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## Rheology and vibration of fresh concrete

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1 **Rheology and vibration of fresh concrete: predicting the radius of action of**  
2 **poker vibrators from wave propagation**

3  
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13  
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15  
16 **Abstract**

17  
18  
19 The compaction of fresh concrete by an internal poker vibrator has been analysed  
20 using closed-form solutions for the propagation of the shear and compressive  
21 waveforms, assuming that concrete conforms to the Bingham model. In the inner  
22 liquefied zone around the vibrator the flow is due to shear whereas in the outer  
23 unsheared zone propagation is due to compressive waves. The analysis gives a  
24 method of predicting the radial position at which the flow changes, which coincides  
25 with the radius of action of the vibrator. Theory and experiment agree well and  
26 confirm that the peak velocity of the vibration governs its efficacy, with radius of  
27 action increasing with increasing velocity. The radius of action increases with  
28 decreasing yield stress and with increasing plastic viscosity. The work offers the  
29 potential to optimise the design and use of vibrators.

30  
31  
32 **Keywords**

33  
34  
35 Fresh concrete; vibration; Bingham model; rheology

39 **1. Introduction**

40

41 For decades it has been worldwide industrial practice to use vibration to compact  
42 fresh concrete into formwork and around reinforcement, releasing air bubbles and  
43 producing concrete of the highest density, strength and durability [1]. Even with the  
44 increasing utilization of self-compacting concrete, vibrators are still in widespread  
45 use, so it is justifiable to seek improvements in the efficiency of the vibration process.  
46 This paper presents a new analysis of the behaviour of concrete under the action of  
47 immersed internal vibrators which has the potential to deliver those improvements.

48

49 **2. Previous work**

50

51 It has long been known that fresh concrete conforms to the Bingham model [2],  
52 confirmed by the ordinary everyday observation that it can stand unsupported without  
53 flowing under its own weight (as in the slump test). This model can be expressed as:

54 
$$\tau = \tau_0 + \mu\dot{\gamma} \tag{1}$$

55 where concrete can support shear stresses  $\tau < \tau_0$ , the yield stress, without flowing (i.e.  
56 shear rate  $\dot{\gamma} = 0$ ) but flows at higher stresses. In common with all yield stress  
57 materials fresh concrete is a weak solid below the yield stress while above the yield  
58 stress it flows as a liquid with a plastic viscosity  $\mu$ .

59

60 Phenomenologically, vibration appears to remove or overcome the yield stress of  
61 concrete, which then flows under its own weight. The phenomena have been  
62 described empirically and there is an extensive literature on the role of frequency,  
63 amplitude and acceleration of the imposed vibration on its efficacy [1], but in most

64 cases the characteristics of the concrete have taken second place in importance to  
65 those of the vibration. In research reports and practical guidelines workability has  
66 generally been defined in terms of single point tests, which, as has been pointed out  
67 before, are fundamentally incapable of reliably distinguishing different concretes [2].  
68 Tattersall and Baker were the first to attempt to relate the rheology of fresh concrete  
69 to its behaviour under vibration. They used an electromagnetic vibrating table as a  
70 well-characterised source and found that the governing characteristic of the vibration  
71 is its peak velocity [3, 4]. They showed that the fluidity of vibrated concrete, defined  
72 as the reciprocal of its low shear rate viscosity, is proportional to peak vibrational  
73 velocity up to a critical value, above which it remains constant. With fresh concretes  
74 of different rheological characteristics the viscosity of the vibrated concrete is  
75 proportional to the plastic viscosity of the unvibrated concrete [5].

76

77 When an internal poker vibrator is used there is a clearly visible liquefied region near  
78 the vibrator, from which air bubbles are released, while at greater distances the  
79 concrete seems unaffected. The radius of action of the vibrator is a parameter of  
80 considerable practical importance which governs the productivity with which concrete  
81 can be compacted. Many empirical studies on the effects of internal vibrators on fresh  
82 concrete have been reported [6-9] but knowledge of the theory and controlling  
83 mechanisms for the flow around a vibrator is limited. Taylor [9] investigated the  
84 influence of frequency and amplitude on the efficacy of internal vibrators, as shown  
85 by the radius of action within which the vibrator was capable of compacting the  
86 concrete to 2% air content, as determined by gamma ray attenuation in the hardened  
87 concrete. He found that the efficacy is influenced by frequency  $f$  and amplitude  $A$  and  
88 that for a given acceleration ( $\propto f^2 A$ ), a vibrator with high amplitude is more effective

89 than one with low amplitude but higher frequency. This is consistent with the peak  
90 velocity criterion ( $\propto fA$ ) mentioned above, as shown by the following example.  
91 Consider a vibrator of amplitude 0.5 mm and frequency 100 Hz. To maintain a  
92 constant acceleration when the amplitude is doubled to 1 mm the frequency must drop  
93 to 70.7 Hz, but in so doing the velocity increases by a factor of  $\sqrt{2}$  and the vibrator is  
94 seen to be more effective. Similarly, to maintain a constant acceleration when the  
95 amplitude is halved to 0.25 mm the frequency must rise to 141.4 Hz, but in doing so  
96 the velocity is reduced by a factor of  $\sqrt{2}$  and the vibrator is consequently less  
97 effective.

