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Determination of nonlinear refractive index of large area monolayer MoS₂ at telecommunication wavelength using time-resolved Z-scan technique

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The nonlinear refractive index of large area monolayer molybdenum disulphide (MoS₂) thin film at the 1550 nm wavelength band is determined using time resolved z-scan technique. MoS₂ exhibits positive nonlinear phase shift with a large nonlinear refractive index, n_2 value of $1.40 \times 10^{-13} \text{ m}^2 \text{ W}^{-1}$. The obtained value is similar to the n_2 value of monolayer MoS₂ film measured at 2.0 μm , and is approximately 5 and 7 orders of magnitude larger than silicon and common bulk dielectrics. The large n_2 indicates that MoS₂ can be used as nonlinear refractive 2D material for photonics applications operating in the 1550 nm optical telecommunications wavelength band.

Keywords: nonlinear refractive index; molybdenum disulphide; Z-Scan; telecommunication wavelength

1. Introduction

The successful exfoliation of atomically thin graphene sheets at the turn of the 21st century has kick-started a new research field on two-dimensional (2D) materials.¹ It is found that monolayer or few-layer graphene exhibit excellent electronic and optical properties that is not present in its bulk form.² Since then, rapid progress has been made in the development and characterization of new families of 2D materials such as hexagonal boron nitride,

transition metal dichalcogenides (TMDs) and black phosphorus.³⁻⁵ Their unique properties have opened up the possibilities of new applications in nanoscale electronics, photonics sensing and signal processing when integrated with silicon complementary metal-oxide-semiconductor and optical waveguide platforms.⁶

In the photonics field, the nonlinear optical (NLO) properties of 2D materials are widely studied. They exist in two distinct forms: 1) nano- and micro-flakes dispersed in solution, and 2) uniform single- or few-layer thin film with large coverage area, depending on the methods of fabrication.⁷ Their NLO properties play an important role in pulsed laser generation, all-optical switching, frequency conversion applications, etc. In particular, a monolayer molybdenum disulphide (MoS₂) is found to have the ability to produce pulsed lasers due to its saturable absorption characteristic.⁸⁻¹² NLO properties in the visible and near infrared wavelengths are normally studied using Z-scan technique.^{9, 13-16} Wang et al.¹⁷ investigated the n_2 of liquid-phase-exfoliated MoS₂ nano-flakes dispersion in the wavelength of 532 nm and 1064 nm using Z-scan technique. Wu et al.¹⁸ provided a comprehensive study of the third order nonlinear susceptibility ($\chi^{(3)}$) of MoS₂ nano-flakes dispersed solution in a wide wavelength range from 473 nm to 830 nm using spatial self-phase modulation method. More recently, the n_2 of few-layer MoS₂ thin film – grown using chemical vapour deposition (CVD) – was investigated using four-wave mixing method at 1550 nm¹⁹. The studies above were carried out on few-layer MoS₂ with flake size of not larger than 500 μm in size. However, with the advent of MoS₂ fabrication and transfer techniques, large-area MoS₂ thin films are made available on optical substrates such as silica and polymers.^{3, 20} It would be interesting to study the NLO properties of large-area monolayer MoS₂ as it is expected to provide a more accurate NLO response compared to dispersed MoS₂ flakes. Despite this, there is no report on the study of the NLO properties of large area monolayer MoS₂ thin film using Z-scan technique at the 1550 nm telecommunication wavelength band.

In this work, we investigate the third order nonlinearities of large area monolayer MoS₂ thin film using time-resolved Z-scan technique and determine the value of n_2 . We compare the value of n_2 obtained from large area monolayer MoS₂ film with silicon and other common bulk dielectric materials. This result would encourage more development of nonlinear photonics applications, such as all-optical switching and wavelength conversion that operate in the 1550 nm optical telecommunication wavelength band.

2. Methods

2.1. Characterization of atomically thin MoS₂

A CVD-grown monolayer MoS₂ thin film on a polydimethylsiloxane (PDMS) substrate is purchased from 6Carbon Technology. The monolayer MoS₂ thin film has a coverage area of 1 cm \times 1 cm. Figure 1 shows the optical micrograph of the sample. In Fig. 1(a), it can be seen that there is an optical contrast between the MoS₂ thin film and the PDMS substrate, which allows one to visually determine the position of the film on the substrate. Besides, from Fig. 1(b), it can be inferred that the deposited MoS₂ forms good coverage on the substrate, which will ensure good overlapping of MoS₂ sample with the incident laser beam during the Z-scan measurement. Atomic force microscopy (AFM) is carried out using NT-MDT with semi-contact mode to measure the thickness of the MoS₂ film. From Fig. 1(c), the apparent height measured is \sim 0.8 nm, which indicates the monolayer thickness of the MoS₂ film.²¹

