Polarization and image rotation induced by a rotating dielectric rod

Polarization and image rotation induced by a rotating dielectric rod: an optical angular momentum interpretation

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When light is transmitted along the axis of a rotating glass rod, the polarization of the light is rotated through a small angle [Proc. R. Soc. London, Ser. A **349**, 423 (1976)]. Under the same conditions, we predict a rotation of the transmitted image by exactly the same angle. The treatment of the two effects in terms of light's spin and orbital angular momentum suggests that they share a common origin. © 2006 Optical Society of America

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In the mid-1970s Jones[1–3] published a number of papers relating to the effect that moving media had on the properties of transmitted light. Initially, Jones examined the lateral displacement of a light beam upon transmission through a linear dielectric with transverse velocity v, described as ether drag. He then demonstrated a rotational equivalent where the polarization of the light propagating along the axis of a rotating rod was subject to a small rotation, $\Delta\theta_{\text{pol}}$, given by

$$\Delta\theta_{\text{pol}} = \Omega(n_g - 1/n_\phi)L/c. \qquad (1)$$

Here Ω is the angular velocity of the dielectric medium, L is its length, and n_g and n_ϕ are the refractive indices corresponding to the group and phase velocities of the light in the medium. The work of Jones was supported by the theoretical analysis of Player,[4] who applied Maxwell's equations within a moving reference frame[5] and carefully included the effects of dispersion. Later work by Nienhuis *et al.*[6] examined the problem on an atomic scale and drew compatible conclusions.

In this Letter we argue that an equivalent mechanically induced rotation should be observed on the transmitted image and that, if treated in terms of the light's angular momentum,[7,8] the rotation of the polarization and images correspond to phase shifts that are dependent, respectively, on the spin and orbital components of the optical angular momentum.

The expression for the transverse photon drag, observed by Jones, resulting from a medium with a transverse velocity v, is readily derived by considering the frame of reference in which the medium is stationary (Fig. 1). We shall assume that the velocity of the medium is very much less than that of the light so that the nonrelativistic Gallilean transformation can be used. The transformation into the rest frame of the medium results in a small angle of incidence, α, of the beam given by

$$\alpha = v/c. \qquad (2)$$

The resulting lateral displacement, $\Delta_{\text{refraction}}$, of the beam due to refraction in the dielectric medium of thickness L is

$$\Delta_{\text{refraction}} = \frac{vL}{c}(1 - 1/n_\phi). \qquad (3)$$

Note that we have taken care to distinguish the refractive index associated with the phase velocity, n_ϕ, from that associated with the group velocity, n_g. Here the phenomenon responsible for the displacement is refraction; so it is n_ϕ that appears.

An additional contribution to the lateral displacement of the beam arises from the delay experienced by the light traveling through the dielectric
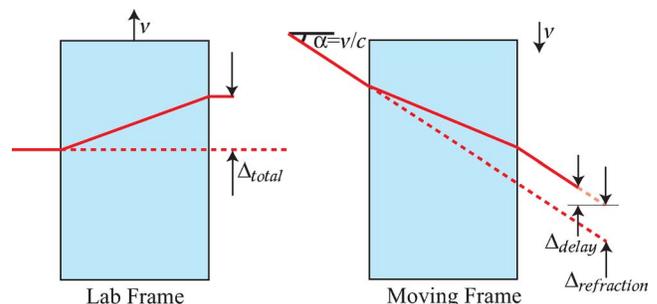

Fig. 1. (Color online) Transverse shift imparted to a light beam by a moving medium can be calculated in the frame of the medium.

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$$t_{\text{delay}} = \frac{L}{\cos(\alpha/n_\phi)} \frac{(n_g - 1)}{c} \approx L \frac{(n_g - 1)}{c}. \quad (4)$$

In the rest frame of the dielectric, this delay results in a lateral displacement of

$$\Delta_{\text{delay}} = \alpha L(n_g - 1) = \frac{vL}{c}(n_g - 1). \quad (5)$$

The phenomenon responsible for the delay is the time taken for the light to traverse the dielectric, and so it is n_g that appears. The total lateral displacement of the beam is given by the sum of these refractive and delay terms:

$$\Delta_{\text{total}} = \frac{vL}{c}(n_g - 1/n_\phi). \quad (6)$$

This displacement, in conjunction with the transit time through the dielectric, is equivalent to a transverse drag velocity, $u_{\text{drag}} = v[1 - 1/(n_\phi n_g)]$, acting on the light and is, therefore, in agreement with earlier work.[9–11]

For a rotating medium we may consider the transverse velocity of the medium to arise from the tangential component of the rotary motion of a glass rod, coaxial with the axis of the light beam. Under such conditions, at a radius r from the axis of the rod rotating with angular velocity Ω, the angular shift $\Delta\theta$ is given by

$$r\Delta\theta = (n_g - 1/n_\phi)r\Omega\frac{L}{c}. \quad (7)$$

If an image is considered to be made up of a number of such light beams or rays, we can conclude that an image will be rotated through the angle

$$\Delta\theta_{\text{image}} = (n_g - 1/n_\phi)\Omega\frac{L}{c}, \quad (8)$$

which is exactly equal to that through which the polarization is rotated.