98

99 Asserting that the radius of action is due to attenuation, ACI Committee 309's state-  
100 of-the-art review [1] recommends a formula first presented by Dessoiff in 1937 [10]  
101 for estimating the geometrical energy distribution due to the radial generation of  
102 compressive waves around an internal vibrator:

$$103 \quad u_r = u_o \sqrt{\frac{r_i}{r}} \exp\left[-\frac{\Omega}{2}(r - r_i)\right] \quad (2)$$

104 where  $u_r$  is the radial velocity at radius  $r$ ,  $u_o$  is the velocity of vibration of the vibrator  
105 surface and  $r_i$  its radius.  $\Omega$  is the coefficient of damping, and for concrete of  
106 consistency ranging from flowing to plastic, a value of between 0.04 and 0.08 is  
107 suggested [1]. Dessoiff's formula was originally presented as an approximate  
108 procedure for the study of compact soil, and its application to concrete can be  
109 criticised on the grounds that compressive waves do not propagate through liquids,  
110 and therefore its use would be restricted to the outer region where the concrete is not  
111 liquid, a restriction that is not mentioned by ACI Committee 309. The damping is due  
112 to internal friction between the solid particles (1). If the formula is not applicable to

113 the liquid region surrounding the vibrator a new approach based on shear wave  
114 propagation is needed. Teixeira *et al* [11] presented a preliminary analysis in terms of  
115 the propagation of shear waveforms outward from the surface of the vibrator. The  
116 amplitude of the wave decays with distance and at a critical distance has fallen to a  
117 level that is insufficient to exceed the yield stress. Beyond this distance the concrete is  
118 solid and in this region the motion is controlled by the compressive waveforms. This  
119 critical point corresponds to the radius of action of the vibrator and this paper  
120 develops this alternative analysis of wave propagation in the two regions.

121

122

123 The main objective of this paper is therefore to analyse the radius of action of  
124 vibrators in relation to the rheology of the fresh concrete and the characteristics of the  
125 vibration. A subsidiary objective is to investigate the possibility that the decay of  
126 acceleration in the liquid region is simply a consequence of the shear wave  
127 propagation.

128

### 129 **3. Theory**

130

#### 131 **3.1 Problem definition and research approach**

132

133 In the proposed approach, the vibrational process for a poker vibrator in fresh  
134 concrete is analysed as two distinct cases, namely: (i) the oscillating two-dimensional  
135 incompressible viscous fluid motion around a cylinder in a confining volume of  
136 material, i.e. a shear waveform, and (ii) the acoustic motion of a cylindrical travelling

137 wave with dissipation of energy, i.e. a compressive waveform. The theoretical  
138 analysis is also investigated experimentally and a prediction approach is developed.  
139  
140 The construction of a poker-type internal vibrator for concrete is shown schematically  
141 in Figure 1. An eccentric mass inside a fixed cylindrical casing of radius  $r_i$  rotates  
142 about the point  $O$  and makes the casing oscillate. The entire assembly moves in such a  
143 way that a point  $P$  on the surface of the casing describes a circular path of a radius  
144 that is small compared to  $r_i$  but the casing itself does not rotate. During operation, at  
145 any instant  $t$ , point  $P$  imparts to the surrounding medium a compressive force in the  
146 direction  $\phi$ , while points  $P'$  and  $P''$ , at angles  $\phi \pm \pi/2$ , impart a shear force in the  
147 directions  $\pm \phi$ . Since a compressive waveform cannot propagate through a liquid  
148 medium only the shear excitation needs to be considered.

149

### 150 **3.2 Shear waveform**

151

152 Alexander [12] studied the mechanics of motion of fresh concrete during vibration  
153 using a mechanical driving point impedance technique. He found different mechanical  
154 impedance curves depending on whether the dynamic stress applied is above or below  
155 a threshold level, i.e. the yield stress, although he did not call it this. Fresh concrete  
156 below the yield stress possesses the mass, damping and stiffness characteristics of a  
157 solid, while above the yield stress it is a liquid. Various combinations of force and  
158 frequency were found to cause liquefaction, which was associated with a  
159 simultaneous sharp drop in impedance. His experimental results showed that  
160 concretes of normal consistencies behave like a fluid during vibration, as confirmed

161 by the results of Tattersall and Baker [3, 4] and Banfill et al [5]. Therefore the use of  
 162 hydrodynamic theory to analyse the liquefaction process is justified.

163

164 Chen et al [13] presented an analytical and experimental study of a cylindrical rod  
 165 vibrating in a viscous liquid enclosed by a rigid concentric cylindrical shell. Figure 2  
 166 shows the coordinate system they used and the vibrator casing is represented by an  
 167 infinitely long cylinder of radius  $r_i$  oscillating with velocities:

$$168 \quad u_r = u_0 \cos \theta (\cos \omega t + i \sin \omega t) \quad \text{and} \quad u_\theta = -u_0 \sin \theta (\cos \omega t + i \sin \omega t) \quad (3)$$

169 where  $u_r$  and  $u_\theta$  are the velocity components in the radial and tangential directions at  
 170 an arbitrary point on the casing which subtends an angle  $\theta$  to the coordinate axis,

171  $i^2 = -1$ ,  $\omega = 2\pi f$  is the angular velocity,  $u_0$  is the peak velocity and  $f$  is the

172 frequency. Where the amplitude of oscillation of the source is small compared to its  
 173 dimensions, the equations for the conservation of mass and momentum may be

174 linearised [14] as:

$$175 \quad \nabla^4 \psi - \frac{1}{\nu} \frac{\partial}{\partial t} \nabla^2 \psi = 0 \quad (4)$$

176 where  $\psi$  is the stream function,  $\nabla^2$  is the Laplacian operator and  $\nu$  is the kinematic

177 viscosity of the fluid. This assumption is reasonable for most internal vibrators, for

178 which the amplitude is less than 1 mm and  $r_i$  is typically 25 mm. The velocity

179 components for the fluid are given by:

$$180 \quad u_r = -\frac{\partial \psi}{r \partial \theta} \quad \text{and} \quad u_\theta = \frac{\partial \psi}{\partial r} \quad (5)$$

181 giving the solution of equation (4) as:

$$182 \quad \psi = u_o \left[ A \left( \frac{r_i^2}{r} \right) + Br + Cr_i I_1(kt) + Dr_i K_1(kr) \right] \sin \theta \exp(i\omega t) \quad (6)$$



