-method, which is based on effective stress analysis (Pham 2022b). As a result, skin resistance is influenced by the accuracy of effective stress prediction of soils. Pham and Sutman (2022b) proposed an effective stress equation for both saturated and unsaturated soils that took into account the particle contact area ratio, which is therefore used for the analysis of energy pile in this study.

Friction resistance of pile in saturated zone

Because the behaviour of the soil-structure interface is quite similar to one of the fundamental soils surrounding the pile surface, it is frequently easier to calculate the shear strength of the pile-soil interface by multiplying the soil shear strength with the friction interaction coefficient that represents the effect of surface roughness. The skin resistance of the energy pile at the original temperature T_0 therefore can be predicted by:

$$Q_{fs} = \sum_{i=1}^{j} C_i.L_i.\beta_i.\sigma'_{is} = \sum_{i=1}^{j} C_i.L_i.\beta_{is}.(\sigma_{is} - u_w.[1-D]) \quad (52)$$

$$\beta_i = K.tan\delta_i = (1 - sin\varphi').OCR^{0.5}.tan\delta_i \quad (53)$$

Where C_i = perimeter of pile section i, L_i = length of pile section i, β_i = interface resistance factor of soils within saturated zone, K = static earth pressure coefficient, OCR = over-consolidation ratio, u_w = pore-water pressure ($u_w = \gamma_w.z_{iw}$), φ'= friction angle of soils surrounded by pile section i, δ_i = interfacial effective friction angle ($\delta_i \approx 0.8\varphi' - 0.95\varphi'$), σ_{is} and σ'_{is} = total and effective stress of soils respectively at the center of section i within the saturated zone, D = particle contact area ratio ($D \cong 0.05 - 0.15$).

It should be noted that the temperature of the surrounding soil varies in response to the temperature inside the pile. As a result, total skin friction at the pile-soil interface comes from interface shear stress induced by self-weight of soil and an additional stress imposed on by the pile's radial thermal expansion/contraction (σ_{rT}). The temperature variation is therefore expected to have an influence on the skin resistance of pile-soil interface through variation in friction coefficient (β_i), thermal expansion of pile material ($C_i.L_i$), the pile–soil radial thermal stresses (σ_{rT}), and effective stress (σ'_{is}). Thus, the skin resistance of energy pile within saturated zones can be expressed as follows, taking into account the influence of temperature T:

$$\left(Q_{fs}\right)_T = \sum_{i=1}^{j} C_{iT}.L_{iT}.\beta_{iT}.\left[(\sigma_{is})_T - u_{wT}.(1-D_T) + \frac{\sigma_{rT}}{\beta_{iT}} tan\delta_{iT}\right] \quad (54)$$

$$\beta_{iT} = (1 - sin\varphi'_T).(OCR)_T^{0.5}.tan\delta_{iT} \quad (55)$$

Where σ_{rT} = pile-soil radial interface stress induced by radial thermal expansion/contraction of the pile

Temperature dependency of friction coefficient

It is noted that the friction coefficient is influenced by the friction angle (φ'_T), the roughness coefficient (δ_{iT}), and the over-consolidation ratio $(OCR)_T$. But numerous laboratory studies have demonstrated that temperature has a negligible impact on the friction angle (Yavari et al. 2016; Di Donna et al. 2016; Yazdani et al.

2019; Masoodi et al. 2020). While some researchers discovered that the pre-consolidation pressure (or OCR) decreases as temperature increases (Sultan et al. 2002; Laloui and Cekerevac 2004; Abuel-Naga et al. 2007). The dependence of OCR on temperature is thus taken into account in the proposed approach, and the impact of temperature on the pile-soil interface was indirectly integrated. The OCR at arbitrary temperature is defined as follows:

$$(OCR)_T = \frac{p_{cT}}{\sigma'_z} = (OCR)_{T_0} \times exp\left(-3\frac{1+e_0}{\lambda-\kappa}\alpha_p\Delta T\right) \tag{56}$$

Where p_{cT}= pre-consolidation pressure, σ'_z = overburden effective stress, κ = swelling index, λ = compression index, e_0 = initial void ratio, α_p = is a material parameter, having the unit of a thermal dilation coefficient (1/^{0}C and was determined as $\alpha_p = 10^{-4}$ by Picard (1994).

