Evaluating alternative low carbon fuel technologies using a stakeholder participation-based q-rung orthopair linguistic multi-criteria framework

Citation for published version:

Digital Object Identifier (DOI):
10.1016/j.apenergy.2022.120492

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Applied Energy

Publisher Rights Statement:
© 2022 The Author(s).

General rights
Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.
Evaluating alternative low carbon fuel technologies using a stakeholder participation-based q-rung orthopair linguistic multi-criteria framework

Zaoli Yang a, Salman Ahmad b, Andrea Bernardi c, Wen-long Shang d, Jin Xuan e, Bing Xu f, * 

a College of Economics and Management, Beijing University of Technology, China 
b School of Business and Creative Industries, University of the West of Scotland, UK 
c Department of Chemical Engineering, Imperial College London, UK 
d Beijing Key Laboratory of Traffic Engineering, College of Metropolitan Transportation, Beijing University of Technology, Beijing, China 
e Department of Chemical and Process Engineering, University of Surrey, UK 
f Edinburgh Business School, Heriot-Watt University, UK

HIGHLIGHTS
- A novel multicriteria (MCDM) framework using AHP and q-ROLPBM operator is presented.
- We incorporate stakeholders’ degree of pessimistic and optimistic rate for all criteria.
- Environmental and economic are the most crucial dimensions followed by social and technical ones.
- e-fuel and e-biofuel are found to be the two top ranked production pathways.
- Findings are valuable for policy/investment development for sustainable fuel production.

ARTICLE INFO
Keywords:
- e-fuels
- Alternative low carbon fuels (ALCF)
- UK transportation sector
- Analytics hierarchy process (AHP)
- q-rung orthopair linguistic partition Bonferroni mean operator
- MCDM

ABSTRACT
It is widely believed that alternative low carbon fuels (ALCF) can be instrumental in achieving the transportation sector’s decarbonization goal. Unlike conventional fossil-based fuels, ALCF can be produced through a combination of different chemical processes and feedstocks. The inherent complexity of the problem justifies the multi-criteria decision-making (MCDM) approach to support decision-making in the presence of multiple criteria and data uncertainty. In this paper, we propose a novel stakeholder participation-based MCDM framework integrating experts’ perspectives on ALCF production pathways using the analytics hierarchy process (AHP) and the q-rung orthopair linguistic partition Bonferroni mean (q-ROLPBM) operator. The key merit of our approach lies in treating criteria of different dimensions as heterogeneous indicators while considering the mutual influence between criteria within the same dimension. The proposed framework is applied to evaluate four ALCF production pathways against 13 criteria categorised under economic, environmental, technical, and social dimensions for the case of the United Kingdom (UK). Our analysis revealed the environmental and the economic dimensions to be the most important, followed by the social and technical evaluation dimensions. The e-fuel followed by the e-biofuel are found to be the two top-ranked production pathways that utilise the electrochemical reduction process and its combination with anaerobic digestion. These findings, along with our recommendations, provide decision-makers with guidelines on ALCF production pathway selection and formulate effective policies for investment.

1. Introduction
Global transportation accounts for 57 % of oil demand [1] and is responsible for 24 % of the direct carbon dioxide (CO₂) emissions from fuel combustion [2]. Although the COVID-19 travel restrictions reduced the emissions (7.2 Gt CO₂) in 2020 compared to (8.5 Gt CO₂) 2019, the rebound in passenger and cargo transport demand would result in emissions growth [3]. Technology lock-in to use fossil oil for various transport modes has made it difficult for the sector to decarbonise [4]. In response to the global climate change challenge, alternative
more specific, the total biofuel production surged from 142.6 million and play a crucial role in decarbonizing the transportation sector. To be specific, alternative low carbon fuels (ALCFs) have the largest share [10] and play a crucial role in decarbonizing the transportation sector. To be more specific, the total biofuel production surged from 142.6 million litres to 160.9 million litres in 2019 (78 % bioethanol and 22 % biodiesel) [11]. Along with emission savings, the use of renewable fuels ensures energy security and rural development and achieves circular economy goals [12].

The ALCF follows a waste-to-energy approach. The key merit of this approach is to produce useful products (methane, methanol) by reducing waste material, CO_2 emissions to the atmosphere, and consumption of non-renewable resources. Typical feedstocks used in waste-to-energy pathways include but are not limited to fats, oil, and grease (FOG), sludge, manure, forestry, and agricultural waste. Similarly, capturing CO_2 to make fuel is an innovative and emerging topic attracting worldwide attention [13,14], hence the focus of our current research. The resultant products produced following the waste-to-energy approach then require further upgrading to generate transport fuels (also known as “drop-in” or “synthetic” fuels).

Despite its potential to enable the transportation sector to achieve carbon neutrality, ALCF also faces significant barriers to scale-up [15,16]. The foremost of these is the technical challenge of energy efficiency. For example, Ganesh [17] highlighted the low energy efficiency of production processes as one of the key hurdles to converting CO_2 into sustainable fuels. This view is reiterated by Montazery et al. [18] with research being performed to improve the production system efficiency of converting CO_2 to produce methanol. For biogas production, Mahmudul et al. [19] reviewed several technologies and suggested using solar energy to improve production efficiency. Likewise, Kargbo et al. [20] pointed out that technical inefficiencies due to a low level of technical readiness in drop-in fuel production methods can potentially extend into the economic domain, manifesting themselves in the form of high cost in comparison to fossil fuels, thus holding back ALCF commercialization. Apart from the technical uncertainty of the sustainable fuel production pathway, there are complex non-technical barriers that need to be resolved, including the social perception of ALCFs [21,22], the environmental impact of drop-in fuel production and distribution [23], and economic considerations [20,24].

### 1.1. Motivation

The selection of ALCF production pathways is a multi-faceted tactical decision problem amid a high level of uncertainty in the sector. Most studies have focused on assessing the attractiveness of ALCF using techno-economic analysis (TEA) [25–28]. Despite TEA being instrumental in optimising process design and quantifying final product selling price [29], the evaluation of ALCF involves a multi-dimensional (e.g., economics, environmental, technical, social) and multi-hierarchical structure characteristic. Therefore, there is a need to utilise the multi-criteria decision-making (MCDM) framework to effectively aggregate multiple and conflicting criteria, incorporating data uncertainty, supporting data in different forms, and reflecting stakeholders’ perspectives. Our paper aims to fill this gap by proposing a stakeholder participatory approach based on the MCDM framework to assess ALCF production pathways.

One strand of studies focused on applying standard MCDM methods (e.g., Analytics Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)) to evaluate fuel production technologies. However, this class of techniques unable to fully reflect the criteria measurements’ data uncertainty, as most ALCF production pathways are in a relatively early stage and are continuously evolving. To incorporate the issue of data uncertainty, several authors have utilised fuzzy MCDM methods. Ren and Liang [30], for example, used the fuzzy TOPSIS-based MCDM method to measure the sustainability of alternative marine fuels. Similarly, Sehatpour and Kazemi [31] drew a hybrid framework based on fuzzy multi-objective programming and MCDM to determine an optimised sustainable fuel portfolio of six different fuels. Lin et al. [32] assessed the sustainability prioritisation of hydrogen pathways using a Z-number best worst method to address the ambiguity and fuzziness of stakeholders’ opinions.

Despite fuzzy MCDM methods providing a valuable solution to assess uncertain and fuzzy criteria, they can only offer a qualitative grade level. One common approach to quantify qualitative information is to use an intuitionistic fuzzy set (IFS) to set parameters of membership degree and non-membership degree. The main limitation of traditional IFS resides in its inability to handle nonbinary limited evaluation information; that is, the sum of membership degree and non-membership degree in the evaluation value must be less than or equal to 1 [33–35]. As such, Yager [36] proposed the concept of a q-rung orthopair fuzzy set (q-ROFS), which can effectively process the complex evaluation information i.e., a situation where the sum of the degree of membership and the degree of non-membership is greater than 1. Meanwhile, the MCDM method based on q-ROFS has been widely used to address practical problems, such as the evaluation of government strategies [37], energy and environmental assessment [38], and product design optimisation [39].

Note that both IFS and q-ROFS can only express the “good” and “bad” aspects of a thing, but they cannot give the credibility of the “good” and “bad”. Therefore, the evaluation information expressed by IFS or q-ROFS has high subjectivity and randomness, which can adversely affect the robustness of the evaluation result. In practice, decision makers are likely to face self-contradiction in assessing the acceptance and disapproval rate for certain criterion given the early stage of technologies. Under such circumstances, one can utilises Herrera and Herrera-Viedma’s concept of q-rung orthopair linguistic set to capture the contradictory information by evaluators while give the credibility measurement level of it to provide a comprehensive decision support [40].

Several researchers have enriched the q-rung orthopair linguistic set theory [41–48]. For example, Akram et al. [46] applied the q-rung orthopair linguistic model to address the group decision-making problems. Liu and Huang [47] investigated the consensus reaching process in group decision-making based on the q-rung orthopair linguistic set theory. In addition, Akram and Shahzadi [43], Naz et al. [42], and Saha et al. [44] used the q-rung orthopair linguistic set to handle MCDM problems and developed a series of new decision-making algorithms.

