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Mode-mixing quantum gates and entanglement without particle creation in periodically accelerated cavities

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Abstract. We show that mode-mixing quantum gates can be produced by non-uniform relativistic acceleration. Periodic motion in cavities exhibits a series of resonant conditions producing entangling quantum gates between different frequency modes. The resonant condition associated with particle creation is the main feature of the dynamical Casimir effect which has been recently demonstrated in superconducting circuits. We show that a second resonance, which has attracted less attention since it implies negligible particle production,

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produces a beam splitting quantum gate leading to a resonant enhancement of entanglement which can be used as the first evidence of acceleration effects in mechanical oscillators. We propose a desktop experiment where the frequencies associated with this second resonance can be produced mechanically.

Contents

1. Introduction

In relativistic quantum field theory, the particle content of a quantum state is affected by the evolution of the spacetime, including the motion of any boundaries. Further, the very notion of a 'particle' depends on the motion of an observer. In flat spacetime, celebrated examples are the thermality seen in Minkowski vacuum by uniformly accelerated observers, known as the Unruh effect [1, 2], and the creation of particles by moving boundaries, known as the dynamical (or non-stationary) Casimir effect (DCE) [3, 4]. In curved spacetime, a celebrated example is the Hawking radiation emitted by black holes [5]. The DCE is related to a fundamental prediction by Fulling and Davies that a non-uniformly accelerated mirror will excite photons out of the vacuum [6]. It was later realized that this effect may be significantly enhanced if, instead of a simple mirror, a cavity is used in which one or both of the mirrors are in motion [7]. The simplest situation in which to observe the DCE is that of a cavity oscillating sinusoidally with frequency ω_c. The DCE is predicted to exhibit a fundamental resonance condition for the production of quantum entangled photon-pairs, $\omega_c = \omega_1 + \omega_2$, where $\omega_{1,2}$ are the two entangled photon frequencies [7]. The actual number of photons predicted for a mechanically oscillating cavity is strongly limited ($\sim 10^{-9}$ photons s^{-1}) by the maximum achievable ω_c. For this reason a number of alternative systems that also exhibit a periodically varying boundary of some kind have been proposed with the aim of enhancing the DCE. Examples are superconducting quantum interference device (SQUID) mirrors, Bose–Einstein condensates (producing phonon pairs) and cavities controlled using nonlinear optics [3, 4, 8, 9]. Notwithstanding recent breakthroughs, the DCE remains an extremely difficult effect to observe and study experimentally.

In this paper we consider the general case of a rigid cavity undergoing an arbitrary (mechanically induced) acceleration. In the specific cases of a linear sinusoidal or a uniform circular motion, we show that a mode-mixing resonance condition, $\omega_c = |\omega_1 - \omega_2|$ [10], for which no photons are generated, can be brought significantly below the DCE photon generation resonance condition, to apparently experimentally accessible frequencies. We show how this low-frequency resonance leads to the generation of entanglement between existing and previously non-entangled cavity modes. The oscillating cavity can be shown to behave like a generalized beam splitter, thus performing an essential quantum gate functionality. This demonstrates that relativistic effects, in this case non-uniform relativistic acceleration, can be exploited for quantum information. There are many proposals to generate gates in non-relativistic quantum information. Our scheme pioneers on how to implement quantum gates in relativistic quantum information. We then discuss the possibility of performing actual experiments with mechanically oscillating optical cavities.

2. Cavity in (1 + 1) dimensions

2.1. Preliminaries

We first consider the simplified case of a cavity in $(1+1)$-dimensional Minkowski spacetime. The cavity is assumed mechanically rigid, maintaining constant length L in its instantaneous rest frame. The proper acceleration at the centre of the cavity is denoted by $a(\tau)$, where τ is the proper time. To maintain rigidity, the acceleration must be bounded by $|a(\tau)|L/c^2 < 2$ [11]. From now on we set $c = \hbar = 1$.