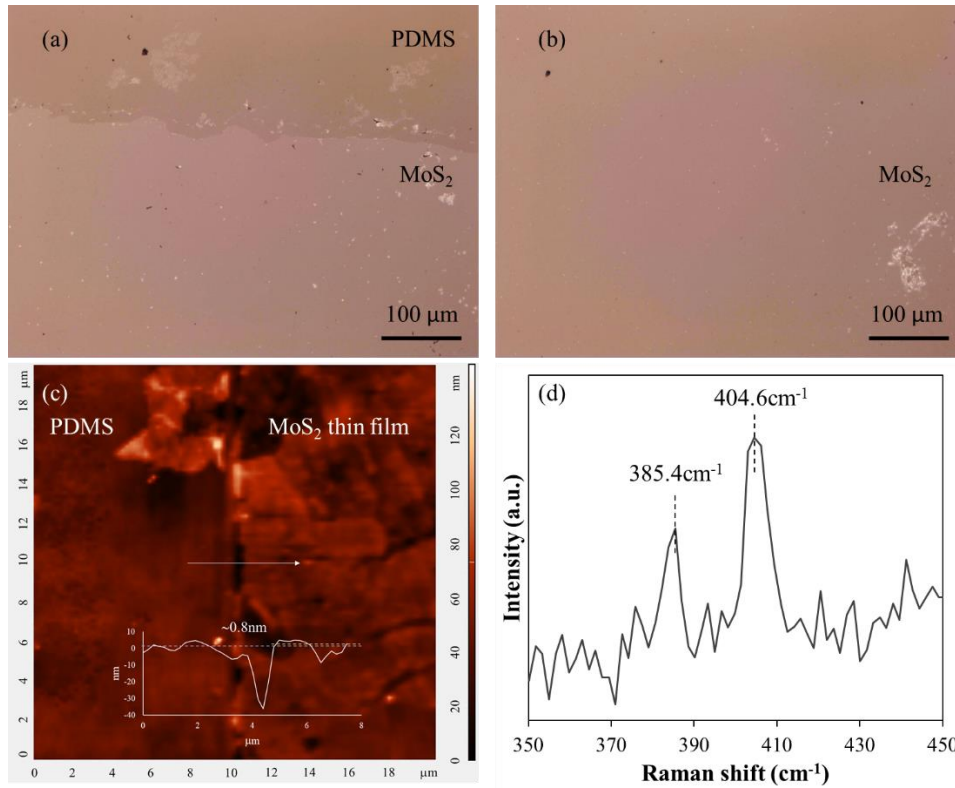


Fig.1. Optical micrographs of MoS₂ sample: (a) boundary of the MoS₂ deposition; (b) continuous layer of the grown MoS₂; (c) AFM image of MoS₂ on PDMS; (d) Raman spectrum of MoS₂ sample with 532 nm excitation wavelength.

Raman spectroscopy is carried out on the sample using Renishaw InVia micro-Raman spectrometer. The wavelength of the Raman laser source is 532 nm with an excitation power of 0.25 mW. The power used is kept low to avoid optical damage to the sample. From Fig. 1(d), two characteristic peaks of 385.4 cm⁻¹ and 404.6 cm⁻¹ are observed, which correspond to the in-plane E_{2g} and out-of-plane A_{1g} modes of monolayer MoS₂, respectively. Both of these vibrational modes are thickness dependent. As the number of layers reduces from bulk to monolayer, the E_{2g} and A_{1g} modes will experience red- and blue-shifts, respectively. This is mainly due to the change in stacking structure of individual MoS₂ sheets, which in turn changes the atomic vibration of these modes.^{22, 23} This would provide another quantitative measure to determine the number of layers of the MoS₂ film. We perform Raman spectroscopy on eight locations, with most of the spectra resembled the result in Fig. 1(d). The values of E_{2g} and A_{1g} modes obtained are consistent with single layer MoS₂.

