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A Common Age Effect Model for the Mortality of Multiple Populations

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We introduce a model for the mortality rates of multiple populations. To build the proposed model we investigate to what extent a common age effect can be found among the mortality experiences of several countries and use a common principal component analysis to estimate a common age effect in an age-period model for multiple populations. The fit of the proposed model is then compared to age-period models fitted to each country individually, and to the fit of the model proposed by Li & Lee (2005).

Although we do not consider stochastic mortality projections in this paper, we argue that the proposed common age effect model can be extended to a stochastic mortality model for multiple populations, which allows to generate mortality scenarios simultaneously for all considered populations. This is particularly relevant when mortality derivatives are used to hedge the longevity risk in an annuity portfolio as this often means that the underlying population for the derivatives is not the same as the population in the annuity portfolio.

Keywords: Mortality of Multiple Populations, Stochastic Mortality model, Longevity, Basis Risk

1 Introduction

A number of stochastic models for mortality rates were developed in recent years. Among those the Lee-Carter (LC) model introduced by Lee & Carter (1992) remains a very popular and widely used model. This model breaks down the mortality experiences at different ages and calendar years into age and period effects. The period effect for a given population can

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then be viewed as a mortality index for all ages. When a LC model is fitted to a number of populations individually, an individual age effect is obtained for each population. This makes it more difficult to compare the period effects observed in different populations as they are fitted to different age effect.

In this paper we consider an extension of the LC model to multiple populations where the age effect is common to all populations. We will call this model a common age effect (CAE) model. In particular, we study the differences in the goodness of fit between individual models and CAE models. The main question we wish to answer is: How important are individual age effects for the goodness of fit of individual LC models compared to the impact of an additional age-period effect in a CAE model.

This study is motivated by the observation that obtained age effects are very similar when they are estimated in different countries of similar socio-economic structure. This suggests that the number of parameters, in particular, age effects, can be reduced when the mortality experiences of several countries or populations are modelled simultaneously. In addition, a CAE model allows for more direct comparison of period effects, since these period effects in different populations are scaled with the same age parameters.

The proposed model can be applied directly to mortality data from different countries or populations, or, alternatively, can be applied to the residuals of other multiple population models, for example, the multiple population model introduced by Kleinow & Cairns (2013) where smoking prevalence is used to explain differences in the mortality experiences in different countries.

In addition to the introduction of the CAE model we also show how to use an estimation method called common principal component analysis to identify common age effects. The proposed model can be fitted using other estimation methods like Maximum Likelihood Estimation. However, using common PCA has some advantages, which we discuss in Section 3.

In our empirical study we will apply the model to the mortality rates observed for males aged 18-87 in the following ten countries: Austria, Australia, Canada, Switzerland, Denmark, France, Great Britain, New Zealand, Sweden and the United States. We choose those ten countries since they are all well developed countries with similar socio-economic characteristics. Therefore, we expect that a mortality model with common factors will allow us to jointly model mortality rates in those countries. The empirical results are based on observed mortality rates for the calendar years 1948 to 2007. We will split the ages into two groups of 35 years each, that is, we separately consider males aged 18-52 and 53-87. This is necessary since we require the number of calendar years to exceed the number of ages. All observed mortality rates are obtained from the Human Mortality Database.

In Section 2 we will review the LC model including a straight forward extension to p age-period effects. This also includes a brief review of the estimation of parameters using principal component analysis (PCA) rather than maximum likelihood methods. We concentrate here on the PCA as we wish to use a modification of this method, called common principal component analysis (cPCA) in Section 3 to obtain estimates of the common age effects. In the following Section 4 we will then compare the estimated age and period effects resulting from the individual models and the CAE model. In the same section we will also compare the goodness of fit of the two models.

