3-D Printed Filtering Rat-Race Couplers Using Hemispherical Cavity Resonator

Citation for published version:

Digital Object Identifier (DOI):
10.1109/TMTT.2023.3284266

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
IEEE Transactions on Microwave Theory and Techniques

Publisher Rights Statement:
© 2023 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

General rights
Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.
3-D Printed Filtering Rat-Race Couplers Using Hemispherical Cavity Resonator

Gang Zhang®, Senior Member, IEEE, Shuai Feng®, Yuan Chen, Yang Yang®, Senior Member, IEEE, He Zhu®, Member, IEEE, Xin Zhou®, Graduate Student Member, IEEE, Jiasheng Hong®, Fellow, IEEE, Wanchun Tang®, Member, IEEE, and Jiquan Yang®

Abstract—In this article, a new approach for designing 3-D printed single- and dual-band filtering rat-race couplers based on hemispherical cavities is presented, for the first time. By constructing two types of multiplexing multimode topologies in dual-cavity configurations, both single- and dual-band filtering rat-race couplers are developed. In these designs, a pair of TM$_{101}$ degenerate modes form the passband for single-band operation, while dual-band operation is realized by exciting an additional pair of TE$_{101}$ degenerate modes. Due to the natural E-field distributions of these modes, the desired 0° and 180° phase differences and isolations can be realized between output ports without additional matching circuits. By elaborately placing the internal coupling slots and input–output ports, desired filtering responses and phase requirements can be flexibly attained for both single- and dual-band filtering rat-race coupler designs. For demonstration, two prototypes are well implemented as monoblock circuits using 3-D-printing technology. Compared with the up-to-date designs, the proposed works exhibit the merits of compact size, high Q-factor, lightweight, good single- and dual-band filtering responses with controllable bandwidths and center frequencies, nice phase difference characteristics as well as flexible extendable topologies.

Index Terms—Degenerate mode, dual-band, filtering rat-race coupler, hemispherical cavity (HC) resonator.

I. INTRODUCTION

FILTERING rat-race couplers are multifunction integrated devices that provide both the functions of frequency selectivity as a filter and power division as a coupler. It has been widely explored with different methods and technologies. Microstrip/stripline structures are used for filtering rat-race coupler designs based on planar circuit boards, such as dual-mode stub-loaded resonators [1], stepped impedance resonators (SIRs) [2], shorted parallel-coupled multilines sections [3], ring structures [4], slot-coupled resonators [5], etc. With the help of low-temperature co-fired ceramic (LTCC) technology, researchers have also studied an alternative filtering rat-race coupler based on an eight-line spatially symmetrical coupled multilayer structure as presented in [6]. Yet, it is known that they suffer from limited Q-factor, especially at high frequencies. Benefiting from the high Q-factor of substrate integrated waveguide (SIW) [7], circular patch resonators [8], dielectric resonators (DRs) [9], and rectangular cavity resonators [10] are used in the design of filtering rat-race couplers. More specifically, in [7], two types of single-band filtering rat-race couplers based on TE$_{102}$ and TE$_{201}$ orthogonal degenerate modes are proposed using square and circular SIW cavities. In [8], the E-field distributions of TM$_{11}$, TM$_{11}$-like, and TM$_{12}$-like modes in circular patch resonators are studied. Not only a single-band but also dual-band filtering rat-race couplers are designed. Furthermore, two single-band filtering couplers are proposed in one quad-mode rectangular and cylindrical DR cavity configurations, featuring compact size in [9]. By coupling three rectangular metal cavities, an alternative high Q-factor coupler with single-band operation is also designed in [10].

These above-reported filtering rat-race couplers can be divided into the following five main categories according to the different implementation forms, i.e., transmission line resonators [1], [2], [3], [4], [5], [6], SIW resonators [7], patch resonators [8], DRs [9], and metal cavity resonators [10]. In comparison, the design based on metal cavity resonators exhibits the advantage of a higher Q-factor. For the same building metal material and at the same target resonant frequency, cavities with a higher volume-to-surface-area ratio can achieve a higher Q-factor. Therefore, the spherical metal cavity performs the best among all possible cavity geometries. So far, spherical cavities have been successfully applied in designing filters [11], [12], [13], [14], [15]. For instance, a 5th-order filter [11] is designed with good out-of-band rejection by coupling five TM$_{101}$ mode spherical cavities in an Olympic-shaped configuration. Another 4th-order bandpass filter (BPF) in [13] is also proposed with
two spherical dual-mode cavity resonators. An interesting application can be found in [15] that an ellipsoidal cavity filter is realized with multiple asymmetrical transmission zeros (TZs) by virtue of a pair of degenerate modes in a transverse routing scheme. In addition, noting that a hemispherical cavity (HC) features a relatively high $Q$-factor but has only half size of a spherical cavity exhibits similar EM field distribution and also receives increasing attention for filter designs [16], [17], [18], [19], [20]. However, all the above spherical cavity designs focus on filtering response and there is a rare study on filtering couplers. The reason behind this situation may be that it is very challenging that how to multiplex and control the resonant modes of spherical cavities to realize such functions-integrated filtering component, let alone the dual-band scenario.