2.2. Measurement of nonlinear optical (NLO) properties

A time-resolved Z-scan technique is employed to determine the NLO properties of the sample.²⁴⁻²⁶ It is typically used to solve the cumulative thermal effect in the sample when a high repetition rate laser is used as the laser source in Z-scan measurement. The

experimental setup is illustrated in Fig. 2. A mode-locked fiber laser with a center wavelength of 1560 nm, laser repetition rate of 1.0 MHz and pulse width of 450 fs is used as the laser source. An optical attenuator is used to vary the laser power. The beam is expanded by a factor of 10 using a pair of positive convex lenses (L_1 and L_2) in Keplerian beam expanding configuration. A mechanical shutter is placed at the focal point of L_1 to create an opening window of 5 ms with risetime of 30 μ s. The laser beam is focused by L_3 ($f = 100$ mm) to a beam waist radius of $\omega_0 = 26$ μ m and Rayleigh length, $z_0 = 1.4$ mm. Two identical photodiodes (PDs) are used to measure the signal powers. PD_1 receive the near field open-aperture (OA) signal while PD_2 measures the far-field closed-aperture (CA) signal. The CA Z-Scan is performed at 20 % aperture transmission. The sample is translated along the laser axis using a linear translation stage. The intensity, I_o , of the pulse laser used in time-resolved Z-scan measurement is 6.2 GW cm^{-2} . Each measurement is repeated 3 times.

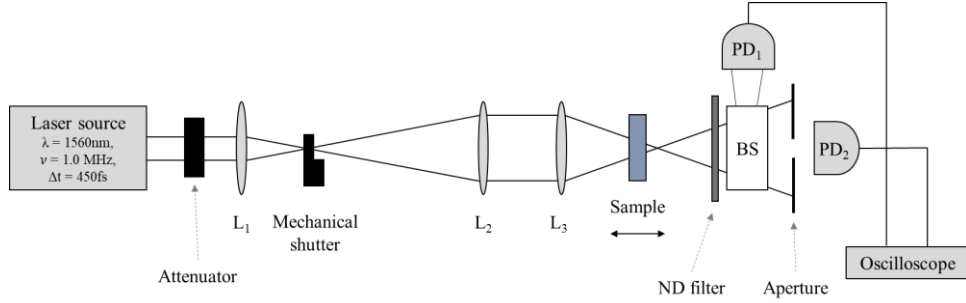


Fig. 2. Experimental setup of the time-resolved Z-scan technique. L_1 , L_2 and L_3 are plano-convex lenses. BS is a beam splitter. PD_1 and PD_2 are photodiodes which measure the open-aperture and far-field closed-aperture signal respectively.

3. Results and discussion

The normalized signal intensity in a time-resolved Z-scan CA measurement can be described using the Falconieri model below:²⁴

$$\frac{I(\zeta, t)}{I(\zeta, 0)} = 1 + \frac{\vartheta(q)}{q} \frac{1}{(1 + \zeta^2)^{q-1}} \tan^{-1} \left(\frac{2q\zeta}{[p^2 + \zeta^2] \frac{t_c(\zeta)}{2qt} + p + \zeta^2} \right), \quad (3.1)$$

where q is the order of absorption, ϑ is the thermal lens strength, $\zeta = z/z_0$ is the normalized z -position, $p = 2q + 1$ and t_c is the characteristic time of the material. The parameter t represents the time when the mechanical shutter is turned on.

Eq. (3.1) demonstrate the change in normalized CA signal intensity due to the thermal lensing effect as a function of ζ and t . The thermal lensing effect takes place when $t > 0$, therefore it is possible to determine the instantaneous electronic nonlinearity (related to n_2) at $t = 0$. In other words, the two optical effects can be separated using Eq. (3.1).²⁴

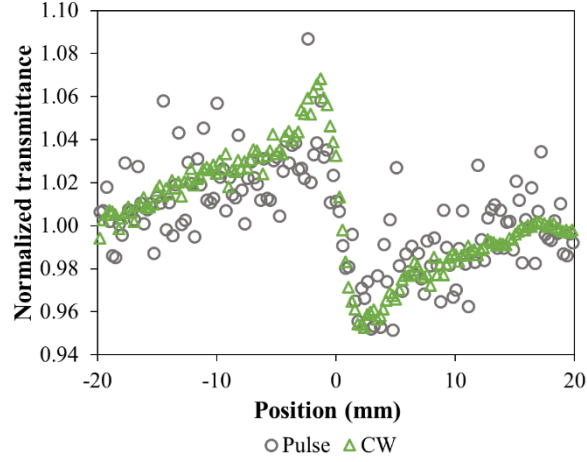


Fig. 3. Conventional Z-scan measurements upon MoS₂ sample using pulse and CW laser sources. The average power of both laser sources is 3 mW. Similar CA trend between pulse and CW laser sources imply that the observed lensing properties originates from cumulative thermal effect.