To describe these contradictions and give the credibility of evaluation information on evaluating ALCF production pathways, it is crucial to introduce the q-rung orthopair linguistic set to characterize the evaluation information. Furthermore, the criteria system in our study has the characteristics of multi-layer structure, existing methods often ignore the independence between the criteria from different dimensions and interrelatedness among the criteria in the same category [33–35,49]. Therefore, we also need to develop an aggregation operator and multi-criteria framework based on q-rung orthopair linguistic set.

### 1.2. Contributions

Our overarching research question is how to provide a comprehensive decision support for ALCF production pathway selections to accelerate transportation sector’s decarbonization goal? The innovative methodological contributions of this paper are summarised as follows. First, we introduce the q-rung orthopair linguistic set that represents the...
complex evaluation information for ALCF. The linguistic term set and q-rung orthopair fuzzy set depict the criterion’s rating level and its credibility/uncertainty, respectively. Second, a new MCDM method based on the q-rung orthopair linguistic weighted partition Bonferroni mean operator is proposed to aggregate criteria information with a multi-level structure for evaluating sustainable fuel production pathways. Following the multi-dimensional characteristics of the criterion system, we use the attribute segmentation approach to highlight the heterogeneity and independence of criteria of different dimensions. To be more specific, we avoid linear addition or multiplication in traditional weighted average operators that prevent treating criteria of different dimensions as homogenous indicators.

Our practical contributions including the design of a United Kingdom (UK) case study to assess competing ALCF technologies using our novel framework. Note that one of the main challenges in assessing ALCF technologies is to identify and apply criteria that are trustworthy among stakeholders. Thus, we developed a thorough criteria system for evaluating ALCF production pathways and validated it through both an in-depth literature review and expert consultations. Each pathway is assessed against 13 criteria encompassing the economic, environmental, technical, and social dimensions. For every criterion, we gather the relative importance/weights together with ratings of each production pathway via an online expert survey. Finally, by evaluating four competing sustainable fuel production pathways based on expert opinion and preferences, we address another important research question; namely, why certain pathways are doing better in specific aspects and globally, respectively? These findings are important in production location selection decisions, identifying and developing technology clusters, and planning for future energy infrastructure flexibility and resilience. The rankings provided could be instrumental in unfolding the complexity around the ALCF supply chain and towards defining a long-term road map for energy and transport system decarbonization and creating local and international collaborations as co-benefits. The proposed approach is a generic framework that could be used for any benchmarking activities for businesses, countries, or regions looking for developing their alternative fuel productions.

1.3. Outline

The rest of this paper is organised as follows. Section 2 describes our proposed MCDM framework for ranking alternative low carbon drop-in fuel production pathways and data collection. Section 3 introduces the q-rung orthopair linguistic partition Bonferroni mean operator. Section 4 presents and discusses the results in detail, while Section 5 reports the sensitivity analysis. Finally, Section 6 concludes and sets the direction for future work.

2. Framework design

In this study, we propose a novel framework to evaluate the attractiveness of four competing ALCF production pathways and formulate a multi-criterion ranking to assist stakeholders in making informed decisions. Fig. 1 illustrates the key steps to operationalise our proposed framework.

2.1. Choice of alternative and performance criteria

In terms of the choice of an alternative, we opt to evaluate four competing low carbon fuel production pathways that cover a wide spectrum of technology readiness levels as follows: e-fuel, solar-fuel, biofuel, and e-biofuel – see Fig. 2 and Table 1 for details. More specifically, we include fuel production with captured CO$_2$ from industrial processes or from the atmosphere, as in the case of e-fuel, solar-fuel, and e-biofuel, or from biological feedstock for e-biofuel and biofuel production.
pathways. The prefix ‘e’ signifies electricity, for electrochemical CO\textsubscript{2} reduction purposes, from sustainable sources such as wind, solar, or nuclear power. Furthermore, the solar-fuel production pathway considered in this study uses sunlight to activate the photocatalyst for the conversion process. On the other hand, the biofuel production pathway employs a conventional anaerobic digestion (AD) process, which entails a biological breakdown of organic material by bacteria. Finally, the novel e-biofuel production pathway integrates the conventional biofuel and e-fuel production pathways. For a detailed description of these production pathways, the reader is referred to [50,51], and [52].

Note that four ALCF production pathways are considered based on expert consultation. They agreed that the evaluation should focus on innovative and emerging fuel production technologies. Furthermore, the limitation was motivated by the simplicity and feasibility of the expert preference data elicitation process. For example, say 10 ALCF production pathways on the social dimension (e.g., contribution to economy; public acceptance; job creation) would require 136 pairwise comparisons\textsuperscript{2} to be performed by each expert. This carries a risk that experts might not be able to perform these comparisons in a reliable and meaningful way.

One of the key issues in assessing emerging technologies is to use performance criteria and measurements with credibility among different stakeholders. Therefore, we conducted an in-depth review of the literature and initially identified a total of 38 evaluation criteria under the technical, economic, environmental, and social dimensions. Next, we narrowed the initial set of criteria to 13 by considering data availability and relevance to our study. Finally, an expert workshop was organised to validate the criteria and their measures. Table 2 reports a detailed description and references, whether it is a cost (i.e., to be minimised, the smaller the better) or a benefit (i.e., to be maximised, the larger the better) criterion.

2.2. Data and preference gathering

The first part of this process involves collecting stakeholders’ preferences and weights on all measures of the criteria. The second part gathers technology-specific data on the measures of the final set of performance criteria.

2.2.1. Stakeholder preferences and weights

To extract experts’ preferences on different criteria, we prepared a
Table 2
A comprehensive multidimensional evaluation criteria framework.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Criteria</th>
<th>Cost vs. Benefit</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technical</td>
<td>Technology maturity (C1)</td>
<td>Benefit</td>
<td>This measures the technological maturity of the process pathway based upon technology readiness level (TRL 1-9), where TRL 9 represents a mature technology.</td>
<td>[54]</td>
</tr>
<tr>
<td>Energetic</td>
<td>content (C2)</td>
<td>Benefit</td>
<td>This refers to the energy intensity level of an e-fuel, measured by MJ/kg.</td>
<td>[55,56]</td>
</tr>
<tr>
<td>Process</td>
<td>efficiency (C3)</td>
<td>Benefit</td>
<td>This refers to the overall conversion efficiency % (conversion plus/minus refining), the higher level the better.</td>
<td>[57]</td>
</tr>
<tr>
<td>Fuel</td>
<td>production system complexity (C4)</td>
<td>Cost</td>
<td>This refers to the level of fuel production system complexity, the lower level the better.</td>
<td>Expert panel, [20]</td>
</tr>
<tr>
<td>Economic</td>
<td>Operational cost (C5)</td>
<td>Cost</td>
<td>This measures the cost of fuel production and plant maintenance for a particular pathway.</td>
<td>[56,58]</td>
</tr>
<tr>
<td>Investment</td>
<td>cost (C6)</td>
<td>Cost</td>
<td>This refers to the capital required in setting up a commercial level production facility.</td>
<td>[24,58]</td>
</tr>
<tr>
<td>Market</td>
<td>maturity (C7)</td>
<td>Benefit</td>
<td>The market availability, commercial competitiveness and compatibility with the existing economic system leading to financing opportunities.</td>
<td>Expert panel, [59]</td>
</tr>
<tr>
<td>Environmental</td>
<td>Net water use (C8)</td>
<td>Cost</td>
<td>This measures the quantity of freshwater required.</td>
<td>[60,61]</td>
</tr>
<tr>
<td>Carbon</td>
<td>footprint (C9)</td>
<td>Cost</td>
<td>This measures the life cycle carbon emission from a sustainable fuel production pathway option.</td>
<td>[62]</td>
</tr>
<tr>
<td>Land use</td>
<td>change (C10)</td>
<td>Cost</td>
<td>This refers to both direct and indirect land use change due to the introduction of sustainable fuel production.</td>
<td>[8,63]</td>
</tr>
<tr>
<td>Social</td>
<td>Contribution to economy (C11)</td>
<td>Benefit</td>
<td>This refers to the wealth creation from activities such as new enterprise establishments,</td>
<td>[15,62]</td>
</tr>
</tbody>
</table>

Table 2 (continued)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Criteria</th>
<th>Cost vs. Benefit</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social</td>
<td>Contribution to economy (C11)</td>
<td>Benefit</td>
<td>This refers to the wealth creation from activities such as new enterprise establishments,</td>
<td>[15,62]</td>
</tr>
<tr>
<td>Environmental</td>
<td>Net water use (C8)</td>
<td>Cost</td>
<td>This measures the quantity of freshwater required.</td>
<td>[60,61]</td>
</tr>
<tr>
<td>Carbon</td>
<td>footprint (C9)</td>
<td>Cost</td>
<td>This measures the life cycle carbon emission from a sustainable fuel production pathway option.</td>
<td>[62]</td>
</tr>
<tr>
<td>Land use</td>
<td>change (C10)</td>
<td>Cost</td>
<td>This refers to both direct and indirect land use change due to the introduction of sustainable fuel production.</td>
<td>[8,63]</td>
</tr>
</tbody>
</table>

2.2.2. Choice of a system for rating low carbon fuel production pathways

We need to establish the evaluation criterion system and collect evaluation information. The ratings of the competing sustainable fuel production pathways against each criterion were obtained from a combination of empirical computations and expert opinions by applying a simple rating system.