In this article, a novel design approach for developing single-band and dual-band filtering rat-race couplers based on HC is proposed, which exhibits good flexible controllable filtering responses and nice phase difference characteristics. The initial work, as presented at the IEEE MTT-S International Microwave Filter Workshop conference (IMFW) 2021 [21] by us, briefly introduced a single-band filtering coupler concept but with asymmetrical and inflexible design arrangement, no analysis, and only simulated results. In this article, we further significantly extend our previous work by providing additional necessary details including new design strategies with both single- and dual-band cases, thorough theoretical analysis, design flexibility and prototypes, experimental demonstration, and extended discussion. By stacking dual cavities to form desired multiplexing multimode topologies, two types of single- and dual-band filtering rat-race couplers are developed. The proposal has achieved the following distinctive features, including: 1) for the first time, dual-band waveguide filtering coupler has been realized and demonstrated; 2) flexibly controllable bandwidth and center frequencies have been attained for both single- and dual-band filtering couplers; 3) low loss at sub-mm-wave frequency bands and very compact size, benefiting from the intrinsic high $Q$-factor and miniaturization properties of hemispherical resonators; and 4) easy for fabrication and lightweight owing to 3-D printed technique.

The rest of this article is organized as follows. Section II describes the fundamental theories of the hemispherical resonator cavity. Section III provides the analysis and design of the proposed single- and dual-band filtering rat-race couplers. Subsequently, Section IV gives the comparison of the proposed work and other the state of the art as well as some extended discussion of our work. Finally, Section V draws the conclusion of the article.

II. HIGH-Q HEMISPHERICAL RESONATOR

An air-filled HC is chosen as the fundamental resonant unit of the filtering rat-race couplers to realize the desired single-/ dual-passband response and phase difference characteristics. First, the resonance characteristics and $E$-field distributions of the HC are studied to clearly describe the principle of our works.

The physical structure of the HC is half in size of the spherical one, and they exhibit similarities in the resonant modes and corresponding $E$-field distributions. There are the first three resonance modes of a spherical cavity resonator with the TM$_{101}$, TM$_{201}$, and TE$_{101}$ modes [13]. Because of the structural symmetry, there are many highly degenerate eigenmodes in a spherical cavity resonator. But for a HC, only one direction of polarization exists for the TM$_{101}$ mode, and two directions of polarization for the TM$_{201}$ and TE$_{101}$ modes, respectively. To avoid confusion, these modes in the HC are still referred to by the same names as the whole spherical cavity though they exhibit half $E$-fielded distributions compared with the whole spherical cavity as shown in Fig. 1. The eigenmode frequencies of the first three modes are determined by the radius ($r$) of the HC and can be mathematically obtained from the following equations:

\[
f_{\text{TM}101} = \frac{\omega_{\text{TM}101}}{2\pi} = \frac{y_{11}}{2\pi r \sqrt{\mu \varepsilon}} = \frac{2.744}{2\pi r \sqrt{\mu \varepsilon}}
\]

\[
f_{\text{TM}201} = \frac{\omega_{\text{TM}201}}{2\pi} = \frac{y_{21}}{2\pi r \sqrt{\mu \varepsilon}} = \frac{3.870}{2\pi r \sqrt{\mu \varepsilon}}
\]

\[
f_{\text{TE}101} = \frac{\omega_{\text{TE}101}}{2\pi} = \frac{x_{11}}{2\pi r \sqrt{\mu \varepsilon}} = \frac{4.493}{2\pi r \sqrt{\mu \varepsilon}}
\]

where $\omega_{\text{TM}101}$, $\omega_{\text{TM}201}$, and $\omega_{\text{TE}101}$, are the corresponding angular frequencies, and the roots $y_{11}$, $y_{21}$, and $x_{11}$ of the eigenvalue equation equal to 2.744, 3.870, and 4.493 for TM$_{101}$, TM$_{201}$, and TE$_{101}$ modes, respectively [22]. $\mu$ and $\varepsilon$ are permeability and permittivity of the air-filled cavity, and
for the TM
as plotted in Fig. 2(b). It shows a high
the simulated unloaded
resonant frequencies of the first three modes of the HC are the
mapped in Fig. 2(a). As observed, for the same radius
elicity, the white circles
of relative high
benefits of using HC as the resonant unit over others in terms
and compact size.
where
(1)–(3) reduces to
\[
 f_{TM101} \approx \frac{1.3102 \times 10^2}{r} \text{ (GHz)} \tag{4}
\]
\[
 f_{TM201} \approx \frac{1.8478 \times 10^2}{r} \text{ (GHz)} \tag{5}
\]
\[
 f_{TE101} \approx \frac{2.1452 \times 10^2}{r} \text{ (GHz)} \tag{6}
\]

where \( r \) in millimeters.

