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A theory of single-electron shuttling in an external magnetic field in nanoelectromechanical system with magnetic leads is presented. We consider partially spin-polarized electrons in the leads and electron transport in both the Coulomb blockade regime and in the limit of large bias voltages when the Coulomb blockade is lifted. The influence of the degree of spin polarization on shuttle instability is considered. It is shown that there is certain degree of spin polarization above which the magnetic field ceases to control electron transport. In the Coulomb blockade regime the dependence of the threshold magnetic field, which separates the "shuttle" and vibron regimes, on the degree of polarization is evaluated. The possibility of re-entrant transitions to the shuttle phase is discussed. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4904445]

1. Introduction

Electron transport through a quantum dot (QD) in mechanically "soft" systems can be realized as shuttling of electrons[1] (see also the reviews[2–4]). The shuttle regime of charge transport is characterized by a strong enhancement of the electrical current at a certain bias voltage, which determines the threshold of shuttle instability. In an ideal nonmagnetic system the threshold voltage in the weak tunneling limit under certain conditions is determined only by the frequency of QD vibrations.[5] In realistic systems, when dissipation and defects in the mechanical subsystem are present, the threshold voltage depends on the friction coefficient and the characteristics of the pinning potential. In this case the shuttle electrical current abruptly appears when the QD is de-pinned by external sources (microwave electromagnetic or acoustic fields) and a small bias voltage drives the system to the regime of self-sustained mechanical vibrations (see, e.g., the experiment[6]). Although electron shuttling in the Coulomb blockade regime has not been observed yet, the experimental realization of a single electron shuttle is expected in the nearest future.

It has been predicted[7,8] that in magnetic nano-electromechanical systems the tunnelling of spin-polarized electrons could be sensitive to an external magnetic field. In particular, in an idealized situation, when electrons in the leads are 100% spin-polarized and the source and drain leads have opposite polarization, the electrical current is blocked in the absence of an external magnetic field ("spin blockade"[9]). It was shown[7] that even a small magnetic field can trigger a shuttle instability in magnetic nano-electromechanical systems. In principle, this mechanism allows one to realize magnetically controlled electron transport in single-electron transistors. In Ref. 7 the calculations were performed in the limit of high voltages $V \to \infty$ when the Coulomb blockade is lifted. In this case the shuttle instability, in the absence of dissipation, appears in arbitrarily small magnetic fields. The magnetic field, however, strongly influences the increment $r(h) > 0$ of the exponential growth of classical shuttle coordinate $x_c \propto \exp(rt)$.[7]

The purpose of the present paper is to generalize the model of Ref. 7 to a more realistic situation when electrons in the leads are not fully spin-polarized (Fig. 1). We also consider both the Coulomb blockade ($eV \ll U$, U is the charging

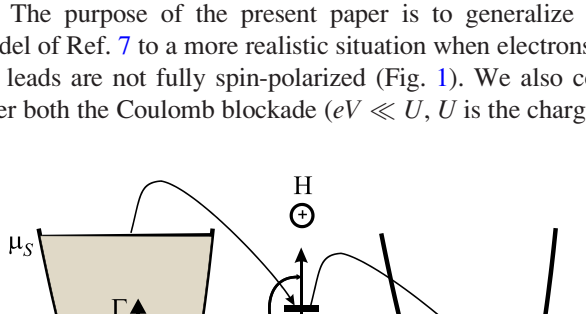

FIG. 1. Sketch of the nanomagnetic device studied: a movable quantum dot with a single spin-degenerate electron level is coupled to two partially spin-polarized leads. Γ and γ are the tunneling rates for the two spin projections (we assume that $\gamma \leq \Gamma$). The potential difference $\mu_S - \mu_D = |e|V$ between the leads is due to a bias voltage V. An external magnetic field H induces flips between the spin-up and spin-down states on the dot.

energy) and $V \to \infty$ regimes of electron transport. We show that in the Coulomb blockade regime there is a threshold magnetic field which separates the vi-bronic (small oscillations around the equilibrium position of QD), $H < H_{\text{th}}$, and shuttling, $H > H_{\text{th}}$, regimes of quantum dot vibrations.