Regarding the environmental impact category, the functional unit (FU) is 19.9 MJ of fuel produced (corresponding to 1 kg of methanol based on the lower heating value) and the life cycle inventories for each pathway are derived from the literature [28,66-68]. The processes are assumed to be located in the UK, and the main source of biomass is assumed to be wood chips. ReCiPe2016 [69] is the impact assessment methodology applied in this work, and the comparison is based on three midpoint indicators: global warming potential (kg CO2-eq), water consumption (m³ water), and land use (m²a). For the multi-product processes, system expansion via substitution was adopted to solve the multifunctionality, as recommended by von der Assen et al. [70]. The environmental assessment is conducted in Simapro (version 9.1.1.1), using ecoinvent 3.6 for the background process inventories. In particular, the electricity UK grid mix in ecoinvent 3.6 refers to 2016, as reported in the International Energy Agency (IEA) World Energy Balance report [71], while wood chip production is representative of the average European production, as more regionalised data were not available.

Note that not all measures can be quantified straightforwardly (e.g., technology maturity, public acceptability). Hence, we opt for an in-depth interview with five experts in the field to collect data and transform it into discrete measures for all criteria. We use a numeric scale of 1

---

4 https://hwsml.eu.qualtrics.com/jfe/form/SV_7P0KgUWrex7eT0F.
5 Note that ReCiPe2016 provides midpoint indicators, which quantify the effects of resource utilisation and emissions on a specific environmental category (e.g., global warming), and endpoint indicators, which represent the three areas of protection: human health, ecosystems quality, and resources.
6 When system expansion via substitution is considered, a process receives environmental credits for the by-products that are produced along with the main product.
to 9" to enable experts to communicate their ratings for each pathway and criterion combination. In addition, for each measure, we also obtain the level of optimistic support and pessimistic support towards the score they have provided. The reader is referred to Appendix A Tables A.2 and A.3 for the full dataset.

3. Methodology

In this section, we introduce the q-rung orthopair linguistic partition Bonferroni mean operator. Note that the q-rung orthopair linguistic set handles experts' rating information and corresponding confidence level. The experts' rating value is represented by the linguistic term, and the confidence level is made up of the optimistic support degree and pessimistic support degree that experts place on their evaluation, corresponding to the membership degree and non-membership degree in q-ROFs. We leverage attribute partition theory to process data with multiple aspects and the Bonferroni mean operator to examine the mutual influence relationship among different impact dimensions to rank order alternatives.

3.1. Preliminaries

Definition 1. [40]. Let \( L = \{0, \theta, 1\} \) be a linguistic term set with odd cardinality, and let \( t \) be a positive whole number. \( s \) is a possible value for a linguistic variable. The linguistic term is regarded as follows:

\[
L = \{s_0 = \text{worst}, s_1 = \text{very bad}, s_2 = \text{bad}, s_3 = \text{relatively bad}, s_4 = \text{not bad}, s_5 = \text{medium}, s_6 = \text{not good}, s_7 = \text{relatively good}, s_8 = \text{good}, s_9 = \text{very good}, s_{10} = \text{best}\}.
\]

The linguistic term sets have the following properties:

i. If \( i \geq j \), then \( \max(l_i, l_j) = l_i \);
ii. If \( i \geq j \), then \( \min(l_i, l_j) = l_j \);
iii. If \( i \geq j \), \( l_i \leq l_j \).

Definition 2. [36]. Let \( X \) be a fixed set; a q-ROFS\( A \) on \( X \) can be represented as:

\[
A = \{(x, \mu_A(x), \nu_A(x), \pi_A(x))|x \in X\}
\]

where \( \mu_A: X \rightarrow [0, 1] \) denotes the degree of membership and \( \nu_A: X \rightarrow [0, 1] \) denotes the degree of non-membership of element \( x \in X \) to set \( A \), respectively, with the condition that \( 0 \leq (\mu_A(x) + \nu_A(x)) \leq 1 \) (q≥1). The degree of indeterminacy is given as \( \pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \); for convenience, we call \( (\mu_A(x), \nu_A(x), \pi_A(x)) \) a q-ROLN denoted by \( A = (\mu_A, \nu_A, \pi_A) \).

The evaluation information of the sub-criteria in this paper is composed of a linguistic term set and its positive support degree and negative support degree, and the sum of positive and negative support degrees is greater than 1. The traditional intuitionistic fuzzy set has difficulty dealing with such complex evaluation information. Therefore, the q-ROLS is introduced to characterise the index evaluation information in this study. The details are as follows:

Definition 3. [41]. Let \( X \) be a fixed set; a q-ROFS\( A \) on \( X \) can be represented as:

\[
A = \{(x, (\mu_A(x), \nu_A(x), \pi_A(x)))|x \in X\}
\]

where \( \mu_A(x) \) is the linguistic term set, \( \mu_A: X \rightarrow [0, 1] \) denotes the support degree of membership, and \( \nu_A: X \rightarrow [0, 1] \) denotes the support degree of non-membership of the element \( x \in X \) to the set \( A \), respectively, with the condition that \( 0 \leq (\mu_A(x) + \nu_A(x)) \leq 1 \) (q≥1). The degree of indeterminacy is given as \( \pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \); for convenience, we call \( (\mu_A(x), \nu_A(x), \pi_A(x)) \) a q-ROLN denoted by \( A = (\mu_A, \nu_A, \pi_A) \).

Definition 4. [41]. Let \( a_1 = (l_{q_1}, \mu_1, \nu_1, \pi_1) \) and \( a_2 = (l_{q_2}, \mu_2, \nu_2, \pi_2) \) be two q-ROLS, and let \( \lambda \) be a non-negative real number; then:

i. \( a_1 + a_2 = (l_{q_1}, \mu_1, \nu_1, \pi_1) \times \lambda + (l_{q_2}, \mu_2, \nu_2, \pi_2) \)
ii. \( a_1 \times a_2 = (l_{q_1}, \mu_1, \nu_1, \pi_1) \times \lambda \times (l_{q_2}, \mu_2, \nu_2, \pi_2) \)
iii. \( a_1 \lambda = (l_{q_1}, \mu_1, \nu_1, \pi_1) \lambda \)
iv. \( a_1^\lambda = (l_{q_1}, \mu_1, \nu_1, \pi_1) \lambda\)

Definition 5. [41]. Let \( a = (l_{q_1}, \mu_1, \nu_1, \pi_1) \) be a q-ROLN; then, the score function of a is defined as \( S(a) = l_0 \times (u_0 - \nu_0 + 1) \), and the accuracy function of a is defined as \( H(a) = l_0 \times (u_0 + \nu_0) \). For any two q-ROLS \( a_1 = (l_{q_1}, \mu_1, \nu_1, \pi_1) \) and \( a_2 = (l_{q_2}, \mu_2, \nu_2, \pi_2) \), we have the following scenarios:

1) If \( S(a_1) > S(a_2) \), then \( a_1 > a_2 \);
2) If \( S(a_1) = S(a_2) \), we compare the \( H(a_1) \) and \( H(a_2) \); if \( H(a_1) > H(a_2) \), then \( a_1 > a_2 \); if \( H(a_1) > H(a_2) \), then \( a_1 = a_2 \).

To investigate the potential interrelation among evaluation indicators, we introduce the Bonferroni mean (BM) operator to capture this interaction relationship instead of the traditional weighted average operator, which ignores correlations among indicators.

Definition 6. [72]. Let \( t \geq 0 \), and let \( a_k (k = 1, 2, \ldots, m) \) be a collection of non-negative real numbers; then, the BM aggregation function is expressed as follows:

\[
BM^{\text{q}}(a_1, a_2, \ldots, a_m) = \left( \frac{1}{m(m-1)} \sum_{i,j=1}^{m} a_i \cdot a_j \right)^\frac{1}{t}
\]

3.2. The q-rung orthopair linguistic weighted Bonferroni mean operator

This paper combines the Bonferroni mean operator and q-rung orthopair linguistic set to present the q-rung orthopair linguistic weighted Bonferroni mean (q-ROLWBM) operator. Thus, criteria under a single dimension are aggregated to obtain scores and rankings of different dimensions of each scheme.

Definition 7. Suppose \( a_k = (l_{q_k}, \mu_1, \nu_1, \pi_1) (k = 1, 2, \ldots, m) \) is a collection of q-ROLS, and \( s, t \geq 0, q \geq 1 \); then, the q-ROLWBM operator can be defined as:

\[
q - \text{ROBWBM}^{\text{q}}(a_1, a_2, \ldots, a_m) = \left( \frac{1}{m(m-1)} \sum_{i,j=1}^{m} (w_i \mu_i \cdot w_j \nu_j) \right)^\frac{1}{t}
\]

Theorem 1. Suppose \( a_k = (\mu_1, \nu_1) (k = 1, 2, \ldots, m) \) is a collection of q-ROLS, and \( s, t \geq 0, q \geq 1 \); then, the result aggregated from Definition 5 is still a q-ROLN – see Appendix B for its specific form.
3.3. The q-rung orthopair linguistic partitioned Bonferroni mean operator

Considering the multidimensional index system in this paper, which is composed of four levels, we introduce attribute segmentation theory to construct the partitioned Bonferroni mean operator as follows:

**Definition 8.** [73]. For any \( r, s > 0 \) with \( r + s > 0 \), and \( T = \{a_1, a_2, \ldots, a_n\} \) with \( a_n \geq 0 \) \((k = 1, 2, \ldots, m)\), which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \), where \( \bigcup_{t=1}^{d} P_t = T \), the partitioned Bonferroni mean (PBM) aggregation operator of dimension \( m \) is a mapping PBm:

\[
PBM^{q}(a_1, a_2, \ldots, a_m) = \frac{1}{d} \left( \sum_{i=1}^{d} \left( \frac{1}{|P_i|} \sum_{a \in P_i} a_i \left( \frac{1}{|P_i| - 1} \sum_{a \in P_i, a \neq a_i} a_i \right) \right) \right)^{\frac{1}{q}}
\]

where \( |P_i| \) denotes the cardinality of \( P_i \) and \( d \) is the number of partitioned sorts.

