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Technical Paper

Effect of temperature on the soil–water retention characteristics in unsaturated soils: Analytical and experimental approaches

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Abstract

In unsaturated soil mechanics, the soil–water retention curve (SWRC) continues to play an important role, since it provides the necessary links between the properties and behaviour of unsaturated soils with a variety of engineering challenges. The temperature has been identified as the main factor influencing SWRC as compared to a variety of other parameters. The goal of this research is to describe theoretical and experimental aspects of the temperature effect on unsaturated soil water retention phenomena. Theoretically, a brief review of the constitutive laws governing the thermal-hydro-mechanical (THM) behaviour of unsaturated soils is presented, along with links between variations in suction with water content, temperature, and void ratio. It also provides a broad framework that would be well adapted to describing many specific circumstances. Through a closed-form predictive relationship that is developed in this framework, the effect of temperature is examined. By using this relationship, the soil–water retention curve at arbitrary temperature could be determined from one at a reference temperature, therefore significantly decreasing the number of tests necessary to describe the thermo-hydro-mechanical behaviour of a soil. Besides, the SWRC of kaolinite clay was also measured at three different temperatures in an experimental program. The test findings reveal that when the temperature rises, the SWRC decreases significantly. The experimental results were then integrated with sixteen other available data sets covering a wide range of soil types, densities, and suction to create a complete verification program for analytical models. The proposed model has a good performance and reliability in forecasting the fluctuation of non-isothermal SWRC than any existing model, according to statistical assessment results. The analytical model can be used to examine the thermo-hydro-mechanical characteristics of unsaturated soils in numerical simulations.

Keywords: Unsaturated soil; Soil–water retention curve; Soil suction; Thermo-hydro-mechanical; Temperature; Non-isothermal models; Geological transition

1. Introduction

Unsaturated soils can be found commonly in practice, such as within embankments, roadbeds, and earth dams, as well as in the form of compacted soils. Furthermore, climatic change may alter the surface conditions from saturated to unsaturated, due to modifications in rainfall patterns, resulting in a number of new geologically significant process transitions. Therefore, the interest of the geotechnical and geological community in unsaturated soils has increased in recent years. Unsaturated soils, unlike saturated soils, contain pore air, resulting in the presence of both pore-air and pore-water pressure in soils. The differential between pore-air pressure and pore-water pressure is known as “matric suction”, and thus unsaturated soil behaviour is far more complicated than saturated soil behaviour (Pham and Sutman 2022a). The soil–water retention curve (SWRC), defined as the relation between

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matric suction and soil water content, plays a key role in determining hydraulic conductivity, dealing with different aspects of hydrogeology (Nguyen et al., 2022; Zhai et al., 2021), shear strength (Hashemi et al., 2023; Pham, 2022; Hoyos et al., 2014), structure stability (Pham et al., 2022; Pham, 2020a; Saffari et al., 2020; Houston et al., 2015), analysing seepage (Dang et al., 2020; Dye et al., 2011), investigating liquefaction (Mele and Flora, 2019; Mele et al., 2022), and simulating landslide of slopes (Filho and Fernandes, 2019). The general shape of the SWRC is influenced by soil density, pore geometry, grain size distribution, soil type (granular or clayey), loading rate, particularly, and temperature (Pham and Sutman, 2023a; Lahoori et al., 2021; Alsherif et al., 2016).

However, there has been a significant number of situations and applications in practice where the behaviour of unsaturated soils is associated with temperature variation. Heat storage application, geothermal constructions (energy pile, tunnel, or wall), and stability analysis of unsaturated slopes are several examples of this case (Pham and Sutman, 2023a; Hashemi et al., 2022; Thota and Vahedifard, 2021; Pham and Dias, 2021a; Zhang et al., 2020; Gu et al., 2014; Delage, 2013). In all of these circumstances, the soils are subjected to thermal cycles in an unsaturated state, making it crucial to understand the temperature impacts on SWRC in order to predict variations in the thermo-hydro-mechanical behaviour of soils. In this interest, several researchers have exerted to investigate the non-isothermal SWRC through experimental campaigns (Wan et al., 2015; Laloui et al., 2013; Ye et al., 2012; Schneider and Goss, 2011; Uchaipichat and Khalili, 2009; Birle et al., 2008; Villar and Gómez-Espina, 2007; Villar and Lloret, 2004; Romero et al., 2001; Constantz, 1991). On the other hand, a group of non-isothermal models were also proposed to predict the variation in SWRC with temperature (Grant and Salehzadeh, 1996; Salager et al., 2010; Tong et al., 2012; Zhou et al., 2014; Rosheni and Sedano, 2016; Vahedifard et al., 2018; Liu et al., 2020). However, there are several shortcomings and limitations among existing models that need to be addressed.

The first issue is that the majority of studies focus on the role of temperature in governing matric suction. However,
there is not a broad framework that completely takes into account the impact of temperature on matric suction and SWRC. Secondly, the majority of existing non-isothermal models investigated the effect of temperature on SWRC by considering the variation in surface tension, disregarding other key variables such as contact angle, void ratio, or expansion of water and solids, which may lead to inaccurate results (Bachmann and van der Ploeg, 2002). This could be due to the fact that soils have a complicated mineralogical composition, and the temperature dependence of the latter elements was difficult to determine. Furthermore, the complex nature of the soil and the variability of environmental factors result in a range of scenarios in practice, raising the necessity for a reliable model for estimating SWRC fluctuation with temperature. Nonetheless, almost all empirical models were established and verified using a limited number of data sets for low-temperature ranges. The applicability of these empirical models has not been always fully validated. It is hence always desirable to verify the non-isothermal SWRC models with other experimental data sets under a variety of soil conditions, considering different soil types with various soil structures, hydraulic conductivity, and dry density.

This study aims to analyse the effect of temperature on the SWRC, using both experimental and analytical models. In this regard, a brief review of the constitutive laws governing the THM behaviour of unsaturated soils is provided in the first section. Then, a general law reflecting the relationship between variations in matric suction and temperature is proposed. A general framework is also provided that appears to be well-adapted to describe many particular cases. An analytical model accounting for the effect of temperature on soil–water retention is then provided. Relying on theoretical features, the model determines the SWRC at different temperatures considering the SWRC at the reference temperature, significantly reducing the number of tests needed to characterise the thermo-hydraulic behaviour of the soil. In addition, several existing non-isothermal models are reviewed to discuss their advantages and disadvantages. In the following section, the details of the experimental apparatus and methods to determine the non-isothermal SWRC of soils, employing the filter paper method, are well discussed, supplemented with the outcomes on the non-isothermal SWRC of kaolinite. Finally, the performance of non-isothermal SWRC models is assessed by validating the model against seventeen measured data sets and six other analytical models.

2. Theoretical analysis of non-isothermal SWRC

2.1. Thermo-hydro-mechanical evolution of matric suction

The difference between pore-air pressure \( (u_a) \) and pore-water pressure \( (u_w) \), which can be stated as follows, is typically used to determine matric suction:

\[
\psi = u_a - u_w
\]

It should be noted that temperature changes \( (\Delta T) \), volumetric water content variations \( (\Delta \theta) \), and void ratio variations \( (\Delta e) \) are commonly used to characterize variations in the THM condition of unsaturated soil (Liu et al., 2021; Salager et al., 2010). As a result, the total matric suction differential with regard to the three independent variables \( T, \theta, \) and \( e \) can be stated as follows:

\[
d\psi = \left( \frac{\partial \psi}{\partial T} \right)_{\theta, e} dT + \left( \frac{\partial \psi}{\partial \theta} \right)_{T, e} d\theta + \left( \frac{\partial \psi}{\partial e} \right)_{T, \theta} de
\]

It is noted on the right side of Eq. (2) that the first term represents the matric suction variation with temperature (thermal state), the second term represents the matric suction variation with volumetric water content (hydro state), and the last term represents the matric suction variation with void ratio (mechanical state). Temperature, however, has a far greater impact on matric suction than other elements since a temperature change may also cause a combined change in water content and void ratio.