The dependence of the threshold magnetic field on the degree of electron polarization is the main result of our paper. It is shown that the threshold magnetic field decreases with the decrease of the degree of polarization, and at certain value of polarization (numerically ~64%) the threshold field vanishes.

The paper is organized as follows. In Sec. 2 we generalize the model of Ref. 7 to the case of partially spin-polarized electrons in the leads. In Sec. 3 an analytic solution for the increment of exponential growth of shuttle coordinate is obtained. We discuss the shuttle instability in the absence of a Coulomb blockade and in the Coulomb blockade regime in Sec. 4. In the Conclusion section we summarize the main results of our paper.

2. Hamiltonian and equations of motion

The Hamiltonian $\hat{\mathcal{H}} = \hat{\mathcal{H}}_l + \hat{\mathcal{H}}_d + \hat{\mathcal{H}}_v + \hat{\mathcal{H}}_t$ of our system consists of four terms. The first term, $\hat{\mathcal{H}}_l$ describes non-interacting electrons in the leads,

$$\hat{\mathcal{H}}_l = \sum_{k,\sigma,\kappa} \varepsilon_{k,\sigma,\kappa} a^\dagger_{k,\sigma,\kappa} a_{k,\sigma,\kappa}. \tag{1}$$

Here the operator $a^\dagger_{k,\sigma,\kappa}(a_{k,\sigma,\kappa})$ creates (destroys) an electron with momentum k and the spin projection $\sigma = (\uparrow, \downarrow)$ in the lead $\kappa = (S, D) = (-1, 1)$; $\varepsilon_{k,\sigma,\kappa}$ is the electron energy.

The second term is the quantum dot Hamiltonian, $\hat{\mathcal{H}}_d$ which reads

$$\hat{\mathcal{H}}_d = (\varepsilon_0 - e\mathcal{E}x) \sum_\sigma a^\dagger_\sigma a_\sigma - \frac{g\mu H}{2}\left(a^\dagger_\uparrow a_\downarrow + a^\dagger_\downarrow a_\uparrow\right) - U a^\dagger_\uparrow a^\dagger_\downarrow a_\uparrow a_\downarrow. \tag{2}$$

It describes the single electron state in the dot and its coupling to an electric field $\mathcal{E}$ and a magnetic field H (μ is the Bohr magneton, g is the gyromagnetic ratio). In Eq. (2) $a^\dagger_\sigma(a_\sigma)$ is the creation (annihilation) operator for electron on the dot. The intra-dot electron correlations are characterized by the Coulomb energy U.

Vibrations of the dot are described by the harmonic oscillator Hamiltonian

$$\hat{\mathcal{H}}_v = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 \hat{x}^2}{2}, \tag{3}$$

where $\hat{x}$ is the dot displacement operator, $\hat{p}$ is the canonical conjugated momentum ($[\hat{x}, \hat{p}] = i\hbar$), m is the mass and ω is the vibrational frequency of the dot. The last term in our Hamiltonian represents spin-conserving tunnelling of electrons between dot and leads,

$$\hat{\mathcal{H}}_t = \sum_{k,\sigma,\kappa} T_{\sigma,\kappa}(\hat{x}) a^\dagger_{k,\sigma,\kappa} a_\sigma + \text{H.c.} \tag{4}$$

Here $T_{\sigma,\kappa}(\hat{x}) = T_{\sigma,\kappa} \exp(\kappa \hat{x}/\lambda)$ is the position-dependent tunnelling amplitude, λ being the tunnelling length. The electrons in each lead are held at a constant electrochemical potential $\mu_{S,D} = \pm|e|V/2$ (relative to the Fermi level), where $V > 0$ is the bias voltage. The electron density of states $\nu_j = \nu$ in the leads is assumed to be independent of energy.

