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Finance and Mathematics: where is the ethical malaise?

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Discussions of the role of mathematics in finance appearing in *The Mathematical Intelligencer* can be split into two classes. Marc Rogalski [26] and Jonathan Korman [18] capture a widespread fury at a collapse in commercial ethics while Ivar Ekeland [6] and Peter Haggstrom [13] offer economic facts. The conclusions of Rogalski and Korman can be summarised as that mathematicians should spurn the financial world; Haggstrom and Ekeland point to technocratic solutions, characterised by better regulation. I do not buy into the argument that the problems of finance can be solved by regulations, it is, as both the U.K. and U.S. governments have identified[1], an ethical problem. But I also do not think it is virtuous for mathematicians to spurn finance, so I am not completely aligned with Rogalski or Korman. My position is that mathematicians should be forthright in presenting financial mathematics as a discipline centred on the concept of justice, making it explicit that successful finance must be moral finance.

During the Financial Crisis of 2007-2009 I was the U.K. Research Council's 'Academic Fellow' in Financial Mathematics, meaning my background is, like Ekeland and Haggestrom, that of a financial mathematics 'insider'. In this role I was expected to explain the discipline I represented to U.K. policy makers, both in government and in the media. As I attempted to meet these expectations I took an unconventional step for a mathematician and started

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[1] In the U.S. Financial Crisis Inquiry Commission's report of 2011 and the U.K. 'Changing Banking for Good' report of 2013.

looking into the origins of mathematical probability, both technically and the cultural context. I noticed that in solving the Problem of Points, in 1654, Pascal and Fermat were pricing a derivative contract on a binomial tree, and their solution would today be recognised as the Cox-Ross-Rubinstein option pricing model, published in 1978. There was a difference between the 1654 and 1978 models, CRR give a methodology for identifying the branch probabilities on the tree, Pascal and Fermat assume they are a half. This raised the question: how did Pascal and Fermat conceive the probabilities they used?

The answer came, initially, in some work the historian Edith Dudley Sylla did in the process of translating the *Ars Conjectandi*. Sylla observes that

> equity among associates or partners rather than probabilities in the sense of relative frequencies provided the foundation for the earliest mathematical probability theory.[28, p 13]

and that

> the foundations (...) [were] not chance (frequentist probability), but rather *sors* (expectation) in so far as it was involved in implicit contracts and the just treatment of partners.[28, p 28]

Intrigued by this point, I followed the path of mathematical probability from the origins of western mathematics in Fibonacci's text on financial mathematics, the *Liber Abaci*, to contemporary mathematics' *Fundamental Theorem of Asset Pricing*. The *Fundamental Theorem* is a consequence of Black and Scholes' paper on pricing options [2] that is based on the arbitrage argument, which originates in Aristotle's discussion of justice in commercial exchange in *Nicomachean Ethics* and features in the *Liber Abaci*. The novelty of contemporary financial mathematics is not in the techniques used, or the products traded[2], but in the fact

[2]Most of these products existed in medieval times, the 'Triple Contract' shares the features of 'structured products' prominent in the crisis. 'Mortgage Backed Securities' were introduced in the U.S. in the late nineteenth century – see [19, Ch 5] for an enlightening account. It is not in the interests of well dressed bankers to tell their clients that what they are charging fat fees for existed before Columbus.

that, today, mathematicians approach the problem of one of 'positive' science, not 'normative' ethics. For example, Black and Scholes opens with the observation that "it should not be possible to make sure profits"[3], appealing to a consequentialist argument that if you get your price wrong[4] someone will bankrupt you, where as medieval merchants were conscious of the Catholic Church's injunction that a riskless profit was *turpe lucrum* (filthy money).

Back in 2009, at the start of this journey, I took a position similar to Ekeland: there are economic laws that "outweigh the puny might of mathematicians" and the solution is in the hands of regulators. Today I have a darker view of the role of mathematics in the markets.