Thermal expansion of pile materials

Because the pile often expands or contracts due to the temperature difference $(\Delta T = T - T_0)$, the perimeter and length of pile section *i* at the temperature T considering thermal characteristics is shown as follows:

$$C_{iT} = C_i.(1+\alpha_c\Delta T) = \pi d_p.(1+\alpha_c\Delta T) \tag{57}$$

$$L_{iT} = L_i.(1+\alpha_c\Delta T) \tag{58}$$

Where α_c = coefficient of thermal expansion/contraction of concrete (10 με/°C) and ΔT is the net change in temperature of the pile, d_p = pile diameter.

Temperature dependency of additional radial thermal stress

The pile–soil radial contact stresses, σ_{rT}, resulting from the radial thermal expansion/contraction of the pile could be estimated using a cavity expansion analysis as follows (Faizal et al. 2018):

$$\sigma_{rT} = \frac{E_s.\Delta r}{(1+\nu_s).r} \tag{59}$$

Taking into account a change in pile volume ΔV_T caused by a change in temperature ΔT as follows:

$$\Delta V_T = \pi L_0[r_T^2.(1+\alpha_c\Delta T) - r_0^2] \tag{60}$$

Using the thermal volumetric expansion relationship, meanwhile, results in:

$$\Delta V_T = V_T - V_0 = \pi L_0 r_0^2[3\alpha_c\Delta T] \tag{61}$$

Combining Eqs. (60) and (61) gives:

$$\Delta r = r_T - r_0 = \sqrt{\frac{1+3\alpha_c\Delta T}{1+\alpha_c\Delta T}} - 1 \tag{62}$$

Returning Eq. (62) to Eq. (59) results in:

$$\sigma_{rT} = \frac{E_s}{(1+\nu_s)}\left(\sqrt{\frac{1+3\alpha_c\Delta T}{1+\alpha_c\Delta T}} - 1\right) \tag{63}$$

It should be noted that there may be variations in the temperature distribution along with the pile depth. The temperature variation of the pile-soil interface can be calculated by: $\Delta T = T(z) - T_0$ where $T(z)$ = temperature of pile-soil interface at a depth z; and T_0 = initial temperature of pile-soil interface. Based on several experiment results, the temperature variation with pile depth could be described by a linear relationship (Singh et al. 2015; Wang et al. 2015; Fang et al. 2020) or nonlinear relationship (Caulk et al. 2016) although field measurement or numerical simulation are required to establish this relationship. In the absence of measurement data, it is possible to assume that the temperature distribution and pile length will remain constant (Bourne-Webb et al. 2009).

$$(\Delta T)_z = (\Delta T)_{z=0}.(pz + q) \qquad \text{for linear function} \qquad (64a)$$

or

$$(\Delta T)_z = (\Delta T)_{z=0}.(pe^z + q) \qquad \text{for nonlinear function} \qquad (64b)$$

Where $(\Delta T)_z$ = temperature variation of pile-soil interface at depth z, $(\Delta T)_{z=0}$ = temperature variation of pile-soil interface at ground surface, p and q = correlated coefficients

Temperature dependency of total stress

It is well known that as temperature rises, the void ratio of soil and the density of water tends to change. As a result, it was possible to recalculate the total stress $(\sigma_{is})_T$ at the specified temperature T that was applied to the center of pile section i within the saturated zone as follows:

$$(\sigma_{is})_T = \gamma z_w + \gamma_{sat}(z_i - z_w) = \left(\frac{G_s+S.e_T}{1+.e_T}\right)\gamma_{wT}.z_w + \left(\frac{G_s+e_T}{1+.e_T}\right)(z_i - z_w).\gamma_{wT} \qquad (65)$$

Where, G_s = specific gravity, γ = bulk unit weight of water, γ_{sat}=saturated unit weight, γ_{wT} = water unit weight at temperature T ($\gamma_{wT} = g.\rho_{wT}$), g = gravity acceleration, ρ_{wT} = water density at temperature T, e_0 and e_T= void ratio at original temperature T_0 and arbitrary temperature T, respectively, z_{iw} = depth from groundwater table to center of pile section i within the saturated zone, z_i = depth from ground surface to center of pile section i. It is noted that the change of void ratio with temperature can be calculated by using Equation (42) while the change of water density (ρ_{wT}) with temperature can be determined using Eq. (44).