183 where

$$184 \quad k = \sqrt{i \frac{\omega}{\nu}}.$$

185  $A, B, C, D$  are arbitrary constants that can be determined as [13]:

$$186 \quad A = \left\{ -\alpha^2 [I_0(\alpha)K_0(\beta) - I_0(\beta)K_0(\alpha)] + 2\alpha [I_1(\alpha)K_0(\beta) + I_0(\beta)K_1(\alpha)] \right\}$$
$$187 \quad - 2\alpha\delta [I_0(\alpha)K_1(\beta) + I_1(\beta)K_0(\alpha)] + 4\delta [I_1(\alpha)K_1(\beta) - I_1(\beta)K_1(\alpha)] \} / \Delta \quad (7)$$

$$188 \quad B = \{ 2\alpha\delta [I_1(\alpha)K_0(\beta) - I_0(\beta)K_1(\beta)] + \alpha^2 \delta^2 [I_0(\alpha)K_0(\beta) - I_0(\beta)K_0(\alpha)]$$
$$189 \quad - 2\alpha\delta^2 [I_1(\alpha)K_0(\beta) + I_0(\beta)K_1(\alpha)] \} / \Delta \quad (8)$$

$$190 \quad C = \left\{ -2\alpha K_0(\beta) - 4\delta K_1(\beta) + \delta^2 [2\alpha K_0(\alpha) + 4K_1(\alpha)] \right\} / \Delta \quad (9)$$

$$191 \quad D = \left\{ -2\alpha I_0(\beta) + 4\delta I_1(\beta) + \delta^2 [2\alpha I_0(\alpha) - 4I_1(\alpha)] \right\} / \Delta \quad (10)$$

192 where

$$193 \quad \alpha = kr_i$$

$$194 \quad \beta = kr_o$$

$$195 \quad \delta = r_i/r_o$$

196 and

$$197 \quad \Delta = \alpha^2 (1 - \delta^2) [I_0(\alpha)K_0(\beta) - I_0(\beta)K_0(\alpha)]$$
$$198 \quad + 2\alpha\delta [I_0(\alpha)K_1(\beta) - I_1(\beta)K_0(\alpha) + I_1(\beta)K_0(\alpha) - I_0(\beta)K_1(\beta)]$$
$$199 \quad + 2\alpha\delta^2 [I_0(\beta)K_1(\alpha) - I_0(\alpha)K_1(\alpha) + I_1(\alpha)K_0(\beta) - I_1(\alpha)K_0(\alpha)]. \quad (11)$$

200  $I_0$  and  $I_1$  are modified Bessel functions of the first kind and  $K_0$  and  $K_1$  are modified

201 Bessel functions of the second kind.

202

203 Equations (5) and (6) can be used to calculate the velocity components in the radial  
204 and tangential directions as a function of distance from the source and this can be  
205 used to predict the decay of vibration within the inner flow region.

206

### 207 **3.3 Compressive waveform**

208

209 Beyond the critical distance where the amplitude of the oscillatory shear has  
210 decreased to the point where the shear stress is less than the yield stress the concrete is  
211 unsheared. In this outer region where the effects of vibration are not sufficient to  
212 liquefy the Bingham material, the principles of hydrodynamics are no longer  
213 applicable. Here fresh concrete behaves as an elastic solid and instead structural  
214 vibration theory can be used to describe the motion.

215

216 A complete description of the compressive wave motion in a Bingham material at  
217 stresses below the yield stress is not available and a simplified first order equation of  
218 motion is adopted in this analysis. Assuming that a cylindrical wave spreads outwards  
219 from the radial position of the interface between liquid and solid zones,  $r_{ls}$ , the  
220 amplitude depends only on the radial distance  $r$  and the wave equation in cylindrical  
221 coordinates for this case is [15]:

$$222 \quad \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) = \frac{1}{c^2} \frac{\partial^2 u_r}{\partial t^2} \quad (12)$$

223 where  $u_r$  is the particle velocity component in the radial direction,  $c$  is the velocity of  
224 propagation of compressive waves in the material and  $t$  is time.

225

226 If  $r_{ls}$  is small compared to the wavelength the particle velocity component in the radial  
227 direction at large distances  $r$  is given by [15]:

$$228 \quad u_r = u_{ls} \pi r_{ls} \sqrt{\frac{f}{cr}} \exp \left[ i \frac{\omega}{c} (r - ct) - i \left( \frac{\pi}{4} \right) \right] \quad (13)$$

229 where  $u_{ls}$  is the velocity of oscillation at the interface between liquid and solid zones.

230 The assumption that  $r_{ls}$  is small compared to the wavelength is reasonable because the  
231 velocity of wave propagation in fresh concrete is approximately 500 m/s and the  
232 wavelength at a typical vibrator frequency of 200 Hz is therefore 2.5 m, which is  
233 sufficiently greater than the typically observed radius of action of an internal vibrator  
234 of about 0.2 m. Thus equation (13) can be used to calculate the velocity distribution as  
235 a function of distance from the source and to predict the decay of vibration outside the  
236 liquid region where the Bingham materials behaves as a solid.

237

238 Since the vibrational velocity is of interest the ratio of the velocity at any point  $r$  to  
239 that at the interface between solid and liquid is given by:

$$240 \quad \frac{u_r}{u_{ls}} = \pi r_{ls} \sqrt{\frac{f}{cr}} \quad (14)$$

241 It should be noted that the radial position of the interface is not known *a priori* and  
242 therefore the calculations presented here are based on reference values obtained  
243 experimentally for  $u$  and  $r$  that were well inside the solid region beyond the interface.

244 Equation (14) describes the motion in the solid region and any value of  $u_{ls}$  can be used  
245 to generate a curve of  $u_r$  as a function of distance.