2 Individual Model

We consider the mortality rates in k populations. For each population $i = 1, \ldots, k$ we observe the realised log mortality rates $\tilde{m}_i(x,t)$ at age $x \in \{x_1, \ldots, x_n\}$ in year $t = 1, \ldots, T$, that is,

$$\tilde{m}_i(x,t) = \log \frac{D_i(x,t)}{E_i(x,t)}$$

where $D_i(x,t)$ is the observed number of deaths in country i at age x during year t and $E_i(x,t)$ is the corresponding exposure to risk. These rates are observed for n different ages and a total of T years. We assume that $T > n$, and the ages $x_1, \ldots, x_n$ and the years $1, \ldots, T$ are the same for all populations.

In the following we will consider centralised log mortality rates. Therefore, we first calculate the average log mortality rate, $\bar{m}_i(x)$, for a life aged x in population i, that is,

$$\bar{m}_i(x) = \frac{1}{T} \sum_{t=1}^{T} \tilde{m}_i(x,t).$$

and define the centralised log mortality rates

$$m_i(x,t) = \tilde{m}_i(x,t) - \bar{m}_i(x).$$

We denote by m_i the matrix of the observed centralised log mortality rates, that is,

$$m_i = \begin{pmatrix} m_i(x_1,1) & \cdots & m_i(x_1,T) \\ \vdots & & \vdots \\ m_i(x_n,1) & \cdots & m_i(x_n,T) \end{pmatrix}$$

The individual model of order p for the centralised mortality rates m_i in each country i is an extension of the Lee-Carter model to p age and period effects, that is,

$$m_i(x,t) = \beta_i^{(1)}(x)\kappa_i^{(1)}(t) + \ldots + \beta_i^{(p)}(x)\kappa_i^{(p)}(t) + \varepsilon_i(x,t)$$

which can be written in matrix form as:

$$m_i = {}_p\beta_i {}_p\kappa_i + \varepsilon_i \tag{1}$$

with

$${}_p\beta_i = \begin{pmatrix} \beta_i^{(1)}(x_1) & \cdots & \beta_i^{(p)}(x_1) \\ \vdots & & \vdots \\ \beta_i^{(1)}(x_n) & \cdots & \beta_i^{(p)}(x_n) \end{pmatrix} \quad \text{and} \quad {}_p\kappa_i = \begin{pmatrix} \kappa_i^{(1)}(1) & \cdots & \kappa_i^{(1)}(T) \\ \vdots & & \vdots \\ \kappa_i^{(p)}(1) & \cdots & \kappa_i^{(p)}(T) \end{pmatrix} \tag{2}$$

The residuals $\varepsilon_i = \Big(\varepsilon_i(x,t)\Big)$ form a $n \times T$ matrix, and we assume that $\mathrm{E}[\varepsilon_i(x,t)] = 0$ for all populations i. To avoid identifiability problems we also assume that $||\beta_i^{(j)}|| = 1$ for all i and j, where $||.||$ denotes the Euclidean norm, that is, $||x|| = x^\top x$ for any vector x. The maximum number of age effects is $p = n$ since there are only n ages. To simplify notation we define

$$\beta_i = {}_n\beta_i.$$

The individual model can be fitted in different ways. In the actuarial literature, methods based on Maximum Likelihood Estimation (assuming a particular distribution for the number of deaths) are widely used. Alternatively, methods based on generalised linear models could also be applied. Since those methods are based on models for the number of deaths rather than models for the mortality rates, the obtained estimates for the age and period effects are strongly dependent on those ages and periods in which large numbers of deaths have been observed, and less dependent on ages and periods in which relatively few deaths have been observed. This is often seen as an advantage. However, we wish to extend the individual model to a model for multiple populations that are of different sizes. We therefore prefer a method that attaches the same weight to all observed mortality rates.