European science is often distinguished from other cultures' science by the fact that it is mathematicised and there is an argument, first offered in 1934[5] but developed more recently [12, 17], that this came out of Aristotle's examination of ethics in commerce. Justice in exchange is distinguished from distributive and restorative justice by Aristotle as being characterised by equality, "there is no giving in exchange", it is a reciprocal arrangement essential in binding society together and for social cohesion [17, p 51; 3, 1133a15–30]. It is notable that Aristotle approached this ethical problem mathematically, since he rarely applied mathematics to the physical world elsewhere [12, p 75; 4, p 13; 3, 1094b15–28]. On this basis, the medieval Scholastic scholars realised that money was a universal measure, up until then Hellenic thought (including Islamic scholarship of the time) had considered different physical properties, such as time and space, to be 'incommensurable' – the idea of inertia was impossible – and it was this property of money that enabled the development of modern physics based on mathematics [4, 17]. To appreciate the point, Copernicus wrote on money before he wrote on the planets; Stevin, founder of the influential Dutch Mathematical School, was a financier; the financier Gresham endowed the first chair of mathematics in England and laid the foundations for the Royal Society. Recently, Bernard Bru has explained the significance

[3] This is the basis of Ekeland's argument.

[4] Ramsey's 'Dutch book' argument, which has been described as a modern version of the 'Golden Rule', "Do unto others as you would have them do unto you", Luke 6:31.

[5] By the Marxist theoretician Borkenau in *The Transition from the Feudal to the Bourgeois World View.*

of Bachelier's experience of stock-markets in the development of Kolmogorov's ideas on probability [30, pp 20–21]. The close relationship between mathematics and finance is born out of the fact that finance is concerned with relations, measured as prices, between objects. Finance informs mathematics on measurement and uncertainty while mathematics is critical to finance because we cannot perform experiments in the economy. It might not be possible to divorce the two disciplines, even if we wanted to.

The classicist Richard Seaford offers some insight into this account when he goes into the roots of western thought and argues that Greek philosophy, including democracy and mathematics, are a consequence of Archaic Greece's use of money [27]. He notes that other ancient civilisations were based on centralised re-distribution, where as pre-Socratic Greek society was based on exchange, reliant on a conception of equality and reciprocity. He suggests that when the Pythagoreans assigned a number to every object, they were, in fact, pricing the object.

The view that finance is socially corrosive is more novel than the practices of finance. One way of approaching Shakespeare's *The Merchant of Venice* is as a study of the four natures of love – erotic, familial, friendship and the highest form of love – charity/*caritas*/$\alpha\gamma\alpha\pi\eta$ – and Shakespeare personifies charity in the form of Antonio, the merchant of Venice. Throughout the seventeenth and eighteenth century, commerce was considered a civilising influence, in *The Rights of Man* (1792) Thomas Paine writes "commerce is a pacific system, operating to cordialise mankind" following a path laid by Montesquieu, Hume, Condorcet and Adam Smith [15, 8]. After the Industrial Revolution, these attitudes all but disappeared and today it would be inconceivable to personify Christian love in the form of a merchant. An explanation for this cultural shift can be found in *Dialectic of Enlightenment* [1] where it is argued that the Enlightenment led to the objectification of nature and its mathematisation, which in turn leads to 'instrumental mindsets' that look to optimally achieve predetermined ends in the context of an underlying need to control external events. Where as during the seventeenth and eighteenth centuries public spaces emerged, the public sphere, which facilitated rational discussion that sought the truth in support of the public good, through the nineteenth century, mass circulation mechanisms came to dominate the public sphere and these were controlled by private interests. As a consequence, the public became consumers of information rather than creators of a consensus through engagement with information [11].