We integrate the partitioned Bonferroni mean operator and the q-ROFS to present the q-rung orthopair linguistic partitioned Bonferroni mean operator as follows:

**Definition 9.** Let \( T = \{a_1, a_2, \ldots, a_n\} \) be a collection of q-ROLNs, which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \), and \( \bigcup_{t=1}^{d} P_t = T \). The q-ROLPBM operator of dimension \( m \) is a mapping q-ROLPBM:

\[
q - \text{ROLPBM}^{q}(a_1, a_2, \ldots, a_m) = \frac{1}{d} \left( \sum_{i=1}^{d} \left( \frac{1}{|P_i|} \sum_{a \in P_i} a_i \left( \frac{1}{|P_i| - 1} \sum_{a \in P_i, a \neq a_i} a_i \right) \right) \right)^{\frac{1}{q}}
\]

where \( |P_i| \) denotes the cardinality of \( P_i \) and \( d \) is the number of partitioned sorts.

**Theorem 3.** Suppose \( a_k = (u_k, v_k)|k = 1, 2, \ldots, m| \) is a collection of q-ROLNs, and \( s, t \geq 0, q \geq 1 \); then, the result aggregated from Definition 7 is still a q-ROLN. Its specific form and the proof are shown in Appendix B.

3.4. The q-rung orthopair linguistic weighted partitioned Bonferroni mean operator

To investigate the influence of index weight on information aggregation and ranking results, based on Definition 7, we propose the q-rung orthopair linguistic weighted partitioned Bonferroni mean operator as follows:

**Definition 10.** Let \( T = \{a_1, a_2, \ldots, a_n\} \) be a collection of q-ROLNs, which is partitioned into \( d \) distinct sorts \( P_1, P_2, \ldots, P_d \), and let \( \sum_{t=1}^{d} P_t = T \), \( w_i \) denote the weight of each argument, satisfying \( 0 \leq w_i \leq 1 \) and \( \sum_{i=1}^{d} w_i = 1 \). For any \( s, t \geq 0 \) and \( s + t > 0 \):

\[
q - \text{ROLWPBM}^{q}(a_1, a_2, \ldots, a_m) = \frac{1}{d} \left( \sum_{i=1}^{d} \left( \frac{1}{|P_i|} \sum_{a \in P_i} w_i a_i \left( \frac{1}{|P_i| - 1} \sum_{a \in P_i, a \neq a_i} w_i a_i \right) \right) \right)^{\frac{1}{q}}
\]

where \( |P_i| \) denotes the cardinality of \( P_i \) and \( d \) is the number of partitioned sorts.

**Step 1:** Normalise the original criteria as follows [74]:

\[
a_i = \begin{cases} 
  \langle b_{ij}, (u_{ij}, v_{ij}) \rangle, & \text{if the criterion } a_i \text{ is benefit type} \\
  \langle b_{ij}, (v_{ij}, u_{ij}) \rangle, & \text{if the criterion } a_i \text{ is cost type}
\end{cases}
\]

**Step 2:** Apply our proposed aggregation method to obtain the comprehensive evaluation value:

1. Use the q-rung orthopair linguistic set to represent the rating
In this section, we first report and examine performance weights obtained from the experts followed by a mono-criterion ranking of the production pathways. Next, we discuss the dimensional rankings. Finally, we present the global rankings of these pathways.

4.1. Performance criteria weights

Based on our online survey, we obtain experts’ preferences as expressed by the relative importance assigned to each criterion. Table 3 shows that the environmental and social dimensions are the most important, with relative weights of 31 % and 27 %, respectively. With a relative weight of 23 %, the economic dimension is ranked third, while the technical dimension with a relative weight of 19 % is ranked last. Within each impact dimension, local weights define the importance of a single criterion. Under the environmental dimension, net water use is rated higher (35.8 %) than the land use change (32.7 %) and the carbon footprint (31.5 %). Within the social dimension, contribution to economy (35.2 %), public acceptability (33 %), and job creation (31.7 %) were all considered important. Similarly, investment cost has the highest importance (40.0 %) over operational costs (33 %) and market maturity (26.9 %) within the economic dimension. Finally, process efficiency (34.5 %) is desired over technology maturity (26.5 %), while fuel production system complexity and energetic content receive the least preference (19.5 % each) within the technical dimension. In addition, for each criterion, we calculate the global weights using the product of each criterion’s local weight and its respective dimension’s relative weight – see Table 3 for details.

The global criteria ranking analysis reveals that net water use has the highest importance (11.2 %), followed by land use change (10.2 %) and carbon footprint (9.8 %). It is worth noting here that the top three global criteria are from the environmental dimension. This importance ranking by experts is plausible, as sustainable low carbon fuels are considered environmentally friendly options for the transportation sector. Furthermore, contribution to economy with a global weight of 9.6 % is the most important criterion from the social dimension. Likewise, investment cost and process efficiency are considered the most important from the economic and technical dimensions, with global criterion weights of 9.0 % and 6.6 %, respectively.

4.2. Mono-criterion ranking

Although our primary goal is to estimate performance against multiple criteria, it is useful to have a good understanding of whether a production pathway performs well on a specific aspect. Table 4 shows the unidimensional rankings based on each of the measures, where competing production pathways are ranked from the best (1, in bold) to the worst.

In the technical dimension, process efficiency (0.345) is considered the most important criterion, and we find biofuel and our proposed e-biofuel to be equally ranked the best. This could be due to the similarities in the underlying conversion process (anaerobic digestion) for both biofuel and e-biofuel production pathways. AD is a mature technology optimised over decades of R&D and commercial applications. Likewise, e-biofuel is ranked higher than e-fuel and solar-fuel because it couples two electrochemical processes together [18], thereby minimising energy loss and achieving a higher process efficiency. The solar-fuel production pathway is ranked as one of the worst production pathways concerning the technical aspect. The main reason for this low ranking is that solar-fuel and e-biofuel rely on carbon capture and storage and...
direct air capture of CO₂, which at the moment are expensive technologies [77]. By corollary, the same can also be said for the e-biofuel production pathway.

Moving on to the environmental evaluation dimension, the minimal amount of water used can be achieved by e-biofuel, while the biofuel production pathway consumed the highest amount of water. Note that the lesser water use can be attributed to additional CO₂ being used as a feedstock and therefore the e-biofuel production pathway obtains a higher ranking from the panel of experts. Likewise, the biofuel production pathway is also ranked last under the carbon footprint and land use criteria. This is not surprising given that biomass cultivation directly impacts the conversion of forestland to cropland and indirectly changes land use from food to fuel-crop cultivation [63]. Biomass production, harvesting, treatment, and transportation are all energy-intensive processes resulting in high carbon emissions [20].

Finally, in the social dimension, contribution to the economy is considered the most important criterion. Our results reveal that e-fuel is ranked the best alternative, followed by e-biofuel, while solar-fuel is regarded as the worst production pathway. For this criterion, we can attribute expert panel propensity to a more established chemical process for fuel production compared to the novel idea of using a photocatalytic conversion process [78].

In summary, based on unidimensional rankings, we find that no production pathway consistently outperformed others for all criteria. For instance, despite the biofuel production option offering the best technical maturity, it also ranked the worst for carbon footprint and public acceptability. Furthermore, we also find ties for some criteria given that some pathways share parallel technical and investment environments. Hence, decision-makers may face a challenge in making an informed decision regarding the best production pathway while considering all criteria simultaneously. To improve alternative ranking performance, we incorporate data uncertainty and consider simultaneous multiple criteria evaluations. Note that these rankings do not reflect the potential uncertainties within the dataset.

4.3. Production pathway multi-criteria ranking

Before we dive into the global multi-criteria ranking, we first analyse and discuss each ALCF pathway performance based on the technical, economic, social, and environmental impact categories. By applying the q-ROLWPBM (Definition 7), we incorporate the performance data/rating and potential uncertainties within the dataset (i.e., membership and non-membership) for each criterion. Fig. 3 summarises the categorical rankings after computing dimension-level scores for each production pathway alternative.

The score function of production pathways’ technical dimension is presented in the top left panel of Fig. 3. The results reveal that biofuel is ranked the best alternative, followed by e-fuel, while e-biofuel and solar-fuel are the lowest ranked pathways. The biofuel production pathway is based on an anaerobic digestion process, which is a technically mature process [79], in comparison to the solar-fuel process, which is still developing [80].