The capillary tube theory is frequently used to determine the matric suction since it is linked to the capillary phenomena. Consider about a small glass tube that is submerged in water under the atmospheric condition (Fig. 1). The vertical force balance of capillary water in the tube is:

\[
2\pi r_i \sigma_{so} \cos \alpha = \pi r_i^2 \rho g h_c
\]
Where, \( h_c \) = capillary height, \( g \) = gravitational acceleration, \( \rho_w \) = water density, \( r_i \) = tube radius or pore radius, \( \sigma_s \) = surface tension, \( \cos \alpha \) = wetting coefficient, \( z \) = air-water contact angle.

Thus, the following formula is used to determine the capillary height of pure water in a clean glass tube:

\[
h_c = \frac{2 \sigma \cos \alpha}{\rho_w g r_i}
\]  

(4)

At point C, the air pressure is atmospheric \( (u_a = 0) \) and the water pressure is negative \( (u_w = -\rho_w g h_c) \). The matric suction \( (\psi = u_a - u_w) \) at point C can then be expressed as follows:

\[
\psi = \rho_w g h_c = \frac{2 \sigma \cos \alpha}{r_i}
\]  

(5)

The differential component of matric suction concerning temperature can thus be expressed as follows:

\[
\frac{d\psi}{dT} = \frac{\psi}{\rho_w} \left( \frac{\partial \sigma}{\partial T} \cos \alpha + \frac{\partial \cos \alpha}{\partial T} \right) - \frac{\psi}{r_i} \left( \frac{\partial \sigma}{\partial T} \right)
\]  

(6)

On the other hand, the soil-water retention curve is also influenced by the volumetric water content of soils, which is a function of water content, void ratio, and the densities of the solid and liquid phases. This relationship is illustrated as follows:

\[
\theta = \frac{\rho_s}{\rho_w} \left( 1 + e \right)
\]  

(7)

Where \( \rho_s \) = density of solid, \( w \) = water content, and \( e \) = void ratio.

The differential of volumetric water content is therefore derived:

\[
d\theta = \frac{\rho_s}{\rho_w} \left( 1 + e \right) dw + \frac{w}{\rho_w} d\rho_s - \frac{\rho_s}{\rho_w} \left( 1 + e \right) d\rho_w
\]

\[
- \frac{\rho_s}{\rho_w} \frac{w}{\rho_w} d\rho_s
\]  

(8)

The second and third terms of the right side in Eq. (8) contain the density parameters of solid and water phases, respectively. It is worthy to note that \( \rho_s \) and \( \rho_w \) are not constant but they may vary with temperature. The temperature-dependent relationship of \( \rho_s \) and \( \rho_w \) can be described by integrating volumetric thermal expansion coefficient of solid and water phases as follows:

\[
\beta_s = -\frac{1}{\rho_s} \frac{d\rho_s}{dT}
\]  

(9a)

\[
\beta_w = -\frac{1}{\rho_w} \frac{d\rho_w}{dT}
\]  

(9b)

Substituting Eq. (9) into Eq. (8), we obtain:

\[
d\theta = \frac{\rho_s}{\rho_w} \left( 1 + e \right) dw - \frac{\rho_s}{\rho_w} \left( 1 + e \right) \frac{\beta_s}{\rho_w} dT - \frac{w}{\rho_w} \frac{\beta_w}{\rho_w} \frac{w}{\rho_w} d\rho_s
\]

\[
- \frac{w}{\rho_w} \frac{\beta_s}{\rho_w} \frac{w}{\rho_w} d\rho_s
\]  

(10)

It can be observed from Eq. (10) that the volumetric water content can shift with a variation in water content (because of drainage or evaporation), temperature (because of thermal contraction or dilation), and void ratio (because of void volume change). The differential component of matric suction concerning volumetric water content can be expressed as:

\[
\frac{d\psi}{dT} = \frac{\rho_w}{\rho_s} \left( 1 + e \right) \frac{1}{\rho_s} \frac{\partial \beta_s}{\partial T} - \frac{\rho_s}{\rho_w} \left( 1 + e \right) \frac{\rho_s}{\rho_w} \left( 1 + e \right) \frac{\beta_w}{\rho_w} + \frac{\beta_w}{\rho_w} \frac{w}{\rho_w} \frac{\partial w}{\partial T}
\]

(11)

The changes in the THM state of unsaturated soil are typically expressed in terms of void ratio \( (e) \), changes in water content \( (w) \), and changes in temperature \( (T) \). This section provides a general framework for understanding how temperature, water content, and void ratio affect the evolution of matric suction. It should be noted that the link between these three variables is one of reciprocity, whereby changes in temperature may have an indirect impact on matric suction through the variation of water content, and void ratio. As a result, any potential model could be also developed using this foundation; however, depending on the studied case, some aspects may be considered.

2.2. Pore radius conversion formulation

Equation (6) states that the temperature can affect matric suction through changes in pore radius \( (r_i) \). To apply mathematical rules and make precise theoretical deductions, the pore radius must be transformed in relation to the particle radius and void ratio of soils. The relation between pore and particle radii can be written as follows:

\[
r_i = R \sqrt{\frac{2e n_i^{1-\delta}}{3}}
\]  

(12)

Where, \( n_i = \) number of soil particles, \( R = \) particle radius, \( r_i = \) pore radius, and \( \delta = \) an empirical constant that takes into account the effect of particle size, shape, and orientation in an actual soil sample. The value \( \delta \) can be identified precisely by doing calibration with measured data. In this case, the measured SWRCs at two different initial void ratios must be provided. On the other hand, Arya and Paris (1981) tested five different materials and discovered that the best-fit values varied between 1.3 and 1.43. It is therefore assumed to be approximately 1.4 in this study.

Substituting Eq. (12) back into Eq. (5) gives:

\[
\psi = \frac{2 \sigma \cos \alpha}{R} \sqrt{\frac{3}{2e n_i^{1-\delta}}}
\]  

(13)

The differential of matric suction concerning temperature therefore can be re-written as follows:
\[
\frac{d\psi}{dT} = \frac{\psi}{\sigma_s} \left( \frac{\partial \sigma_s}{\partial T} \right) + \frac{\psi}{\cos \alpha} \left( \frac{\partial \cos \alpha}{\partial T} \right) - \frac{\psi}{R} \left( \frac{\partial R}{\partial T} \right) - \frac{\psi}{e} \left( \frac{\partial e}{\partial T} \right)
\]

Equation (14) derives that the matric suction can vary with temperature through variation in (i) surface tension, (ii) air–water contact angle, (iii) particle size expansion, (iv) void ratio. The influence of temperature on the soil–water retention curve can be predicted using a variety of models. Almost all existing models, however, concentrated solely on the fluctuation in surface tension with temperature, while other crucial elements were frequently overlooked. The next section of this study, thus, proposes a new model for non-isothermal SWRC of unsaturated soils.