246

### 247 **3.4 Radius of action**

248

249 By definition, the radius of action of the vibrator is the radial position of the interface  
 250 between the liquid and solid regions  $r_{ls}$ . Referring to Figure 3, at all radii  $r$  where  $r_i <$   
 251  $r < r_{ls}$  the concrete is fluidified and the radius of action defines the size of the fully  
 252 compacted region. Since there can be no consolidation in the solid region the radius of  
 253 action cannot be larger than the position of the interface between the two zones, but in  
 254 practice it may appear somewhat smaller if the shear waveform is decaying only  
 255 slowly as it approaches the interface. Based on the preceding analysis of the shear and  
 256 compressive waveforms, it is expected that a radial distribution of velocity will show  
 257 two zones. The velocity will decrease relatively rapidly with increasing radius through  
 258 the liquid zone as far as the interface, beyond which it will decrease more slowly with  
 259 radius into the solid zone. In principle, the point where the two curves cross coincides  
 260 with the interface between liquid and solid regions and defines the radius of action.

261

262 In the liquefied zone the concrete is confined between two concentric cylinders (the  
 263 vibrator and the unsheared concrete) so the shear stress at radius  $r$  decreases from a  
 264 maximum value  $\tau_w$  at the surface of the vibrator, radius  $r_i$ , to the yield stress  $\tau_0$  at the  
 265 interface between solid and liquid. This is the radius of action and is given by:

$$266 \quad r_{ls} = \sqrt{\frac{\tau_w}{\tau_0} r_i^2} \quad (15)$$

267 The shear stress at the surface of the vibrator  $\tau_w$  is given by the Bingham model  
 268 (equation 1):

$$269 \quad \tau_w = \tau_0 + \mu \dot{\gamma}_w \quad (16)$$

270 where  $\tau_0$  and  $\mu$  are the yield stress and plastic viscosity of the concrete, respectively,

271 and the shear rate at the surface of the vibrator  $\dot{\gamma}_w$  is given by:

272 
$$\dot{\gamma}_w = r \frac{\partial}{\partial r} \left( \frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \quad (17)$$

273 where  $u_r$  and  $u_\theta$  may be calculated using equations (4) and (6). With this analysis it  
274 becomes possible to predict the radius of action from a knowledge of the  
275 characteristics of the vibrator and the rheology of the concrete.

276

#### 277 **4. Experimental work**

278

279 The aim of the experimental work was to investigate the applicability of the  
280 prediction equations to the practical situation of an internal vibrator immersed in fresh  
281 concrete and to identify the liquid and solid zones and the radius of action.

282

283 All experimental work was carried out using an electrically driven vibrator  
284 (Rotopoka, Fyne Machinery and Engineering Ltd, London) of 28 mm external  
285 diameter. Vibrational measurements used piezoelectric accelerometers (Bruel & Kjaer  
286 Type 4344), calibrated with a vibration calibrator (Bruel & Kjaer Type 4294), driven  
287 by charge amplifiers (Bruel & Kjaer Type 2635) and analysed with a dual channel  
288 frequency analyser (Bruel & Kjaer Type 2032). The acceleration levels in the radial,  
289 tangential and axial directions were measured at different positions along the vibrator,  
290 as well as the magnitude and phase difference between the radial and tangential  
291 acceleration levels. The accelerometer was attached to the vibrator with a 20x20x40  
292 mm aluminium block held in place by a circular screw clip. In all tests the vibrator  
293 was fully immersed in the fresh concrete sample in order to prevent overheating, as  
294 recommended by the manufacturer, and the vibrator and its attached accelerometer  
295 were removed from the concrete before it had a chance to set and thoroughly cleaned.

296

297 Measurements in fresh concrete were carried out in the apparatus shown  
298 schematically in Figure 4. Accelerometers capable of measuring the acceleration in  
299 radial, tangential and axial directions were immersed at 25 mm increments of distance  
300 from the vibrator. Two containers were used: (1) a steel cylinder 640 mm internal  
301 diameter and 400 mm high, closed at the bottom and (2) a cuboidal timber mould  
302 1500x1500 mm and 500 mm high. Container 1 was a compromise between the need  
303 to be larger in diameter than the anticipated size of the zone of liquefaction and the  
304 capacity of the concrete mixer available in the laboratory. Container 2 was much  
305 larger so as to avoid any possible interference of the walls of the mould with wave  
306 propagation. In each case the vibrator was held vertically in the centre of the container  
307 by a frame.

308

309 Two ordinary concretes were used in the tests and one further concrete was used for  
310 predictions of the radius of action. Concrete A was prepared in a 0.2 m<sup>3</sup> laboratory  
311 pan mixer and was used in the smaller container 1. Concrete B was obtained from a  
312 ready-mixed concrete supplier and was used in the much larger container 2. Concrete  
313 C was prepared in a 0.2 m<sup>3</sup> laboratory pan mixer with the sole purpose of providing  
314 the rheological data upon which the predictions of radius of action could be made for  
315 comparison with Taylor's results [9]. All concretes used aggregate of maximum  
316 particle size 20 mm but unfortunately details of the mixture proportions have been  
317 lost. The concretes were characterised by the slump test and the two-point workability  
318 test, using the apparatus described by Domone *et al* [16]. Density was determined  
319 according to BS EN 12350-6 [17]. Velocity of sound was determined for each  
320 concrete using a time of flight measurement. Transient plane wave impulses were  
321 generated by a frequency analyser (dual channel Bruel & Kjaer Type 2032) and

322 imparted to the fresh concrete by a small shaker / vibrator (LDS Type 406) driven by  
323 a power amplifier (Bruel & Kjaer Type 2706) at levels too low to cause liquefaction  
324 and detected by an accelerometer (Bruel & Kjaer Type 4500) connected to a storage  
325 oscilloscope (Gould type 1421).