Recall that an economic evaluation represents the commercial
viability and uptake of drop-in fuel produced from the considered pathways. We find that biofuel is the most preferred production pathway with respect to the economic dimension over the other three options, namely, e-biofuel, solar-fuel, and e-fuel. This ranking comes as no surprise, as the market familiarity and availability of commercial-level biofuel production facilities play a crucial role in making this pathway an economically attractive option. Although the e-biofuel pathway has a similar conversion process (electrochemical reduction) as well as closeness in the feedstock (CO₂ and water), the additional anaerobic digestion process and biomass feedstock requirement make e-biofuel a less economically attractive option. Furthermore, on the economic dimension, our analysis is consistent with the International Renewable Energy Agency (IRENA) claim that bio-based production pathways are less expensive than electricity-dependent pathways, such as e-fuels [81]. In particular, the technical complications involving electricity production and feedstock (CO₂ and water) availability make these pathways a more expensive option [58,80].

For the environmental dimension, the bottom left figure in Fig. 3 shows the ranking of four competing production pathways. The e-biofuel pathway performs the best, followed by solar-fuel and e-fuel, by utilising captured CO₂, while biofuel is found to be the worst-performing pathway. The inclinations towards the top three production pathways can be attributed to low land use change, as they do not require any fertile land for feedstock and they also provide a quick carbon recycling potential [80]. Likewise, it seems that the expert panel held the biomass-based production pathway, biofuel, as more detrimental to the environment than others. It has been established that the cultivation, transportation, and conversion of biomass to final fuel contribute substantially to carbon emissions [82], and this is reflected in expert...
Proposed CO present in the bottom right panel of Fig. 3. We find that evaluations.

Table 6
The influence of criterion weight on ranking results.

<table>
<thead>
<tr>
<th>Weight vector</th>
<th>Score</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>w=(0.7,0.1,0.1,0.1)</td>
<td>S(e-fuel) = 0.255, S(solar-fuel) = 0.166, S(biofuel) = 0.29, S(e-biofuel) = 0.221</td>
<td>biofuel &gt; e-fuel &gt; e-biofuel &gt; solar-fuel</td>
</tr>
<tr>
<td>w=(0.1,0.7,0.1,0.1)</td>
<td>S(e-fuel) = 0.215, S(solar-fuel) = 0.213, S(biofuel) = 0.342, S(e-biofuel) = 0.192</td>
<td>biofuel &gt; e-fuel &gt; solar-fuel &gt; e-biofuel</td>
</tr>
<tr>
<td>w=(0.1,0.1,0.7,0.1)</td>
<td>S(e-fuel) = 0.323, S(solar-fuel) = 0.305, S(biofuel) = 0.149, S(e-biofuel) = 0.365</td>
<td>e-biofuel &gt; e-fuel &gt; solar-fuel &gt; biofuel</td>
</tr>
<tr>
<td>w=(0.1,0.1,0.1,0.7)</td>
<td>S(e-fuel) = 0.321, S(solar-fuel) = 0.232, S(biofuel) = 0.27, S(e-biofuel) = 0.205</td>
<td>e-fuel &gt; biofuel &gt; solar-fuel &gt; e-biofuel</td>
</tr>
</tbody>
</table>

4.4. Production pathway global ranking

To overcome the decision-maker choice dilemma identified in the previous section, for each alternative, we apply our proposed q-ROLWPBM operator in Definition 10 to aggregate all evaluation information. To incorporate the sum of optimistic membership degree ($u_1$) and pessimistic ($u_2$) non-membership degree less than 1 and to comply with the operation rules, as stipulated in Definition 4, we set the parameter $q = 3$. Meanwhile, for definiteness and without loss of generality [84], we set the criteria correlation parameters $s = t = 1$ to assume the relationship intensity of the criteria is equal. As such, the degree of pessimism corresponds to the membership degree in q-ROFS, the degree of optimism corresponds to the nonmembership degree, and the linguistic term value is the parameter $l_1$ in Definition 3. Finally, according to the score function in Definition 5, we obtain the final score of each candidate scheme, as shown in Table 5 and Fig. 5.

The final scores suggest e-fuel to be the best alternative, followed by e-biofuel, with scores of 0.2944 and 0.2671, respectively. According to our proposed MCDM method, the two least attractive options of sustainable fuel production are biofuel with a score of 0.2490 and solar fuel with a score of 0.2481.

The top two ALCF production pathways show that there is agreement among the experts regarding sustainable fuel production technology. The e-fuel and e-biofuel pathways both use electrochemical reduction. This finding contradicts the literature, which places the electrochemical process in development and thus underperforms [50]. It was expected that the second-best production pathway, e-biofuel, would be higher in ranking due to its underlying process of combining AD and electrochemical reduction; however, this is found not to be the case. One possible explanation could be the uncertainty in the possibility of integration of technologies, while another could be a lack of industrial-level concerns at the lowest. This shows that biofuel has a high social reputation, but its environmental implications are not fully comprehended. This will make it difficult for biofuel to become a popular solution in the market when countries are trying to achieve carbon neutrality. By comparison, the scores of the four dimensions of solar fuel and e-biofuel are relatively concentrated, but their scores are all at a low level.

This analysis further highlights the decision-makers’ choice dilemma in identifying the best (or rank order) ALCF production pathway integrating all criteria. Therefore, it is necessary to further synthesize the scores of each criterion dimension to obtain a comprehensive decision result.

Table 7
Proposed q-ROLWPBM approach comparison with other methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Whether complex decision information can be represented</th>
<th>Whether the relationship between indicators is examined</th>
<th>Whether it reflects the independence of indicators in different dimensions</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>The MCDM method based on the IFS [88]</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>/</td>
</tr>
<tr>
<td>The MCDM method based on the complex IFS [89]</td>
<td>No</td>
<td>yes</td>
<td>No</td>
<td>/</td>
</tr>
<tr>
<td>The MCDM method based on the q-ROLWA operator [90]</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Biofuels &gt; e-fuel &gt; Solar fuels &gt; e-bio-fuels</td>
</tr>
<tr>
<td>The MCDM method based on the q-ROLWG operator [90]</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Biofuels &gt; e-fuel &gt; Solar fuels &gt; e-bio-fuels</td>
</tr>
<tr>
<td>The proposed method- q-ROLWPBM</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Biofuels &gt; e-fuel &gt; Solar fuels &gt; e-bio-fuels</td>
</tr>
</tbody>
</table>
and prefer to divert from conventional processes (AD). Thus, our findings (electrochemical reduction) but with caution (photocatalytic reduction) underlying technology suggests that experts prefer novel technologies therefore making lack of interest by experts could be the intermittence in solar energy, -implementation [18]. The third ranked biofuel production pathway suggests that AD can be a viable conversion process either as the main conversion process (this pathway) or in combination (e-biofuel). Finally, the solar-fuel pathway is the least preferred pathway, indicating the shortcomings of the photocatalysis process. One possible reason for this lack of interest by experts could be the intermittence in solar energy, therefore making solar-fuel as a less feasible production pathway, as also pointed out by Falter et al. [58]. Overall, the production pathway underlyling technology suggests that experts prefer novel technologies (electrochemical reduction) but with caution (photocatalytic reduction) and prefer to divert from conventional processes (AD). Thus, our findings provide different perspectives compared to the general literature, which suggests that novel technology is an impediment to upscaling sustainable fuel production (see Neuling and Kaltschmitt [85]).

Regarding feedstock, our analysis reveals that direct conversion of CO₂ to sustainable drop-in fuel (using the e-fuel pathway) is deemed the best option. This preference indicates that experts perceive capturing CO₂ to be more beneficial than using biomass (biofuel and/or e-biofuel). The availability of CO₂ from the electricity, cement, chemical, and steel industries can be ensured [86]. CO₂ is a harmful by-product from these industries that needs to be handled amicably. Biomass, on the other hand, comes in many types and forms (manure, biosolids, agricultural or forestry waste). Each type of biomass requires its own handling requirements before the conversion process [8, 67]. Furthermore, biomass requires extensive establishment of supply chains [81] as opposed to tapping the CO₂ on site for the e-fuel, solar-fuel, and e-biofuel production pathways. This simplifies the operations and improves the production economics.

5. Sensitivity analysis

In this section, we carry out a battery of sensitivity analyses to ascertain the robustness of our findings on ALCF production pathway ranking. To be more specific, we executed three main approaches: 1) vary our models’ initial parameters; 2) change the criteria weights; and 3) use four alternative MCDM methods.

5.1. Initial parameter experiments

To check the reliability of our empirical results, we perform several robustness analyses by varying our three key initial parameters in our proposed method: q (information parameter), s, and t (criteria correlation parameters).

First, the sensitivity analysis is performed by altering q by replacing q = 2 with 3, 5, 10 & 15 and validating the final rankings. 8 Recall that the parameter q represents the complexity of the information environment such that the larger q is, the more complex the information envi-

---

8 Note that here we fixed s=t =1 and only change the value of q.

---

Table A1
Initial List of Criteria.