To solve the problem, one needs to know the evolution of a reduced density matrix operator ρ, which describes the vibrational degree of freedom coupled to a single electronic dot state. The electronic state is spanned by the four basis vectors $|0\rangle$, $|\uparrow\rangle = a^\dagger_\uparrow|0\rangle$, $|\downarrow\rangle = a^\dagger_\downarrow|0\rangle$ and $|2\rangle = a^\dagger_\uparrow a^\dagger_\downarrow|0\rangle$. We first consider the Coulomb blockade regime, $eV < U$, where the tunnelling of a second electron onto the dot is blocked by the Coulomb interaction ($\rho_2 \to 0$).

It is convenient to introduce dimensionless variables for time, $t\omega \to t$, dot displacement $x/x_0 \to x$ (where $x_0 = \sqrt{\hbar/m\omega}$ is the zero-point oscillation amplitude), momentum $px_0/\hbar \to p$ and various characteristic energies, $\hbar\omega \to 1$, $g\mu H/\hbar\omega \to h$, $eEx_0/\hbar\omega \to d$, $\Gamma^\sigma_\kappa(x)/\hbar\omega \to \Gamma^\sigma_\kappa(x)$ ($\Gamma^\sigma_\kappa(x) = 2\pi\nu|T_{\sigma,\kappa}(x)|^2 \equiv \Gamma^\sigma_\kappa \exp(2\kappa x/\lambda)$ are partial level widths).

Following Ref. 7 one gets the equations of motion for the reduced density matrix operators $\rho_0 \equiv \langle 0|\rho|0\rangle$, $\rho_\uparrow \equiv \langle\uparrow|\rho|\uparrow\rangle$, $\rho_\downarrow \equiv \langle\downarrow|\rho|\downarrow\rangle$ and $\rho_{\uparrow\downarrow} \equiv \langle\uparrow|\rho|\downarrow\rangle$. These equations are

$$\frac{\partial\rho_0}{\partial t} = -i\left[\hat{\mathcal{H}}_v + dx, \rho_0\right] - \frac{1}{2}\left\{\Gamma^\uparrow_S(x) + \Gamma^\downarrow_S(x), \rho_0\right\} + \sqrt{\Gamma^\uparrow_D(x)}\rho_\uparrow\sqrt{\Gamma^\uparrow_D(x)} + \sqrt{\Gamma^\uparrow_D(x)}\rho_\downarrow\sqrt{\Gamma^\downarrow_D(x)}, \tag{5}$$

$$\frac{\partial\rho_\uparrow}{\partial t} = -i\left[\hat{\mathcal{H}}_v, \rho_\uparrow\right] - \frac{ih}{2}\left(\rho_{\uparrow\downarrow} - \rho^\dagger_{\uparrow\downarrow}\right) + \sqrt{\Gamma^\uparrow_S(x)}\rho_0\sqrt{\Gamma^\uparrow_S(x)} - \frac{1}{2}\left\{\Gamma^\uparrow_D(x), \rho_\uparrow\right\}, \tag{6}$$

$$\frac{\partial\rho_\uparrow}{\partial t} = -i\left[\hat{\mathcal{H}}_v, \rho_\uparrow\right] + \frac{ih}{2}\left(\rho_{\uparrow\downarrow} - \rho^\dagger_{\uparrow\downarrow}\right) + \sqrt{\Gamma^\downarrow_S(x)}\rho_0\sqrt{\Gamma^\downarrow_S(x)} - \frac{1}{2}\left\{\Gamma^\downarrow_D(x), \rho_\downarrow\right\}, \tag{7}$$

$$\frac{\partial\rho_{\uparrow\downarrow}}{\partial t} = -i\left[\hat{\mathcal{H}}_v, \rho_{\uparrow\downarrow}\right] + \frac{ih}{2}(\rho_\uparrow - \rho_\downarrow) - \frac{\rho_{\uparrow\downarrow}}{2}\left(\Gamma^\uparrow_S(x) + \Gamma^\downarrow_D(x)\right) - \left(\Gamma^\downarrow_S(x) + \Gamma^\uparrow_D(x)\right)\frac{\rho_{\uparrow\downarrow}}{2}. \tag{8}$$