Temperature dependency of pore-water pressure

The pore-water pressure (u_{wT}) at various temperatures is consequently different because of the variation in water density with temperature, and may be expressed as follows:

$$u_{wT} = (0.6582 + 0.002509T - 0.000004606T^2).g.(z_i - z_w) \qquad (66)$$

Furthermore, it should be noted that fine-grained soils, particularly saturated clays, may behave under undrained conditions. Due to the differential expansion of the pore water and soil particles, heating saturated clays cause the formation of excess pore water pressure. It is a crucial scenario to be considered as the generation of thermally induced excess pore water pressure frequently results in a reduction in effective stress. Using the principles of thermoelasticity, Campanella and Mitchell (1968) developed a theoretical method to calculate the generation of excess pore water pressure in a sample of saturated soil during undrained heating. The pore water pressure at an arbitrary temperature under undrained conditions is calculated by the following equation:

$$u_{wT} = (0.6582 + 0.002509T - 0.000004606T^2).g.z_{iw} + \Delta u_T \qquad (67)$$

$$\Delta u_T = \frac{n\Delta T.(\alpha_s - \alpha_w) + \alpha_{st}}{m_v + n.m_w} \qquad (68)$$

$$m_v = \frac{1}{1+e_0} . \frac{\kappa}{p^{\prime}} = \frac{1}{E^{\prime}} \quad (69)$$

Where, Δu_T = the change in pore water pressure due to temperature difference ΔT, n = porosity, m_v = coefficient of volume compressibility of the solid skeleton, m_w = coefficient of volume compressibility of pore water, $p^{\prime}$ = mean effective stress, $E^{\prime}$ = Young's modulus, α_s= volumetric thermal expansion coefficient of mineral solids, α_w = volumetric thermal expansion coefficient of pore water ($\approx$ 170 με/°C), α_{st} = physicochemical coefficient of soil structure volume change, which can be determined using the empirical equation proposed by Ghaaowd et al. (2017) as follows:

$$\alpha_{st} = 10^{-4} . e^{-0.014PI} \quad (70)$$

Temperature dependency of particle contact area ratio

On the other hand, the particle contact area ratio D_T at temperature T in Equation (54) can be calculated according to the proposed equation by Pham and Sutman (2022b) as follows:

$$D_T = D . \left(\frac{1+e_0}{1+e_T}\right)^{2/3} \quad (71)$$

Friction resistance of pile in unsaturated zone

For the unsaturated soil zone (L_u), the skin resistance of the energy pile at the present temperature T can be predicted by:

$$\left(Q_{fu}\right)_T = \sum_{i=1}^{j} C_{iT} . L_{iT} . \beta_{iT} . \left\{(\sigma - u_a) + \psi_T . [\theta_T + S_T . (1 - \theta_T) + D_T] + \frac{\sigma_{rT}}{\beta_{iT}} tan\delta_{iT}\right\}_i \quad (72)$$

To use the proposed model, the degree of saturation and the corresponding matric suction at different temperatures are determined as a first step. It should be noted that in this work, ψ_T and S_T are obtained by using Eq. (46).

Temperature dependency of volumetric water content

It is also worthy to note that in many cases, there may occur a change in water content due to evaporation or infiltration. As a first step, Eq. (42) is applied to plot SWCCs at various temperatures. The actual volumetric water content accounting for evaporation must then be recalculated at each chosen temperature. The corresponding matric suction is then determined using the interpolation technique. It is highlighted that evaporation-related changes in the volumetric water content of soil could be calculated using the temperature-based energy balance theory (Milly, 1984; Qiu et al. 1999). The following simple linear relationship, on the other hand, was proposed in response to the results of several other researchers' experimental programs (She and Sleep 1998; Grifoll et al. 2005):

$$(\theta_T)_{eva} = \theta_{T=293} - a_T . (T - 293) \quad (73)$$

Where $(\theta_T)_{eva}$ = volumetric water content at temperature T after considering evaporation, $\theta_{T=293}$ = volumetric water content at room temperature (T = 293 K), a_T = an empirical constant that can vary with specific soil under consideration ($a_T \approx 0.004 \pm 0.002$).