<table>
<thead>
<tr>
<th>Technical</th>
<th>Economic</th>
<th>Environmental</th>
<th>Social</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology maturity</td>
<td>Capital cost/ investment cost</td>
<td>CO₂ emission</td>
<td>Contribution to economy</td>
</tr>
<tr>
<td>Energetic content</td>
<td>Feedstock transportation and storage cost</td>
<td>Exhaust emission</td>
<td>Public acceptability</td>
</tr>
<tr>
<td>Process efficiency</td>
<td>Cost parity</td>
<td>Land use change impact</td>
<td>Cost of e-fuel compatible vehicle</td>
</tr>
<tr>
<td>Process Scalability</td>
<td>Minimum selling price</td>
<td>Consumptive water use/net water use</td>
<td>Job creation</td>
</tr>
<tr>
<td>Feedstock availability</td>
<td>Feedstock cost</td>
<td>Soil and water pollution</td>
<td>Availability of e-fuel compatible vehicles</td>
</tr>
<tr>
<td>Process yield</td>
<td>Availability of production incentives</td>
<td>Energy use</td>
<td></td>
</tr>
<tr>
<td>Fuel transportation &amp; storage</td>
<td>Infrastructure development</td>
<td>Traceability</td>
<td></td>
</tr>
<tr>
<td>Combustion Efficiency</td>
<td>Subsidies</td>
<td>Market maturity</td>
<td>Carbon footprint</td>
</tr>
<tr>
<td>Freezing temperature</td>
<td>Flash temperature</td>
<td>Density</td>
<td></td>
</tr>
<tr>
<td>Fuel production system complexity</td>
<td>Viscosity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compatibility</td>
<td>Feedstock quality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with current fuel refineries</td>
<td>Fuel production system complexity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology maturity</td>
<td>Economic complexity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energetic content</td>
<td>(C5)</td>
<td>(C6)</td>
<td>(C7)</td>
</tr>
<tr>
<td>Sub-Criteria</td>
<td>Cost/Benefit</td>
<td>e-fuel-CO₂</td>
<td>Solarfuels-CO₂</td>
</tr>
<tr>
<td>Technology maturity (C1)</td>
<td>Benefit</td>
<td>TRL 4</td>
<td>TRL 2</td>
</tr>
<tr>
<td>Energetic content (C2)</td>
<td>Benefit</td>
<td>15.9 MJ/kg</td>
<td>15.9 MJ/kg</td>
</tr>
<tr>
<td>Process efficiency (C3)</td>
<td>Benefit</td>
<td>c.a. 50 %</td>
<td>less than 5 %</td>
</tr>
<tr>
<td>Fuel production system complexity (C4)</td>
<td>Cost</td>
<td>5 (Fair)</td>
<td>7 (Bad)</td>
</tr>
<tr>
<td>Economic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment cost (C6)</td>
<td>Cost</td>
<td>8 (Very Bad)</td>
<td>8 (Very Bad)</td>
</tr>
<tr>
<td>Market maturity (C7)</td>
<td>Benefit</td>
<td>5 (Fair)</td>
<td>3 (Bad)</td>
</tr>
<tr>
<td>Environmental</td>
<td>Net water use (C8)</td>
<td>Cost</td>
<td>–0.6494</td>
</tr>
<tr>
<td>Carbon footprint (C9)</td>
<td>Cost</td>
<td>–0.6464</td>
<td>–0.0022</td>
</tr>
<tr>
<td>Land use change (C10)</td>
<td>Cost</td>
<td>–0.1779</td>
<td>–0.0646</td>
</tr>
<tr>
<td>Social</td>
<td>Contribution to economy (C11)</td>
<td>Benefit 9 (Very good)</td>
<td>2 (Very Bad)</td>
</tr>
<tr>
<td>Public acceptability (C12)</td>
<td>Benefit</td>
<td>8 (Very Good)</td>
<td>5 (Fair)</td>
</tr>
<tr>
<td>Job creation (C13)</td>
<td>Benefit</td>
<td>2 (Very Bad)</td>
<td>5 (Fair)</td>
</tr>
</tbody>
</table>

---

Table A2
Initial information matrix.

<table>
<thead>
<tr>
<th>Sub-Criteria</th>
<th>Cost/Benefit</th>
<th>e-fuel-CO₂</th>
<th>Solarfuels-CO₂</th>
<th>Biofuels-Biomass</th>
<th>eBiofuels-CO₂-Biomass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology maturity (C1)</td>
<td>Benefit</td>
<td>TRL 4</td>
<td>TRL 2</td>
<td>TRL 7</td>
<td>TRL 2</td>
</tr>
<tr>
<td>Energetic content (C2)</td>
<td>Benefit</td>
<td>15.9 MJ/kg</td>
<td>15.9 MJ/kg</td>
<td>0.038 MJ/L</td>
<td>15.9 MJ/kg</td>
</tr>
<tr>
<td>Process efficiency (C3)</td>
<td>Benefit</td>
<td>c.a. 50 %</td>
<td>less than 5 %</td>
<td>c.a. 60 %</td>
<td>c.a. 60 %</td>
</tr>
<tr>
<td>Fuel production system complexity (C4)</td>
<td>Cost</td>
<td>5 (Fair)</td>
<td>7 (Bad)</td>
<td>3 (Good)</td>
<td>5 (Fair)</td>
</tr>
<tr>
<td>Environmental</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net water use (C8)</td>
<td>Cost</td>
<td>–0.6494</td>
<td>–0.9608</td>
<td>0.1788</td>
<td>–1.0311</td>
</tr>
<tr>
<td>Carbon footprint (C9)</td>
<td>Cost</td>
<td>–0.6464</td>
<td>–0.0022</td>
<td>0.0088</td>
<td></td>
</tr>
<tr>
<td>Land use change (C10)</td>
<td>Cost</td>
<td>–0.1779</td>
<td>–0.0646</td>
<td>–0.0022</td>
<td>0.0088</td>
</tr>
<tr>
<td>Social</td>
<td>Contribution to economy (C11)</td>
<td>Benefit 9 (Very good)</td>
<td>2 (Very Bad)</td>
<td>5 (Fair)</td>
<td>3 (Bad)</td>
</tr>
<tr>
<td>Public acceptability (C12)</td>
<td>Benefit</td>
<td>8 (Very Good)</td>
<td>5 (Fair)</td>
<td>2 (Very Bad)</td>
<td>3 (Bad)</td>
</tr>
<tr>
<td>Job creation (C13)</td>
<td>Benefit</td>
<td>2 (Very Bad)</td>
<td>5 (Fair)</td>
<td>9 (Very good)</td>
<td>5 (Fair)</td>
</tr>
</tbody>
</table>
biofuel, while with $s = t = 5$, the ranking order is $e$-fuel $>$ biofuel $>$ e-biofuel $>$ solar-fuel, and when $s = 8$, $t = 3$, the sequencing results also have corresponding changes as biofuel $>$ e-fuel $>$ solar-fuel $>$ e-biofuel.

Therefore, we can infer that the changes in parameters $s$ and $t$ can significantly affect the ranking results of the four production pathways. Our results show that the greater the mutual influence between criteria is, the more significant the change in the sorting results of the four production pathways. Although the idea of attribute segmentation is used to maximise the mutual independence of criteria in different dimensions, the mutual relations between criteria in the same dimension will still affect the ranking results.

### 5.2. Stakeholder preference scenarios

Each stakeholder involved in the ALCF sector has personalised preference characteristics for different ALCF production pathways. The weights represent the decision-maker’s preferences for each evaluation dimension when selecting ALCF production pathways. Therefore, we set different weight vectors to incorporate different types of stakeholder preferences. Table 6 summarises the multi-criteria rankings with different weighting schema.

First, we analyse our findings from technology-inclined decision-makers with a weighting scheme of $w = (0.7,0.1,0.1,0.1)$ corresponding to the technical, economic, social, and environmental dimensions. The first row of Table 6 provides the ranking with this weight vector. We find that biofuel is seen to be the optimal choice, while solar-fuel is the least preferred choice, with decision-makers preferring technology maturity and production efficiency significantly over economic, social, and environmental impacts. Second, we consider another scenario for commercially motivated stakeholders (e.g., businesses) that emphasise economic factors over the remaining aspects aiming to select the most economically viable pathway. Using a weight vector $w = (0.1,0.7,0.1,0.1)$, we find that biofuel is the most preferred, while e-biofuel is the least desirable option. Similarly, for a socially motivated decision-maker, we assume a weight vector of $w = (0.1,0.1,0.1,0.7,0.1)$, in which social factors become the principal decision variables. Under this preference, the optimal production pathway was determined to be $e$-biofuel. Finally, when the preference is highlighted for environmentally motivated factors with a weight vector of $w = (0.1,0.1,0.7,0.1,0.1)$, the e-fuel becomes the optimal selection, while e-biofuel is ranked the lowest.

Our findings are useful in providing guidance for a tailored selection of ALCF production pathway selection as well as communication for any further development of policy and/or regulations in promoting ALCF uptake.

### 5.3. Comparative analysis

Next, we compare our proposed method with four classical MCDM methods based on intuitionistic fuzzy sets (IFS) [88,89] and q-ROFS [90]. Note that the IFS-based MCDM method can only depict the evaluation information with the sum of membership degree (MD) and non-membership degree (NMD) less than or equal to 1 [91], as shown in Fig. 8. Therefore, this class of methods cannot conduct the decision problems under the complex decision information with the sum of MD and NMD being greater than 1. In contrast, the q-ROFS proposed by [36] can handle more complex decision information than IFS, as it has a wider representation space for evaluation information; that is, the q-ROFS can address the complex evaluation information with the sum of MD and NMD or their squares being greater than 1, as shown in Fig. 8. Another key merit of q-ROFS-based MCDM methods lies in their flexibility and ability to adapt to decisions under any information environment by changing parameter $q$ [92,93]. For example, when $q = 1$, the q-ROFS is reduced to the IFS, $q = 2$, and the q-ROFS can be converted to the Pythagorean fuzzy set.