### 2.3. Extended formulation of non-isothermal SWRC

In this section, the temperature-dependent model presented in Pham and Sutman (2023a, b) is refined by considering the dependency of air-entry value on the water density. For the analysis, a subscript \( T \) indicates variables at arbitrary temperature while subscript “0” indicates that the variables are at the reference temperature, which is defined as the room temperature at which samples are tested. For example, it is assumed that the matric suction at arbitrary temperature \( T \) is \( \psi_T \), and the matric suction at the reference temperature is \( \psi_0 \).

It is assumed that the reference matric suction \( \psi_0 \) has been determined at a reference temperature \( T_0 \) and attempt to express the matric suction \( \psi_T \) at any other temperature \( T \) in terms of \( \psi_0 \). According to the postulated conditions, the variation of \( \psi_T \) with temperature is caused by the variation of the surface tension \( (\sigma_s) \), wetting coefficient \( (\cos \alpha) \), particle size expansion \( (R) \), void ratio \( (e) \), and water density \( (\rho_w) \) with temperature. Due to the fact that the soil moisture content is maintained constant throughout all temperature changes in the current treatment, the soil structure is not expected to change appreciably as the temperature of the soil varies.

Let’s assume that the matric suction of the soil moisture at reference and arbitrary temperatures has been determined and found to be \( \psi_0 \) and \( \psi_T \) respectively as follows:

\[
\psi_0 = \frac{2\sigma_{0,T} \cos \alpha_0 R_0}{2 e_0 n_{1,0}^{\delta}}
\]

\[
\psi_T = \frac{2\sigma_{T,T} \cos \alpha_T R_T}{2 e_T n_{1,T}^{\delta}}
\]

Dividing Eq. (15b) to (15a) side by side gives:

\[
\psi_T = \psi_0 \times \frac{\sigma_{T,T}}{\sigma_{0,T}} \times \frac{\cos \alpha_T}{\cos \alpha_0} \times \frac{R_0}{R_T} \times \frac{e_0}{e_T} \left( \frac{n_{0,T}}{n_{1,T}} \right)^{1-\delta}
\]

For the simplification purpose, Eq. (16a) could be rewritten as follows:

\[
\psi_T = \psi_0 \times f_T = \psi_0 \times f_0 \times f_R \times f_e \times f_v
\]

Where \( f_\sigma = \) temperature-dependent factor of surface tension, \( f_\alpha = \) temperature-dependent factor of air–water contact angle, \( f_R = \) temperature-dependent factor of particle size, \( f_e = \) temperature-dependent factor of void ratio, \( f = \) overall temperature-dependent factor.

#### 2.3.1. Temperature-dependent factor of surface tension

According to Equation (16b), the procedure should start by expressing the surface tension as a function of temperature \( T \). Surface tension is the term used to describe the force distributed along the air–water interface curvature. The laboratory test equipment could be used to accurately determine the relationship between surface tension and temperature. The data regarding the effect of surface tension on temperature was supplied by Vargaftik et al. (1983). With increasing temperature, a highly nonlinear variation in surface tension was observed. Pham and Sutman (2023a) then established the following nonlinear temperature-dependent surface tension function:

\[
\sigma_{T,T} = 96.76 - 0.0125T - 0.000238 T^2
\]

Where the surface tension is expressed in mN/m, and the temperature \( T \) is degrees absolute or so-called Kelvin degree (K).

Therefore, the temperature-dependent factor of surface tension could be expressed by:

\[
f_\sigma = \frac{\sigma_{T,T}}{\sigma_{0,T}} = \frac{96.76 - 0.0125T - 0.000238 T^2}{96.76 - 0.0125T_0 - 0.000238 T_0^2}
\]

#### 2.3.2. Temperature-dependent factor of wetting coefficient

The angle formed by the vertical axis and the direction of surface tension at the air–water interface is known as the contact angle. By taking into account a temperature-dependent contact angle, Grant and Salehzadeh (1996) demonstrated that the temperature derivative of the wetting coefficient, \( \cos \alpha \), may be stated in terms of enthalpy of immersion as follows:

\[
\frac{d\cos \alpha}{dT} = \frac{1}{\sigma_{T,T}} \left( \frac{\sigma_{T,T} \cos \alpha_T + \Delta h}{T} - \cos \alpha_T \frac{d\sigma_{T,T}}{dT} \right)
\]

The immersion enthalpy must be amended with experimentally derived parameters, though, it is particularly challenging to determine accurately. The following equation, which eliminates the immersion enthalpy, is provided by Pham and Sutman (2023a) after they have solved the integration of Equation (19).

\[
\cos \alpha_T = \frac{\sigma_{T,T} \times T_0}{\sigma_{0,T} \times T + (0.000238 T^2 + 96.76) \times \Delta T} \cos \alpha_0
\]

Therefore, the temperature-dependent factor of wetting coefficient can be demonstrated by:
2.3.3 Temperature-dependent factor of particle size

The size of solid particles expands as a result of temperature changes. Assuming that the soil particles are in spherical shape, the volume of a soil particle at reference and arbitrary temperatures can be determined to be \( V_{0} \) and \( V_{aT} \) respectively.

\[
V_{T} = 4\pi \frac{R_{0}^{3}}{3} = 4\pi \left(1 + \beta_{s} \times \Delta T\right)^{3} \frac{R_{0}^{3}}{3}
\]

(22)

The temperature-dependent function of particle size can be stated using the link between volumetric thermal expansion and particle radius as follows:

\[
f_{R} = \frac{R_{0}}{R_{T}} = \frac{1}{\sqrt{1 + \beta_{s} \times \Delta T}}
\]

(23)

Where, \( \beta_{s} \) is volumetric thermal expansion coefficient of soils \( \approx 5.6 \times 10^{-5} \text{ } ^{\circ} \text{C} \) (Shetty et al., 2019).

2.3.4 Temperature-dependent factor of void ratio

It should be noted that an increase in temperature might result in a change in volume. As a result, the soil void ratio of soil sample will change, affecting the matric suction fluctuation. The following is the temperature-dependent function of void ratio (Pham and Sutman 2023 a, b):

\[
f_{e} = \sqrt{\frac{e_{0}}{e_{T}}} \left[\left(\frac{1 + e_{r}}{1 + e_{0}}\right)\left(1 + \beta_{v} \times \Delta T\right)^{1-\delta}\right]
\]

(24)

Where, \( e_{r} \) is the void ratio at current temperature \( T \), and \( e_{0} \) is the void ratio at reference temperature \( T_{0} \).

The change in void ratio with temperature can be appointed using the equation of Pham and Sutman (2023b):

\[
e_{r} = e_{0} - \left[10^{-6} (I_{P})^{2} + 3.10^{-6} (I_{P}) + 0.0002\right](1 + e_{0}) \times \Delta T
\]

(25)

Where, \( I_{P} \) is the index of plasticity of soils,

2.3.5 Temperature-dependent factor of water density

The expansion of water volume caused by a rise in temperature will impact the water density. The capillary tube theory is employed in this study to simulate how temperature affects the soil water retention curve through changes in water density.