It is easy to check that for the case of fully spin-polarized electrons in the leads, Eqs. (5)–(8) are reduced to the equations given in Ref. 7. In what follows we will restrict ourselves to the symmetrical case, $\Gamma^\uparrow_S = \Gamma^\downarrow_D \equiv \Gamma$, $\Gamma^\downarrow_S = \Gamma^\uparrow_D \equiv \gamma$.

We are interested in the classical motion of the dot. By using Eqs. (5)–(8) it is easy to get the classical equations of motion for coordinate and momentum:

$$\frac{\partial x_c}{\partial t} = \text{Tr}\left\{\frac{\partial}{\partial t}(\hat{x}\rho)\right\} = p_c, \tag{9}$$

$$\frac{\partial p_c}{\partial t} = \text{Tr}\left\{\frac{\partial}{\partial t}(\hat{p}\rho)\right\} = -x_c - d\,\text{Tr}\rho_0. \tag{10}$$

Therefore one needs to know the equations of motion for the zeroth moments, $R_i = \text{Tr}\rho_i$, (the index i runs over all the sub-indices in Eqs. (5)–(8)).

The dynamics of the zeroth moments is coupled to the dynamics of the first moments, which in turn are coupled to higher moments. We will decouple at the level of the first moments by using the rule $\text{Tr}\{\hat{x}\rho_i\} \to x_c\text{Tr}\rho_i$, where x_c is

the classical shuttle coordinate. In addition to restricting our study to the vibrational dynamics near the ground state we will assume the parameters d, $1/\lambda$ to be small and linearize all equations with respect to the classical displacement x_c.

It is convenient to introduce the following linear combinations of R_i,

$$R_0 = \mathrm{Tr}\rho_0, \quad R_1 = 1 - \mathrm{Tr}\rho_\uparrow, \quad R_2 = i\mathrm{Tr}(\rho_{\uparrow\downarrow} - \rho_{\uparrow\downarrow}),$$
$$R_3 = \mathrm{Tr}(\rho_{\uparrow\downarrow} + \rho_{\downarrow\uparrow}). \tag{11}$$

Using the approximations described the equations of motion for the zeroth moments take the form

$$\frac{\partial R_0}{\partial t} = -\left(2\Gamma + \gamma - \gamma\frac{2x}{\lambda}\right)R_0 + (\Gamma - \gamma)\left(1 + \frac{2x}{\lambda}\right)R_1 + \gamma\left(1 + \frac{2x}{\lambda}\right), \tag{12}$$

$$\frac{\partial R_1}{\partial t} = -\Gamma\left(1 - \frac{2x}{\lambda}\right)R_0 - \gamma\left(1 + \frac{2x}{\lambda}\right)R_1 + \frac{h}{2}R_2 + \gamma\left(1 + \frac{2x}{\lambda}\right), \tag{13}$$

$$\frac{\partial R_2}{\partial t} = h(1 + R_0 - 2R_1) - \frac{\Gamma + \gamma}{2}\left(1 + \frac{2x}{\lambda}\right)R_2. \tag{14}$$

(Note that the equation for R_3 is decoupled from the other equations and not relevant in what follows.)