It is well known that estimates for $\beta_i^{(j)}$ (column j in matrix β_i) for any individual population i can also be obtained by a principal component analysis using a singular value decomposition of the matrix m_i, that is,

$$m_i = \beta_i L_i U_i^\top$$

where β_i is a $n \times n$ orthogonal matrix, that is, $\beta_i^\top \beta_i$ is the n-dimensional identity matrix, L_i is a $n \times n$ diagonal matrix, and U_i is a $T \times n$ matrix with mutually orthonormal columns, that is, $U_i^\top U_i$ is the n-dimensional identity matrix. We assume that all matrices m_i have full rank, which is then equal to n since we assumed that $n < T$. Note that the singular value decomposition above can also be stated in terms of a $n \times T$ diagonal matrix L_i, and a $T \times T$ orthogonal matrix U_i. Such a decomposition would be equivalent to the one used here.

Also note that the estimated matrix of age effects is now an orthogonal matrix, meaning that the identifiability constraint $||\beta_i^{(j)}|| = 1$ is fulfilled, and, in addition, ${\beta_i^{(j)}}^\top \beta_i^{(l)} = 0$, which is, in general, not the case if age effects are estimated using maximum likelihood methods.

Equivalently, estimates for β_i can also be obtained from computing the eigenvectors of $m_i m_i^\top$:

$$Q_i = m_i m_i^\top = \beta_i \kappa_i \kappa_i^\top \beta_i^\top = \beta_i \Lambda_i \beta_i^\top$$

since

$$m_i m_i^\top = \beta_i L_i U_i^\top U_i L_i^\top \beta_i^\top = \beta_i L_i L_i^\top \beta_i^\top = \beta_i \Lambda_i \beta_i^\top \text{ with } \Lambda_i = L_i L_i^\top.$$

The eigenvalues of $m_i m_i^\top$ are on the diagonal of the matrix Λ_i, and the first estimated age effect $\hat{\beta}_i^{(1)}$ is then the eigenvector corresponding to the largest eigenvalue of $m_i m_i^\top$. For an individual model of order $p \leq n$ we only use the p estimated eigenvectors corresponding to the p largest eigenvalues, that is, the estimated matrix ${}_p\hat{\beta}_i$ contains the first p columns of $\hat{\beta}_i$.

The estimated first age effects $\hat{\beta}_i^{(1)}$ for the ten countries mentioned in the introduction are shown in Figure 1 in grey. In can be seen in this figure that the age effects for ages 53-87 are indeed rather similar for different countries and might therefore be replaced by an age effect that is the same for all countries. For younger ages this is less obvious. We will now turn to a model and a corresponding estimation procedure for such a common age effect. The black line in Figure 1 already shows the estimated first common age effect for these countries based on the CAE model that we will introduce in the following section.

3 Common age-effect model

In this section we will first introduce the CAE model and then discuss the estimation of its parameters in Section 3.2.

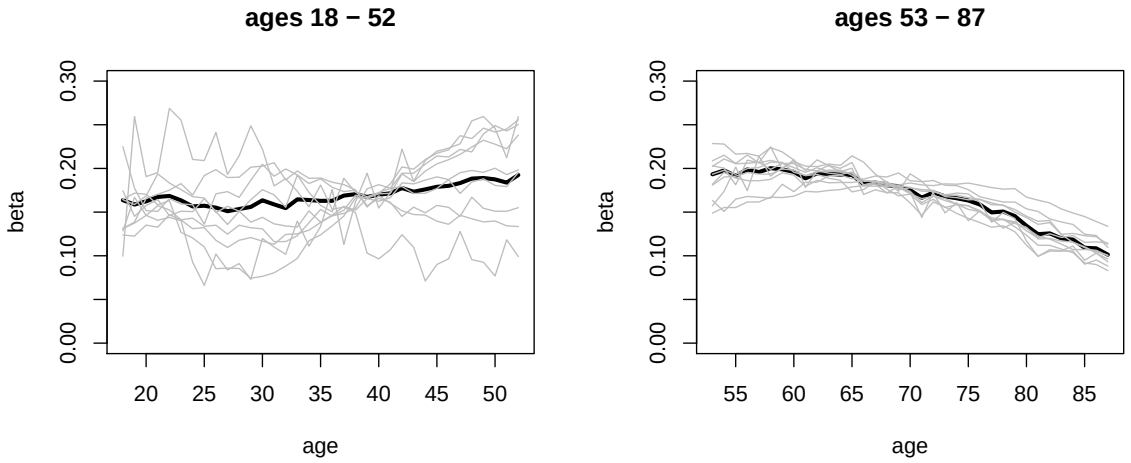

Figure 1: First order age effects $\hat{\beta}_i^{(1)}$ (grey) and first order common age effect (black).