Temperature dependency of hydraulic conductivity

On the other hand, the matric suction and degree of saturation for various depths may vary due to hydraulic gradient and the impact of flow rate. For one-dimensional vertical liquid water flow in isotropic and homogenous materials, the matric suction variation with depth can be described by using Darcy's law as follows:

$$q_s = -k_u\left(\frac{\partial\psi}{\gamma_w.\partial z_{iw}} + 1\right) \tag{74}$$

Where q_s = steady vertical fluid flow rate or flux density function (q_s = 0 for hydrostatic, q_s > 0 for evaporation, q_s < 0 for infiltration), k_u = hydraulic conductivity.

The hydraulic conductivity of unsaturated soils (k_u) can be related to the hydraulic conductivity of saturated soil (k_s) by following equation (Gardner 1958; Constantz 1982; Tindall et al. 1999):

$$k_u = k_s.e^{\psi_{AEV}\times\psi} \tag{75}$$

$$k_s = \frac{k_{in}\times\gamma_{wT}}{\eta_T} \tag{76}$$

Where, ψ_{AEV} = air-entry value, k_{in} = intrinsic permeability that is assumed to be dependent only on pore structure and geometry, η_T = water viscosity. The water viscosity varies with temperature as follows (Lide 1995):

$$\eta_T = \left(0.2601 + 1.517 \times e^{-0.034688\times(T-273)}\right) \times 10^{-3} \tag{77}$$

By extending the analytical solution provided by Lu and Griffiths (2004), the suction variation in unsaturated soil layers is derived for various flow rates as follows (Thota et al. 2021).

$$(\psi)_{q_s} = \frac{\gamma_{wT}}{\psi_{AEV}} ln\left[e^{-\psi_{AEV}.z_i} + \frac{q_s}{k_s}.\left(e^{-\psi_{AEV}.z_i} - 1\right)\right] \tag{78}$$

The effect of flow rates ($q_s \neq 0$) on SWCC can be integrated by replacing Eq. (78) in Eq. (46), which gives:

$$\begin{cases} \theta_T = \theta_0 = \dfrac{c_f.\theta_s}{(ln[2.71828+(\psi_0/a)^n])^m} \\ \psi_T = \dfrac{\gamma_{wT}}{\psi_{AEV}} ln\left[e^{-\psi_{AEV}.z_i} + \dfrac{q_s}{k_s}.\left(e^{-\psi_{AEV}.z_i} - 1\right)\right] \times f_\sigma \times f_\alpha \times f_r \times f_e \times f_\rho \end{cases} \tag{79}$$

End bearing resistance of energy pile

In the simplified form, the end bearing capacity is computed using the same formula as the bearing capacity of shallow footings. Equation (80) should be used if the soil at the base pile is saturated, while Equation (81) should be used if the soil at the base pile is unsaturated.

$$(Q_b)_T = [(\sigma_{is})_T - u_{wT}.(1 - D_T)].N_q.A_{bT} + c_T.N_c.A_b \tag{80}$$

$$(Q_b)_T = (\sigma - u_a).N_q.A_{bT} + (c_T + \psi_T.[\theta_T + S_T.(1 - \theta_T) + D_T]).N_c.A_{bT} \tag{81}$$

In which,

$$N_q = e^{\pi tan\varphi'_T}.tan^2(45^0 + \varphi'_T/2) \tag{82}$$

$$N_c = 2(N_q - 1).cot\varphi'_T \tag{83}$$

$$A_{bT} = \frac{\pi d_p^2}{4}.(1 + \alpha_c\Delta T)^2 \tag{84}$$

Where, A_{bT} = the cross-sectional area of the pile base considering thermal expansion/contraction, c_T = cohesion of soils at temperature T, N_b and N_c = bearing capacity factors of soil at pile base.

Discussion of proposed method

According to the above analysis, pile resistance will decrease if the pile is not undrained or if the water content remains constant when the temperature increases (i.e., there is no evaporation). Otherwise, the pile resistance can be increased by raising the temperature under different conditions.

