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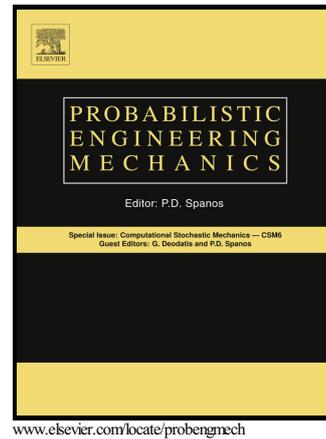
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Enhancing energy harvesting by a linear stochastic oscillator

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Abstract

A mathematical model of a linear electromechanical oscillator with a randomly fluctuating damping parameter as an energy harvester is considered. To demonstrate the idea and present an explicit analytical solution the variations of the damping are taken in a form of a telegraphic process. It is shown that substantial enhancement of the harvested energy can be reached if the proposed process has specially selected characteristics. A close relationship with the mean square stability of the oscillator is established. An analytical result for stationary electrical net mean power is presented.

1. Introduction

Energy harvesting is a conversion of ambient energy presented in the environment into electrical energy that can be used or stored. The energy harvesters can be used as a replacement for batteries in low-power wireless electronic devices (e.g. sensor networks used in structural health monitoring). Among the power harvesting methods, the conversion of mechanical energy from vibrations into electrical energy via the piezoelectric effect is particularly effective and has received a great attention over the last ten years (see e.g. books [1, 2] and reviews [3–5]). Vibration-based energy harvesters are

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typically implemented as linear mechanical resonators based on the following well-known phenomenon which is usually described at undergraduate texts in differential equations.

Consider the ubiquitous harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2x = \eta(t), \quad (1)$$

where $\gamma > 0$ is a damping parameter, ω is a natural frequency, the excitation $\eta(t)$ is harmonic, $\eta(t) = a \sin(\Omega t)$. Then the solutions of Eq. (1) grow dramatically if $\omega \approx \Omega$ and γ is small (the resonance phenomenon). A crucial limitation of the mechanical resonators is that the best performance of the devices is limited to a very narrow bandwidth around the natural frequency. If the excitation frequency deviates slightly from the resonance condition, the power output is drastically reduced, rendering linear resonant harvesters unsuitable for most practical applications. In a lot of cases the excitation exhibits random rather than deterministic behavior with a broad-band frequency spectrum.

When a broad-band external excitations is modeled by a zero-mean Gaussian white noise with intensity D , the power gained by the system is proportional to the noise intensity $P_w = D$. The latter is a very interesting result, since the harvested energy of a linear system is independent of the system's parameters. This fact has been extended to the case of nonlinear systems [6] and it has been shown that the mass of the system and the noise intensity are the two factors influencing the harvesting capabilities of any dynamical system, except the bistable systems, for instance [7–10]. It should be stressed that the above result is the upper boundary for the considered class of systems which may or may not be reached, whereas in the real life environment, where the excitation is rather broad band with finite power, any system has to be adjusted for its best performance. For instance if one considers a linear system (1) under the Ornstein-Uhlenbeck process then power of (1) can be obtained exactly analytically using the method of moments:

$$P_\mu = \frac{D(1 + \gamma\mu)}{(1 + \gamma\mu + \omega^2\mu^2)} = P_w \frac{1}{1 + \frac{\omega^2\mu^2}{1 + \gamma\mu}}$$

where D, μ -are parameters of the Ornstein-Uhlenbeck process. The above formula clearly shows that the harvested power depends on both the damping coefficient and natural frequency of the system.

In this paper we propose a promising approach for the vibration-based energy harvesting staying within the linear mechanical system framework. The vibratory system has a random time-varying variations of the damping parameter with specially selected characteristics that allow significantly enhance the energy harvesting. It can be achieved by using a magnetorheological (MR) damper [11]. The major idea behind the MR damper is the MR fluid which can change its viscosity under the influence of a magnetic field. It has been reported that MR dampers required relatively low amount of energy, outperform passive devices and SMAs, as well as have a fast response [12–15].