In our design, \( r = 16.0 \) mm and thus the frequency
of the first three modes can be deduced as about 8.19, 11.55, and 13.41 GHz, respectively. These values are verified
by selecting an eigen-mode solution in the High-Frequency
Structure Simulator (HFSS) and the corresponding results are
mapped in Fig. 2(a). As observed, for the same radius \( r \), the resonant frequencies of the first three modes of the HC are the same as those of the spherical one. The relationship between
the simulated unloaded \( Q \)-factor (\( Q_u \)) and \( r \) is also investigated
as plotted in Fig. 2(b). It shows a high \( Q_u \) of HC, e.g., 10,418
for the TM\(_{201}\) mode and 12,246 for the TE\(_{101}\) mode with \( r = 16 \) mm. Although these \( Q_u \) are around 42.4\%, 31.1\%, and 55\%
less than those of a spherical cavity in these corresponding
modes, respectively, the volume of the HC is reduced by half
and they are still enough to meet the engineering applications
in these bands of interest. Table I further shows the comparison
of the HC with the other three types of cavities. It shows the benefits of using HC as the resonant unit over others in terms
of relative high \( Q_u \) and compact size.

The simulated \( E \)-field distributions of the first two higher
modes in HC are illustrated in Fig. 1 where the left column
graphs are 3-D views of these modes, the middle ones are
\( E \)-field vector illustrations on the bottom of the HC, and the
right ones are planar \( E \)-field vectors. For the single-band case,

**TABLE I**

<table>
<thead>
<tr>
<th>Type</th>
<th>Hemispherical</th>
<th>Spherical</th>
<th>Rectangle</th>
<th>Cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size(mm)</td>
<td>15.5(R)</td>
<td>15.5(R)</td>
<td>26X26</td>
<td>15(R)</td>
</tr>
<tr>
<td>Volume(mm(^3))</td>
<td>7795</td>
<td>15590</td>
<td>11492</td>
<td>15190</td>
</tr>
<tr>
<td>( Q_u @12GHz )</td>
<td>10267</td>
<td>15025</td>
<td>9427</td>
<td>12986</td>
</tr>
<tr>
<td>Mode 1</td>
<td>TM(_{201})</td>
<td>TM(_{201})</td>
<td>TE(_{111})</td>
<td>TE(_{221})</td>
</tr>
<tr>
<td>( Q_u @ GHz )</td>
<td>12073(@)</td>
<td>26840(@)</td>
<td>12647(@)</td>
<td>14547(@)</td>
</tr>
<tr>
<td>Mode 2</td>
<td>TE(_{101})</td>
<td>TE(_{101})</td>
<td>TE(_{102})</td>
<td>TM(_{116})</td>
</tr>
</tbody>
</table>

the orthogonal degenerate TM\(_{201}\) modes emerge as a pair of
resonant modes. Here, to distinguish the pair of degenerate
modes, we refer to them as TM\(_{201A}\) and TM\(_{201B}\) modes. As illustrated in Fig. 1(a), the \( E \)-field distribution of TM\(_{201A}\) mode is divided into two parts (i.e., parts \( a \) and \( b \)), the
\( E \)-field in part \( b \) expands along \( z \)-direction, while that of part \( a \) expands along \( -z \)-direction. Similarly, the \( E \)-field distribution of TM\(_{201B}\) is shown in Fig. 1(b) with parts \( a' \) and part \( b' \) in terms of equal magnitudes and out-of-phase phases. By comparing TM\(_{201A}\) and TM\(_{201B}\) modes, it is noticed that the \( E \)-fields of the pair of orthogonal degenerate modes show opposed trends at the same place.

For the conception of a dual-band filtering rat-race coupler,
we deploy additional pair of orthogonal degenerate modes at
higher frequencies, denoted by TE\(_{101A}\) and TE\(_{101B}\) modes,
respectively. These two degenerate TE\(_{101}\) modes also have
two distinguished orientations of polarization. As illustrated in Fig. 1(c), the \( E \)-field distribution of TE\(_{101A}\) mode at the
bottom of HC (i.e., parts \( c \) and \( d \)) is perpendicular to the
\( x-o-y \) plane. From the directions of the electric field, it can
be seen that the signals in the two parts are differential with
a phase difference of 180\(^\circ\). The \( E \)-field distribution of TE\(_{101B}\) mode is depicted in Fig. 1(d), which is perpendicular to that of TE\(_{101A}\) mode. In Fig. 1(d), the \( E \)-field vector at the bottom is divided into two distinctive parts (parts \( c' \) and \( d' \)) according to the identical magnitude and out-of-phase difference. Owing
to that these modes are all orthogonal degenerate modes that
appear in pairs, the \( E \)-field of TE\(_{101A}\) mode is strong at a place
where that of TE\(_{101B}\) mode is weak or vice versa. Without
additional perturbations in our design, these interested TM
modes can be excited in the HC, and they are fully exploited
to implement single-/dual-band magnitude and phase
responses for the presented filtering rat-race couplers, which
will be introduced in Section III.