On the other hand, Fredlund and Xing (1994) provided the following equation for the soil–water retention curve:

\[
\theta = \frac{\theta_{T}}{\left(ln[2.7127 + (\psi/a)^{n}]^{m}\right)}
\]

(26)

In this equation, the parameter \( a \) representing air-entry value, could be converted by:

\[
a = \psi_{0} \left[(\psi/a)^{n} - 2.7127\right]^{-1/n}
\]

(27)

Replacing Eq. (5) in Eq. (27) gives:

\[
a = \rho_{w} g h_{c} \left[(\psi/a)^{n} - 2.7127\right]^{-1/n}
\]

(28)

The capillary height and volumetric water content are consequently constant because the water content is considered to be constant with temperature variation. As a result, the following relationship exists between the parameter \( a_{r} \), or air-entry value, at the current temperature \( T \) and the one at the reference temperature \( a_{0} \):

\[
a_{r} = a_{0} \times \frac{\rho_{wT}}{\rho_{w0}}
\]

(29)

Where \( a_{r} \) is the air-entry value at current temperature, \( a_{0} \) is the air-entry value at reference temperature, \( \rho_{wT} \) is the water density at current temperature, \( \rho_{w0} \) is the water density at reference temperature.

Water density is defined as the mass-to-volume ratio of water. For most geotechnical engineering applications, the density of water under isothermal circumstances is generally assumed to be 1000 kg/m³. However, it was discovered that the density of water varies with temperature fluctuations due to water volume expansion. The following is the closed-form equation for the temperature-dependent component of water density:

\[
\rho_{wT} = 658.2 + 2.509T - 0.004606T^{2}
\]

(30)

Substituting Eq. (30) back into Eq. (29) leads to:

\[
a_{r} = a_{0} \times \frac{658.2 + 2.509T - 0.004606T^{2}}{658.2 + 2.509T_{0} - 0.004606T_{0}^{2}}
\]

(31)

At the reference temperature, the reference SWRC can be plotted by using one of the existing isothermal forms (Fredlund and Xing, 1994; van Genuchten, 1980; Brooks and Corey, 1964). It should be noted that for the same water content, the matric suction at various temperatures is different. The SWRC at an arbitrary temperature, thus, can be predicted by considering variation in matric suction with temperature for the same water content. The non-isothermal SWRC model is, therefore, proposed as follows:

\[
\left\{\begin{array}{l}
\theta_{T} = \frac{\theta_{0}}{\left[ln[2.7127 + (\psi/a)^{n}]^{m}\right]} \\
\psi_{T} = \psi_{0} \times f_{a} \times f_{g} \times f_{R} \times f_{e}
\end{array}\right.
\]

(32)

Where, \( a, n, m = \) fitting parameters, \( \theta_{r} = \) volumetric water content at the saturated condition.

In order to better illustrate the temperature influence on matric suction, Fig. 2a shows matric suction versus pore size at 200 °C considering five temperature-dependent functions. It should be noted that a wide range of pore diameters reflecting diverse soil types was described, ranging from very coarse sand (1.5 mm), fine sand (0.175 mm), silt (0.02 mm), to clay (0.0015 mm). It can be observed that the
following elements, in descending order, are involved in the sensitivity of the matric suction to temperature: air–water contact angle, surface tension, water density, particle size, and void ratio. For example, at a pore size radius of 1 mm and when the temperature is increased from 20 °C to 200 °C, the matric suction decreases 6.5% when only considering temperature-dependent void ratio, decreases 11.9% when only considering temperature-dependent particle size, decreases 18.4% when only considering temperature-dependent water density, decreases 48.3% when only considering temperature-dependent surface tension, and decreases 82.1% when only considering temperature-dependent contact angle. The findings show that the temperature-dependent function of contact angle has the greatest impact on the reduction of matric suction as temperature rises.

Fig. 2b presents the variations in matric suction as a function of temperature. When compared to contact angle and surface tension, the effects of the thermal void ratio and thermal expansion of water and solids are quite minor. Interestingly, it is found that the matric suction reduction in a low-temperature range is more significant than that of a high-temperature range. For \( r_i = 1 \) mm, the matric suction reduction per increase of 1 °C is approximately by 111, 80, 62, and 50% by increasing the temperature from 20 °C to 50 °C, 100 °C, 150 °C, and 200 °C, respectively.
3. Review of several non-isothermal SWRC models

Several non-isothermal SWRC models are described in this section. Six models, representing a diversity of theoretical perspectives, have been chosen for comparison.

3.1. Model proposed by Liu et al. (2020)

The voids in unsaturated soils are idealized as similar circular pores and spread uniformly over the two-dimensional cross-section, according to Liu et al. (2020). The total normal force of water acting in the normal direction of the cross-section is computed using this ideal assumption by:

\[
F_w = -\sigma_s \int_0^L \sin zdz + \frac{\Delta T}{\rho_w} u_w dA_w
\]  

(33)

where \(F_w\) = total normal force of water, \(\sigma_s\) = surface tension, \(L\) = length of the water–air interface, \(A_w\) = water area.

Considering continuous, isotropic, and homogeneous materials, the first and second parts of the right side in Eq. (33) can be expressed as follows:

\[
-\sigma_s \int_0^L \sin zdz = -\sigma_s \int_0^L \sin \left(\frac{\pi L}{T}\right) dl = -\sigma_s \frac{2L}{\pi}
\]  

(34a)

\[
\frac{\Delta T}{\rho_w} u_w dA_w = u_w A_w
\]  

(34b)

By several integration procedures, the final equation of temperature-dependent matric suction is derived:

\[
\psi_T = \frac{1}{S} \frac{(1 - S)^{0.5}}{S^{1.5} + 1.5T/647} \left(1 - T/647\right)^{1.256} \left[1 - 0.625(1 - T/647)\right] \psi_0
\]  

(35)

where, \(S\) = degree of saturation, \(\psi_0\), \(\psi_1\) and \(\psi_2\) = fitting parameters for the model of Liu et al. (2020).

3.2. Model proposed by Vahedifard et al. (2018)

According to this model, the temperature-dependent function of matric suction can be expressed as follows:

\[
\psi_T = \psi_0 \left(\frac{\beta_T + T}{\beta_0 + T}\right)
\]  

(36)

The coefficients \(\beta_T\) and \(\beta_0\) corresponds to current and reference temperatures are determined as follows:

\[
\beta_T = \frac{\Delta h_T}{C_1}
\]  

(37)

\[
\beta_0 = \frac{\Delta h_0}{C_1}
\]  

(38)

In which, \(\Delta h_T\) = enthalpy of immersion per unit area at the current temperature and can be determined by using the equation of Watson (1943) as follows:

\[
\Delta h_T = \Delta h_0 \left(\frac{1 - T_0}{1 - T}\right)^{0.38}
\]  

(39)

where \(\Delta h_0\) = enthalpy of immersion per unit area at a reference temperature.

The constant \(C_1\) can be determined as:

\[
C_1 = \frac{\Delta h_{T_0} + (0.11766 - 0.0001535T_0) \cos \zeta_0}{T_0}
\]  

(40)

where, \(\cos \zeta_0\) = wetting coefficient at the reference temperature, and \(\zeta_0\) = air–water contact angle at the reference temperature.

3.3. Model proposed by Roshani and Sedano (2016)

Roshani and Sedano (2016) suggested that the influence of temperature on matric suction variation can be considered through the change of surface tension and water density, which are expressed as follows:

\[
\sigma_s = 0.117 - 0.000153T
\]  

(41)

\[
\rho_w = 658.2 + 2.509T - 0.004606T^2
\]  

(42)

where, \(T\) = the current temperature in Kelvin.