2. Mathematical model of the energy harvester

The vibration-based energy harvester comprises two parts. The key part is a mechanical oscillator but the second one is a capacitor that stores the electrical energy and is coupled with the oscillator by a transducer mechanism based on the piezoelectricity. A simplified representation of the piezoelectric vibration - based energy harvester is shown in Fig. 1 where the left part is the capacitive energy harvesting circuit (capacitor) but the right one is the base accelerated mechanical oscillator. Using Newton's second law and Ohm's law one can obtain for the mechanical states and electric voltage the following coupled equations [16]:

$$m \frac{d^2 \bar{x}(\tau)}{d\tau^2} + b(\tau) \frac{d\bar{x}(\tau)}{d\tau} + k\bar{x}(\tau) + \theta \bar{V}(\tau) = -m \frac{d^2 \bar{Z}(\tau)}{d\tau^2}, \quad (2)$$

$$RC_p \frac{d\bar{V}(\tau)}{d\tau} + \bar{V}(\tau) = R\theta \frac{d\bar{x}(\tau)}{d\tau}, \quad (3)$$

where \bar{x} is a deflection of mass m , $b(\tau)$ is a time-varying damping coefficient, k is a stiffness coefficient, θ is a linear electromechanical coupling, C_p is a capacitance of the piezoelectric element, \bar{V} is an electric voltage measured across the equivalent resistive load R , and $\frac{d^2 \bar{Z}(\tau)}{d\tau^2}$ is a base acceleration. We suppose that the damping coefficient has form

$$b(\tau) = a[1 + \sigma \zeta(\omega_1 \tau)],$$

where $\zeta(\omega_1 \tau)$ is some narrow band random variations of the constant damping coefficient a with a frequency ω_1 and noise intensity $0 \leq \sigma < 1$. Introduce the substitutions

$$\begin{aligned}
\omega &= \sqrt{\frac{k}{m}}, t = \omega\tau, \gamma = \frac{a}{\sqrt{mk}}, k_1 = \frac{\theta^2}{kC_p}, k_2 = \frac{1}{RC_p\omega}, \\
x_1 &= \frac{\bar{x}}{l_c}, Z = \frac{\bar{Z}}{l_c}, x_3 = \frac{\bar{V}C_p}{\theta l_c} \\
x_2 &= \frac{dx_1}{dt}, \eta = -\frac{d^2Z}{dt^2}, \xi(t) = \zeta\left(\frac{\omega_1 t}{\omega}\right),
\end{aligned} \tag{4}$$

where l_c is a length scale. Then Eqs. (2), (3) are reduced to the following system in the non-dimensional form:

$$\begin{aligned}
\frac{dx_1}{dt} &= x_2, \\
\frac{dx_2}{dt} &= -x_1 - \gamma[1 + \sigma\xi(t)]x_2 - k_1x_3 + \eta(t), \\
\frac{dx_3}{dt} &= x_2 - k_2x_3,
\end{aligned} \tag{5}$$

We suppose that the process $\eta(t)$ is a Gaussian white noise with intensity D . This process is considered as a good model of a broadband ambient excitation (see e.g. [16–19]). The random process $\xi(t)$ is a zero-mean telegraphic process with the state $\{-1, 1\}$ and the transition rate $\alpha/2$. This process is an ergodic Markov process and it is often used in applications (see e.g. books [20, 21]). The spectral characteristics of system (5) can be obtained exactly using the measuring filters approach [22]. It should be stressed that such a choice of the process was dictated by the ability of deriving an exact analytical solution to set of equations (5) rather than anything else, thereby illustrating clearly the influence of parameters on the energy harvesting process. It is possible to consider any other narrow band exponentially correlated process excitation process, but an explicit analytical solution may not be available resulting in a lack of understanding the phenomenon behind the proposed idea. The processes $\eta(t)$ and $\xi(t)$ are supposed independent. System (4) can be presented as a linear system of Itô stochastic differential equations

$$\begin{aligned}
dx_1 &= x_2 dt, \\
dx_2 &= -x_1 dt - \gamma[1 + \sigma\xi(t)]x_2 dt - k_1x_3 dt + Ddw(t), \\
dx_3 &= x_2 dt - k_2x_3 dt,
\end{aligned} \tag{6}$$

where $w(t)$ is a standard Wiener process.

3. Analytical development and results

First consider the system (4) with $\eta \equiv 0$. The trivial solution of this system is called mean square asymptotically stable if all second moments of any solution of the system tends to zero if $t \rightarrow \infty$.