Thota et al. (2021) recently presented a calculation method for resistance analysis of energy piles. However, the main focus of their model was to estimate the energy pile resistance by taking the impact of temperature variation on suction into account. This model is therefore limited to energy pile in unsaturated soils. The proposed model, in contrast to the model of Thota et al. (2021), evaluates the energy pile resistance by correlating the variation in *(i)* shear strength and *(ii)* pile expansion with temperature. Specifically, the relationship between shear strength and temperature was examined using six components: Suction (ψ_T), undrained pore-water pressure under heating (u_{wT}), water density (ρ_{wT}), friction coefficient (β_{iT}), hydraulic conductivity (K_{wT}), and particle contact area ratio (D_T). Thus, the temperature-dependent suction was only a component that effects on the resistance of pile. Meanwhile, the expansion of pile with temperature are considered through the thermal expansion in both longitudinal and radial directions. As a result, the proposed model is therefore applicable to energy pile in both saturated and unsaturated soils, which must be practical where groundwater table may arise above the pile base. On the other hand, Liu et al. (2022) recently presented a simple method for the analysis of a single energy pile, in which the thermal expansion theory of the pile was simply employed to evaluate the impact of temperature on the pile-soil interface. The relationship between soil behaviour and temperature, however, was not investigated. The proposed method clearly displays a more comprehensive performance for the analysis of energy piles because it takes into account the effect of temperature on interface through both soil behaviour and thermal expansion of the pile.

7. Model validity for analysis of energy piles

Summary of experimental data sets

In order to validate the analytical model, the test data must include some important components, such as SWCC, skin or end-bearing resistance, and the ultimate bearing capacity of the energy piles. Even though there have been some reported experimental studies of energy piles in unsaturated soils (Behbehani and McCartney 2022; Sani and Singh 2020; Akrouch et al. 2016; You et al. 2016), these studies mainly concentrated on the thermal response and efficiency of the energy piles without providing SWCC and bearing capacity information of piles. Because there is a limitation of field test data on the ultimate bearing capacity of energy piles in unsaturated soils at various temperatures, the proposed model is validated for unsaturated soils using data from several scaled-model tests. Additionally, due to the suitability of the proposed method for the analysis of energy piles in both saturated and unsaturated soils, a case study of a field test on the ultimate carrying capacity of energy piles in saturated soil is also taken into consideration for comparison.

The first data set is obtained from model tests by Gu et al. (2014) to assess the effect of temperature on the resistance of piles in clayey soil using a modified micropenetrometer. The variation in skin resistance, end-bearing resistance, and total resistance at various temperatures and degrees of saturation are thoroughly presented, this case is thus helpful for comparison. It is also interesting that the water content is maintained at a constant level as temperature increases from 20°C to 50°C. This situation is obtained by placing a thin top cover on the surface of the specimen and fixing by bars in order to prevent evaporation. As a result, the effect of temperature on the resistance of the pile at different degrees of saturation was observed. It should be noted that this model was built up with roughly 100 probes, each sized 3mm in diameter. In order to ensure an approximation to the realistic situation, the pile dimension of the prototype is calculated by converting the size of 100 probes into an equivalent pile diameter by scaling the parameter in the scaled model 100 times. As a result, the dimensions of 0.3 m in diameter and 10 m in length were used in the theoretical calculation. The second data set selected for comparison is from centrifuge tests performed by Goode and McCartney (2015). The semi-floating energy pile in the scaled model was 342.9 mm in length and 63.5 mm in diameter, which was embedded in a layer of unsaturated Bonny silt. This is equivalent to a prototype energy pile with an 8.2 m length and 1.5 m diameter when scaled by a centripetal acceleration of 24g. To examine the output of the model, the prototype dimensions were employed. The initial dry unit weight and water content of the silty layer were both uniforms. The load-settlement curves were then measured for various temperatures of 21°C, 32°C, and 40°C. These data sets are very useful when the variation in water content with temperature was measured. It should be noted that the ultimate bearing capacity in both model tests is assessed under no-flow conditions (q = 0), as the initial suction of compacted soil in the model tests is uniform with depth. The field experiment results for the case study of an energy pile in saturated soils, on the other hand, were reported by Sutman et al. (2019). For different temperatures, mobilized skin resistance along the piles was reported. Two test piles, named TP-1 and TP-3 selected for comparison in this study. Each pile penetrated 9 m into the clay layer, and the remaining portion rested on a dense sand layer. The main difference between the two test piles is that TP-1 is free at the head and TP-3 has a maintained mechanical load during induced temperature variations. As a result, the locations of the null point (NP), which is defined as the location of zero thermal displacements were different for the two piles. For TP-1 and TP-3, the NP location was 8.7 and 5.7 meters, respectively. It should be noticed that the mobilized shaft resistance of the pile part above the NP point is frequently represented by a negative sign to indicate that this pile portion is upwards and a positive sign for the pile portion beneath the NP point to indicate that this pile portion is downwards. The design parameters used in calculation using the analytical method for three test models are summarized in Table 4.