A temperature-dependent model for SWRC was then proposed by modifying the air-entry value with temperature as:

\[
S = \frac{1}{\left(\ln[2.72 + (\psi/\alpha_T)^m]\right)^m}
\]  

(43)

The effect of temperature on the SWRC is controlled by the fitting parameter \(\alpha_T\), which is suggested to link with surface tension and water density:

\[
\alpha_T = \frac{13840.(0.117 - 0.000153T)}{652.8 + 2.509T - 0.004606T^2}
\]  

(44)

3.4. Model proposed by Wan et al. (2015)

Wan et al. (2015) proposed an empirical model to predict the effect of temperature on the SWRC. The approach of this model is similar to the model of Roshani and Sedano (2016), in which the air-entry value is changed with temperature. In this model, the air-entry value and water density are suggested to decrease when the temperature is increased. The expression for the model of Wan et al. (2015) is as follows:

\[
\theta = \frac{\theta_0 + \xi (T - T_0)}{\left(\ln[2.72 + (\psi/\alpha_T)^m]\right)^m}
\]  

(45)

\[
\xi = a_T \text{ a temperature-related coefficient with a fitted value of } -0.000096 \text{ for clays.}
\]

The fitting parameter \(a_T\) is assumed as a function of temperature by:

\[
a_T = AEV \cdot \ln(T - 273) + a_1
\]  

(46)

where, \(AEV\) = the air-entry value of reference SWRC, and \(a_T\) = calibrated constantly.

Based on the test results for compacted bentonite with several different temperatures, an empirical function was proposed for the fitting parameter \(a\) is:

\[
a_T = -4.1474 \ln(T) + 20.395
\]  

(47)
3.5. Model proposed by Salager et al. (2010)

Similar to many other existing models, Salager et al. (2010) also proposed a non-isothermal SWRC model, in which the variation of surface tension and thermal volumetric expansion of water were considered.

According to this model, the variation in the thermo-hydraulic interaction can be expressed:

\[ d\eta = -\left[ w\beta_\eta + \frac{\psi}{F_w} \frac{d\sigma_i}{dT} \right] dT \]  

(48)

The variation in thermal expansion coefficient and surface tension of water with temperature is suggested as follows:

\[ \beta_\eta = \left[ -0.0042(T - 273)^2 + 1.17(T - 273) - 1.87 \right] \times 10^{-3} \]  

(49)

\[ \sigma_i = -2.73 \times 10^{-7} \times (T - 273)^2 - 1.4 \times 10^{-4} \times (T - 273) + 7.56 \times 10^{-2} \]  

(50)

The temperature-dependent model of SWRC therefore can be written as:

\[
\frac{\theta}{\theta_0} = \left[ \frac{1 + \beta_\psi (T - 273)}{1 + \beta_\psi \sigma_i} \right]^{\beta_1} \\
\psi_T = \psi_0 \times \frac{\theta}{\theta_0} \]  

(51)

3.6. Model proposed by Grant and Salehzadeh (1996)

Grant and Salehzadeh (1996) provided an empirical model based on the interfacial energy theory that includes the influence of temperature on the matric suction as follows:

\[ \psi_T = \psi_0 \times \left( \frac{\beta_0 + \beta_1 T}{\beta_0 + \beta_1 T_0} \right)^{\beta_1} \]  

(52)

However, the parameter \( \beta_1 \) was suggested to equal 1 according to their empirical theory. Therefore, Eq. (52) can be re-written as follows:

\[ \psi_T = \psi_0 \times \left( \frac{\beta_0 + T}{\beta_0 + T_0} \right) \]  

(53)

where, \( \beta_0 \) is an empirical constant and can be determined by using Eq. (16).

An expression for the temperature-dependent SWRC model of Grant and Salehzadeh (1996) is:

\[ \theta = \left[ \ln \left( 2.72 + \left( \psi_0 \times \left( \frac{\beta_0 + T}{\beta_0 + \sigma_i} \right) / a \right)^n \right) \right]^m \]  

(54)

4. Experimental program

4.1. Test set-up and methodology

The impact of temperature on SWRC is primarily determined by its effect on pore water viscosity, pore water thermal expansion, and thermal volumetric strains of soil (Vega and McCartney, 2014). This section investigates the temperature impact on the SWRC of kaolinite by employing the filter paper method. To better understand the impact of temperature, three groups of samples were prepared initially at 24 °C (i.e., room temperature) and kept in a chamber at 5 °C, 24 °C, and 45 °C to reach equilibrium. In this regard, the SWRC of Polwhite E kaolinite, with properties shown in Table 1, has been investigated along the wetting path. Matric suction was measured using the "contact" filter paper technique recommended by ASTM D5298-16. The contact filter paper technique indirectly estimates matric suction by measuring the amount of moisture transferred from an unsaturated soil specimen to initially dry filter paper (Garakani et al., 2018).

For each set of tests, eight specimens with varying moisture contents were prepared at room temperature in ring-shaped moulds of 3 cm in height and 5.5 cm in diameter. In this regard, completely oven-dried kaolinite powder was mixed with distilled water to make a homogeneous mixture with an initial moisture content of 10%. The mixture was then compacted inside PVC moulds using a static compaction device to reach the dry density of 1.31 g/cm³. Then, the remaining required amount of water was added to each specimen to get to the pre-determined moisture content. This procedure was followed for six sample groups with water contents of 2%, 10%, 18%, 22%, 26%, and 34% (two samples to check repeatability). Additionally, the two samples with an initial water content of 2% were prepared separately to capture the residual part of the curve.

In this study, matric suction was measured using Whatman No. 42 filter paper. The target filter paper was sandwiched between two larger protective filter papers and placed between the two soil samples to ensure hydraulic continuity between the water in the soil pore and the water absorbed by the filter papers, as shown in Fig. 3. Before sandwiching, the filter papers were treated with a 2% formaldehyde solution to prevent organism growth or biological decomposition (particularly at high temperatures), and then oven-dried (ASTM D5298-16, 2016). The two soil samples, with filter papers in between, were placed inside a glass jar, and the jar was properly sealed with several layers of parafilm (Garakani et al., 2018). Afterwards, the glass jars were placed inside a temperature-controlled chamber, maintaining the desired temperature within ± 0.1 °C. After

<table>
<thead>
<tr>
<th>Physical properties of Polwhite E kaolinite.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Liquid Limit (%)</td>
</tr>
<tr>
<td>Plastic Limit (%)</td>
</tr>
<tr>
<td>Plasticity index</td>
</tr>
<tr>
<td>Specific Gravity, G_s</td>
</tr>
<tr>
<td>Initial void ratio e_0</td>
</tr>
<tr>
<td>Mean particle diameter, D_32 (μm)</td>
</tr>
<tr>
<td>Mean particle diameter D_42 (μm)</td>
</tr>
<tr>
<td>Classification in accordance with USCS</td>
</tr>
</tbody>
</table>
14 days, an acceptable agreement between the calibration equation and the experimental results was observed.

The equilibration time for the calibration of Whatman No. 42 filter paper using the vapour equilibrium technique is dependent on the imposed level of matric suction (Marinho, 1994). The lower the matric suction, the longer the equilibration time required to achieve matric suction equilibrium. On the other hand, extending the equilibrium period may result in fungal growth and measurement errors. As a result, the glass jars were left in the environmental chamber for approximately 30 days to ensure equilibrium between the soil samples and the target filter paper, recommended for matric suctions less than 250 kPa (Marinho, 1994; Khosravi et al., 2020). As removed from the glass jars, filter papers were quickly placed inside plastic containers to minimize moisture loss during the weighing process. The filter papers were then precisely weighed to 0.0001 g to determine the water content.