Let

$$\begin{aligned}
\alpha_1 &= \alpha + \gamma \\
a_0 &= \alpha_1(1 + \alpha(1 + \alpha_1 + k_1) + [\alpha_1(2\alpha + \gamma) + k_1]k_2 + \\
&\quad + \alpha_1 k_2^2 \times [\gamma^2 k_2 + k_1 k_2 + \gamma(1 + k_1 + k_2^2)] [8k_2 + 4\alpha(1 + k_1) + \\
&\quad + \alpha(\alpha + 2\gamma)(\alpha + 2k_2)], \\
a_1 &= b_0 + b_1 k_2 + b_2 k_2^2 + b_3 k_2^3 + b_4 k_2^4, \\
b_0 &= 2\alpha(1 + k_1)[\alpha^4 + 2\alpha^3 \gamma + 3\alpha \gamma(1 + k_1) + 2(1 + k_1)^2 + \alpha^2(3 + \gamma^2 + 3k_1)], \\
b_1 &= 3\alpha^5 \gamma + 8(1 + k_1)^2 + \alpha^4(8 + 7\gamma^2 + 4k_1) + \alpha^3 \gamma \times (24 + 4\gamma^2 + 17k_1) + \\
&\quad + 4\alpha \gamma(5 + 8k_1 + 3k_1^2) + 2\alpha^2(10 + 13k_1 + 3k_1^2 + 8\gamma^2 + 7\gamma^2 k_1), \\
b_2 &= 2\alpha^5 + 15\alpha^4 \gamma + 16\gamma(1 + k_1) + 4\alpha^2 \gamma(13 + 3\gamma^2 + 5k_1) + 8\alpha[3 + 2k_1 + \\
&\quad + 2\gamma^2(2 + k_1)] + \alpha^3[25\gamma^2 + 4(4 + k_1)], \\
b_3 &= 2[4\alpha^4 + 13\alpha^3 \gamma + 8(1 + \gamma^2) + 4\alpha \gamma(6 + \gamma^2 + k_1) + \alpha^2 \times (13\gamma^2 + 12 + 3k_1)], \\
b_4 &= 2[5\alpha^3 + 8\gamma + 9\alpha^2 \gamma + 2\alpha(5 + 2\gamma^2 + k_1)], \\
a_2 &= 2k_2(\alpha + 2k_2)[\alpha^3 + 2k_2 + \alpha^2(\gamma + k_2) + \alpha(2 + k_1 + \gamma k_2)].
\end{aligned} \tag{7}$$

It should be emphasized that the damping coefficient γ is always positive because of our original assumption for $0 < \sigma < 1$.

Proposition. The trivial solution of system (4) with $\eta \equiv 0$ is mean square asymptotically stable iff

$$\sigma < \sigma_* = \sqrt{\frac{a_1 - \sqrt{a_1^2 - 4a_0 a_2}}{2a_2}}. \tag{8}$$

Proof. According to (4) the vector $\mathbf{y} = (x_1^2, x_2^2, x_3^2, x_1 x_2, x_1 x_3, x_2 x_3)^T$ satisfies the following equation in R^6 :

$$\frac{d\mathbf{y}}{dt} = A\mathbf{y} + \xi(t)B\mathbf{y}, \tag{9}$$

where

$$A = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & -2\gamma & 0 & -2 & 0 & -2k_1 \\ 0 & 0 & -2k_2 & 0 & 0 & 2 \\ -1 & 1 & 0 & -\gamma & -k_1 & 0 \\ 0 & 0 & 0 & 1 & -k_2 & 1 \\ 0 & 1 & -k_1 & 0 & -1 & -\gamma - k_2 \end{pmatrix}, \quad (10)$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sigma \end{pmatrix}.$$

Then for the mean value $E[\mathbf{y}(t)]$ we have the coupled system [23, 24]

$$\frac{dE[\mathbf{y}(t)]}{dt} = A\mathbf{y} + B\mathbf{y}_1,$$

$$\frac{d\mathbf{y}_1(t)}{dt} = -\alpha\mathbf{y}_1 + A\mathbf{y}_1 + BE[\mathbf{y}], \quad (11)$$

where $\mathbf{y}_1(t) = E[\xi(t)\mathbf{y}(t)]$. The mean square asymptotic stability of the trivial solution of system (4) with $\eta \equiv 0$ is equivalent to asymptotic stability of the trivial solution of system (9). This system is a set of 12 linear differential equations of first order with the constant coefficients. It is well known that asymptotic stability of its trivial solution can be investigated by inspecting the signs of the real parts of the eigenvalues of the coefficient matrix. Using well-known Routh-Hurwitz criterion and symbolic computation packages one can obtain the following necessary and sufficient condition of the stability:

$$g(\sigma) = a_2\sigma^4 - a_1\sigma^2 + a_0 > 0. \quad (12)$$

There is a more simple approach to get this condition which was proposed in Ref. [25]. Note that $8k_2g(\sigma)$ is a determinant of the coefficient matrix for system (9). Solving the inequality (12) one arrives to condition (8). \square

Let's consider now system (5) with an electromechanical coupling. The main quantity of interest in the energy harvesting is the electrical net mean power [16–19]

$$\bar{P}(t) = \frac{E[V^2(t)]}{R}.$$

Its non-dimensional counterpart is

$$P(t) = \frac{E[x_3^2]}{RC_p^2} = k_1 k_2 E[x_3(t)]^2$$

but in many application the long-term behavior of this value (stationary mean power)

$$P_{st} = k_1 k_2 \lim_{t \rightarrow \infty} E[x_3(t)]^2 \quad (13)$$

is very important.