III. Analysis and Design of Proposed Filtering Rat-Race Couplers

Fig. 3 shows the coupling topologies for the designed
single-/dual-band filtering rat-race couplers. In these topolo-
gies, the white circles \( S_1 \) and \( S_2 \) denote input port 1 and
port 4, while \( L_1 \) and \( L_2 \) represent the output port 2 and port 3,
respectively. Subscripts 1 and 2 represent the upper and lower
HCs. In addition, rectangular signifies the corresponding HC
resonator. The A, B, C, and D represent the TM\(_{201A}\), TM\(_{201B}\),
TE$_{101A}$ and TE$_{101B}$ modes, respectively. Superscripts I and II are utilized to differentiate the two passbands in Fig. 3(b), i.e., first- and second-band, respectively. For four-port filtering rat-race couplers, when port 1 is excited, port 2 and port 3 receive output signals with equal amplitude and in-phase, whereas port 2 and port 3 can get inverse signals when the input port 4 is excited. Based on the designed topologies above, two types of filtering rat-race couplers are developed using HCs, the details of which are described below in Sections III-A and III-B.

A. Proposed Single-Band Filtering Rat-Race Coupler

According to the topology of Fig. 3(a), a single-band HC filtering rat-race coupler is proposed and its layout is shown in Fig. 4. The proposed design is constructed by stacking two internal-coupled hemispherical resonators in a back-to-back manner. Port 1 and port 4, which are perpendicular to each other, connect to the upper HC and extend from the edge to the center. Ports 2 and 3 are spatial symmetric with respect to port 1 at ±135°, and are linked with the lower HC. Four pairwise symmetrical coupling slots are etched on the middle metal wall which is different from the asymmetrical and inflexible slots in [21], more flexibly controlling the internal couplings between the two HCs 1 and 2, leading to adjustable filtering coupler response.

From Fig. 5(a), it can see that TM$_{201A}$ mode is excited in both the upper and lower HCs. Signals can be transmitted from input port 1 to upper cavity 1, and further coupled to lower cavity 2 through a pair of rectangular slots 1, then equally transmitted to two output ports with 0° phase difference, owing to the natural in-phase equal-magnitude E-field distributions of TM$_{201A}$ modes with respect to the symmetrical line T. As shown in the figure, the E-field of TM$_{201A}$ mode at port 4 is almost zero and thus port 4 cannot receive the signal from port 1. Moreover, when the signals at port 4 are injected, TM$_{201B}$ mode is successfully excited in this dual-cavity structure in Fig. 5(b). Through another pair of rectangular slots 2, the upper and lower cavities exhibit the same field distribution which shows the inherent out-of-phase equal-magnitude characteristics with respect to the symmetrical line T. Therefore, the input signal from port 4 will be equally separated into output port 2 and port 3 but with inverse signals. Likewise, signals cannot be transmitted between port 1 and port 4.

The desired passband center frequency (CF) for the HC-based single-band filtering rat-race coupler is set as 11.50 GHz, and the remaining parameters are set as 0.95% 0-dB fractional bandwidth (FBW) and 20 dB in-band return loss (RL). Using the classic synthesis procedure provided in [24], the coupling coefficients $M$ and external quality factors $Q_e$ are extracted as $M_{1A,2A} = M_{1B,2B} = 0.0158$, and $Q_{e,5} = 70.18$. The desired coupling coefficients ($M$) between upper and lower HCs and external quality factor ($Q_{e,5}$) can be extracted from the following equations:

$$|M| = |M_{1j,2j}| = \frac{f^2_{1j} - f^2_{2j}}{f^2_{1j} + f^2_{2j}}, \quad (j = A, B, C, D) \quad (7)$$

$$Q_{e,5} = \pi f_i \cdot \tau_{511} (f_i)/2, \quad (i = I, II) \quad (8)$$

where $j$ denotes the corresponding mode $j$ while $i$ denotes the first-/second-band case.

Fig. 6(a) exhibits the extracted $M$ against varied coupling slot lengths $l$ and the distances $d$ of the coupling slot from the center. As observed, when $l$ increases, the internal couplings rise. There is a similar tendency for distance $d$. Fig. 6(b) illustrates the extracted curves of the $Q_{e,5}$ from group delay versus the input probe length $l$ with height $h$ as a variable.
Fig. 6. Variation trend versus varied physical parameters. (a) Internal coupling $M$ against $l$ with $d$. (b) External quality factors $Q_{e,S}$ against $t$ with $h$.

Fig. 7. Fabricated prototype of the single-band filtering rat-race coupler.

When the probe length increases by about 11.5 mm, the $Q_{e,S}$ reaches the maximum value and then gradually decreases as the height increases.