Filter paper tests have been carried out under zero external stress conditions. To compensate for this shortcoming, the unsaturated oedometer device was used to measure the degree of saturation at 24 °C for matric suction values of 0, 100, and 300 kPa. The schematic of the suction-controlled oedometer cell is illustrated in Fig. 4. The results in Fig. 5 shows that the data obtained from the unsaturated oedometer tests (circle markers) fit well on the SWRC captured by the filter paper tests under zero external stress conditions. It is worth noting that assuming a constant void ratio and calculating the degree of saturation of the specimen based on the water content of the sample is a valid premise due to the negligible change in the void ratio induced by thermal loading (Hashemi and Sutman, 2022) and no significant volume change observed during the imbibition stage of unsaturated oedometer tests.

4.2. Calibration procedure of filter paper technique

The gravitational moisture content of the filter paper at equilibrium is measured and related to soil matric suction using the calibration curve. However, calibration curves for different filter papers may differ significantly from batch to batch, even among the same type of filter paper (Likos and Lu, 2002). Thus, the accuracy of the calibration curve for the filter paper batch in use is critical. In this case, at least one point on the curve must be obtained to verify the batch-specific calibration (Haghighi, 2011).

The proposed calibration equation by Haghighi et al. (2012) was used and validated to capture the matric suction evolution as a function of temperature and water content in this study.

\[
\ln \psi = 33.97 \times w_{fp}^{-0.33} - 4.55 \times T^{0.04} \text{ For } \psi \leq 500 \text{ kPa}
\]

\[
\ln \psi = -1.23 \times (w_{fp}^{0.56} + T^{0.19}) + 16.48 \text{ For } \psi > 500 \text{ kPa}
\]

where, \( w_{fp} \) = water content of filter paper.

4.3. Results and discussion

The measured SWRC of kaolinite at reference temperature (24 °C) in terms of the degree of saturation and volumetric water content is shown in Fig. 6. Three SWRC equations, namely, BC equation (Brooks and Corey, 1964), VG equation (van Genuchten, 1980), and FX (Fredlund and Xing, 1994) were selected to plot the reference SWRC. Each of the SWRC equations contains two variables: one to describe the air-entry value and the second one to present the rate at which soils desaturate (residual conditions). The VG and FX equations, unlike the BC equation, have a third variable that can be used to calibrate the shape of the SWRC in the low-matric suction range. The fitting parameters of these three equations are summarized in Table 2. It can be observed that the VG and FX equations matched with measured data better than the BC equation. This is because the three-parameter SWRC equations as VG and FX provide greater flexibility for the best-fitting analysis than the two-parameter SWRC.
equations (BC model). However, it is also found that the SWRC of the FX equation connected with measured data better than one of the VG equations, particularly in the residual regime. One of the problems with the VG or other empirical equations is that at high suction values (i.e., beyond the residual value), the results become asymptotic to a horizontal line as matric suction goes to infinity. The reason behind this tendency is due to a residual matric suction value (RSV) of 1500 kPa imposed in the VG equation and matric suctions with higher RSV thus will approach infinity. Meanwhile, the FX equation overcomes this problem by using a log–log format as well as applying a correction factor that directs the SWRC equation to a soil matric suction of 10^6 kPa at zero degree of saturation.

Fig. 4. Schematic of the suction-controlled oedometer test (after Haghighi, 2011).

Fig. 5. Test data obtained from oedometer test and filter paper test.

Fig. 6. Measured data and predicted SWRC at reference temperature (24°C): a) suction versus volumetric water content; b) suction versus saturation degree.
A comparison between predicted SWRC by the proposed model and measured data at different temperatures ranging from 5 °C to 45 °C is shown in Fig. 7. Both experimental and analytical models agree that the higher temperature produces a lower SWRC. The results proved that for the identical degree of saturation, the matric suction is reduced with increasing temperature. One of the most important factors that should be considered in predicting the matric suction variation with temperature is the air–water contact angle, which was neglected in existing models.

To consider the influence of contact angle on the model performance, two cases are specified: (1) temperature-independent contact angle (dashed lines), generated by assuming $f_{ad} = 1$; and (2) temperature-dependent contact angle (solid lines), generated by considering all five factors in Eq. (24). It can be observed that the variations in matric suction for the case of temperature-independent contact angle are less significant. However, considering the temperature-dependent contact angle led to significantly larger variations in matric suction with temperature fluctuation. Taking into account an example at a volumetric water content of 0.365, the matric suction decreases as much as 9.7% for the temperature-independent contact angle case and 26% for the temperature-dependent contact angle case as the temperature increases from 24 °C to 45 °C. Similarly, the matric suction increased by about 7% for the temperature-independent contact angle case and 33% for the temperature-dependent contact angle case as the temperature decreased from 24 °C to 5 °C. Comparing the results of the two examined cases indicate that accounting only for the temperature-dependent surface tension is not sufficient to predict the SWRC under non-isothermal conditions. The results highlight the importance of considering a temperature-dependent contact angle for numerical simulations involving non-isothermal processes in unsaturated soils, which is generally neglected in most previous non-isothermal models.

Fig. 8 shows the comparison between analytical and experimental results at two different temperatures. All analytical models, except the model proposed by Wan et al. (2015), agree with the experimental results that a lower temperature leads to a shift to higher matric suctions of SWRC. However, the models of Liu et al. (2020) show a significant overprediction while the other remaining models demonstrate a high underprediction. Additionally, the effect of temperature on the SWRC is nearly negligible.

### Table 2

<table>
<thead>
<tr>
<th>Model name</th>
<th>BC model (Brooks and Corey, 1964)</th>
<th>VG model (Van Genuchten, 1980)</th>
<th>FX model (Fredlund and Xing, 1994)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$\theta = \left[ \frac{h}{p^b} \right]^{1/m}$</td>
<td>$\theta = \frac{\theta_s}{1 + \left( \frac{\theta_s}{\theta_s \theta_{AEV} \theta_{BC}^{1/3}} \right)^n}$</td>
<td>$\theta = \frac{\theta_s \theta_{AEV} \theta_{BC}^{1/3}}{\ln(2 + \left( \theta_s \theta_{AEV} \theta_{BC}^{1/3} \right)^n)}$</td>
</tr>
<tr>
<td>Fitting parameters</td>
<td>$\theta_s$ for $\psi \leq \theta_{AEV}$</td>
<td>$m_{VG} = 0.508 \theta_{AEV} = 2.461 \theta_{AEV} = 0.009$ kPa</td>
<td>$m_{FX} = 0.963$</td>
</tr>
<tr>
<td>$\theta_{AEV} = 95$ kPa</td>
<td>$\theta_{BC} = 0.9$</td>
<td>$n_{VG} = 2.461$</td>
<td>$a_{FX} = 4.006$</td>
</tr>
<tr>
<td>$\theta_{BC} = 0.9$</td>
<td>$\rho^b = \theta_{AEV} = 95$ kPa</td>
<td>$m_{FX} = 0.963$</td>
<td>$a_{FX} = 117$ kPa</td>
</tr>
</tbody>
</table>
according to the predicted results by models of Roshani and Sedano (2016) and Salager et al. (2010). While the former considered only the temperature-dependent water density, the latter considered the effect of temperature only on surface tension and water expansion. The variation of these factors with temperature is quite small, which might be the reason behind the low performance of both models. It is interesting to note that the model of Grant and Salehzadeh (1996) considering temperature-dependent contact angle produces a good agreement with measured data.

Fig. 8. Comparison of analytical and experimental results: a) at 5 °C, b) at 45 °C.
for the case with the temperature of 45 °C. However, this model underpredicts significantly for the case with a temperature of 5 °C. Comparatively, the proposed model in this study performs much better in predicting the effect of temperature on SWRC than other existing models.