3.1 The CAE model

Using the approach in Section 2 we obtain age and period effects for each population $i = 1, \ldots, k$ individually. We now aim to reduce the overall number of parameters. To this end we introduce a model in which age has the same effect on the centralised log mortality rates for all countries.

Our common age-effect (CAE) model of order p has the same structure as the individual model, but we now assume that the impact of age is independent of the population i, that is,

$$m_i = {}_p\beta \, {}_p\kappa_i^c + \varepsilon_i \qquad i = 1, \ldots, k. \tag{3}$$

where ${}_p\beta$ is a matrix with n rows and p columns, and the ${}_p\kappa_i^c$ are $p \times T$ matrices for all populations i. These matrices are defined as in (2).

Note that in the CAE model the period effects ${}_p\kappa_i^c$ are still dependent on the specific population. This is in contrast to the model proposed by Li & Lee (2005) where there is a common period effect associated with the common age effect, see (6). We use here the notation ${}_p\kappa^c$ for the period effects in the CAE model in (3) to distinguish them from the period effects ${}_p\kappa$ obtained in the individual models.

We remark that the individual model and the common age-effect model can be combined by choosing the matrices ${}_p\beta_i$ in the individual model (1) such that some of their columns are the same for all i. Therefore, while all period effects are population specific, some age effects are the same for all populations. The estimation of such a model is not considered in this paper, but it will consist of estimating a CAE model of an order smaller than p combined with a singular value decomposition applied to the residuals.

A further extension of the CAE model would be a model in which the populations are grouped such that each group has common age effects but age effects between groups are different. However, any extensions of the proposed model are left for further research.

3.2 Estimation of common age effects

For the estimation of the common age effect ${}_p\beta$ in (3) we apply a methodology called common principal component analysis (cPCA) which was first introduced by Flury (1984). Instead of using the estimators proposed by Flury (1984), which are based on Maximum Likelihood estimation, we use here a modification based on least squares estimation. To simplify notation we define $\beta = {}_n\beta$ as in the previous section. The numerical algorithm to obtain estimates $\hat{\beta}$ of β is the F-G-algorithm, see Flury & Constantine (1985) with a modification by Clarkson (1988). In the following we outline the basic ideas underlying this method.

Assuming the CAE model of order p in (3) and following the approach outlined in the previous section, we wish to find an orthogonal matrix $\beta = {}_n\beta$ and diagonal matrices Λ_i such that

$$Q_i := m_i m_i^\top = \beta \Lambda_i \beta^\top \qquad \forall\, i = 1, \ldots, k.$$

This is equivalent to finding an orthogonal matrix β such that $\beta^\top Q_i \beta = \Lambda_i$ is a diagonal matrix for all $i = 1, \ldots, k$. In general, it is not possible to find such a β.

However, our estimate $\hat{\beta}$ for the CAE matrix β is the orthogonal matrix that makes all matrices $\beta^\top Q_i \beta$ as close to diagonal matrices as possible. To make this statement precise we denote by $||A|| = \sqrt{\sum_{i,j} a_{ij}^2}$ the Frobinus-norm of a matrix $A = (a_{jl})_{j=1,\ldots,J, l=1,\ldots,L}$ where a_{jl} is the element in row j and column l. We now estimate β by minimizing the statistic

$$T(\beta) \;=\; \sum_{i=1}^{k} ||\beta^\top Q_i \beta - \mathrm{diag}(\beta^\top Q_i \beta)||^2 = \sum_{i=1}^{k} \sum_{j \neq l} (\beta^\top Q_i \beta)_{jl}^2$$

which is the sum of the squares of the off-diagonal elements of $\beta' Q_i \beta$. Our estimate $\hat{\beta}$ is then

$$\hat{\beta} = \arg\min_{\beta} T(\beta)$$

where the minimum is taken over all $n \times n$ orthogonal matrices β.