Herein, the HC-based single-band filtering rat-race coupler is simulated via the High-Frequency Simulation Software. Copper is used for the cavity boundaries with an electrical conductivity of $5.8 \times 10^7$ S/m in the simulation. Based on the above analysis, the initial parameters can be fixed, and some fine-tuning is then carried out to optimize the filtering coupler performance. The final physical design dimensions of the designed single-band HC filtering rat-race coupler are determined as (all unit: mm):

- $r_0 = 16$
- $w_1 = w_2 = 2$
- $l_1 = l_2 = 8$
- $d_1 = 8.8$
- $d_2 = 8.6$
- $t_1 = t_2 = t_3 = t_4 = 4.5$
- $h = 3.1$

The dimension of the designed circuit excluding ports is $1.41\lambda_g \times 1.41\lambda_g \times 1.43\lambda_g$, where $\lambda_g$ is the guided wavelength at 11.50 GHz.

To verify this design concept, a prototype has been made by making full use of selective laser sintering (SLS) 3-D-printing technology with the PA12 (nylon material) as a whole structure and then plated with a 6 $\mu$m thick copper layer. It is worth mentioning that this thickness increment has been taken into consideration during the design. The surface of the HCs was slotted where appropriate to facilitate the plating process requiring access to the inner surface. The slotted surface can also further reduce the weight of the prototype. The fabricated prototype of the single-band filtering rat-race coupler based on 3-D printed HC is shown in Fig. 7.

Fig. 8(a)–(d) provides the simulated and measured frequency response results and a good agreement can be observed among these curves. Fig. 8(a) shows the simulated and measured $S_{11}$, $S_{21}$, and $S_{31}$. The measured CF is 11.58 GHz with FBW = 0.91%. The measured passband RL is greater than 20 dB and insertion loss (IL) is less than 0.87 dB without consideration of 3-dB power distribution. The resonance that occurred at the lower stopband in the measurement is mainly attributed to the deviation of the SMA connection to port 1 when the signal is injected from the vector network analyzer (VNA). Referring to the measured $S_{44}$, $S_{24}$, and $S_{34}$ in Fig. 8(b), almost the same center frequencies, bandwidth, RL, and IL are obtained as predicted. Moreover, the isolation characteristics between port 1 and port 4 are verified to be higher than 25 dB, while the RLs of output port 2 and port 3 are both great than 20 dB, featuring high isolation and good output port matching. In addition, when signals are injected from port 1 and port 4, the measured passband phase differences between the two output ports are $0^\circ \pm 4.3^\circ$ and $180^\circ \pm 1.5^\circ$, respectively. As seen from the figures, a TZ also appears on the upper stopband due to the higher order mode.
B. Proposed Dual-Band Filtering Rat-Race Coupler

As presented earlier, a single-band filtering rat-race coupler using dual-mode HC has been fabricated and measured with good phase and filtering performances. To further verify the design approach, a dual-band filtering rat-race coupler exploiting two pairs of orthogonal degenerate modes in one HC is proposed and designed, the topology of which is presented in Fig. 3(b). Unlike work I, two more pairs of rectangular coupled slots are added in the middle wall as shown in Fig. 9 and more details are analyzed below. When the input port 1 and port 4 are excited, the E-field directions at the bottom of two HCs at the dual-band operating frequencies are described in Fig. 10, respectively. When signal is injected from input port 1, both TM$_{201A}$ and TE$_{101A}$ modes in cavity 1 will be excited, simultaneously. Then, through these rectangular slots, the signal will be coupled to cavity 2. Correspondingly, cavity 2 will also generate TM$_{201A}$ and TE$_{101A}$ modes and exhibit the same E-field distributions. Due to the symmetry of two outputs with respect to the symmetrical line $T$, identical output signals can be obtained at port 2 and port 3. In this context, the lower passband consists of the TM$_{201A}$ modes in two HCs, while TE$_{101A}$ modes build up the upper passband.

It is worth mentioning that the four pairs of designed rectangular slots realize and control the internal couplings of the dual-band design. Referring to Fig. 10, the E-field of
TM<sub>201</sub> mode is stronger than the E-field of TE<sub>101</sub> mode at slots 1 and 2. Therefore, slots 1 and 2 mainly control the lower band. In contrast, the E-field of TE<sub>101</sub> mode is stronger than the E-field of TM<sub>201</sub> mode along the inner pairs of slots 3 and 4, which can control the internal coupling for the upper band.

To validate this dual-band design concept, the required coupling coefficients and external quality factors for the dual-band circuit can be synthesized according to the required CFs of \( f_1 = 11.61 \) GHz, \( f_2 = 13.29 \) GHz, and 0-dB FBWs of 0.71% and 0.60% with 20 and 24 dB RLs. The corresponding internal coupling coefficients \( M \) and external quality factors \( Q_e \) for the dual-band can be accordingly derived as [24], \( M_{1A,2A} = M_{1B,2B} = 0.0124 \), \( M_{1C,2C} = M_{1D,2D} = 0.0123 \), \( Q_{e,S}^I = 88.11 \), and \( Q_{e,S}^{II} = 86.50 \).