326

## 327 **5. Results**

### 328 **5.1 Characterisation of concrete**

329

330 Table 1 summarises the properties of the experimental concretes. Concretes A and B  
331 were similar, though not identical, and while of a fairly soft consistency they are  
332 representative of concretes that would require vibratory compaction in practice. The  
333 lower slump of concrete B is consistent with its higher yield stress but the plastic  
334 viscosities were significantly different, as a result of the different constituent materials  
335 [2]. The velocity of sound in the fresh concrete is consistent with values reported by  
336 other authors who have used shear wave or pulse propagation techniques to monitor  
337 setting processes [18]. Concrete C was chosen to be similar to that used in Taylor's  
338 investigations [9].

339

### 340 **5.2 Characterisation of the vibrator**

341

342 Tested in free air, the accelerations of the vibrator in the radial and tangential  
343 directions were identical, with a phase angle of  $90^\circ$ , confirming that the vibrator  
344 performs an oscillatory motion in a circular path. The measured frequency was 246  
345 Hz and the acceleration was  $1122 \text{ m/s}^2$  RMS, providing a peak velocity of oscillation

346  $u_o = 1.03$  m/s. The acceleration in the axial direction was negligibly small and can be  
347 ignored in comparison to that in the other directions.

348

### 349 **5.3 Propagation of vibration**

350

351 Figure 5 shows the results for concrete 1 in container A. The symbols represent the  
352 measured data and are the average of 10 tests, while the lines show the predictions  
353 from the shear and compressive waveform equations. In the liquid zone the radial and  
354 tangential velocity components for the shear wave (equation (5)) are almost identical.  
355 Only the radial velocity is available for the compressive waveform (equation (13)).

356 Figure 5 also shows a curve plotted using Dessoff's equation [1, 10] and the predicted  
357 value for the radius of action, calculated from equation (15). The radius of action was  
358 also determined visually from a cross-section cut through the concrete after it had  
359 been allowed to harden and found to be approximately 200 mm, in good agreement  
360 with the predicted value.

361

362 Figure 6 presents the results for concrete 2 in container B, where again the symbols  
363 represent the measured data and are the mean of three tests, while the lines show the  
364 predictions.

365

### 366 **5.4 Radius of action**

367

368 Figure 7 shows a comparison between the radius of action results determined  
369 experimentally by Taylor [9] and those obtained from the prediction method  
370 introduced in this paper. Taylor used concrete of very low workability (6 mm slump)



371 but gave no other information on the rheological properties. The prediction values  
372 therefore use the properties determined for concrete C (table 1).

373

## 374 **6. Discussion**

375

376 The experimental results (figure 5) show a rapid decay in velocity from the surface  
377 value of 1.03 m/s as the distance from the vibrator increased. The experimental  
378 velocity distribution in the liquid region agrees well with the prediction from the shear  
379 wave equation as it drops towards the prediction from the compressive wave equation.  
380 At a radius of about 0.2 m the shear and compressive curves cross and the  
381 experimental points start to follow the upper compressive curve. The excellent  
382 agreement between the simple theoretical model and experimental data in the region  
383 near the vibrator confirms that concrete behaves as a liquid in this region. Further  
384 from the vibrator, outside the liquid region, the decline of the measured velocity is  
385 significantly reduced and is in good agreement with the compressive equation,  
386 confirming that the concrete behaves as a solid in this region.

387

388 Figure 5 also shows the Dessoiff curve (equation (2)), which considerably over-  
389 estimates the experimental velocity and is unable to account for the rapid decay in  
390 velocity near the vibrator. This confirms that it is unsuitable for the liquid region.  
391 However, the shape of the curve is very similar to that of equation (14) for the solid  
392 region but displaced to velocities which are nearly 100-fold higher, which confirms  
393 that Dessoiff's original formula applies to solid materials.

394

395 In figure 5 the two curves predicting the shear and compressive waveform velocity  
396 distributions intersect at about 200 mm. The predicted value for the radius of action  
397 (equation (15)) is 209 mm and the value determined experimentally by visual  
398 inspection of the compaction visible in a radially cut section through the hardened  
399 concrete is  $200 \pm 10$  mm. Clearly the position of the interface between liquid and  
400 solid may be represented by the intersection of the curves and it follows that equation  
401 (15) may be used to predict the radius of action.

402

403 The results with the large mould (container B) shown in figure 6 reinforce the  
404 previous experiments in the cylinder (container A) but are somewhat less clearly  
405 defined, perhaps due to inhomogeneities in the larger volume of concrete used in this  
406 test. There is again good agreement between experimental and predicted velocity for  
407 the shear waveform within the liquid zone and between experimental and predicted  
408 compressive wave velocity in the solid zone towards the extremity of the mould but  
409 the transition between the curves is less clearly defined by the experimental points.  
410 Equation (15) predicts the radius of action to be 231 mm in this case, whereas the  
411 curves intersect at about 300 mm. Again the Dessoiff formula considerably over-  
412 estimates the velocities.

413

414 The effect of velocity on the radius of action, both as measured by Taylor and  
415 predicted by equation (15), is shown in figure 7. Taylor's experiments were  
416 performed in wall-shaped moulds 1200 mm long by 200 mm wide and 600 mm high  
417 with the vibrator held vertically on the centre line 300 mm from one end.  
418 Consequently the results are very scattered, probably due to internal reflections from  
419 the mould surfaces and the possibility of assisted propagation along the wall.

420 Additionally, Taylor's concrete had unknown rheology. While the yield stress is  
421 correct for a slump of 5 mm, it is impossible to confirm the plastic viscosity. The fact  
422 that the experimental points are mostly above the prediction curve suggests that the  
423 plastic viscosity of his concrete may be higher than the 150 Pa s assumed in the  
424 prediction. This is quite possible since Taylor describes his concrete as very stiff. It  
425 should also be pointed out that Taylor's data in Figure 7 is duplicated: for each value  
426 of velocity there is one radius of action from the visual inspection and one from the  
427 gamma ray densitometer measurements, and in most cases the former is lower than  
428 the latter. The predicted values are given for the corresponding peak velocities,  
429 calculated from Taylor's data.