3. Analytical solution

For small vibrations an analytical solution can be found by perturbation theory in terms of the small parameters $\varepsilon = \{d, 1/\lambda\}$. We solve these equations by perturbation expansions,

$$R_i(t) = R_i^{(0)} + R_i^{(1)}(t) + \cdots, \tag{15}$$

where $R_i^{(n)}$ is of nth order in ε. It is evident from Eqs. (12)–(14) that the functions $R_i^{(0)}$ do not depend on time. Hence,

$$R_0^{(0)} = \frac{\Gamma\gamma + h^2}{\Delta}, \quad R_1^{(0)} = \frac{\Gamma\gamma + \gamma^2 + 2h^2}{\Delta}, \quad R_2^{(0)} = \frac{2(\Gamma - \gamma)h}{\Delta}, \tag{16}$$

where

$$\Delta = \Gamma^2 + \Gamma\gamma + \gamma^2 + 3h^2.$$

It is convenient to define the vector-function $|\mathbf{R}\rangle = (R_0^{(1)}, R_1^{(1)}, R_2^{(1)})^T$. Then to first order in perturbation theory one has

$$\frac{\partial|\mathbf{R}\rangle}{\partial t} = \hat{A}|\mathbf{R}\rangle + \frac{2}{\lambda}x_c(t)|\mathbf{e}\rangle, \tag{17}$$

where

$$\hat{A} = \begin{pmatrix} -(2\Gamma + \gamma) & \Gamma - \gamma & 0 \\ -\Gamma & -\gamma & h/2 \\ h & -2h & -(\Gamma + \gamma)/2 \end{pmatrix}, \tag{18}$$

and the vector $|\mathbf{e}\rangle$ is defined as follows:

$$|\mathbf{e}\rangle = \frac{1}{\Delta}\begin{pmatrix} 2(\Gamma + \gamma)(\Gamma\gamma + h^2) \\ 2\Gamma^2\gamma + (\Gamma + \gamma)h^2 \\ (\gamma^2 - \Gamma^2)h \end{pmatrix}. \tag{19}$$

Consequently, the eigenfrequencies of the shuttle vibrations can be found from the equation

$$\frac{\partial^2 x_c(t)}{\partial t^2} + x_c(t) = -\frac{2d}{\lambda}\int_{-\infty}^{t} dt' x_c(t')\langle\mathbf{e}_0|e^{\hat{A}(t-t')}|\mathbf{e}\rangle, \tag{20}$$

where $|\mathbf{e}_0\rangle = (1, 0, 0)^T$.

We are interested in the sign of the imaginary part of the correction to the shuttle eigenfrequency, $x_c \sim \exp(i\Omega t)$, $\Omega = 1 + (d/\lambda)\tilde{\omega}$, due to coupling with the leads (the increment of exponential growth is $r = -i\mathrm{Im}\Omega > 0$). It follows from Eq. (20) that this correction takes the form

$$\tilde{\omega} = \frac{1}{\Delta}\frac{\mathcal{D}_1}{\mathcal{D}_0}, \tag{21}$$

where

$$\mathcal{D}_1 = (\Gamma + \gamma)[2(\Gamma\gamma + h^2) - 2h^2(h^2 + 2\Gamma\gamma) - \Gamma\gamma(\Gamma^2 + \gamma^2)] - i[3\Gamma\gamma(\Gamma^2 + \gamma^2) + 2\Gamma^2\gamma^2 + 2h^2(\Gamma + \gamma^2)], \tag{22}$$

and

$$\mathcal{D}_0 = \frac{\Gamma + \gamma}{2}(5 - \Gamma^2 - \Gamma\gamma - \gamma^2 - 3h^2) + i(1 - 2\Gamma^2 - 3\Gamma\gamma - 2\gamma^2 - h^2). \tag{23}$$