We also obtain estimates for the diagonal matrices Λ_i, which are

$$\hat{\Lambda}_i = diag(\hat{\beta}^\top Q_i \hat{\beta}).$$

As mentioned earlier, the modified F-G-Algorithm by Clarkson (1988) is used to obtain the estimates for β numerically.

As in the individual model, we only take the first p columns of $\hat{\beta}$ to obtain a common age-effect model of order p with estimated CAE matrix ${}_p\hat{\beta}$. Note that the first p columns are here the columns that correspond to the largest p values in the diagonal of one of the Λ_i matrices, but the order of elements in the diagonal of Λ_i might be different from the order in other matrices Λ_j. There is no general solution for this issue, but it turns out in our empirical study that this is not a major issue for the mortality rates in those countries that we consider.

After obtaining the estimate ${}_p\hat{\beta}$ for the CAE matrix ${}_p\beta$, we estimate ${}_p\kappa_i^c$ in the usual way treating ${}_p\hat{\beta}$ as given. Since ${}_p\hat{\beta}$ is an orthogonal matrix, we obtain ${}_p\hat{\kappa}_i^c$ as

$${}_p\hat{\kappa}_i^c = {}_p\hat{\beta}^\top m_i$$

and the observed residuals ε_i are given by $\varepsilon_i = m_i - {}_p\hat{\beta}\, {}_p\hat{\kappa}_i^c$.

As mentioned in Section 2 other estimation methods could be applied. Assuming that the number of deaths in each population has a specific distribution, we can obtain Maximum

Likelihood estimators for the parameters in the CAE model. However, the obtained estimators of the common age effects would strongly depend on the mortality in larger populations. In this paper, we see this as a disadvantage since we are interested in common features (age effects) across mortality rates in a number of populations that are of very different sizes. We, therefore, suggest to consider an estimation method based on the observed rates rather than the observed numbers of deaths and exposures.

4 Empirical results and model comparison

As mentioned earlier, Figure 1 shows the obtained estimates $\hat{\beta}_i^{(1)}$ for the individual age effects and the estimated common age effect $\hat{\beta}^{(1)}$ for the ten countries in our empirical study. These appear to be rather close at least for high ages, but this is clearly a weak argument for suggesting that the differences do not matter. To decide whether we can indeed replace individual β_i with a common β we will now study the impact of a common age effect on the estimated period effects and compare the goodness of fit of individual models with the goodness of fit of the CAE model.