430

431 Despite these reservations, the broad trend is a clear increase in the radius of action  
432 with increasing peak velocity, as predicted. It confirms Tattersall and Baker's findings  
433 that the peak velocity is the most important characteristic of the vibration. Moreover,  
434 Taylor's experimental observation that for a given acceleration a vibrator with large  
435 amplitude is likely to perform better than one with lower amplitude and higher  
436 frequency is confirmed by the predictions. For example, a vibrator of 30 mm radius  
437 giving an acceleration of  $395 \text{ m/s}^2$  has a radius of action of 273 mm if operated at 200  
438 Hz and 0.25 mm amplitude, compared to a radius of action of 385 mm if operated at  
439 100 Hz and 1.0 mm amplitude.

440

## 441 **7. Implications for concrete practice**

442

443 The prediction equations for the radius of action of an immersed poker vibrator in a  
444 given situation require information on both the concrete properties – yield stress,

445 plastic viscosity and density – and the properties of the poker – diameter, frequency,  
446 amplitude – as well as the size of the container. The complexity of these seven  
447 variables makes it difficult to answer questions like “What is the radius of action in  
448 this situation?” or its converse “What conditions are needed to achieve a given radius  
449 of action?” and “What concrete should be used for a particular vibrator and size of  
450 container?”, and therefore a small computer program (POKER) was written in C++.  
451 This requests the user to enter values for yield stress, plastic viscosity, density, poker  
452 diameter, frequency and amplitude, and container size and gives the radius of action.  
453 The user interface offers a range of preset values for each variable, but these can be  
454 over-written with user-selected values if required. The “container size” box offers a  
455 “free field” value to deal with the situation where the mould is effectively of infinite  
456 size. Radius of action is then calculated using equation (15).

457

458 Table 2 shows the results of a parametric survey of the effect of each variable on the  
459 predicted radius of action of the vibrator, in the form of a  $2^7$  factorial design using two  
460 levels of each variable (one high and one low). The low and high values are: (i) yield  
461 stress 250 and 3000 Pa, (ii) plastic viscosity 25 and 200 Pa s, (iii) density 1800 and  
462 2600 kg/m<sup>3</sup>, (iv) poker diameter 20 and 80 mm, (v) frequency 50 and 300 Hz, (vi)  
463 amplitude 0.5 and 1.0 mm, and (vii) container size 0.5 m and free field. These values  
464 represent the extremes that might be encountered in practice. Comparing rows 1-64  
465 with 65-128 shows that container size has an insignificant effect on the radius of  
466 action (i.e. less than 0.01 m) between 0.5 m and free field conditions, except for four  
467 combinations at low plastic viscosity (compare row 61 with 125 and row 62 with  
468 126). Comparing successive groups of four rows, e.g. rows 1-4 and 5-8, shows that  
469 concrete density has an negligible effect on the radius of action (i.e. some differences

470 of 0.01 m), except for four combinations at low plastic viscosity (compare row 57  
471 with 61 and row 58 with 62). All the other variables have a strong effect: radius of  
472 action decreases with increasing yield stress but increases with increasing plastic  
473 viscosity (except for eight combinations at high density (compare row 29 with row 31,  
474 row 30 with 32, row 61 with 63, and row 62 with 64). Radius of action increases with  
475 increasing vibrator diameter, increasing frequency and increasing amplitude, although  
476 in some cases the increase is small (e.g. compare row 2 with row 34 (amplitude) and  
477 with row 82 (frequency)).

478

479 The principal effects identified in table 2 are amplified graphically, with intermediate  
480 values to demonstrate the trends, in figures 8, 9 and 10. Figure 8 shows the effect of  
481 poker diameter and frequency on the radius of action at a moderate yield stress of  
482 1500 Pa with plastic viscosity from 25 to 250 Pa s. Figure 9 shows the same at a  
483 moderate plastic viscosity of 100 Pa s with yield stress from 250 to 2500 Pa. These  
484 two graphs show the opposing effects of yield stress and plastic viscosity, which is  
485 shown more clearly in figure 10, which takes points at the approximate centre of the  
486 grids in figures 8 and 9. One point is omitted from figure 8 because the calculation  
487 became unstable. Points at high radius of action may be less certain because of the  
488 assumption that the radius at the interface is small compared to the wavelength.

489

490 The importance of the rheology of the fresh concrete being vibrated has not  
491 previously been quantified, although ordinary practical observation shows that  
492 workability is important. Two important issues emerge from figure 10. The first is  
493 that yield stress and plastic viscosity have opposite effects on the radius of action of a  
494 given vibrator. This is a further reason for using two-point tests to characterise the

495 concrete: a single point measurement (slump, flow, etc), no matter how precise and  
496 sophisticated, cannot provide the necessary minimum of information, since an infinite  
497 number of combinations of yield stress and plastic viscosity can give the same single  
498 point result [2]. The second issue is that the combination of low yield stress and high  
499 plastic viscosity that gives the maximum radius of action (figure 10) is the same  
500 combination that is needed to ensure that concrete is self-compacting [19].

501

502 This work has not studied the rate of compaction. Since the viscosity of the vibrated  
503 concrete is proportional to the plastic viscosity of the unvibrated concrete [5] the flow  
504 and release of air bubbles during compaction is slower with higher plastic viscosities.  
505 However, the results presented here show that a high radius of action requires a high  
506 plastic viscosity so the productivity in practice is a compromise between the two  
507 requirements. A low plastic viscosity permits rapid compaction but the small radius of  
508 action requires the vibrator to be inserted many times at close spacing in the form,  
509 while a high plastic viscosity requires the vibrator to be held in one place for longer  
510 but without so many insertions.

511

## 512 **8. Conclusions**

513

514 An analysis of the compaction of fresh concrete by an internal poker vibrator has been  
515 developed using closed-form solutions for the shear and compressive waveforms  
516 based on the assumption that concrete conforms to the Bingham model. Theory and  
517 experiment agree well.