Therefore, the condition for being in the shuttle domain is that the inequality

$$h^6 + C_4h^4 + C_2h^2 + C_0 > 0, \tag{24}$$

is fulfilled. Here the coefficients C_i take the form

$$C_4 = \frac{\Gamma^2 + \gamma^2}{2} + 2(\Gamma\gamma - 1), \tag{25}$$

$$C_2 = \frac{3\Gamma}{4}(\Gamma^2 + \gamma^2) - \frac{\Gamma^4 + \gamma^4 - \Gamma^2 - \gamma^2}{2} + 5\Gamma^2\gamma^2 - \Gamma\gamma + 1, \tag{26}$$

$$C_0 = \frac{\Gamma\gamma}{4}(\Gamma^4 + \gamma^4) + \frac{\Gamma^2\gamma^2 + 5\Gamma\gamma}{4}(\Gamma^2 + \gamma^2) - \frac{\Gamma^2\gamma^2}{2} + \Gamma\gamma. \tag{27}$$

In the next section, based on this inequality, we will discuss the specific features of the shuttle domain.

4. Analysis of the solution

4.1. Shuttle dynamics in the Coulomb blockade regime

The inequality (24) defines the shuttle instability domain. In Fig. 2 we plot the extent of this domain in the (Γ, h)-plane for several values of γ. The case of fully spin-polarized leads ($\gamma = 0$) was considered in detail in Ref. 10:

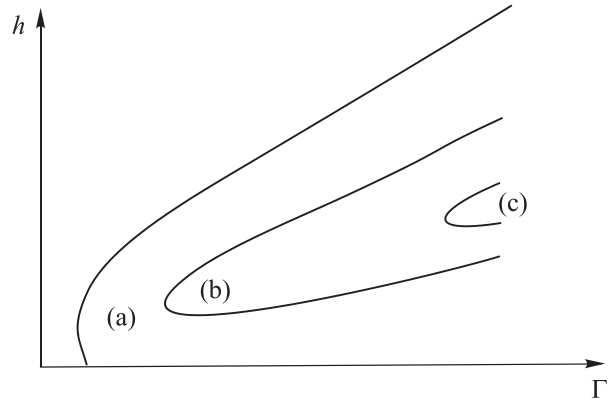

FIG. 2. Schematic dependence of magnetic field h on the tunneling rate Γ (solid lines) at the border between the shuttle- and vibronic domains for (a) $\gamma = 0$ (full spin polarization), (b) $\gamma = \gamma_b$, and (c) $\gamma = \gamma_c$, where $\gamma_c > \gamma_b$. In each case the vibronic domain is below (above) the upper (lower) branch of the border line. With an increase of γ the shuttle domain expands and eventually the vibronic domain vanishes.

When $\Gamma < (4/3)\hbar\omega_0$ (now we return to dimensional variables), only the "shuttle phase" is stable (for arbitrary values of h). If $(4/3)\hbar\omega_0 < \Gamma < \sqrt{2}\hbar\omega_0$, there is certain interval in h when the "vibronic phase" is stable. When $\Gamma > \sqrt{2}\Gamma\hbar\omega_0$, the transformation from the vibronic to the shuttle region occurs at the threshold magnetic field, $h_{\rm th} \propto \Gamma$ (for $\Gamma \gg 1$).

Increasing γ from zero, the shuttle domain of electron transport expands while the vibronic domain becomes narrower and vanishes completely for a definite value $(\gamma/\Gamma)_m = 0.22$ (see Fig. 3). The corresponding critical degree of spin polarization, defined as

$$\xi = \frac{1-\gamma/\Gamma}{1+\gamma/\Gamma}, \tag{28}$$

is therefore $\xi_m \simeq 64\%$. If the spin polarization is lower than this value the magnetic field ceases to cause any transition between the vibronic and shuttle phases.