Since the basic theory of the MCDM method based on the q-ROLWA operator, the q-ROLWG operator [90], and the proposed method is q-ROFS, these three methods all deal with complex decision information. As shown in Table 7, the ranking results based on the q-ROLWA and q-ROLWG operators are biofuels $>$ e-fuel $>$ solar-fuels $>$ e-biofuel, which are consistent with the results by our method, but there are significant differences between the proposed method and the methods based on the q-ROLWA and q-ROLWG operators. The methods based on the q-ROLWA and q-ROLWG operators [90] do not examine the correlation between criteria. The correlation between criteria will have a significant impact on the evaluation results. In this paper, the BM operator is introduced to analyse the relationship between indicators, making the evaluation results more consistent with the objective situation. Furthermore, the findings show that when the q-ROLWA and q-ROLWG operators deal with the evaluation problem with a multi-hierarchy structure criterion system, they do not regard criteria of different dimensions as independent of each other, which also affects the unreliability of the results. In contrast, this study introduced the attribute segmentation theory to examine the independent characteristics of different dimensional criteria. Overall, the proposed method takes the

---

**Table A3**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Technical</td>
<td>Benefit</td>
<td>5 (0.8, 0.3)</td>
<td>1 (0.7,0.3)</td>
<td>7 (0.9,0.2)</td>
<td>1 (0.7,0.4)</td>
</tr>
<tr>
<td>(0.19) Technology maturity (0.050)</td>
<td>Benefit</td>
<td>8 (0.9,0.2)</td>
<td>8 (0.7,0.1)</td>
<td>8 (0.9,0.1)</td>
<td>8 (0.7,0.4)</td>
</tr>
<tr>
<td>Energetic content (0.037)</td>
<td>Benefit</td>
<td>5 (0.8,0.2)</td>
<td>2 (0.7,0.2)</td>
<td>7 (0.8,0.3)</td>
<td>7 (0.6,0.4)</td>
</tr>
<tr>
<td>Process efficiency (0.066)</td>
<td>Cost</td>
<td>5 (0.7,0.2)</td>
<td>7 (0.8,0.2)</td>
<td>3 (0.8,0.2)</td>
<td>5 (0.7,0.3)</td>
</tr>
<tr>
<td>Fuel production system complexity</td>
<td>Cost</td>
<td>5 (0.7,0.2)</td>
<td>7 (0.8,0.2)</td>
<td>3 (0.8,0.2)</td>
<td>5 (0.7,0.3)</td>
</tr>
<tr>
<td>(0.037)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economics</td>
<td>Cost</td>
<td>5 (0.8,0.2)</td>
<td>3 (0.7,0.2)</td>
<td>8 (0.8,0.3)</td>
<td>2 (0.7,0.2)</td>
</tr>
<tr>
<td>(0.226) Capital cost/ investment cost</td>
<td>Cost</td>
<td>8 (0.9,0.2)</td>
<td>8 (0.7,0.2)</td>
<td>3 (0.8,0.3)</td>
<td>5 (0.7,0.4)</td>
</tr>
<tr>
<td>(0.090) Production cost/operational</td>
<td>Cost</td>
<td>8 (0.9,0.2)</td>
<td>8 (0.7,0.2)</td>
<td>3 (0.8,0.3)</td>
<td>5 (0.7,0.4)</td>
</tr>
<tr>
<td>(0.075)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environmental (0.313)</td>
<td>Benefit</td>
<td>1 (0.8,0.2)</td>
<td>5 (0.7,0.3)</td>
<td>9 (0.9,0.1)</td>
<td>1 (0.8,0.3)</td>
</tr>
<tr>
<td>Market maturity (0.061)</td>
<td>Benefit</td>
<td>4 (0.8,0.3)</td>
<td>2 (0.7,0.2)</td>
<td>7 (0.9,0.05)</td>
<td>4 (0.8,0.1)</td>
</tr>
<tr>
<td>Net water use (0.112)</td>
<td>Cost</td>
<td>5 (0.8,0.2)</td>
<td>3 (0.8,0.3)</td>
<td>9 (0.9,0.1)</td>
<td>1 (0.8,0.3)</td>
</tr>
<tr>
<td>Carbon footprint (0.099)</td>
<td>Cost</td>
<td>2 (0.7,0.2)</td>
<td>7 (0.8,0.2)</td>
<td>3 (0.7,0.3)</td>
<td>5 (0.7,0.3)</td>
</tr>
<tr>
<td>Land use change (0.102)</td>
<td>Cost</td>
<td>1 (0.8,0.2)</td>
<td>5 (0.7,0.3)</td>
<td>7 (0.9,0.05)</td>
<td>3 (0.7,0.3)</td>
</tr>
<tr>
<td>Social</td>
<td>Benefit</td>
<td>8 (0.8,0.2)</td>
<td>2 (0.7,0.3)</td>
<td>5 (0.8,0.2)</td>
<td>3 (0.7,0.3)</td>
</tr>
<tr>
<td>Contribution to economy (0.096)</td>
<td>Benefit</td>
<td>8 (0.8,0.2)</td>
<td>2 (0.7,0.3)</td>
<td>5 (0.8,0.2)</td>
<td>3 (0.7,0.3)</td>
</tr>
<tr>
<td>(0.271)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public acceptability (0.090)</td>
<td>Benefit</td>
<td>8 (0.9,0.1)</td>
<td>5 (0.8,0.05)</td>
<td>2 (0.8,0.3)</td>
<td>3 (0.8,0.3)</td>
</tr>
<tr>
<td>Job Creation</td>
<td>Benefit</td>
<td>2 (0.8,0.2)</td>
<td>5 (0.8,0.2)</td>
<td>8 (0.8,0.2)</td>
<td>3 (0.7,0.2)</td>
</tr>
<tr>
<td>(0.086)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
multi-layer structure characteristics of the evaluation index system and the mutual influence relationship among the indicators into account. The evaluation process based on our method is more in line with reality, and the corresponding evaluation results are more objective and reasonable.

6. Conclusion

It has been scientifically established that on the path to limit the rise in global temperature by curbing GHG emissions, particularly in the transport sector, alternating low carbon fuels, such as methanol, can play a central role. However, numerous ways to produce low carbon fuel have presented a significant challenge in selecting a particular production pathway to focus upon. Most studies rely on TEA or LCA or standard MCDM models to assess the relative performances of competing production pathways. There is a lack of multi-criteria decision-making frameworks to evaluate ALCF production pathways that reflect data uncertainty due to the early stage of technological readiness, interrelationships among criteria, and stakeholders’ perspectives.

6.1. Theoretical contributions

Our study contributes to this line of research by leveraging experts’ participatory approach in developing a holistic evaluation framework based on technical, economic, environmental, and social dimensions. A hybrid AHP and q-ROLWPBM approach is presented for evaluating four low-carbon drop-in fuel production pathways. The AHP is employed to rate the selected evaluation criteria, while the q-ROLWPBM set handles experts’ rating information and corresponding confidence level. This arrangement approached the imprecision and uncertainty in experts’ evaluation and examined the mutual influence among different impact dimensions to rank order the competing alternatives more accurately. Furthermore, we performed sensitivity analysis to emphasise the robustness of our approach’s generated rankings. Likewise, we compared our approach with similar methods and obtain a ranking similar to that of our approach. However, we argue that the proposed approach is superior to other methods, as it can represent complex information, efficiently examine the relationship between indicators, and reflect the independence of indicators in different dimensions (see Table 7).

6.2. Practical contributions

The empirical results show that stakeholders perceive environmental and economic issues to be more important than social and technical issues. The emphasis on these two categories is credible, as drop-in fuels are seen as environmentally friendly rather than conventional fossil-derived fuels but are also relatively expensive to produce. Although the least emphasis is given to the technical dimension, it shows confidence in the scientific community to invent and enhance novel production pathways. Furthermore, net water use, land use change, carbon footprint, contribution to economy, investment cost, and public acceptability are considered to be the most important factors when assessing drop-in sustainable fuel production pathways. We also find that no one production pathway dominates all 13 evaluation criteria based on the mono-criterion rankings. However, by considering each impact category alone in the mono-criterion ranking, we see a mix of results except for biofuel. Unlike other pathways, biofuel consistently outranks other production pathways against technical and economic dimensions. However, we advise caution on this finding, as biofuel seems not to be the best option for the environmental and social impact categories. The global ranking reveals that electrochemical reduction used in e-fuel and e-biofuel pathways is the best conversion process, followed by AD and the photocatalysis process used in biofuel and solar-fuel production pathways, respectively. One of the main takeaways from this result is that there should be a set of strong policies to increase renewable electricity generation capacity to maximise ALCF production and its overall benefits.

With regard to feedstock, the results reveal that captured CO2 and water are a better option than biomass alone or in combination with CO2 and water. Therefore, to accelerate the development and deployment of low carbon fuels, we urge that both technical and regulatory efforts be made to develop and integrate the CO2 supply chain. By co-locating sustainable fuel production plants, a steady stream of CO2 can be ensured by capturing it from the exhaust gases of fuel or biomass plants [50].