5. Performance and reliability of non-isothermal SWRC models

5.1. Summary of collected data sets

It is important to conduct comparisons between the analytical and experimental models for more test data sets in order to obtain a full evaluation of the reliability and performance of non-isothermal SWRC models. For this reason, 16 published data sets in the literature were gathered. These selected data sets cover a wide range of different soil types including hard clays, swelling clays, silt, clayey-silty sands, sands, bentonite, and geosynthetic clay-liner, which therefore allow evaluating sufficiently the sensitivity of SWRC with temperature. The gathered data sets also cover a broad variation in matric suction, from a low matric suction range to a very high matric suction range. It is noted that the temperature range among test sets varies commonly between 4 °C and 80 °C. The characteristic of tested soils and the input parameters of models are presented in Table 3. It should be noted that among the selected data sets, different types of equipment and test procedures have been used for measuring the SWRC, in which axis translation technique and vapor equilibrium technique were commonly used to control matric suction.

The full performance of a model must be proved through both accuracy (validity) and consistency (reliability). Therefore, the performance of the non-isothermal SWRC models is assessed using three statistical criteria in this study. The first criterion is the effectiveness of the measured versus predicted curve to the linear line 1:1, which reveals the reliability of a prediction model.

However, the first criterion does not measure directly the overprediction or underprediction of a SWRC model. Therefore, the average relative error (ARE) is used as the second criterion to assess the tendency of the model, in which positive values of ARE express an overestimation tendency while negative values of ARE reveal an underestimation tendency (Pham et al. 2023; Pham and Dias 2021b; Pham 2020b). The smaller the ARE value, the better the prediction performance. The validity of a prediction model is represented by this criterion. The formula for the calculation of value ARE is as follows:

Table 3
Summary of collected data sets for comparison.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Case number</th>
<th>Soil type</th>
<th>Void ratio</th>
<th>Saturated volumetric water content</th>
<th>Plasticity index Ip</th>
<th>Tested temperature</th>
<th>Reference temperature</th>
<th>Fitting parameters of reference SWRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constantz (1991)</td>
<td>1</td>
<td>Oakley Sand</td>
<td>0.52</td>
<td>0.34</td>
<td>0</td>
<td>20 °C, 80 °C</td>
<td>20 °C</td>
<td>δ = 1.3, α = 7.0, m = 1.8, n = 1.25</td>
</tr>
<tr>
<td>Constantz (1991)</td>
<td>2</td>
<td>Nonwelded tuff</td>
<td>1.08</td>
<td>0.52</td>
<td>5</td>
<td>20 °C, 80 °C</td>
<td>20 °C</td>
<td>δ = 1.3, α = 50, m = 1.6, n = 1.1</td>
</tr>
<tr>
<td>Romero et al. (2001)</td>
<td>3</td>
<td>Loose swelling (Boom clay)</td>
<td>0.97</td>
<td>0.49</td>
<td>27</td>
<td>22 °C, 80 °C</td>
<td>22 °C</td>
<td>δ = 1.45, 7686, m = 0.991, n = 0.65</td>
</tr>
<tr>
<td>Romero et al. (2001)</td>
<td>4</td>
<td>Dense swelling (Boom clay)</td>
<td>0.62</td>
<td>0.38</td>
<td>27</td>
<td>22 °C, 80 °C</td>
<td>22 °C</td>
<td>δ = 1.45, 172.3, m = 0.562, n = 1.2</td>
</tr>
<tr>
<td>Villar and Lloret (2004)</td>
<td>5</td>
<td>FEBEX bentonite</td>
<td>0.63</td>
<td>0.39</td>
<td>52</td>
<td>20 °C, 40 °C, 60 °C</td>
<td>20 °C</td>
<td>δ = 1.45, 30,000, m = 0.91, n = 1.3</td>
</tr>
<tr>
<td>Imbert et al. (2005)</td>
<td>6</td>
<td>FoCa compacted clay</td>
<td>0.42</td>
<td>0.30</td>
<td>62</td>
<td>20 °C, 50 °C</td>
<td>20 °C</td>
<td>δ = 1.45, 114, m = 1.59, n = 0.50</td>
</tr>
<tr>
<td>Uchaipichat and Khaliti (2009)</td>
<td>7</td>
<td>Bourke silt</td>
<td>0.41</td>
<td>0.29</td>
<td>6</td>
<td>20 °C, 40 °C, 60 °C</td>
<td>20 °C</td>
<td>δ = 1.35, 100, m = 1.6, n = 1.3</td>
</tr>
<tr>
<td>Salager et al. (2010)</td>
<td>8</td>
<td>Terracotta ceramic</td>
<td>0.46</td>
<td>0.32</td>
<td>0</td>
<td>20 °C, 60 °C</td>
<td>20 °C</td>
<td>δ = 1.3, 200, m = 0.6, n = 2.3</td>
</tr>
<tr>
<td>Salager et al. (2010)</td>
<td>9</td>
<td>Clayey-silty sand</td>
<td>0.77</td>
<td>0.44</td>
<td>5</td>
<td>20 °C, 60 °C</td>
<td>20 °C</td>
<td>δ = 1.35, 5.9, m = 0.725, n = 0.8</td>
</tr>
<tr>
<td>Haghighi, 2011</td>
<td>10</td>
<td>Kaolin clay</td>
<td>1.5</td>
<td>0.6</td>
<td>23.6</td>
<td>10 °C, 25 °C, 50 °C</td>
<td>25 °C</td>
<td>δ = 1.35, 500, m = 1.9, n = 1.6</td>
</tr>
<tr>
<td>Lalouei et al. (2013)</td>
<td>11</td>
<td>Hard clay (Opalinus clay)</td>
<td>0.29</td>
<td>0.23</td>
<td>42</td>
<td>21 °C, 80 °C</td>
<td>21 °C</td>
<td>δ = 1.4, 550, m = 0.95, n = 0.85</td>
</tr>
<tr>
<td>Wan et al. (2015)</td>
<td>12</td>
<td>GMZ01 bentonite</td>
<td>0.56</td>
<td>0.36</td>
<td>239</td>
<td>20 °C, 40 °C, 60 °C</td>
<td>80 °C</td>
<td>δ = 1.45, 100,000, m = 1.6, n = 0.6</td>
</tr>
<tr>
<td>Roshani and Sedano (2016)</td>
<td>13</td>
<td>Unimin silica sand</td>
<td>0.63</td>
<td>0.39</td>
<td>0</td>
<td>4 °C, 20 °C, 49 °C</td>
<td>4 °C</td>
<td>δ = 1.3, 4.5, m = 2.0, n = 3.0</td>
</tr>
<tr>
<td>Roshani and Sedano (2016)</td>
<td>14</td>
<td>Super fine sand</td>
<td>0.56</td>
<td>0.36</td>
<td>0</td>
<td>4 °C, 20 °C, 49 °C</td>
<td>4 °C</td>
<td>δ = 1.3, 4.0, m = 1.2, n = 3.0</td>
</tr>
<tr>
<td>Ghavam-Nasiri et al. (2019)</td>
<td>15</td>
<td>Geosynthetic Clay-liner</td>
<td>4.8</td>
<td>0.83</td>
<td>120</td>
<td>20 °C, 35 °C</td>
<td>20 °C</td>
<td>δ = 1.4, 120, m = 0.48, n = 2.27</td>
</tr>
<tr>
<td>Liu et al. (2020)</td>
<td>16</td>
<td>Loamy sand</td>
<td>0.8</td>
<td>0.44</td>
<td>4</td>
<td>10 °C, 40 °C</td>
<td>20 °C</td>
<td>δ = 1.3, 10, m = 0.48, n = 1.6</td>
</tr>
</tbody>
</table>
Fig. 9. Comparison of analytical and experimental models for sixteen selected data sets.
Fig 9. (continued)
\[ ARE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{V_{mi} - V_{pi}}{V_{mi}} \right) \times 100 \]  

(57)

where \( V_{mi} \) = measured value of the ith data point, \( V_{pi} \) = predicted value of the ith data point, \( n \) = numbers of measured points.