The cavity contains a real scalar field ϕ of mass $\mu_0 \geqslant 0$, with Dirichlet boundary conditions. We assume that the cavity is initially inertial, and we denote by u_n, $n = 1, 2, \ldots$, a standard basis of cavity field modes that are of positive frequency $\omega_n = \sqrt{\mu_0^2 + (\pi n/L)^2}$ with respect to the cavity's proper time before the acceleration. We also assume that the cavity's final state is inertial, and we denote by $\tilde{u}_n$, $n = 1, 2, \ldots$, a standard basis of cavity field modes that are of positive frequency ω_n with respect to the cavity's proper time after the acceleration. Because of the acceleration at intermediate times, the two sets of modes need not coincide, but the completeness of each set allows the sets to be related by the Bogoliubov transformation [12, 13]

$$\tilde{u}_m = \sum_n (\alpha_{mn} u_n + \beta_{mn} u_n^*), \tag{1}$$

where the star denotes complex conjugation. The Bogoliubov coefficient matrices α and β are determined by solving the field equation in the cavity during the acceleration.

In the initial and final inertial regions the field operator ϕ has the respective expansions $\phi = \sum_n (a_n u_n + a_n^\dagger u_n^*)$ and $\phi = \sum_n (\tilde{a}_n \tilde{u}_n + \tilde{a}_n^\dagger \tilde{u}_n^*)$, where the non-vanishing commutators of the early (respectively late) time creation and annihilation operators are $[a_n, a_m^\dagger] = \delta_{nm}$ ($[\tilde{a}_n, \tilde{a}_m^\dagger] = \delta_{nm}$). The early and late time creation and annihilation operators need not coincide, but it follows from (1) that they can be expressed in terms of each other in terms of the Bogoliubov coefficients [12, 13]. In particular, the transformation mixes creation and annihilation operators if and only if some of the β-coefficients are non-vanishing.

Now, working in the Heisenberg picture, the quantum state of the field in the cavity does not change in time. However, given a state $|\Psi\rangle$, we interpret its particle content at early times in terms of the early time vacuum $|0\rangle$, which satisfies $a_n|0\rangle = 0$, and the early time excitations created by $a_n^\dagger$. At late times, we similarly interpret the particle content of $|\Psi\rangle$ in terms of the late time vacuum $|\tilde{0}\rangle$, which satisfies $\tilde{a}_n|\tilde{0}\rangle = 0$, and the late time excitations created by $\tilde{a}_n^\dagger$. The acceleration hence affects the particle content of the cavity whenever the Bogoliubov transformation (1) differs from the identity transformation. The β-coefficients are responsible for creation and annihilation of particles, while the α-coefficients are responsible for mode mixing. In particular, the vacua $|0\rangle$ and $|\tilde{0}\rangle$ coincide if and only if all the β-coefficients vanish [12, 13].

2.2. Bogoliubov coefficients for general time-dependent acceleration

We shall express the Bogoliubov coefficients as a time-ordered integral, allowing both the magnitude and the time-dependence of the acceleration to remain general within the rigidity bound $|a(\tau)|L < 2$.

We encode α and β into the matrix $U = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}$, so that the composition of Bogoliubov transformations amounts to matrix multiplication of the corresponding U-matrices. The Bogoliubov identities [12] are then encoded in the matrix equation $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = U \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U^\dagger$.

When the acceleration between the initial and final inertial regions is uniform and lasts for proper time $\bar{\tau}$, we have [11] $U_h(\bar{\tau}) = K_h^{-1} \tilde{Z}_h(\bar{\tau}) K_h$, where $K_h = \begin{pmatrix} {}_{\text{o}}\alpha_h & {}_{\text{o}}\beta_h \\ {}_{\text{o}}\beta_h & {}_{\text{o}}\alpha_h \end{pmatrix}$, $\tilde{Z}_h(\bar{\tau}) = \begin{pmatrix} Z_h(\bar{\tau}) & 0 \\ 0 & Z_h^*(\bar{\tau}) \end{pmatrix}$, $Z_h(\bar{\tau}) = \operatorname{diag}(\mathrm{e}^{\mathrm{i}\Omega_1(h)\bar{\tau}}, \mathrm{e}^{\mathrm{i}\Omega_2(h)\bar{\tau}}, \cdots)$, $\Omega_n(h)$ are the angular frequencies during the acceleration, ${}_{\text{o}}\alpha_h$ and ${}_{\text{o}}\beta_h$ are the Bogoliubov coefficient matrices from the initial inertial segment to the uniformly accelerated segment, and the acceleration has been encoded in the dimensionless parameter $h = aL$. The field modes and the angular frequencies during the acceleration have elementary expressions for $\mu_0 = 0$ and are given in terms of modified Bessel functions for $\mu_0 > 0$. The coefficients encoded in ${}_{\text{o}}\alpha_h$ and ${}_{\text{o}}\beta_h$ do not have elementary expressions, but they can be written as integrals involving the inertial and accelerated mode functions over the constant time surface where the acceleration begins.