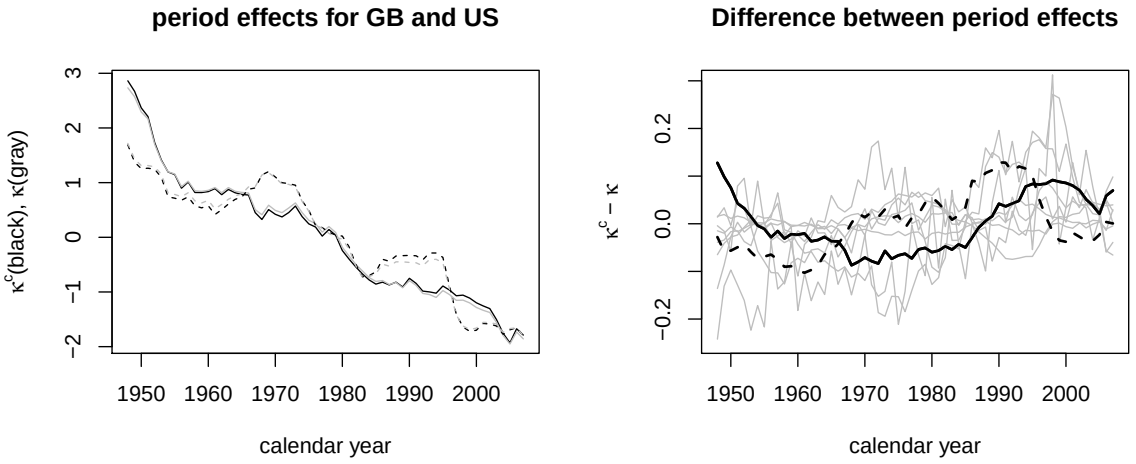

Figure 2: First period effects estimated in individual models ($\hat{\kappa}$, grey lines) and in the CAE model ($\hat{\kappa}^c$, black lines) for the United Kingdom (solid lines) and the United States (dashed lines). The age range is 18-52.

The plot on the left of Figure 2 shows the estimated first period effects $\hat{\kappa}_i$ (grey) and $\hat{\kappa}_i^c$ (black) for the UK (solid lines) and for the US (dashed lines). These are the first period effects in models fitted to the age range 18-52. The plot on the right hand side shows the difference $\hat{\kappa}_i - \hat{\kappa}_i^c$ for the UK (black solid line), the US (dashed line) and the other eight countries is our empirical study (grey lines). It appears that the first period effects for these two countries change very little when the individual age effects are replace by a common age effect. We observe a very similar picture for all countries. The result is also similar when we fit the models to other age ranges.

In a next step we investigate the goodness of fit of the CAE model compared to individual

models. To this end we first plot the observed log mortality rates together with the fitted log mortality rates for the UK at ages 50 and 70 in Figure 3.

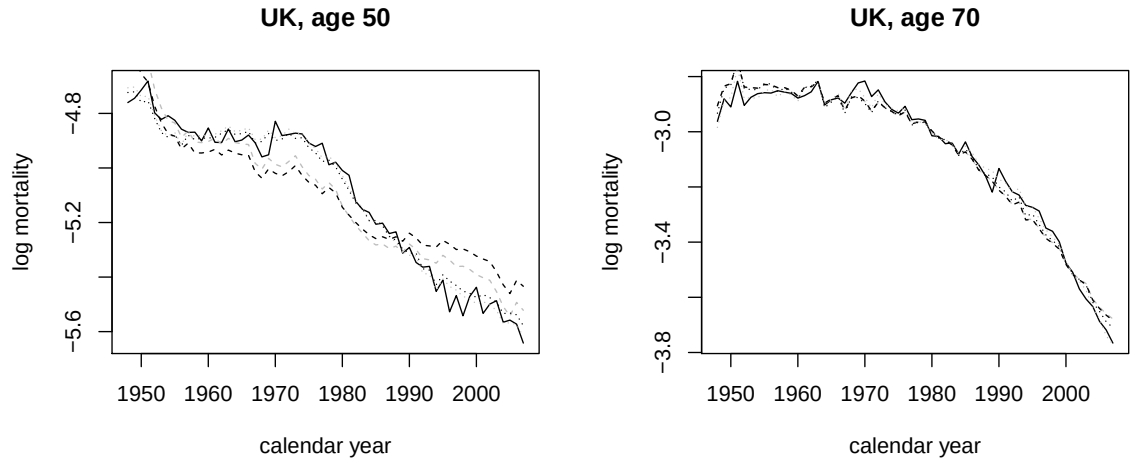

Figure 3: Observed (solid line) and fitted log mortality rates for the UK at ages 50 and 70. The dashed lines correspond to fitted rates with one age-period effect ($p = 1$), and the dotted lines are fitted rates for two age-period effects ($p = 2$). The grey lines are fitted rates for the individual model, and the black lines show fitted rates for the CAE model.