Fig. 11 plots the extracted \( M \) and \( Q_{e,S} \) versus varied physical parameters to attain the initial physical dimensions. As the length of slot 1 and slot 2 (\( l_1 \) and \( l_2 \)) increases, \( M_{1A,2A} \) becomes larger but \( M_{1C,2C} \) almost remain zero shown in Fig. 11(a). Similarly, \( M_{1C,2C} \) increases as the length of slot 3 and slot 4 (\( l_3 \) and \( l_4 \)) increases and \( M_{1A,2A} \) is almost fixed at a constant value as shown in Fig. 11(b). Moreover, when slot 3 and slot 4 are moved away in parallel from the center with increasing distances, \( M_{1C,2C} \) decreases while \( M_{1A,2A} \) is almost unchanged as shown in Fig. 11(c). The results also confirm the independent control of the internal couplings by the four pairs of coupling slots as mentioned above. In addition, \( Q_{e,S}^I \) is extracted as similarly as that in section A and omitted here for brevity. The influence of \( Q_{e,S}^{II} \) for the second passband is investigated here with the varied probe length, as described in Fig. 11(d). When the probe length increases, the \( Q_{e,S}^{II} \) decreases while the same case can be found for \( h \). A dual-band filtering rat-race coupler prototype is simulated, fabricated, and measured according to the above analysis. The final dimensions of the designed filtering rat-race coupler are ultimately selected as: \( r_0 = 16 \), \( t_1 = t_2 = t_3 = t_4 = 13.0 \), \( l_1 = 8.5 \), \( l_2 = 8.5 \), \( l_3 = 5.1 \), \( l_4 = 5.6 \),
### Table II

<table>
<thead>
<tr>
<th>References</th>
<th>Techniques</th>
<th>CFs (GHz)</th>
<th>3-dB FBW(%)</th>
<th>ILs (dB)</th>
<th>RLs (dB)</th>
<th>Isolation (dB)</th>
<th>Phase difference</th>
<th>Q-factor</th>
<th>Number of TZs</th>
<th>Dimension $(\lambda_x \times \lambda_y \times \lambda_z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>SIR</td>
<td>2.24/5.84</td>
<td>4.6/5.2</td>
<td>1.0/1.4</td>
<td>15/15</td>
<td>23.5/32.8</td>
<td>9.5°</td>
<td>—</td>
<td>1</td>
<td>0.41×0.39×0.0138</td>
</tr>
<tr>
<td>[6]</td>
<td>LTCC</td>
<td>3.5</td>
<td>14</td>
<td>2</td>
<td>20</td>
<td>20</td>
<td>4.5°</td>
<td>~35</td>
<td>2</td>
<td>0.108×0.113×0.059</td>
</tr>
<tr>
<td>[7]</td>
<td>SW</td>
<td>10.94</td>
<td>3.1</td>
<td>1.3</td>
<td>17</td>
<td>25</td>
<td>5.5°</td>
<td>~230</td>
<td>0</td>
<td>0.88×1.64×0.00048</td>
</tr>
<tr>
<td>[9]</td>
<td>DR</td>
<td>3.51</td>
<td>1.7/1.3</td>
<td>0.36</td>
<td>17</td>
<td>23</td>
<td>2.0°</td>
<td>~2500</td>
<td>3</td>
<td>0.42×0.42×0.32</td>
</tr>
<tr>
<td><strong>Work I</strong></td>
<td>HC</td>
<td>11.58</td>
<td>2.15</td>
<td>0.87</td>
<td>20</td>
<td>25</td>
<td>&lt;5°</td>
<td>4917</td>
<td>2</td>
<td>1.41×1.41×1.43</td>
</tr>
<tr>
<td><strong>Work II</strong></td>
<td>HC</td>
<td>11.61/13.29</td>
<td>1.88</td>
<td>1.07/0.60</td>
<td>19/18</td>
<td>25</td>
<td>&lt;10°</td>
<td>4917/3264</td>
<td>3</td>
<td>1.65×1.65×1.53</td>
</tr>
</tbody>
</table>


**Fig. 16.** Resonant frequencies for TM$_{201}$ and TE$_{001}$ modes versus the height of the loaded internal metallic post.

**Fig. 17.** Center frequencies adjustability (a) lower passband. (b) upper passband.

$w_1 = w_2 = w_3 = w_4 = 2.0$, $d_1 = 8.0$, $d_2 = 7.9$, $d_3 = 2.8$, $d_4 = 3.5$, and $h = 3.5$ (all unit: mm). It is also fabricated by utilizing the SLS 3-D-printing technology. The fabricated prototype of the dual-band HC filtering rat-race coupler is exhibited in Fig. 12.