Comparison with model test in clayey soils

It should be noted that to use the proposed model for unsaturated soils, the degree of saturation at different references must be reported. The influence of temperature on the SWCC of clayey soils is depicted in Figure 19a. As the temperature increases, the SWCC shifts significantly. It should be noted that the nature of matric suction is greatly influenced by the water viscosity, which decreases as temperature rises. As a result, as the temperature rises, the matric suction of soils reduces. Figures 19b-d depict the comparison between predicted and measured results of skin resistance, end-bearing resistance, and total resistance, respectively. It can be observed that the present method produces a good agreement with experimental data and shows a better performance than the model of Thota et al. (2021) in predicting pile resistance with various temperatures. This is due to the fact that the Thota et al. (2021) model only took into account the temperature dependence of suction and ignored all other aspects. The pile resistance reduced with rising temperature, according to both experimental and analytical results, implying that increasing the temperature decreases the structural strength of the soil. It should be noted that the soil samples were controlled in the model test by Gu et al. (2014) to best restrict the reduction of water content by water evaporation. In reality, the soils in every model were managed to maintain constant water content. After the experiments, the water contents of samples taken from various depths were measured, confirming the change in water content was minimal. Because water content remains constant while the matric suction decreases with an increase in temperature, the shear strength of soils is decreased with a temperature increase. As a result, the bearing capacity of the pile may be decreased with increasing temperature for a given degree of saturation, which is in good agreement with some previous studies (Guo et al. 2020; Fuentes et al. 2016; Zhang et al. 2012; Uchaipichat, and Khalili 2009). Furthermore, when the degrees of saturation were smaller, the rate of decrease in resistance with increasing temperature was higher. It is also worth noting that rising temperature might cause water migration, resulting in lower water content and higher matric suction.

Comparison with model test in silty soils

The predicted SWCCs for Bonny silt at different temperatures are shown in Figure 20a. The SWRC shifts lower as the temperature is increased. This implies that the matric suction will decrease with increasing temperature at a given degree of saturation, and the degree of saturation will decrease at a given matric suction. However, it was also reported for this test model that there had been a change in water content with an increase in temperature (Behbehani and McCartney, 2020). The corresponding matric suction is obtained by using the nonisothermal SWCCs for various temperatures. The degree of saturation and matric suction relate to three distinct temperatures, which are represented by point A ($S = 0.595$, $\psi = 20$ kPa) for $T = 21°C$, point B ($S = 0.539$, $\psi = 30$ kPa) for $T = 30.5°C$, and point C ($S = 0.493$, $\psi = 45$ kPa) for $T = 38°C$). The results of the resistance analysis of an energy pile at various temperatures are shown in Figure 20b. The measured and predicted values of the ultimate bearing capacity of the pile in unsaturated Bonny silt in relation to temperature change show a reasonable agreement. Additionally, it is also observed that the ultimate bearing capacity of energy piles in unsaturated silt increases when the temperature at the soil-pile interface rises. Although matric suction reduces with rising temperature for a given degree of saturation, the simulated decrease in water content does, however, result in greater matric suction. As a result, the ultimate bearing capacity of the pile increases with increasing temperature. This tendency is contrary to what was observed in the first test model in unsaturated clays. This is because water content was designed to be constant in the test model by Gu et al. (2014) whereas the test model of Goode and McCartney (2015) showed that water content decreased as temperature increased.

Comparison with field test in clays

Figure 21(a) and (b) show, respectively, the predicted and measured mobilized skin resistance over different depths of TP-1 and TP-3 during the heating. It has been found that as the temperature changes, the skin resistance mobilization above and below the NP depth is different. The predicted and measured results showed a good agreement. The comparison of calculated and measured total skin resistance is shown in Figure 21c. It has been observed that the analytical method performs well in forecasting the variation of skin resistance of energy piles with temperature. It is also observed that the overall skin resistance increased with increasing temperature for both piles TP-1 and TP-3. However, the increase in overall skin resistance with the temperature of TP-1 is higher than that of TP-3. As previously mentioned, the NP position was deeper than that of TP-3 because TP-1 was heated without any mechanical stress on the pile head. Additionally, while the head of TP-1 was free, the mechanical load limited the thermal expansion of TP-3.