For accelerations that may vary arbitrarily between the initial time τ_0 and final time τ, $U(\tau, \tau_0)$ is given by the limit of $U_{h_N}(\bar{\tau}_N) U_{h_{N-1}}(\bar{\tau}_{N-1}) \cdots U_{h_2}(\bar{\tau}_2) U_{h_1}(\bar{\tau}_1)$ as $N \to \infty$, such that $\tau - \tau_0 = \sum_{k=1}^N \bar{\tau}_k$ is fixed and each $\bar{\tau}_k \to 0$. As an infinitesimal increase in τ amounts to multiplying $U(\tau, \tau_0)$ from the left by $U_h(\bar{\tau})$ with infinitesimal $\bar{\tau}$, $U(\tau, \tau_0)$ satisfies the differential equation

$$\dot{U}(\tau, \tau_0) = \mathrm{i} K_{h(\tau)}^{-1} \tilde{\Omega}_{h(\tau)} K_{h(\tau)} U(\tau, \tau_0), \tag{2}$$

where $\tilde{\Omega}_{h(\tau)} = \begin{pmatrix} \Omega_{h(\tau)} & 0 \\ 0 & -\Omega_{h(\tau)} \end{pmatrix}$, $\Omega_{h(\tau)} = \operatorname{diag}(\Omega_1(h(\tau)), \Omega_2(h(\tau)), \ldots)$, and the overdot denotes derivative with respect to τ. The solution is

$$U(\tau_{\mathrm{f}}, \tau_0) = T \exp\left(\mathrm{i} \int_{\tau_0}^{\tau_{\mathrm{f}}} K_{h(\tau)}^{-1} \tilde{\Omega}_{h(\tau)} K_{h(\tau)} \, \mathrm{d}\tau \right), \tag{3}$$

where τ_{f} denotes the moment at which the acceleration ends and T denotes the time-ordered exponential.

To summarize: the Bogoliubov transformation between the inertial initial segment ending at proper time τ_0 and the final inertial segment starting at proper time τ_{f} is given by (3). $h(\tau)$ may vary arbitrarily for $\tau_0 \leqslant \tau \leqslant \tau_{\mathrm{f}}$, within the rigidity constraint $|h(\tau)| < 2$: in particular, no small acceleration approximation has been made. For piecewise constant $h(\tau)$, (3) reduces to a product of the matrices $U_h(\bar{\tau}_k)$ from each constant h segment.

A direct consequence of (3) is that the Bogoliubov coefficients evolve by pure phases over any time interval in which h is constant. Particles in the cavity are hence created by *changes* in the acceleration, not by acceleration itself, as can be argued on general adiabaticity grounds [14, 15]. The cavity is in this respect similar to a single accelerating mirror, which excites photons from the vacuum only when its acceleration is non-uniform [6].

2.3. Small acceleration limit

At small accelerations, $\Omega_n(h)$, ${}_{\mathrm{o}}\alpha_h$ and ${}_{\mathrm{o}}\beta_h$ have the expansions [11]

$$\Omega_n = \omega_n + O(h^2), \qquad n = 1, 2, \ldots, \tag{4a}$$

$${}_{\mathrm{o}}\alpha_h = 1 + h\hat{\alpha} + O(h^2), \qquad {}_{\mathrm{o}}\beta_h = h\hat{\beta} + O(h^2), \tag{4b}$$

where

$$\hat{\alpha}_{nn} = 0, \tag{5a}$$

$$\hat{\alpha}_{mn} = \frac{\pi^2 mn(-1 + (-1)^{m+n})}{L^4\,(\omega_m - \omega_n)^3\,\sqrt{\omega_m \omega_n}} \qquad \text{for } m \neq n, \tag{5b}$$

$$\hat{\beta}_{mn} = \frac{\pi^2 mn(1 - (-1)^{m+n})}{L^4\,(\omega_m + \omega_n)^3\,\sqrt{\omega_m \omega_n}}. \tag{5c}$$

Note that $\hat{\alpha}_{mn}$ and $\hat{\beta}_{mn}$ depend on μ_0 and L only via the dimensionless quantity $\mu_0 L$. Formulae (5) are obtained from equations (7) in [11] by an elementary rearrangement.