We observe in Figure 3 that there is hardly any difference in the fitted curves at age 70 for the four models (individual and CAE model, each with $p = 1$ and $p = 2$). However, at age 50 (left plot) we find that the fit improves if we consider two age-period effects ($p = 2$) for both the individual model (grey lines) and the CAE model (black lines). More importantly we observe that the CAE model with $p = 2$ seems to fit the data better than the individual model with $p = 1$. This is a first indication that the additional age-period effect seems to be more important for the goodness of fit than the country-specific differences in the mortality rates. In other words, it seems that the fit of a CAE model with one age-period effect ($p = 1$) can be improved more by adding an additional common age-period effect than by considering an individual age effect (LC model) for each country. It should be noted that the fitted mortality rates at age 50 are calculated on the basis of observed rates for ages 18-52, and the fitted rates for age 70 are produced using rates for ages 53-87. They are therefore based on different data sets.

To investigate this further we calculate the overall mean squared error as a function of the number of the age-period effects p for both the individual model and the CAE model. With the notation

$$\hat{m}_i(x,t) = {}_p\hat{\beta}_i \, {}_p\hat{\kappa}_i \ , \text{ and } \hat{m}_i(x,t) = {}_p\hat{\beta} \, {}_p\hat{\kappa}_i^c$$

respectively, the MSE is defined as

$$\text{MSE}(p) = \frac{1}{10nT} \sum_{i=1}^{10} \sum_{j=1}^{n} \sum_{t=1}^{T} \Big(m_i(x_j,t) - \hat{m}_i(x,t) \Big)^2. \tag{4}$$

ages	18 - 52		53 - 87	
	$p=1$	$p=2$	$p=1$	$p=2$
Ind. Mod.	14.157	10.285	2.139	1.592
CAE Mod.	16.061	11.758	2.556	1.822

Table 1: The table shows MSE(p) $\times 10^3$ for the individual model and the CAE model.

ages	18 - 52			53 - 87		
	$p=1$	$p=2$	$p=3$	$p=1$	$p=2$	$p=3$
Ind. Mod.	-79,953	-77,209	-72,489	-119,639	-116,388	-111,213
CAE Mod.	-80,438	**-80,669**	-79,601	-119,037	**-119,828**	-117,670

Table 2: Approximate BIC for the individual model and the CAE model.

Note that our empirical study is based on two age groups with 35 ages each ($n = 35$), and 60 years of observations, 1948-2007 ($T = 60$), for ten countries. The obtained results for the two age groups are shown in Table 1.

We observe in Table 1 that individual models fit the data better as we would expect. However, we also observe that the MSE for the CAE model with $p = 2$ is less than the MSE of individual models with $p = 1$. Again, this shows that the fit of a CAE model with $p = 1$ is more improved by adding a second common age effect (and the corresponding κ_i^c) than by considering individual age effects for each country.

Clearly, a CAE model with $p = 2$ has more parameters than an individual model with $p = 1$ since the number of additional country-specific period effects in the former exceeds the number of additional country-specific age-effects in the latter. To penalise the MSE for the number of parameters we consider an approximation of the Bayesian Information Criterion (BIC). We did not make an explicit assumption about the distribution of the error terms ε_i in the two models in (1) and (3). However, we can approximate the BIC with

$$\text{BIC}(p) = 10nT \log(\text{MSE}(p)) + k \log(10nT). \tag{5}$$

This approximation is justified if we assume that the error terms ε_i in (3) are approximately normally distributed. Even if the distribution of ε_i is not normal, BIC(p) corrects MSE(p) for the number of parameters and will therefore provide a good measure for the goodness of fit of the models.

In our empirical study, $10nT = 21000$ is the total number of observed mortality rates in the ten countries at n ages in T calendar years, and k is the number of parameters in the models for the centralised log rates. For the individual model we have $k = 10p(n + T)$ since there are 10 countries with p age effects and p period effects each. For the CAE we have $k = p(10T + n)$. The numerical results for our empirical data are shown in Table 2.