518

519 There are two distinct regions around the vibrating source. Near the vibrator the flow  
520 is controlled by the shear waveform and hydrodynamic theory may be used in the  
521 analysis, whereas outside this region the material is solid and the motion is governed  
522 by the compressive waveforms which can be solved by structural vibration theory.

523

524 The rapid decay of energy near the internal vibrator is due to the liquefaction and flow  
525 of the Bingham material and Dessoiff's equation for estimating the radial distribution  
526 of vibrational energy is restricted to the case of the solid material outside the liquefied  
527 zone and cannot be used to predict the size of that zone.

528

529 The analysis developed in this study gives a method of predicting the radial position  
530 of the interface between the liquid and solid regions, i.e. the radius of action of the  
531 vibrator, as a function of the characteristics of the vibration and the rheology of the  
532 concrete. The radius of action increases with increasing plastic viscosity but decreases  
533 with increasing yield stress, with the optimum combination predicted to be a low yield  
534 stress with a high plastic viscosity. The work confirms the importance of velocity as  
535 the most important characteristic of the vibration governing efficacy. This work offers  
536 the potential to optimise the design and use of internal vibrators to achieve the most  
537 efficient and productive compaction of a concrete during production of constructional  
538 elements.

539

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541

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602 **Figure captions**

603

604 1. Construction of a poker type vibrator.

605 2. The coordinate system used in the equations.

606 3. Definition of the radius of action of a vibrator.

607 4. Experimental set-up for vibration tests.

608 5. Radial velocity results for concrete A in container 1.

609 6. Radial velocity results for concrete B in container 2.

610 7. Effect of peak velocity on the radius of action.

611 8. Effect of poker diameter and frequency on the calculated radius of action in

612 concrete of yield stress 1500 Pa, vibration amplitude 1.0 mm, concrete density 2400

613  $\text{kg/m}^3$ . Plastic viscosity (a) 25 Pa s, (b) 100 Pa s, (c) 175 Pa s, (d) 250 Pa s.

614 9. Effect of poker diameter and frequency on the calculated radius of action in

615 concrete of plastic viscosity 100 Pa s, vibration amplitude 1.0 mm, concrete density

616  $2400 \text{ kg/m}^3$ . Yield stress (a) 250 Pa, (b) 1000 Pa, (c) 1750 Pa, (d) 2500 Pa.

617 10. Effect of yield stress and plastic viscosity on the calculated radius of action of a

618 50 mm diameter poker operating at frequency 75 Hz, amplitude 1.0 mm, in concrete

619 of density  $2400 \text{ kg/m}^3$ .

620

621

622 **Tables**

623

624 **Table 1. Properties of the concrete mixtures**

625

Concrete	Slump mm	Yield stress Pa	Plastic viscosity Pa.s	Plastic density kg/m <sup>3</sup>	Sound velocity m/s
A	180	570	15	2300	445
B	150	620	26	2200	515
C	5	2200	150	2200	-

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630

**Table 2. Parametric survey of the effect of concrete properties and vibrator characteristics on the calculated radius of action of a vibrating poker.**

Row No.	Yield stress Pa	Plastic viscosity Pa.s	Density kg/m <sup>3</sup>	Poker diameter mm	Frequency Hz	Amplitude mm	Container size* m	Predicted radius of action m
1	250	25	1800	20	50	0.5	0.5	0.13
2	3000	25	1800	20	50	0.5	0.5	0.04
3	250	200	1800	20	50	0.5	0.5	0.38
4	3000	200	1800	20	50	0.5	0.5	0.11
5	250	25	2600	20	50	0.5	0.5	0.13
6	3000	25	2600	20	50	0.5	0.5	0.04
7	250	200	2600	20	50	0.5	0.5	0.38
8	3000	200	2600	20	50	0.5	0.5	0.11
9	250	25	1800	80	50	0.5	0.5	0.27
10	3000	25	1800	80	50	0.5	0.5	0.09
11	250	200	1800	80	50	0.5	0.5	0.76
12	3000	200	1800	80	50	0.5	0.5	0.22
13	250	25	2600	80	50	0.5	0.5	0.27
14	3000	25	2600	80	50	0.5	0.5	0.09
15	250	200	2600	80	50	0.5	0.5	0.76
16	3000	200	2600	80	50	0.5	0.5	0.22
17	250	25	1800	20	300	0.5	0.5	0.33
18	3000	25	1800	20	300	0.5	0.5	0.1
19	250	200	1800	20	300	0.5	0.5	0.93
20	3000	200	1800	20	300	0.5	0.5	0.27
21	250	25	2600	20	300	0.5	0.5	0.33
22	3000	25	2600	20	300	0.5	0.5	0.1
23	250	200	2600	20	300	0.5	0.5	0.93
24	3000	200	2600	20	300	0.5	0.5	0.27
25	250	25	1800	80	300	0.5	0.5	0.66
26	3000	25	1800	80	300	0.5	0.5	0.19
27	250	200	1800	80	300	0.5	0.5	1.86
28	3000	200	1800	80	300	0.5	0.5	0.54
29	250	25	2600	80	300	0.5	0.5	2.7
30	3000	25	2600	80	300	0.5	0.5	0.78
31	250	200	2600	80	300	0.5	0.5	1.86
32	3000	200	2600	80	300	0.5	0.5	0.54
33	250	25	1800	20	50	1.0	0.5	0.19
34	3000	25	1800	20	50	1.0	0.5	0.06
35	250	200	1800	20	50	1.0	0.5	0.54
36	3000	200	1800	20	50	1.0	0.5	0.16
37	250	25	2600	20	50	1.0	0.5	0.19
38	3000	25	2600	20	50	1.0	0.5	0.06
39	250	200	2600	20	50	1.0	0.5	0.54
40	3000	200	2600	20	50	1.0	0.5	0.16
41	250	25	1800	80	50	1.0	0.5	0.38
42	3000	25	1800	80	50	1.0	0.5	0.12
43	250	200	1800	80	50	1.0	0.5	1.07
44	3000	200	1800	80	50	1.0	0.5	0.31
45	250	25	2600	80	50	1.0	0.5	0.38
46	3000	25	2600	80	50	1.0	0.5	0.12
47	250	200	2600	80	50	1.0	0.5	1.07