We seek $U(\tau_{\mathrm{f}}, \tau_0)$ in the form

$$\alpha = \mathrm{e}^{\mathrm{i}\boldsymbol{\omega}(\tau_{\mathrm{f}} - \tau_0)}(1 + \hat{A} + O(h^2)), \tag{6a}$$

$$\beta = \mathrm{e}^{\mathrm{i}\boldsymbol{\omega}(\tau_{\mathrm{f}} - \tau_0)}\hat{B} + O(h^2), \tag{6b}$$

where $\boldsymbol{\omega} = \mathrm{diag}(\omega_1, \omega_2, \ldots)$, $\hat{A}$ and $\hat{B}$ are of first order in h, and τ_{f} again denotes the moment at which the acceleration ends. Using (2), (4) and (5), we find

$$\hat{A}_{mn} = \mathrm{i}(\omega_m - \omega_n)\hat{\alpha}_{mn} \int_{\tau_0}^{\tau_{\mathrm{f}}} \mathrm{e}^{-\mathrm{i}(\omega_m - \omega_n)(\tau - \tau_0)}\, h(\tau)\,\mathrm{d}\tau, \tag{7a}$$

$$\hat{B}_{mn} = \mathrm{i}(\omega_m + \omega_n)\hat{\beta}_{mn} \int_{\tau_0}^{\tau_{\mathrm{f}}} \mathrm{e}^{-\mathrm{i}(\omega_m + \omega_n)(\tau - \tau_0)}\, h(\tau)\,\mathrm{d}\tau. \tag{7b}$$

To linear order in h, the Bogoliubov coefficients are hence obtained by just Fourier transforming the acceleration.

Two comments are in order. Firstly, while the perturbative solution (7) assumes the acceleration to be so small that $|h| \ll 1$, the velocities, travel times and travel distances remain unrestricted, and the solution remains valid even when the velocities are relativistic. Our perturbative treatment is hence complementary to the small distance approximations often considered in the DCE literature [3, 4], while of course overlapping in the common domain of validity.

Secondly, $\hat{A}_{mn}$ and $\hat{B}_{mn}$ scale linearly in h, but their magnitudes depend also crucially on whether h changes slowly or rapidly compared with the oscillating integral kernels in (7). In the limit of slowly varying h both $\hat{A}_{mn}$ and $\hat{B}_{mn}$ vanish, in agreement with the adiabaticity arguments of [14, 15]. In the limit of piecewise constant h, the changes in the magnitudes of $\hat{A}_{mn}$ and $\hat{B}_{mn}$ come entirely from the discontinuous jumps in h [11, 16, 17]. The limit of piecewise constant h may be difficult to realize experimentally with a material cavity, and we emphasize that no such rapid changes in the acceleration are involved in the experimental scenario considered in section 4. This limit can however be simulated by a cavity whose walls are mechanically static dc SQUIDs undergoing electric modulation [8, 18].

2.4. Resonances

Suppose now that $|h| \ll 1$, so that the solutions (6) and (7) are valid. Suppose that h is sinusoidal with angular frequency ω_c. For generic values of ω_c the integrals in (7) are oscillatory and have no net growth as τ_f increases. However, when ω_c equals the angular frequency of an oscillating integral kernel in (7), there is a resonance and the corresponding Bogoliubov coefficient grows linearly in τ_f. These resonance conditions read

$$\hat{A}_{mn}: \quad \omega_c = |\omega_m - \omega_n|, \tag{8a}$$

$$\hat{B}_{mn}: \quad \omega_c = \omega_m + \omega_n, \tag{8b}$$

where in each case $m - n$ needs to be odd in order for the coefficient to be non-vanishing.

The particle creation resonance (8*b*) is well known in the DCE literature [3, 4, 7, 10, 19–27]. The mode-mixing resonance (8*a*) has been noted [10, 19–24] but seems to have received attention mainly in situations where it happens to coincide with a particle creation resonance. As the case of interest in the experimental scenario of section 4 will be mode mixing without significant particle creation, we recall here some relevant properties of mode mixing in quantum optics.

Mode mixing without particle creation is known in quantum optics as a passive transformation [28], implemented experimentally by passive optical elements such as beam splitters and phase plates. While mode mixing is present already in classical wave optics, its significance in quantum optics is that the mixing can be harnessed to quantum information tasks. The entangling power of passive transformations is well understood: for example, the mixing generates entanglement from an initial Gaussian state only if this state is squeezed [17, 29–31].