The normalized sum of square error (SSE) is the third criterion used to assess the performance of analytical models. The lower the SSE value indicates the better the predictive capability of the model. The definition of SSE is expressed as follows:

\[ SSE = \sum_{i=1}^{n} \left( \frac{V_{mi} - V_{pi}}{V_{mi}} \right)^2 \]  

(58)

5.2. Performance of non-isothermal SWRC models

Fig. 9 shows the comparison between analytical models and measured data for 16 collected data sets. It should be noted that the models of Liu et al. (2020) and Wan et al. (2015) are empirical models, which contain some fitting parameters. To find reasonable values of these parameters, a calibration procedure must be conducted for two models. As a result, more than three different temperatures must be provided in the observed data. The majority of published data sets, however, were confined to only two different temperatures due to the difficulties of completing the test program. As a result of this disadvantage, it is not possible to perform a comprehensive verification process for these two models, and they are thus not considered further in this section. It should be noted that the results predicted by the models of Vahedifard et al. (2018) and Grant and Salehzadeh (1996) are much sensitive to the enthalpy immersion value (\( \Delta h_0 \)), which is often determined empirically. However, this parameter is relatively difficult to be measured precisely, while conducting this test type is often time-consuming and costly. In practice, the enthalpy of immersion is not provided along with collected data sets. The value \( \Delta h_0 \) is therefore assumed to be as follows: \( \Delta h_0 = -0.285 \text{ J/m}^2 \) for sandy soil, \( \Delta h_0 = -0.40 \text{ J/m}^2 \) for silty soil and soft clays, and \( \Delta h_0 = -0.516 \text{ J/m}^2 \) for hard clays and compacted bentonite (Bachmann et al., 2002; Vahedifard et al., 2018).

Fig. 9 also shows that a higher temperature results in a lower SWRC, indicating that the matric suction reduces as the temperature increases. The variation in SWRC with temperature, however, is highly dependent on the density and matric suction range of each soil type. The influence of temperature on SWRC of dense soils is found to be more pronounced than that of loose soils. In addition, the SWRCs of cohesive soils (swelling clays, bentonite, and hard clays) alter more dramatically with temperature than those of granular soils (sands).

Regarding the performance of analytical models, a good agreement between the proposed model and experimental
data was observed for the majority of selected data sets. This observation proved that the proposed model shows an efficient prediction of the temperature effect on the SWRC. The model of Vahedifard et al. (2018) shows a good agreement for three cases while underpredicting significantly for almost all other remaining cases. Conversely, the model of Grant and Salehzadeh (1996) overpredicted highly for five of all cases while a relatively well agreement was observed for some other cases. The temperature has a minor effect on the SWRC in the models of Roshani and Sedano (2016) and Vahedifard et al. (2018) generally remain above the linear line 1:1, which explains a high underprediction of this property of soils. The results predicted by the model of Vahedifard et al. (2018) generally remain above the linear line 1:1, which indicates a high reliability of the observed that almost predicted points remained close to the linear line 1:1, which indicates a high underprediction of this model. The models of Roshani and Sedano (2016) and Salager et al. (2010) produce a low prediction performance. It might be because the influence of temperature was limited to water expansion.

The measured against predicted volumetric water content is presented in Fig. 10 for five analytical models. It is observed that almost predicted points remained close to the linear line 1:1, which indicates a high reliability of the proposed model. The satisfactory agreement between the proposed and experimental model is consistent along many various cases considering different soil types and physical properties of soils. The results predicted by the model of Grant and Salehzadeh (1996) is advanced in comparison to other models as the temperature-dependent contact angle is considered. However, the final solution of this model was too sensitive to the value of immersion enthalpy, which is difficult to be determined precisely in practice.

The performance of the analytical models is specialized by the value ARE and SSE, as shown in Table 4. It is noted that the proposed model has a value ARE smaller than 10% among 94% number of comparison cases. The overall ARE and SSE of the proposed model were only 1.95%, and 0.01. It should be noted that the model of Vahedifard et al. (2018) has the value ARE smaller than 10% for 53% number of comparison cases but also has the value ARE larger than 20% for 20% number of selected cases. Meanwhile, the model of Grant and Salehzadeh (1996) produces the value ARE smaller than 10% for only 20% in total comparison cases. However, the difference in the overall ARE and SSE of the two models is small, with 13.6% for the Vahedifard et al. (2018) model, and 14.9% for Grant and Salehzadeh (1996) model. Nonetheless, the value SSE of the Grant and Salehzadeh’s model (SSE = 0.28) was much lower than that of Vahedifard et al.’s model (SSE = 0.68), which reveals that a higher consistency in results predicted by the Grant and Salehzadeh’s model (1996). It is also found that the models of Salager et al. (2010) and Roshani and Sedano (2016) have relatively high ARE, with

<table>
<thead>
<tr>
<th>Test case</th>
<th>ARE</th>
<th>SSE</th>
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<tr>
<td>Case 1</td>
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<tr>
<td>Case 2</td>
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<tr>
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<tr>
<td>Case 7</td>
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<tr>
<td>Case 8</td>
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</tr>
<tr>
<td>Average</td>
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<td>0.012</td>
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</table>

Note: (+) = overprediction; (-) = underprediction.
6. Conclusions

The theoretical and experimental features of the soil–water retention phenomenon in thermally affected soils were addressed in this study. The following are some of the main conclusions that can be drawn:

- Theoretically, a brief review of the constitutive laws governing the thermal-hydro-mechanical (THM) behaviour of unsaturated soils is presented, along with links between variations in suction with water content, temperature, and void ratio. It also provides a broad framework that would be well adapted to describing many specific circumstances.

- Through a closed-form predictive relationship that is developed in the proposed framework, the effect of temperature is examined. The proposed model takes into account the fluctuation in matric suction with temperature via five major factors: surface tension, contact angle, void ratio, particle size expansion, and water density. As a result, the proposed model is more effective at reflecting SWRC fluctuation with temperature than models based solely on temperature-dependent surface tension. With the use of the water retention curve at a reference temperature, the proposed model can be used to predict the water retention curve at a given temperature. Hence, the number of experimental tests required to characterise the thermo-hydraulic behaviour of soil is strongly reduced.

- An experimental program was also carried out at three different temperatures to capture the SWRC of kaolin clays in different temperatures. The data revealed that as the temperature increases, the matric suction decreases at constant water content. However, the decrease in matric suction with temperature mostly occurs in the transition zone of the SWRC, with little fluctuation in the residual zone. It was also shown that the proposed model agrees well with measured data and performs better than any existing non-isothermal SWRC models.

- A comprehensive verification program for six analytical models was carried out by validating against the other sixteen data sets, including a wide range of soil types with different densities. Statistical assessment results indicate that the proposed SWRC model is reliable and performs well in predicting variations in non-isothermal SWRC. Different soil types produce inconsistent prediction results in the models of Vahedifard et al. (2018) and Grant and Salehzadeh (1996), while Salager et al. (2010) and Roshani and Sedano (2016) failed to capture well the temperature-induced variation in SWRC.

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