7. Conclusion

The discussion of a simplified method for bearing capacity analysis of energy piles integrating temperature-dependent model of soil-water characteristic curve was presented in this study. Several conclusions are summarized as follows:

An analytical model has been presented for predicting the influence of temperature on the matric suction in this paper. The proposed model has been established from the combination of five temperature-dependent functions: surface tension, air-water contact angle, void ratio, thermal expansion of particles, and water density which were often neglected or were not solved fully in existing models. In particular, the proposed model has the advantage over previous models in that it is simple and does not require the use of many empirical parameters to get the solution.

The results obtained from the proposed models showed that the matric suction depends significantly on the temperature and usually decreases with a temperature increase. It was observed that the air-water contact angle has the most important and sensitive influence on the soil suction reduction with increasing temperature. It is also found that the air-water contact angle increases nonlinearly with an increase in temperature, with a decrease in matric suction and particle radius of the unsaturated soils.

A new soil-water characteristic curve equation was also presented in this paper by the extension of the Fredlund and Xing (1994) equation to introduce a nonisothermal model. The main refinements were the inclusion of a "correction" factor C_f, and the application of the temperature-dependent model of matric suction. The proposed model can be integrated into analytical methods or numerical simulations to consider the thermal-hydro-mechanical behaviour of unsaturated soils.

A comparison of the proposed model outcomes with the measured results for several published experimental studies was conducted to investigate the validation of the proposed model. The statistical analysis results showed a very good agreement between predicted and measured values, emphasizing a good performance of the proposed model in predicting the temperature effect on matric suction variation and the soil-water characteristic curve of unsaturated soils. The proposed model also shows a higher performance as compared to the existing models.

The nonisothermal SWCC model was coupled with the bearing resistance theory to provide a simplified method for analysis and design of energy pile. The proposed method is therefore applicable to energy pile in both saturated and unsaturated soils. Due to the fact that the validity of the proposed SWCC model was demonstrated up to 200°C, whereas the temperature controlled in test models is still below 80°C. As a result, the suggested model is promising for both in-situ test models and laboratory tests.

The analytical model agrees with experimental observation that when the temperature increases for a specific degree of saturation or if the soil behaves in an undrained condition, the pile resistance decreases. Additionally, when the degree of saturation was smaller, the rate of resistance decreased with the temperature increase was faster. However, as the temperature rises, water evaporation could cause a decrease in water content and an increase in matric suction. As a result, as soils get dryer, pile resistance will increase as temperature increases.

Data Availability Statement

All data, models, and code generated or used during the study appear in the submitted article.

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Abbreviation

AEV = Air-entry value
OCR = over-consolidation ratio
NP = null point
SWCC = soil-water characteristic curve
SWRC = soil-water retention curve
SSE = normalized sum of squared error

Notation

The following symbols are used in this paper:

a_T = evaporation-related empirical constant (dimensionless),
A_b = cross-sectional area of the pile base (m^2),
c_T = cohesion of soils at temperature T (Pa),
C_i = perimeter of pile section i (m),