To obtain the q value in Eq. (3.1), conventional Z-scan measurements are performed on the MoS₂/PDMS sample using both pulsed and continuous wave (CW) laser sources. Since the intensity of the CW laser is low ($< 1 \text{ kW/cm}^2$), we do not expect to see nonlinear refraction (NLR) during the CA measurement. However, we found that at the same average power levels, both traces show self-defocusing characteristics, i.e. a peak-to-valley profile as shown in Fig. 3. The normalized peak-valley transmittance difference (ΔT_{p-v}) obtained using both laser sources also show a similar value, which is ~ 0.11 . This implies that the self-defocusing properties originate from the cumulative thermal effect in the sample. Besides that, the normalized peak-valley distance (ΔS_{p-v}) is determined to be $3.7z_0$. This value is close to the theoretical value of $3.46z_0$ that corresponds to one photon absorption thermal lensing effect.²⁴ Therefore, we set $q = 1$ in the following time-resolved Z-scan analysis, and Eq. (3.1) can be reduced to:

$$\frac{I(\zeta, t)}{I(\zeta, 0)} = 1 + \vartheta(1) \tan^{-1} \left(\frac{2\zeta}{[9 + \zeta^2] \frac{t_c(\zeta)}{2t} + 3 + \zeta^2} \right), \quad (3.2)$$

Figure 4 demonstrates two normalized transmittance traces of the MoS₂/PDMS sample over a period of 2 ms. These traces correspond to the normalized transmittance at the pre-focal and post-focal z-position. It can be seen from this figure that there is a discernible change in the normalized transmittance before and after 0.45 ms, inferring that there is an inversion of peak and valley of the CA curve. The pre-focal z-scan signal changes from a valley to a peak, so does the post-focal z-scan signal. This implies that the instantaneous electronic nonlinearity and thermal lensing effect of MoS₂/PDMS sample have opposite signs.

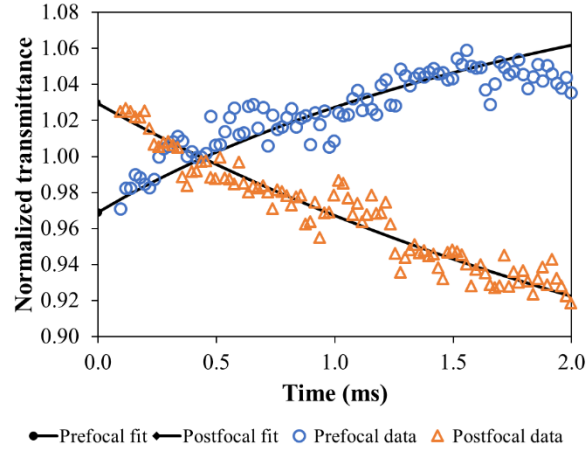


Fig. 4. Normalized transmittance trace of MoS₂ sample measured at pre-focal and post-focal z-positions. The solid curves are the fitting curves generated from Eq. (3.2). The open symbols represent the experimental data, while the solid symbols are the extrapolated values at $t = 0$ which will be used to reconstruct the Z-scan profile.

Figure 5 represents the reconstructed CA Z-scan profiles of MoS₂/PDMS sample at $t = 0$ and 2 ms. As previously suggested, it can be observed that at $t = 0$, the sample presents a NLR with self-focusing characteristic. The ΔS_{p-v} for the NLR trace is calculated to be $1.80z_0$, which is close to the theoretical value of $1.72z_0$ for Kerr induced self-focusing (without thermal lensing). At $t = 2$ ms, the sample exhibits self-defocusing thermal lensing characteristic. It should be noted that at this intensity, we do not observe NLA such as saturable absorption from the MoS₂ from the OA Z-scan measurement (see Fig. 5 inset). This is probably due to the fact that the linear absorption of the monolayer MoS₂ is $\sim 0.2\%$, which is less than the 1% measurement sensitivity of our current measurement setup. Nevertheless, the possibility of the presence of NLA cannot be omitted totally, as it could exist within the noise level.