The proposed framework for evaluating and ranking ALCF production pathways can be leveraged for national or regional policy development. The choice of sustainable drop-in fuel production should focus on a country’s local technical competence, feedstock availability, and market conditions. For example, focusing on solar-fuel production in countries with comparatively high solar irradiation or on e-fuel in regions with significant wind power density would result in better overall returns than other production pathways. Our framework is useful in exploring the future challenges facing maximising energy (electricity and fuel) and transportation infrastructure. Primarily, stakeholders should consider the role of existing or planned infrastructure and adaptations may need to take. As highlighted in our study, investment cost has come up as a crucial criterion for evaluation. In this regard, we propose that schemes should be introduced that ease access to financing with insurance from national governments. We envision that this proposition will lead to increased investor confidence for not only establishing new production facilities but also equally scaling up and modernising existing ones for a higher environmental and financial return.

To conclude, our findings provide useful insights for decision-makers when making investment and/or policy decisions regarding sustainable drop-in fuel production pathways. In the former case, decision-makers can use the global ranking to decide upon which ALCF production pathway to invest in, while in the latter case, policies or strategies can be developed for the cross-border trade, fuel subsidization, and long-term plan to phase out fossil oil from the transportation supply chain. Furthermore, we intend to extend our current work by including other production pathways in evaluation, such as solar thermochemical and expanding feedstock base, particularly municipal solid waste. The benefit of this approach will be in reducing pressure on landfill sites and creating value from waste material. Thus, society is driven towards circular economy development. In addition, the proposed approach can be applied in other decision domains, such as industrial production site selection, partner selection in supply chains, and/or product purchasing.

CRediT authorship contribution statement

Zaoli Yang: Conceptualization, Software, Resources, Data curation, Supervision, Writing – review & editing. Salman Ahmad: Conceptualization, Data curation, Supervision, Software, Resources, Writing – review & editing. Andrea Bernardi: Formal analysis, Data curation, Investigation, Supervision. Wen-long Shang: Software, Investigation,
Data curation, Supervision. Jin Xuan: Conceptualization, Data curation, Supervision, Formal analysis, Investigation. Bing Xu: Conceptualization, Funding acquisition, Formal analysis, Supervision, Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All the data used have been included in the manuscript.

Appendix A. Data and criteria

Appendix B. Specific forms and proofs of our theorems

The specific form of Theorem 1

\[ q - ROLWBM^+(a_1, a_2, \ldots, a_n) = \]

\[ \prod_{i,j=1}^{n} (1 - (1 - (1 - a_i^o)^s_i)(1 - (1 - a_j^o)^s_j))^{1 - \frac{1}{s_j + t}} \]

\[ (1 - \prod_{i,j=1}^{n} (1 - (1 - (1 - v_i^q))(1 - (1 - v_j^q)))^{1 - \frac{1}{s_j + t}} ) \]

Acknowledgments

We are grateful for constructive comments and suggestions from several anonymous reviewers and the editor of this journal. We would like to thank Prof Patricia Thornley and all Supergen Bioenergy Hub members for their valuable comments. We also appreciate for those who have participated in our online survey. The authors wish to acknowledge the financial support provided by the UK Supergen Bioenergy Hub and the Department for Transport (Grant number: SGBH FF Feb2019 1).
The specific form of Theorem 2

\[
q - ROLPBM^{\ast}(a_1, a_2, \ldots, a_n) = \left( \prod_{i=1}^{n} \left( 1 - \prod_{j \neq i} \left( 1 - (1 - \nu_j) (1 - \nu_j) \right) \frac{1}{1 - (1 - \nu_j) (1 - \nu_j)} \right) \right)^{\frac{1}{2}}
\]

Proof of Theorem 2

Based on the algorithm in definition 2, we can get:

\[
a_i^t = \left( l_{ij, (0)}^t, \left( u_i^t (1 - (1 - \nu_j)^{\frac{1}{2}}) \right) \right), \quad a_j^t = \left( l_{ij, (0)}^t, \left( u_j^t (1 - (1 - \nu_j)^{\frac{1}{2}}) \right) \right)
\]

According to the number multiplication and product algorithms, we can get:

\[
w_i^t a_i^t = \left( l_{ij, (0)^t}, \left( (1 - u_i^t)^{\frac{1}{2}}, (1 - (1 - \nu_j)^{\frac{1}{2}}) \right) \right), \quad w_j^t a_j^t = \left( l_{ij, (0)^t}, \left( (1 - u_j^t)^{\frac{1}{2}}, (1 - (1 - \nu_j)^{\frac{1}{2}}) \right) \right)
\]

\[
(w_i^t a_i^t)(w_j^t a_j^t) = \left( l_{ij, (0)^t}, \left( (1 - (1 - u_i^t)^{\frac{1}{2}}) (1 - (1 - \nu_j)^{\frac{1}{2}}) \right) \right)
\]

According to mathematical induction, we can get:

\[
\sum_{i,j \in P_k} \left( l_{ij, (0)^t} \right)^{\frac{1}{2}} = \left( 1 - \prod_{j \neq i} \left( 1 - (1 - \nu_j)^{\frac{1}{2}} \right) \left( 1 - \nu_j \right) \right)^{\frac{1}{2}}
\]

then,
According to the number multiplication algorithm, we can get:

\[
\frac{1}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} (w_i \cdot u'_j) (u'_j \cdot v_i) = \left\{ \begin{array}{l}
\frac{l}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} \begin{pmatrix} w_i \cdot u'_j \end{pmatrix} \begin{pmatrix} v_i \cdot u'_j \end{pmatrix} \\
i \neq j
\end{array} \right\} = \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - u_i^o)^{v_i} \right) \left(1 - (1 - u_i^o)^{v_i} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l} \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - v_j^o)^{u_j} \right) \left(1 - (1 - v_j^o)^{u_j} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l}
\right\}
\]

then,

\[
\frac{1}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} (w_i \cdot u'_j) (u'_j \cdot v_i) = \left\{ \begin{array}{l}
\frac{l}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} \begin{pmatrix} w_i \cdot u'_j \end{pmatrix} \begin{pmatrix} v_i \cdot u'_j \end{pmatrix} \\
i \neq j
\end{array} \right\} = \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - u_i^o)^{v_i} \right) \left(1 - (1 - u_i^o)^{v_i} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l} \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - v_j^o)^{u_j} \right) \left(1 - (1 - v_j^o)^{u_j} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l}
\right\}
\]

According to mathematical induction, we can get (see Tables A1-A3):

\[
\sum_{h=1}^{d} \left( \frac{1}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} (w_i \cdot u'_j) (u'_j \cdot v_i) \right)^\frac{1}{l} = \left\{ \begin{array}{l}
\sum_{h=1}^{d} \frac{l}{|P_h|(|P_h| - 1)} \sum_{i,j \in P_h} \begin{pmatrix} w_i \cdot u'_j \end{pmatrix} \begin{pmatrix} v_i \cdot u'_j \end{pmatrix} \\
i \neq j
\end{array} \right\} = \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - u_i^o)^{v_i} \right) \left(1 - (1 - u_i^o)^{v_i} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l} \\
\left\{ \begin{array}{l}
1 - \prod_{i,j \in P_h} \begin{pmatrix} 1 - \left(1 - (1 - v_j^o)^{u_j} \right) \left(1 - (1 - v_j^o)^{u_j} \right) \end{pmatrix} \frac{1}{|P_h|(|P_h| - 1)}
\end{array} \right\}^\frac{1}{l}
\right\}
\]

According to the number multiplication algorithm, we can get:
\[ q - ROLWPBM^d(a_1, a_2, ..., a_n) = \frac{1}{d} \sum_{i,j} \left( \frac{1}{|P_i| - 1} \sum_{i \neq j} \left( w_i \cdot \theta_i \right) \left( w_j \cdot \theta_j \right) \right)^{\frac{1}{d}} \]

\[ = \left( \frac{1}{d} \sum_{i,j} \left( \frac{1}{|P_i| - 1} \sum_{i \neq j} \left( w_i \cdot \theta_i \right) \left( w_j \cdot \theta_j \right) \right)^{\frac{1}{d}} \right)^{\frac{1}{d}} \]

\[ = \left( \prod_{i=1}^{d} \left( 1 - \prod_{i \neq j}^{d} \left( 1 - \left( 1 - (1 - w_i^j) \cdot \theta_i \right) \left( 1 - (1 - w_j^i) \cdot \theta_j \right) \right)^{\frac{1}{d}} \right) \right)^{\frac{1}{d}} \]
The specific form of Theorem 3

\[ q - ROLWPBM' = \frac{1}{d} \sum_{i=1}^{d} \left( \frac{1}{|P_{a}|(|P_{b}| - 1)} \sum_{i \neq j} (w_{ij}a_{ij}'(w_{ij}'a_{ij}')^T) \right) \]

\[ = \left( 1 - \prod_{i \neq j} \left( 1 - \left( 1 - \left( 1 - \left( \frac{1}{d} \sum_{j \neq i} (w_{ij}a_{ij}'(w_{ij}'a_{ij}')^T) \right) \right) \right) \right) \right) \]

References

https://doi.org/10.1039/c3ee4115f.


Further reading