The threshold magnetic field $h_{\rm th}$ is plotted as a function of γ in Fig. 3. This function has a vertical tangent at the point M, which defines $(\gamma/\Gamma)_m$. The point M divides the plotted curve into an upper and a lower branch. Part of the lower branch is dashed to indicate that for a given dissipation rate

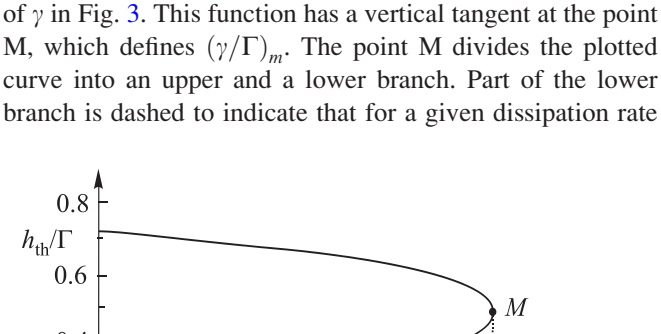

FIG. 3. The threshold magnetic field $h_{\rm th}$ plotted as a function of the minority spin tunnelling rate γ (normalized to the majority spin tunneling rate Γ). Part of the "lower" branch is shown as a dashed curve because for small magnetic fields the rate of instability is small and dissipation prevents the development of instability. Point M defines the maximum value of γ/Γ above which magnetic field does not cause the transition between shuttle and vibronic regime of electron transport.

there can be no shuttle instability for small enough magnetic fields. This is because the rate r (increment) at which the amplitude of the dot oscillations would increase in the absence of dissipation is small ($r \propto h^2$). Therefore any amount of friction will prevent the instability to develop for low enough magnetic fields.

Note that $h_{\rm th}$ tends to zero and $h_{\rm th} \propto \sqrt{\Gamma\gamma}$ as $\gamma \to 0$. In the limits $\gamma/\Gamma \ll 1$ and $\Gamma \gg 1$ it follows that the "large-h" branch is given by the relation $h_{\rm th}/\Gamma \approx (1/\sqrt{2})(1 - 3\gamma/2\Gamma)$. We see that at $\Gamma \gg 1$ threshold magnetic field normalized on Γ depends only on one variable γ/Γ. It is precisely this dependence that is shown in Fig. 3. The maximum value of ratio $(\gamma/\Gamma)_m \approx 0.22$ gives the minimum level of spin polarization, $\xi_m = 0.64$ (see Eq. (28)), below which magnetic field can not induce transition between vibronic and shuttle "phases." It is interesting to note that there is finite interval of polarizations when the increase of magnetic field from small to large values is accompanied by re-entrant transition to the shuttling phase (the vibronic phase is "inside" the curve in Fig. 3).

4.2. Shuttle instability in the regime $V \to \infty$

If $eV > U$ the Coulomb blockade is lifted and there is a finite probability for electrons to occupy both interaction-split energy levels. For fully spin-polarized electrons (i.e., in the case when the leads are half-metals) the shuttle dynamics was considered in Ref. 7, where it was shown that in the absence of dissipation in the mechanical subsystem a shuttle instability takes place for arbitrary values of an external magnetic field. The only condition for the realization of electron shuttling is to direct the external magnetic field not parallel to the direction of magnetization in the leads. The strength of the magnetic field, however, strongly influences the increment of exponential growth of shuttle coordinate.

In this subsection we derive the rate of the development of shuttle instability for partially spin-polarized electrons in the leads and analyze the conditions under which a shuttle instability occurs in the presence of weak dissipation. In the absence of a Coulomb blockade the equations of motion for the matrix elements of the density operator take a form similar to the system of Eqs. (5)–(8). The only distinction is the presence of an additional equation for the matrix element of the doubly occupied state $\rho_2 = \langle 2|\hat{\rho}|2\rangle$,

$$\frac{\partial\rho_2}{\partial t} = -i\left[\hat{\mathcal{H}}_v - dx, \rho_2\right] + \frac{1}{2}\left\{\Gamma_D^{\uparrow} + \Gamma_D^{\uparrow}, \rho_2\right\} - \sqrt{\Gamma_S^{\uparrow}}\rho_{\downarrow}\sqrt{\Gamma_S^{\uparrow}} - \sqrt{\Gamma_S^{\uparrow}}\rho_{\uparrow}\sqrt{\Gamma_S^{\uparrow}}, \tag{29}$$