The simulated and measured $S_{11}$, $S_{21}$, and $S_{31}$ are plotted in Fig. 13(a), which coincides well with each other. The dual-band filtering rat-race coupler shows that the CFs of dual-band are 11.61 and 13.29 GHz with 3-dB BWs of both 250 MHz. The measured minimum passband ILs are better than 1.07 dB for the lower band and 0.60 dB for the upper band. The passband RLs are above 19 and 25 dB, respectively. Fig. 13(b) shows the $S_{44}$, $S_{24}$, and $S_{34}$ with almost the same CFs and BWs as in-phase operation. The measured ILs are 0.41 and 0.46 dB, and the maximum RLs at input port 1 and port 4 are all better than 18 dB. In the passbands, the isolation between input port 1 and port 4 is better than 25 dB and the RLs of output port 2 and port 3 are better than 15 dB in Fig. 13(c), indicating high isolation and good output port matching. Furthermore, three TZs which are distributed between two passbands and the right of the upper passband, named TZ$_1$, TZ$_2$, and TZ$_3$, bring out high-frequency selectivity and band-to-band isolation. Inherently, the TZs are generated due to the higher order mode in the HC. In addition, the in-phase and out-of-phase imbalances of the first and second passbands are limited to $1.3^\circ$, $9.3^\circ$, $1.5^\circ$, and $4.5^\circ$, respectively.

**IV. COMPARISON AND EXTENDED DISCUSSION**

Table II lists the performance comparison between the state-of-the-art works and the proposed filtering rat-race couplers. The contributions of this work include not only the introduction of a new class of cavity-based filtering rat-race couplers, but also the development of a new design method leading to the demonstration of 3-D-printed prototypes with good performance in terms of sharp frequency selectivity, nice port matching, high in-band isolation, satisfactory phase, and magnitude imbalance, as well as compact size and lightweight against others. It is also worth mentioning that the proposed design is fully controllable. For example, Fig. 14 shows independently controllable bandwidths and center frequencies for the proposed dual-band design.
By changing the relative design parameters of feeding probes and coupling slots, the bandwidth of the first band can be adjusted from 240 to 460 MHz without affecting the second one. Similarly, the bandwidth of the second band can be controlled from 210 to 440 MHz without disturbing the first one. In addition, in order to realize independently controllable center frequencies, an additional internal metallic post is creatively introduced as shown in Fig. 15. It is located at the center of two HC resonators to perturb the degenerated TE_{101} mode in two coupled HC resonators. By increasing the height of the internal post \( h_1 \), the resonant frequency of TE_{101} mode will continuously increase while the resonant frequency of TM_{201} mode remains unchanged as verified in Fig. 16. In addition, both of the two center frequencies are decreased with an increased size of the cavity resonator. Therefore, independently controllable center frequencies for the dual-band design can be successfully obtained. Fig. 17(a) shows the CF of the first band can be changed from 11.61 to 11.68 GHz while the second band remains constant. Whereas, in Fig. 17(b), the CF of the second band can be adjusted from 13.3 to 13.4 GHz, while one of the first bands remains unchanged.

On the other hand, the proposed design also has the potential for differential applications. For example, the proposed design method can also be flexibly extended to guide the design of a balanced-to-unbalanced (BTU) filtering coupler, which is displayed in Fig. 18. Since the BTU filtering coupler consists of a BTU in-phase power divider and a BTU out-of-phase power divider, it can be realized by elaborately arranging the two pairs of differential ports. Fig. 19 shows the simulated performance, where the differential-mode circuit has good filtering responses while all the common-mode signals are well suppressed. Similarly, the dual-band filtering coupler can be extended to a BTU design as well. This example further illustrates the flexibility and practicality of the proposed filtering coupler topology and demonstrates its application potential in both single-ended and differential circuits.

V. CONCLUSION

The article has presented a novel design method for building HC-based filtering rat-race couplers, for the first time. By stacking dual cavities to develop flexibly multiplexing multimode topologies, two new types of single- and dual-band filtering rat-race couplers are presented with detailed theoretical analysis, experimental demonstration, and deep discussion. They have been well implemented as monoblock components by 3-D printing technology and investigated with many attractive advantages in terms of excellent filtering power division responses, nice phase difference performance, controllable center frequencies, and bandwidths, simple but extensible topology, compact size as well as lightweight. With these properties, we believe that the proposed design method of the waveguide filtering rat-race couplers has huge potential in practical applications like LEO satellite communication systems.

REFERENCES


---

**IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES**

---

**Gang Zhang** (Senior Member, IEEE) received the Ph.D. degree in electromagnetic field and microwave technology from the Nanjing University of Science and Technology, Nanjing, China, in 2017. From 2013 to 2014, he was an exchanging Ph.D. student with the School of Information Technology and Electrical Engineering, The University of Queensland, Brisbane, QLD, Australia. He has been working as a Lecturer with the School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing, since September 2017, and was promoted exceptionally to Associate Professor, in 2019. His research focuses on comprehensive theory and design technology of functional integrated microwave components.

Dr. Zhang has been an Associate Editor of the *IET Electronics Letters* since 2020 and he has been the organizer and session chair of several international academic conferences on electromagnetic field and microwave technology in recent years.

---

**Shuai Feng** was born in Jiangsu, China. He received the B.S. degree from the Lanzhou University of Technology, Lanzhou, China, in 2020. He is currently pursuing the M.S. degree in electrical engineering at Nanjing Normal University (NNU), Nanjing, China.

His current research interests include 3-D printing technology, waveguide circuits, microwave/millimeter-wave multifunctional filtering circuits, and antennas.