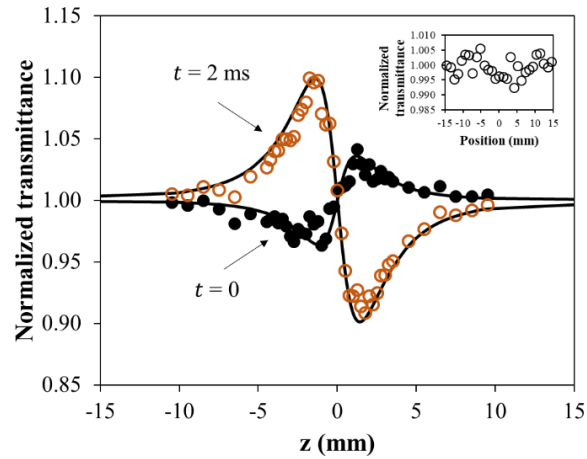


Fig. 5. Reconstructed Z-scan profiles of MoS₂/PDMS sample at $t = 0$ and 2 ms. The solid line traces are corresponding theoretical fit. Inset: OA Z-scan trace of MoS₂, in which no significant NLA can be observed.

From Fig. 5, the normalized transmittance of CA Z-scan at $t = 0$ can be fitted using Sheik-Bahae et al.²⁷ model below:

$$T(z, \Delta\Phi_0) \simeq 1 - \frac{4\Delta\Phi_0\zeta}{(\zeta^2 + 9)(\zeta^2 + 1)}, \quad (3.3)$$

where $T(z, \Delta\Phi_0)$ is the normalized transmittance, $\Delta\Phi_0 = kn_2I_0L_{eff}$ is the on-axis phase shift at the laser focal point, and $\zeta = z/z_0$, with z_0 the Rayleigh length. The model fits the experimental data well as shown in Fig. 5 and enable us to relate the n_2 with $\Delta\Phi_0$ using

$$n_2 = \frac{\Delta\Phi_0}{kI_0L_{eff}}, \quad (3.4)$$

with $k = 2\pi/\lambda$, where λ is the wavelength of laser, and $L_{eff} = 1 - e^{-\alpha L}/\alpha$, with L the thickness of the sample, where $L = L_{MoS_2} + L_{PDMS}$. and α the linear absorption coefficient of the sample. The linear absorption coefficient of the sample at 1550 nm is approximately 2.0 cm^{-1} . Using Eq. (3.4), we have calculated the n_2 of MoS₂/PDMS sample to be $1.49 \times 10^{-18} \text{ m}^2 \text{ W}^{-1}$. However, the measured n_2 refers to the effective n_2 of the sample, which consists of the monolayer MoS₂ and the PDMS substrate. Based on Eq. (3.4), the n_2 obtained from the experiment is evaluated from the on-axis phase shift $\Delta\Phi_0 = kn_2I_0L_{eff}$. Consider the NLR from the PDMS substrate, we can rewrite the $\Delta\Phi_0$ as $\Delta\Phi_0 = kI_0(n_{2,MoS_2}L_{MoS_2} + n_{2,PDMS}L_{PDMS})$. In this way, the n_{2,MoS_2} can be obtained from the equation below:

$$n_{2,MoS_2} = \frac{n_2L_{eff} - n_{2,PDMS}L_{PDMS}}{L_{MoS_2}}, \quad (3.5)$$

where L_{MoS_2} is 0.7 nm, L_{PDMS} is 500 μm , and $n_2 = 1.49 \times 10^{-18} \text{ m}^2 \text{ W}^{-1}$. To determine the contribution of n_2 from PDMS substrate, time-resolved Z-scan is also carried out on a clean PDMS substrate without MoS₂ thin film coating. Our result shows that PDMS possesses a $n_{2,PDMS}$ value of $1.22 \times 10^{-18} \text{ m}^2 \text{ W}^{-1}$. Using Eq. (3.5), the value of n_{2,MoS_2} can be deduced to be $1.40 \times 10^{-13} \text{ m}^2 \text{ W}^{-1}$. This value is similar to the n_2 value of monolayer MoS₂ measured at 2.0 μm , which is $2.5 \times 10^{-13} \text{ m}^2 \text{ W}^{-1}$.¹² In addition, it is approximately 5^{28-30} and $7^{31, 32}$ orders of magnitude larger than that of the silicon and bulk dielectric materials, respectively, making it a potential 2D material for the development of optical nonlinear devices operating in the 1550 nm telecommunication wavelength range.

4. Conclusion

The nonlinear optical properties of large-area monolayer MoS₂ in the 1550 nm wavelength band is successfully determined using time-resolved Z-scan technique. We found that the MoS₂ has a nonlinear refractive index, $n_2 = 1.40 \times 10^{-13} \text{ m}^2 \text{ W}^{-1}$, which is approximately 5 and 7 orders of magnitude larger than the value of silicon and other common bulk dielectrics, respectively. The obtained value is also similar to the value measured at 2.0 μm . This suggests that MoS₂ is a potential 2D material for nonlinear optics application such as in all-optical switching operating in the optical telecommunication wavelength region.

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