Theorem. If condition (7) is fulfilled then

$$P_{st} = k_1 k_2 D \frac{g_1(\sigma)}{g(\sigma)}, \quad (14)$$

where

$$\begin{aligned} g_1(\sigma) &= c_1 \sigma^2 + c_0, \quad c_1 = 2(\alpha + 2k_2)[\alpha^3 + 2k_2 + \alpha^2(\gamma + k_2) + \alpha(1 + k_1 + \gamma k_2)], \\ c_0 &= [(\alpha + \gamma)(1 + \alpha^2 + \alpha\gamma + k_1) + (\alpha + \gamma)(2\alpha + \gamma)k_2 + k_1 k_2 + (\alpha + \gamma)k_2^2] \\ &\quad \times [8k_2 + 4\alpha(1 + k_1) + \alpha(\alpha + 2\gamma)(\alpha + 2k_2)]. \end{aligned}$$

Proof. Because the telegraphic process $\xi(t)$ and the Wiener process $w(t)$ are independent the second moments can be computed in two step

$$E[x_i(t)x_j(t)] = E_\xi[E_w[x_i(t)x_j(t)]], \quad i, j = 1, 2, 3,$$

where E_ξ and E_w are averaging over measure corresponding to the telegraphic process and the Wiener process respectively.

Let introduce the vector $\mathbf{v}(t) =$

$$(E_w[x_1]^2, E_w[x_2]^2, E_w[x_3]^2, E_w[x_1 x_2], E_w[x_1 x_3], E_w[x_2 x_3])^T,$$

where $x_1 = x_1(t)$, $x_2 = x_2(t)$, and $x_3 = x_3(t)$ satisfy set of equations (5). This vector satisfies the following equation in R^6 [26]:

$$\dot{\mathbf{v}} = A\mathbf{v} + \xi(t)B\mathbf{v} + \mathbf{d}, \quad (15)$$

where $\mathbf{d} = (0, 2D, 0, 0, 0, 0)^T$. Using again results from Refs. [23, 24] we have

$$\frac{dE_\xi[\mathbf{v}(t)]}{dt} = AE_\xi[\mathbf{v}(t)] + B\mathbf{v}_1(t) + \mathbf{d},$$

$$\frac{d\mathbf{v}_1(t)}{dt} = -\alpha\mathbf{v}_1(t) + A\mathbf{v}_1(t) + BE_\xi[\mathbf{v}(t)], \quad (16)$$

where $\mathbf{v}_1(t) = E_\xi[\xi(t)\mathbf{v}(t)]$. Therefore we get a system of 12 linear differential equations with constant coefficients. Under condition (7) there exist stationary moments that provide solution to system (5) and they satisfy the system of 12 linear algebraic equations associated with system (14). Applying symbolic computation packages to this system one can obtain expression (12) for P_{st} . \square

From expression (12) follows an important conclusion that the value P tends to infinity if $\sigma \rightarrow \sigma_*-$, i.e. if σ tends to the stability boundary. In the left part of Fig. 2 the mean square stability chart for the system (4) with $\eta \equiv 0$ is presented but the right part is the plot of P_{st} . One can observe substantial enhancement of the mean power near the stability boundary. The approach can be extended to some others models of the ambient excitation $\eta(t)$ but system (14) is much more complicated for them.