One aspect of this process of alienation for the public is the attitude that mathematics is an almost mystical pursuit that can reveal hidden truths, but only for the initiated, a recurring theme in the presentation of mathematics in popular culture. This is nicely captured in a documentary film on the development of the Black-Scholes-Merton equation where the economist Paul Samulelson describes how he 'discovered' Bachelier's thesis, much as Indiana Jones might discover a magical artefact,

> In the early 1950s I was able to locate by chance this unknown book by a French graduate student in 1900 rotting in the library of the University of Paris and when I opened it up it was as if a whole new world was laid out before me.[6]

This trope might seem benign in the context of popularising mathematics, but when combined with the idea that mathematics is immutable and indubitable, themes of traditional histories and philosophies of mathematics, we are given the impression, to paraphrase William Tait, that

> A mathematical proposition is about a certain structure, such as financial markets. It refers to prices and relations among them. If it is true, it is so in virtue of a certain fact about markets. And this fact may obtain even if we do not or cannot know that it does. [29, p 341]

While mathematicians themselves might not make this claim explicitly, mathematics has been used by many to obscure and legitimise financial activity, passing over any consideration of the

[6]The programme is 'The Midas Formula' also known as 'The Trillion Dollar Bet' and is available on YouTube. The relevant section is around 12:20/48:53 minutes. A transcript is available at `http://www.bbc.co.uk/science/horizon/1999/midas_script.shtml`.

ethical implications of those activities. Ekeland might see mathematics as 'puny', but others value its authority and there are too many examples of how mathematics has been employed to prevent democratic oversight of the markets. In their submission to the Parliamentary Commission on Banking Standards in 2013 the Bank of England was highly critical of how some firms have used advanced mathematical techniques to 'pull the wool' over the eyes of the regulator [22, v. II, para. 89] while U.S. authorities identified that this type of mathematical sleight of hand played a part in the 'London whale' episode [23, p 14]. The existence of the Gaussian copula as a 'truth-teller' of the value of complex debt portfolios played a central role in the Crisis of 2007–2009, justifying the actions of banks, despite its short-comings being known to mathematicians [31, 21, 9]. In the early 1970s, the Black-Scholes-Merton framework played an important part in legitimising the re-emergence of financial derivatives markets [20, p 158]. As long ago as 1877 a large, corporate, insurer defended their actions in undermining fraternal/mutual insurers to legislators with the argument that

> There are certain fundamental rules ... which can only be understood by actuaries, and it is impossible for me to go into here [19, p 198]

An antidote to the causes and consequences of 'instrumental mindsets' identified above is to turn away from the philosophical paradigm of Foundationalism, which sees language as being made up of statements that are either true or false and complex statements are valid if they can be deduced from true primitive statements. This approach is exemplified in the standard mathematical technique of axiom-theorem-proof. An alternative approach is to shift the focus from what language says (true or false) to what it does. Specifically, the function of language is to enable different people to come to a shared understanding and achieve a consensus, this is defined as discourse[7] [10]. Because discourse is based on making a claim, the claim being challenged and then justified, to be successful discourse needs to be governed

[7] According to a recent translator of Fibonacci, a key feature of the techniques given in the *Liber Abaci* was that they enabled ideas to be transmitted and improved upon [7, Introduction].

by rules, or norms. The most basic rules are logical and semantic, on top of these are norms governing procedure, such as sincerity, and finally there are norms to ensure that discourse is not subject to coercion or skewed by inequality. This is why reciprocity is central to financial mathematics, it is a norm of market discourse, embedded in the language of mathematics.

Mathematics has not been passive in recent financial crises and I would argue that if mathematicians are not part of the solution, they are part of the problem. For me, the correct response of mathematicians to the financial crises is to work in support of those who wish to redirect finance from regarding markets as competitive arenas to seeing them as centres of cooperative, democratic, discourse[8]. In this vein I have developed the argument [16] that reciprocity is the central message of financial mathematics and it is one of three norms of market discourse, the others being sincerity and charity. For this case to be coherent I have followed Putnam [25] and abandoned the idea of mathematics being a value-neutral truth-teller, rather it is a means of discourse. This is a significant step if you perceive mathematics as being monogamous with the natural and physical sciences, or even celibate. I believe certain twentieth century mathematicians, such as Poincaré[9] [14], Ramsey [5] and Putnam, would have sympathy with the approach I take, particularly in the cases where mathematics is employed in the social and human sciences.

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[9] Poincaré's approach is captured in his observation "these two propositions 'the earth turns round,' and 'it is more convenient to suppose that the earth turns round,' have one and the same meaning" [24, p 91].

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