These entanglement considerations are directly applicable to mode mixing in our cavity, and the entanglement can be determined experimentally by measurements on quanta that are allowed to escape from the cavity [32–35]. We emphasize that while the particle creation resonance (8*b*) can be used to implement two-mode squeezing gates [30, 31, 36] and other multipartite gates [37], the mode-mixing resonance (8*a*) can be used to implement mode-mixing gates even when no particle creation is present.

Specifically, the oscillating cavity can be tuned to act as a beam splitter—a well-studied quantum gate in continuous variable systems [38]. In the following we apply our results to the special class of Gaussian states, characterized by positive Wigner functions, which allow elegant and powerful analytical results [39]. Gaussian states, including coherent and squeezed states, are routinely prepared in the laboratory. For example, two-mode squeezed states are commonly produced by parametric down conversion [40].

In order to calculate the entanglement generated by the mode-mixing gate we have introduced, we adopt the covariance matrix formalism. We consider a family of harmonic oscillators with position and momentum operators q_i, p_i, where $i = 1, 2, \ldots$. We collect these operators in the vector $\mathbb{X} = (q_1, p_1, q_2, p_2, \ldots)$. The canonical commutation relations take the form $[\mathbb{X}_i, \mathbb{X}_j] = \mathrm{i}\Omega_{ij}$, where the only non-vanishing components of the symplectic form $\mathbf{\Omega}$ are $\Omega_{2i-1,2i} = -\Omega_{2i,2i-1} = 1$. The covariance matrix is defined by

$$\sigma_{ij} = \frac{1}{2}\langle \mathbb{X}_i\mathbb{X}_j + \mathbb{X}_j\mathbb{X}_i\rangle - \langle \mathbb{X}_i\rangle\langle \mathbb{X}_j\rangle. \tag{9}$$

This formalism is suitable for Gaussian states since all the relevant information about the state can be encoded in the first moment $\langle \mathbb{X}_i\rangle$ and the covariance matrix. In fact, the quantification of Gaussian state entanglement requires only its covariance matrix [39].

An initial pure state remains pure to first order in the acceleration [36]. This implies that the reduced state σ_{red} of two modes will depend only on the Bogoliubov coefficients that mix these two modes. The contribution of coefficients mixing with other frequencies is negligible to linear order in the acceleration.

From now on we specialize to two-mode Gaussian states for which σ_{red} is symmetric. In the covariance matrix formalism, the entanglement for such states is fully quantified by the smallest symplectic eigenvalue of the partial transpose of the covariance matrix [39]. The partial transpose is given by $\tilde{\sigma}_{\mathrm{red}} = \mathbf{P}\sigma_{\mathrm{red}}\mathbf{P}$, where $\mathbf{P} = \mathrm{diag}(1, 1, 1, -1)$, and its symplectic eigenvalues are the eigenvalues of $\mathrm{i}\mathbf{\Omega}\tilde{\sigma}_{\mathrm{red}}$. The eigenvalue set has the form $\{-\tilde{\nu}_-, \tilde{\nu}_-, -\tilde{\nu}_+, \tilde{\nu}_+\}$, where $0 \leqslant \tilde{\nu}_- \leqslant \tilde{\nu}_+$, and therefore the quantity characterizing the entanglement is $\tilde{\nu}_-$. Within our perturbative small acceleration expansion, it is shown in [30] that $\tilde{\nu}_- = 1 - \tilde{\nu}_-^{(1)} + O(h^2)$, where $\tilde{\nu}_-^{(1)}$ is linear in the acceleration.

While the von Neumann entropy would be a natural measure of entanglement for the reduced two-mode state σ_{red} when the initial state is pure, its small acceleration expansion does not take the form of a power series because of the logarithms involved in its definition [36, 38]. Measures of entanglement based on the partial transpose criterion however do have a power series expansion in the acceleration. We consider the negativity [41], which is in the covariance matrix formalism given by $\mathcal{N} = \max\{0, \frac{1}{2}(\tilde{\nu}_-^{-1} - 1)\}$ [39], and which hence has the small acceleration expansion $\mathcal{N} = \max\{0, \frac{1}{2}\tilde{\nu}_-^{(1)}\} + O(h^2)$.