C_f = correction factor (dimensionless),
d_p = pile diameter (m),
D = disturbance function at reference temperature (dimensionless),
E' = Young's modulus (N/m^2),
e_0 = initial void ratio at reference temperature (dimensionless),
e_T = void ratio at the current temperature (dimensionless),
f_σ = temperature-dependent surface tension factor (dimensionless),
f_α = temperature-dependent contact angle factor (dimensionless),
f_r = temperature-dependent particle size factor (dimensionless),
f_e = temperature-dependent void ratio factor (dimensionless),
f_ρ = temperature-dependent water density factor (dimensionless),
g = gravity acceleration (m/s^2),
G_s = specific gravity (dimensionless),
k_u = hydraulic conductivity of unsaturated soil ($kg.s/m^3$),
k_s = hydraulic conductivity of saturated soil ($kg.s/m^3$),
k_{in} = intrinsic permeability (H/m),
K = static earth pressure coefficient (dimensionless),
m_v = coefficient of volume compressibility of the solid skeleton (m^2/N),
m_w = coefficient of volume compressibility of pore water (m^2/N),
n_s = number of soil particles (dimensionless),
n_{s0} = number of soil particles at reference temperature (dimensionless),
n_{sT} = number of soil particles at current temperature (dimensionless),
n = porosity (dimensionless),
PI = plasticity index (dimensionless),
N = total number of data available (dimensionless),
L = total pile length (m),
L_i = length of pile section i (m),
L_u = pile length of unsaturated soil zone (m),
L_s = pile length of saturated soil zone (m),
u_a = air-pore pressure (Pa),
u_w = water-pore pressure (Pa),
p' = mean effective stress (Pa),
q_s = steady vertical fluid flow rate or flux density function (m^3/s),
Q_{ult} = ultimate bearing capacity (Pa),
Q_f = total skin resistance (Pa),
Q_{fs} = skin resistance along saturated zone (Pa),
Q_{fu} = skin resistance along unsaturated zone (Pa),
Q_b = end-bearing resistance (Pa),
R = meniscus curvature (m),
r = particle radius (m),
r_i = mean pore radius (m),
r_0 = particle radius at reference temperature (m),
r_T = particle radius at current temperature (m),
T_0 = reference temperature (K),
T = current temperature (K),
z_{iw} = depth from groundwater (m),
V_{si}^{0} = volume of a soil particle at reference temperature (m^3),
V_{si}^{T} = volume of a soil particle at current temperature (m^3),
V_s = solid volume (m^3),
V_v = void volume (m^3),
V = total volume of a soil sample (m^3),
V_{s0} = original volume of solid particle at reference temperature (m^3),

W_p = weight of the pile (N),
β_i = friction coefficient of soil-structure interface (dimensionless),
φ' = friction angle of soils surrounded by pile (degree),
δ_i = interfacial effective friction angle (degree),
σ_{is} = total stress (Pa),
σ'_{is} = effective stress (Pa),
η_T = water viscosity (Pa.s)
α = air-water contact angle (degree)
α_T = air-water contact angle at current temperature (degree),
α_0 = air-water contact angle at reference temperature (degree),
α_c = coefficient of thermal expansion/contraction of concrete (με/°C),
α_p = a thermal dilation coefficient of soils (με/°C),
α_s = coefficient of volumetric thermal expansion of solid particle (με/°C),
α_w = volumetric thermal expansion coefficient of pore water (με/°C),
α_{st} = physicochemical coefficient of soil structure volume change (με/°C),
δ = empirical constant (dimensionless),
ω = interface energy-related coefficient (dimensionless),
θ_T = volumetric water content at current temperature (dimensionless),
θ_s = saturated volumetric water content (dimensionless),
θ_i = volumetric water content at an inflection point on SWCC (dimensionless),
σ_s = air-water surface tension (N/m),
σ_{s_0} = air-water surface tension at reference temperature (N/m),
σ_{sT} = air-water surface tension at current temperature (N/m),
σ_{sa} = interfacial tension along with solid-air interface (N/m),
σ_{sw} = interfacial tension along with solid-water interface (N/m),
γ_w = unit weight of water (kg/m^3),
ρ_w = water density (kg/m^3),
ρ_{w0} = water density at reference temperature (kg/m^3),
ρ_{wT} = water density at current temperature (kg/m^3),
λ = compression index (dimensionless),
κ = swelling index (dimensionless),
ψ = matric suction in soils (Pa),
ψ_{AEV} = air-entry value (Pa),
ψ_0 = matric suction at reference temperature (Pa),
ψ_T = matric suction at current temperature (Pa),
ψ_i = matric suction at an inflection point on SWCC (Pa),
ψ_r = matric suction corresponds to residual volumetric water content (Pa),
$\psi_{measured}$= measured suction (Pa),
$\psi_{predicted}$ = predicted suction (Pa),
Δe_T = void ratio variation due to temperature change (dimensionless),
Δh = immersion enthalpy per unit area at temperature T (J/m^2),
Δu_T = change in pore water pressure due to temperature difference (Pa),
ΔT = temperature increment (K),
ΔV = volume increment of solid particle due to temperature (m^3),
$\cos\alpha$ = wetting coefficient (dimensionless),

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