48	3000	200	2600	80	50	1.0	0.5	0.31
49	250	25	1800	20	300	1.0	0.5	0.46
50	3000	25	1800	20	300	1.0	0.5	0.13
51	250	200	1800	20	300	1.0	0.5	1.31
52	3000	200	1800	20	300	1.0	0.5	0.38
53	250	25	2600	20	300	1.0	0.5	0.47
54	3000	25	2600	20	300	1.0	0.5	0.13
55	250	200	2600	20	300	1.0	0.5	1.31
56	3000	200	2600	20	300	1.0	0.5	0.38
57	250	25	1800	80	300	1.0	0.5	0.93
58	3000	25	1800	80	300	1.0	0.5	0.27
59	250	200	1800	80	300	1.0	0.5	2.63
60	3000	200	1800	80	300	1.0	0.5	0.76
61	250	25	2600	80	300	1.0	0.5	3.82
62	3000	25	2600	80	300	1.0	0.5	1.10
63	250	200	2600	80	300	1.0	0.5	2.63
64	3000	200	2600	80	300	1.0	0.5	0.76
65	250	25	1800	20	50	0.5	$\infty$	0.13
66	3000	25	1800	20	50	0.5	$\infty$	0.04
67	250	200	1800	20	50	0.5	$\infty$	0.38
68	3000	200	1800	20	50	0.5	$\infty$	0.11
69	250	25	2600	20	50	0.5	$\infty$	0.13
70	3000	25	2600	20	50	0.5	$\infty$	0.04
71	250	200	2600	20	50	0.5	$\infty$	0.38
72	3000	200	2600	20	50	0.5	$\infty$	0.11
73	250	25	1800	80	50	0.5	$\infty$	0.27
74	3000	25	1800	80	50	0.5	$\infty$	0.09
75	250	200	1800	80	50	0.5	$\infty$	0.76
76	3000	200	1800	80	50	0.5	$\infty$	0.22
77	250	25	2600	80	50	0.5	$\infty$	0.27
78	3000	25	2600	80	50	0.5	$\infty$	0.09
79	250	200	2600	80	50	0.5	$\infty$	0.76
80	3000	200	2600	80	50	0.5	$\infty$	0.22
81	250	25	1800	20	300	0.5	$\infty$	0.33
82	3000	25	1800	20	300	0.5	$\infty$	0.1
83	250	200	1800	20	300	0.5	$\infty$	0.93
84	3000	200	1800	20	300	0.5	$\infty$	0.27
85	250	25	2600	20	300	0.5	$\infty$	0.33
86	3000	25	2600	20	300	0.5	$\infty$	0.1
87	250	200	2600	20	300	0.5	$\infty$	0.93
88	3000	200	2600	20	300	0.5	$\infty$	0.27
89	250	25	1800	80	300	0.5	$\infty$	0.66
90	3000	25	1800	80	300	0.5	$\infty$	0.19
91	250	200	1800	80	300	0.5	$\infty$	1.86
92	3000	200	1800	80	300	0.5	$\infty$	0.54
93	250	25	2600	80	300	0.5	$\infty$	0.66
94	3000	25	2600	80	300	0.5	$\infty$	0.19
95	250	200	2600	80	300	0.5	$\infty$	1.86
96	3000	200	2600	80	300	0.5	$\infty$	0.54
97	250	25	1800	20	50	1.0	$\infty$	0.19
98	3000	25	1800	20	50	1.0	$\infty$	0.06
99	250	200	1800	20	50	1.0	$\infty$	0.5
100	3000	200	1800	20	50	1.0	$\infty$	0.16

101	250	25	2600	20	50	1.0	$\infty$	0.19
102	3000	25	2600	20	50	1.0	$\infty$	0.06
103	250	200	2600	20	50	1.0	$\infty$	0.54
104	3000	200	2600	20	50	1.0	$\infty$	0.16
105	250	25	1800	80	50	1.0	$\infty$	0.38
106	3000	25	1800	80	50	1.0	$\infty$	0.12
107	250	200	1800	80	50	1.0	$\infty$	1.07
108	3000	200	1800	80	50	1.0	$\infty$	0.31
109	250	25	2600	80	50	1.0	$\infty$	0.38
110	3000	25	2600	80	50	1.0	$\infty$	0.12
111	250	200	2600	80	50	1.0	$\infty$	1.07
112	3000	200	2600	80	50	1.0	$\infty$	0.31
113	250	25	1800	20	300	1.0	$\infty$	0.46
114	3000	25	1800	20	300	1.0	$\infty$	0.13
115	250	200	1800	20	300	1.0	$\infty$	1.31
116	3000	200	1800	20	300	1.0	$\infty$	0.38
117	250	25	2600	20	300	1.0	$\infty$	0.47
118	3000	25	2600	20	300	1.0	$\infty$	0.13
119	250	200	2600	20	300	1.0	$\infty$	1.31
120	3000	200	2600	20	300	1.0	$\infty$	0.38
121	250	25	1800	80	300	1.0	$\infty$	0.93
122	3000	25	1800	80	300	1.0	$\infty$	0.27
123	250	200	1800	80	300	1.0	$\infty$	2.63
124	3000	200	1800	80	300	1.0	$\infty$	0.76
125	250	25	2600	80	300	1.0	$\infty$	0.93
126	3000	25	2600	80	300	1.0	$\infty$	0.27
127	250	200	2600	80	300	1.0	$\infty$	2.63
128	3000	200	2600	80	300	1.0	$\infty$	0.76

\* Free field conditions are denoted by the symbol  $\infty$ .

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