Suppose now that the state is a separable state of two modes, m and n, in which each mode is squeezed with the same squeezing parameter $s > 0$. When $\hat{B}_{mn}$ is negligible compared with $\hat{A}_{mn}$, it follows from the expression of $\tilde{\nu}_-^{(1)}$ given in [30] that the leading order contribution to the negativity is linear in the acceleration and given by

$$\mathcal{N} = |\mathrm{Im}(\hat{A}_{mn})| \sinh s. \tag{10}$$

Figure 1 shows a plot of the negativity (10) as a function of the total oscillation time $\Delta\tau = \tau_{\mathrm{f}} - \tau_0$ and the acceleration angular frequency ω_{c}, assuming purely sinusoidal acceleration with phase chosen so that $h(\tau)$ is proportional to $\cos(\omega_{\mathrm{c}}(\tau - \tau_0))$, for fixed $\omega_{\mathrm{r}} = |\omega_m - \omega_n|$ and squeezing parameter s. The linear growth of $\mathcal{N}$ at the resonance, $\omega_{\mathrm{c}} = \omega_{\mathrm{r}}$, is evident from the plot. The scale of the vertical axis depends on m, n and s but also on $\mu_0 L$, in a way that we shall address in section 4.

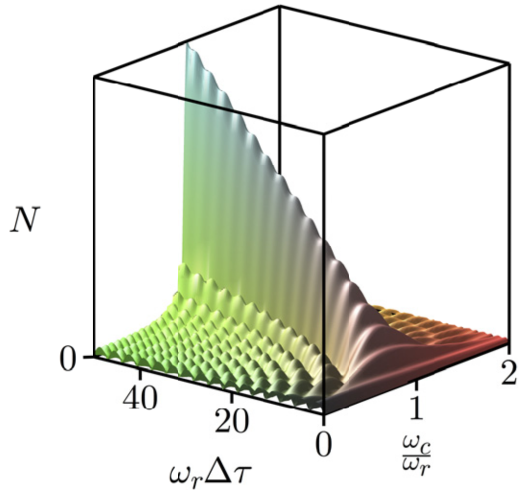

Figure 1. The negativity $\mathcal{N}$ (10) as a function of the cavity oscillation angular frequency ω_{c} and the total oscillation time $\Delta\tau = \tau_{\mathrm{f}} - \tau_0$, for fixed $\omega_{\mathrm{r}} = |\omega_m - \omega_n|$ and squeezing parameter s.

3. Cavity in (3 + 1) dimensions

Let now ϕ be a real scalar field of mass $\mu \geqslant 0$ in a cavity in $(3+1)$-dimensional Minkowski space, with Dirichlet conditions. The inertial cavity is a rectangular parallelepiped with fixed edge lengths L_x, L_y and L_z, and a standard basis of orthonormal field modes is indexed by triples (m, n, p) of positive integers, such that the angular frequencies are $\omega_{mnp} = \sqrt{\mu^2 + (\pi m/L_x)^2 + (\pi n/L_y)^2 + (\pi p/L_z)^2}$.

Acceleration in the cavity's three principal directions can be treated as $(1+1)$-dimensional, with the inert transverse quantum numbers just contributing to the effective mass. Acceleration of unrestricted magnitude and direction would require new input regarding how the shape of the cavity responds to such acceleration [42]. To *linear* order in the acceleration, however, boosts commute, and we can treat acceleration as a vector superposition of accelerations in the three principal directions in the cavity's instantaneous rest frame. Defining $\hat{A}$ and $\hat{B}$ as in (6), and denoting the acceleration three-vector in the cavity's instantaneous rest frame by $(a_x(\tau), a_y(\tau), a_z(\tau))$, equations (7) generalize for changes in the quantum number m to

$$\hat{A}_{mnp,mnp} = 0, \tag{11a}$$

$$\hat{A}_{mnp,m'np} = \mathrm{i}\frac{\pi^2 mm'(-1+(-1)^{m+m'})}{L_x^3\left(\omega_{mnp} - \omega_{m'np}\right)^2\sqrt{\omega_{mnp}\omega_{m'np}}}\int_{\tau_0}^{\tau_{\mathrm{f}}} \mathrm{e}^{-\mathrm{i}(\omega_{mnp}-\omega_{m'np})(\tau-\tau_0)}\, a_x(\tau)\,\mathrm{d}\tau \quad \text{for } m \neq m', \tag{11b}$$

$$\hat{B}_{mnp,m'np} = \mathrm{i}\frac{\pi^2 mm'(1-(-1)^{m+m'})}{L_x^3\left(\omega_{mnp} + \omega_{m'np}\right)^2\sqrt{\omega_{mnp}\omega_{m'np}}}\int_{\tau_0}^{\tau_{\mathrm{f}}} \mathrm{e}^{-\mathrm{i}(\omega_{mnp}+\omega_{m'np})(\tau-\tau_0)}\, a_x(\tau)\,\mathrm{d}\tau \tag{11c}$$

with changes in the quantum numbers n and p given by similar formulae involving respectively a_y and a_z.