A linearly polarized light beam is equivalent to a superposition of right- and left-handed circular polarization ($\sigma = \pm 1$) associated with the spin angular momentum (SAM) of the photon. A rotation of the linear polarization by $\Delta\theta$ is equivalent to a relative phase difference of $2\Delta\theta$ between the constituent circular polarization components. Hence the rotation of the plane of linear polarization is equivalent to a phase shift, $\Delta\phi$, of the circular polarization given by

$$\Delta\phi(\sigma) = \sigma(n_g - 1/n_\phi)\Omega\frac{L}{c}. \quad (9)$$

In a similar way, image rotation can be expressed as phase shifts between the different constituent orbital angular momentum components.[12–14] The azimuthal phase term, $\exp(-il\theta)$, describing the helical phase fronts associated with orbital angular momentum (OAM) means that these modes have an l-fold rotational symmetry about the beam axis. Conse-
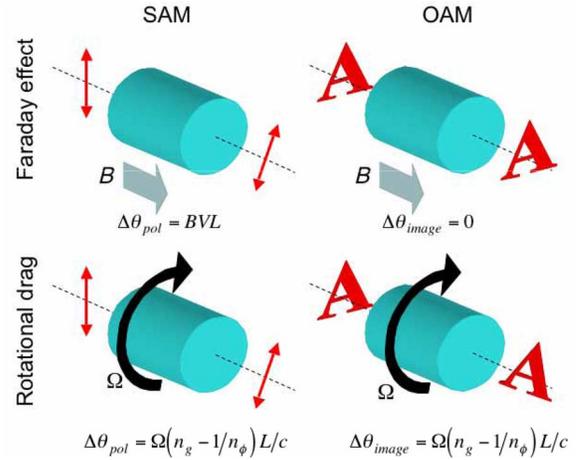

Fig. 2. (Color online) Application of a magnetic field along the axis of the rod leads to a rotation of the transmitted polarization but not of the image. Conversely, the mechanically induced ether drag associated with a rotating cylinder rotates both the polarization and the image through the same angle.

quently, a rotation of the mode about this axis through an angle $\Delta\theta$ changes the phase by $\Delta\phi = l\Delta\theta$. It follows that although the image rotation is independent of l, the associated phase shift will be proportional to l:

$$\Delta\phi(l) = l(n_g - 1/n_\phi)\Omega\frac{L}{c}. \quad (10)$$

For the transmission of light along the axis of a rotating rod we deduce a complete equivalence between the magnitude and direction of the polarization rotation observed by Jones and the predicted image rotation. Furthermore, these expressions can be expressed equivalently in terms of phase changes between the orthogonal spin (polarization) and orbital angular momentum (image) states.

However, whereas we anticipate equivalence between the phase shift for spin and orbital angular momentum arising from mechanical rotation, this is not the case for magnetic Faraday effects. Figure 2 summarizes our understanding of the polarization and image rotations induced by the magnetic Faraday effect and those mechanically induced by the transverse drag. The magnetic Faraday effect gives a rotation of linear polarization proportional to the magnetic field applied along the length of the medium, $\Delta\theta_{\text{pol}} = BVL$, where V is the Verdet constant for the medium. Rotations of many degrees are readily obtainable for the polarization, but a casual inspection of the magnetic Faraday effect reveals that there is no attendant image rotation. Whether there is a more subtle link between the applied magnetic field and a resulting image rotation is perhaps an interesting question that we do not address here.

In summary, we believe that upon transmission along the axis of a rotating medium there is a mechanically induced rotation of both the polarization and the image through precisely the same angle. This might have been expected, as the spin and orbital angular momentum have a common origin in the azi-

muthal component to the Poynting vector,[15] which follows a ray direction. However, whether an image rotation can in fact be induced by the high-speed rotation of a dielectric medium remains to be experimentally verified. Finally, the form of Maxwell's equations in rotating media retains a certain level of controversy,[16] and hence observing image rotation and measuring it accurately would seem to be well worthwhile.

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