---

**Yuan Chen** was born in Anhui, China. She received the B.S. degree in electrical engineering from the Anhui University of Science and Technology, Huainan, China, in 2018, and the M.S. degree in electrical engineering from Nanjing Normal University (NNU), Nanjing, China, in 2022.

Her research interests include 3-D printing filtering circuits.

---

**Yang Yang** (Senior Member, IEEE) was born in Bayan Nur, Inner Mongolia, China. He received the Ph.D. degree in electronic engineering from the Department of Electrical and Computer Systems Engineering, Monash University, Melbourne, VIC, Australia, in 2013. In 2012, he joined Rain Bird Australia, Deer Park, VIC, serving as an Asia Pacific GSP Engineer. In April 2015, he returned to academia, holding the position of Senior Research Associate at Macquarie University, Sydney, NSW, Australia. In April 2016, he was appointed as a Research Fellow with the State Key Laboratory of Terahertz and Millimeter Waves, City University of Hong Kong, Hong Kong. In December 2016, he joined the University of Technology Sydney (UTS), Ultimo, NSW. He is currently an Associate Professor and a Group Leader of 3-D Millimeter-Wave and Terahertz Circuits and Antennas at UTS Tech Laboratory, Botany, NSW. He has over 200 international publications in microwave, millimeter-wave, and terahertz circuits, and antennas. His research interests include 3-D millimeter-wave and sub-terahertz technologies beyond 5G and biomedical applications.

Dr. Yang received the Linkage Infrastructure, Equipment and Facilities (LIEF) Award funded by the Australian Research Council to establish the national laboratory of Australian 3-D Terahertz Beam Measurement Platform (500 GHz), in 2021. He is currently an Associate Editor of *IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES* (from 2022). He is the Chair of IEEE MTT-S Education Resources Development (2023), and the Chair of IEEE NSW AP-S/MTT-S Joint Chapter (2023).
He Zhu (Member, IEEE) received the bachelor’s and master’s degrees from the South China University of Technology, Guangzhou, China, in 2011 and 2014, respectively, and the Ph.D. degree in electrical engineering from the School of ITEE, University of Queensland, Brisbane, QLD, Australia, in 2017. Since July 2017, he has been with the Global Big Data Technologies Center (GBDTC), University of Technology Sydney (UTS), Ultimo, NSW, Australia, as a Post-Doctoral Research Associate and then a Chancellor’s Post-Doctoral Research Fellow. In 2023, he joined the Manufacturing, Commonwealth Scientific and Industrial Research Organization (CSIRO), NSW, Australia, as an IWY-Research Scientist. His research interests include the development of passive and tunable microwave and mm-wave devices, radio frequency integrated circuits and systems, and beam-forming networks for antenna arrays.

Dr. Zhu received the Outstanding Master Thesis Award of Guangdong Province, China, in 2015, the Graduate School International Travel Award (GSITA) from the University of Queensland in 2016, the Dean’s Award for Outstanding HDR Thesis from the University of Queensland in 2018. He is one of the recipients of the Chancellor’s Research Fellowship (CRF) from University of Technology Sydney in 2022. He is also selected as an IWY-researcher via the Impossible Without You (IWY) campaign and the Future Science Platform (FSP) from CSIRO in 2023.

Xin Zhou (Graduate Student Member, IEEE) was born in Henan, China, in 1998. He received B.S. degree in electrical engineering from Henan Polytechnic University (HPU), Jiaozuo, China, in 2020. He is currently pursuing the M.S. degree in electrical engineering at Nanjing Normal University (NNU), Nanjing, China.

His current research interests include multifunctional microwave passive circuits and antennas.

Jiasheng Hong (Fellow, IEEE) received the D.Phil. degree in engineering science from the University of Oxford, Oxford, U.K., in 1994. He then joined the University of Birmingham, Birmingham, U.K., until 2001, when he moved up to Edinburgh, to join Heriot-Watt University, Edinburgh, Scotland, where he is currently a Professor in leading a team for research into advanced radio frequency (RF)/microwave device technologies. He has authored or coauthored over 200 journal articles and conference papers in this field and has published four relevant books: Microstrip Filters for RF/Microwave Applications (Wiley, first edition, 2001, and second edition, 2011), RF and Microwave Coupled-Line Circuits (Artech House, second edition, 2007), Balanced Microwave Filters (Wiley, 2018), and Advances in Planar Filters Design (IET, 2019).

Dr. Hong is a member of the IEEE MTT Technical Committees, the Subject Editor (Microwave) of Electronics Letters, and an Associate Editor of IET Microwaves, Antennas and Propagation and International Journal of RF and Microwave Computer-Aided Engineering.

Jiquan Yang received Ph.D. degree in mechanic technology from the Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2002. Since 2002, he has been with the School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing, where he is currently a Professor and Dean of the college. His research interests include high-performance microwave/millimeter-wave 3-D passive components, circuits and systems realized on printed circuit board (PCB), low-temperature co-fired ceramic (LTCC), multimaterial 3-D printing technology, and so forth.