It is interesting to note that the power of the parametric noise does not have to be high. Indeed, if a ratio of the electrical efficiencies is considered, Θ_p/Θ_L , where the efficiency in the parametrically excited system is Θ_p and efficiency of the linear system $\Theta_L = \Theta_p(\sigma = 0)$ and

$$\Theta = \frac{Power(out)}{Power(in)},$$

then:

$$\frac{\Theta_p}{\Theta_L} = \frac{P_{st}(\sigma)}{W_\eta + W_\xi} \frac{W_\xi}{P_{st}(0)} = \frac{P_{st}(\sigma)}{P_{st}(0)} \frac{1}{1 + W_\eta/W_\xi} \quad (17)$$

where W_ξ and W_η are the power of the external and parametric noise respectively. In the case of unfeasible in reality a white noise, the second fraction is always equal to unity, therefore the efficiency of the parametric system is proportional to the power gained by it, which is much higher than that of the linear system. However, since a white noise is an approximation of a real physical process, the latter has a finite power, and thus the ratio $W_\eta/W_\xi > 0$ making the second fraction less than unity. Hence, keeping the parametric system stable but close to the instability boundary, it is preferable to keep $W_\eta \ll W_\xi$ to maximize the efficiency of the parametric system.

4. Conclusion

The paper proposes a new approach for substantial enhancement of energy harvesting using the electromechanical oscillator with a specially selected

random variations of the damping parameter. The method depends weakly on the ambient excitation because the mean square stability boundary plays crucial role here. It may be considered as an advantage as compared with the frequently used the linear resonator approach. However the random variations of the damping parameter is an active means which require some but not much input power compare to the benefits of harvested energy. It is possible to show that the proposed methodology can be extended to a wider class of exponentially correlated processes.

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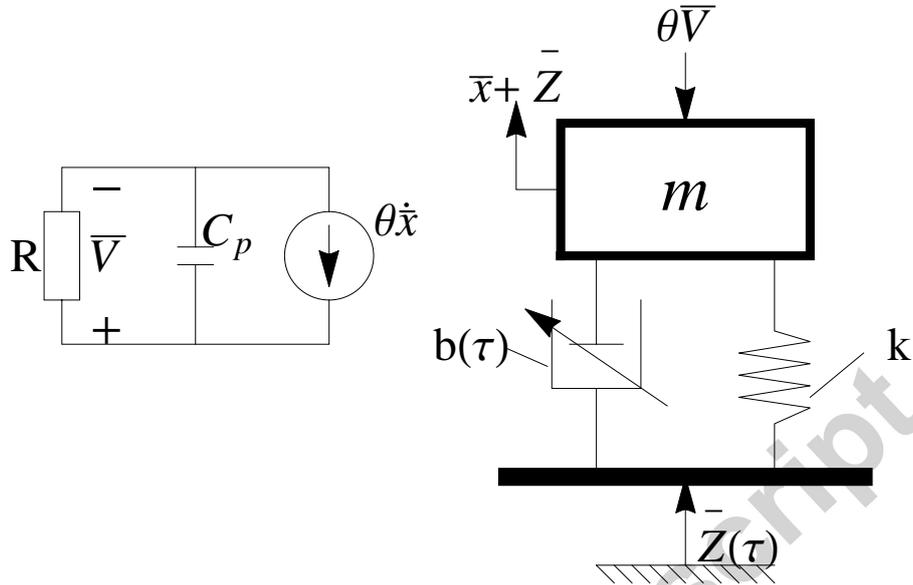


Figure 1: A simplified representation of the vibration - based piezoelectric energy harvester

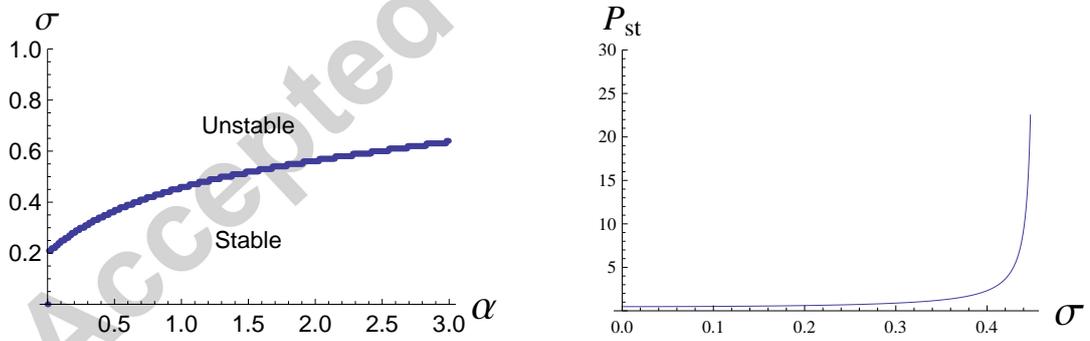


Figure 2: The mean-square stability chart (left) and the stationary mean power (right) for the system (5) with parameter values $\gamma = 0.1$, $k_1 = 0.5$, $k_2 = 0.3$. In the right part $\alpha = D = 1$.