and an additional term $\Gamma_S^{\downarrow(\uparrow)}\rho_2$ in the equations of motion for $\rho_{\uparrow}$ and $\rho_{\downarrow}$. The analysis of the new system of equations is completely analogous to the procedure described in Secs. 2 and 3 and results in the analytical expression for the increment

$$r(h) = \frac{d}{\lambda}\frac{\Gamma_+}{\Gamma_+^2 + 1}\left(1 - \frac{\Gamma_-^2}{\Gamma_+^2 + h^2}\right), \tag{30}$$

where $\Gamma_{\pm} = \Gamma \pm \gamma$. We see that r is never negative since $\Gamma_+^2 + h^2 \geq \Gamma_-^2$. In the limit $\gamma = 0$ (fully spin-polarized electrons) Eq. (30) is reduced to the corresponding formula

derived in Ref. 7 (notice that in Eq. (67) in Ref. 7 there is a misprint: factor $(\Gamma_0^2+1)^{-1}$ is missing). In the opposite limit of unpolarized electrons ($\gamma=\Gamma$) we get

$$r=\frac{2d}{\lambda}\frac{\Gamma}{\Gamma^2+1}. \tag{31}$$

We see that for unpolarized electrons the magnetic field has no influence at all (as it should be). The rate of increase of shuttle amplitude, Eq. (31), for $\Gamma \ll 1$ is reduced to the increment of shuttle instability for unpolarized electrons derived in Ref. 5 (see also[11]) in perturbation theory with T as the small parameter. One of the main conclusions of Ref. 7 was the assertion that for weak dissipation (phenomenologically introduced as friction in the equation of motion for coordinate, γ_f is the friction coefficient) a weak magnetic field can trigger a shuttling instability when $r(h)>\gamma_f$. The corresponding critical magnetic field scales as $h_c \propto (\gamma_f \Gamma^3 \lambda/d)^{1/2}$ for $\Gamma \gg 1$ and we assume that $\gamma_f \to 0$ (otherwise the shuttle instability can not be controlled by the magnetic field).

For partially polarized electrons an external magnetic field influences $r(h)$ when $h^2 \gtrsim \Gamma_+^2-\Gamma_-^2 \propto \Gamma\gamma$ (see Eq. (30)). For a realistic situation ($\Gamma \gg 1$) a magnetic control of electron shuttling can be achieved by external fields obeying the inequality $\sqrt{\gamma/\Gamma} \ll h \ll 1$, i.e., only for a degree of polarization close to 100%.

5. Conclusions

Single-electron shuttling is a nonequilibrium phenomenon, which under certain conditions determines the electron transport in some nanoelectromechanical devices. The possibility to control this shuttle current by external fields is an important problem in nanoelectromechanics.

It is especially interesting to consider magnetically controlled single-electron shuttling—something that can be realized in a magnetic shuttle system. A theory of spin-controlled electron shuttling was formulated in Ref. 7 for the case of 100% spin-polarized electrons in the source-and drain leads. In this ideal case the magnetic control of the electrical current is most effective. Here we have analyzed how the onset of shuttling can be controlled magnetically in the more realistic case of partially spin-polarized leads. Two different regimes of electron transport have been studied: (i) the Coulomb blockade regime and (ii) the regime of large bias voltages when the Coulomb blockade is lifted.

In the Coulomb blockade regime we obtained a universal curve, which for the realistic case that $\Gamma \gg \hbar\omega_0$ determines how the threshold magnetic field (separating the vibron and shuttle domains) depends on the degree of polarization. Using this result a numerical value for the minimum degree of spin polarization ($\sim$64%) was found. For partially polarized leads we also predicted the existence of re-entrant transitions to the shuttle phase as the magnetic field is increased. In the absence of a Coulomb blockade, we showed that magnetic control of electron shuttling can be realized only for almost 100%-polarized leads (so-called half metals).

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