We also compare the fit of the CAE model with the fit of the model proposed by Li & Lee (2005). They suggest

$$\hat{m}_i(x,t) = B(x)K(t) + b(x,i)\kappa(t,i)$$

as a model for the fitted centralised log mortality rates. We estimate B and K using the

ages	18-52	53-87
MSE	12.948	2.104
BIC	-80,883	-119,037

Table 3: The table shows MSE $\times 10^3$ and the approximate BIC for the Li&Lee model.

combined log mortality rates for all countries giving equal weight to each country, that is,

$$\hat{m}(x,t) = \frac{1}{10}\sum_{i=1}^{10} m_i(x,t) \qquad \forall\, x,t.$$

We then apply a singular value decomposition as described in Section 2 for the individual model of order $p = 1$ to obtain estimates $\hat{B}$ and $\hat{K}$.

In a second step we apply the individual model to the residuals

$$r_i(x,t) = m_i(x,t) - \hat{B}(x)\hat{K}(t)$$

to obtain estimates for the country specific $b(x,i)$ and $\kappa(t,i)$ in the Li&Lee model. We then recover the fitted mortality rates as

$$\hat{m}_i(x,t) = \hat{B}(x)\hat{K}(t) + \hat{b}(x,i)\hat{\kappa}(t,i) \tag{6}$$

and calculate the statistics MSE and BIC as in (4) and (5). The number of parameters in the Li&Lee model is $k = 35 + 60 + 10(35 + 60)$ for 35 ages, 10 countries and 60 calendar years.

Table 3 shows the empirical values we obtain for the MSE and the BIC. We find that the MSE of the Li&Lee model is greater than the MSE of the CAE model of order $p = 2$. However, comparing the BIC we also find that the Li&Lee model outperforms the CAE model for the age group 18-52, but for the older ages 53-87 the CAE model performs better than the Li&Lee model. This reinforces our observation in Figure 1 that the age effects of individual countries are closer to each other for old ages, and we would therefore expect, the CAE model to perform better for those ages. This is also reflected in the smaller MSE and BIC for old ages compared to the same model applied to young ages.

Also note that all considered models fit the mortality rates at young ages rather poorly with large mean squared errors compared to older ages. It seems that mortality rates at younger ages are more difficult to model, or that non of the three considered models is appropriate. One way to obtain a better fit might be to increase the order of the models by including more age-period effects. However, this is beyond the scope of this paper.

An alternative way of estimating the Li&Lee model is to combine death and exposure data, that is,

$$D(x,t) = \sum_i D_i(x,t) \qquad \text{and} \qquad E(x,t) = \sum_i E_i(x,t)$$

and then estimate $\hat{B}$ and $\hat{K}$ from the combined mortality rates $D(x,t)/E(x,t)$. This would increase the BIC of the Li&Lee model (since the MSE increases) to -79,921 (ages 18-52) and -118,816 (ages 53-87). However, this is clearly not appropriate in our study since the large exposure of the US would dominate the empirical results.

5 Conclusions and Further Research

We proposed an age-period model for the mortality rates of multiple populations in which age effects are the same for all populations while the period effects are population specific. We find empirical evidence to justify this approach. The empirical results in Section 4 suggest that a second factor in a LC-type model is more important for the fit than the differences in the age effects between individual populations.

The proposed common age effect model allows us to estimate period effects in different countries, which are better comparable than period effects that are influenced by country specific age factors. Although, we did not study the dynamics of these period effects, we argue that the proposed CAE model gives rise to more consistent stochastic mortality models for multiple populations since individual age factors are avoided.

The proposed model could be improved and extended in a number of ways. The extension of the model to include a cohort effect, either common to all countries or country specific, would potentially improve the quality of fit. Considering a cohort effect should be based on a more detailed analysis of the estimated residuals. In addition, further research could focus on developing techniques that can identify age effects which are only common to some countries but not others. An extension of the model in that direction together with a statistical test for the null hypothesis of common age effects would be a further potential development.

Another interesting research question is the identification of common factors for mortality rates in other sets of populations rather than just males in the ten countries considered in this paper. For example, mortality rates for males and/or females in a different set of countries, or the rates for different socio-economic groups in one country, or across different countries, give rise to some relevant research questions. More empirical studies would also allow us to test the robustness of the CAE model, as it might be an appropriate model for some groups of populations, but not for others.

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