For sinusoidal acceleration with angular frequency ω_c, the resonance condition of linear growth is

$$\hat{A}_{mnp,m'n'p'}: \quad \omega_c = |\omega_{mnp} - \omega_{m'n'p'}|, \tag{12a}$$

$$\hat{B}_{mnp,m'n'p'}: \quad \omega_c = \omega_{mnp} + \omega_{m'n'p'}, \tag{12b}$$

where in each case only the quantum number in the direction of the oscillation may differ and this difference needs to be odd.

4. Desktop experiment

The particle creation resonance angular frequency (12b) is always larger than the frequencies of the individual cavity modes. The mode-mixing resonance angular frequency (12a) can however be lower. In $(1+1)$ dimensions, mode-mixing resonances occur significantly below the frequencies of the individual cavity modes if $\mu_0 L \gg 1$, as then $(\Delta\omega_n)/\omega_n \approx \pi^2(\mu_0 L)^{-2} n \Delta n \ll 1$ whenever n and Δn are small compared with $\mu_0 L$. In more than $(1+1)$ dimensions, a similar lowering can be arranged to occur even for a massless field by storing in the cavity quanta whose wave vector is highly transverse to the acceleration, as the transverse momentum then gives rise to a large effective $(1+1)$-dimensional mass. We now outline a $(3+1)$-dimensional experimental scenario that optimizes this lowering of the mode-mixing resonance.

Setting $\mu = 0$, we assume that the quanta in the cavity have wavelength $\lambda \ll \min(L_x, L_y, L_z)$ and have their momenta aligned close to the z-direction, so that $(2/\lambda)^2 \approx (p/L_z)^2 \gg (m/L_x)^2 + (n/L_y)^2$ and $\omega_{mnp} \approx 2\pi/\lambda + \frac{1}{4}\pi\lambda[(m/L_x)^2 + (n/L_y)^2]$. We let the cavity undergo linear or circular harmonic oscillation orthogonal to the z-direction, with amplitude d_x (d_y) in the x-direction (y-direction). For motion in the x-direction, the mode-mixing resonance angular frequency (12a) between modes m and m', with $m - m'$ odd, is

$$\omega_c \approx \frac{1}{4}\pi\lambda L_x^{-2}\, |m^2 - (m')^2| \tag{13}$$

and it follows from (11b) that the mode mixing growth rate is

$$\frac{\mathrm{d}}{\mathrm{d}\tau}|\hat{A}_{\mathrm{res}}| \approx \frac{1}{2}\pi m m' d_x \lambda L_x^{-3}. \tag{14}$$

The lowest resonance occurs for $m = 1$ and $m' = 2$. Similar formulae ensue for the y-resonance, and for circular motion both resonances are present.

As the experimental setup, we first trap one or more quanta in the cavity, in modes whose momenta are aligned close to the z-direction. After a period of linear or circular oscillation perpendicular to the z-direction, a measurement on the quantum state of the cavity is performed, by suitable observations of quanta that are allowed to escape. We assume that the resonance mode mixing dominates any effects due to the initial trapping and the final releasing of the quanta.

A careful choice of the cavity geometry would need to be considered in order to guarantee the success of an experiment. A particular concern would be the mechanical stability of the cavity itself. There are options for creating mechanically very robust cavities based on a monolithic geometry. Examples could be a Bragg grating cavity in an optical fibre. These are mechanically very robust and are a very well developed technology. Another example, closer to the parameters specified below, would be a monolithic Fabry–Perot cavity or etalon filter cavity.

Such cavities can be made with extremely high finesse and are made out of a single solid block of material and hence inherit the robustness of the material itself (typically, glass).

We choose $\lambda = 600\,\text{nm}$ and $L_x = L_y = 1\,\text{cm}$. The lowest resonance angular frequency is then $\omega_{\text{c}} \approx 1.4 \times 10^{-2}\,\text{m}^{-1} \approx 4.2 \times 10^6\,\text{s}^{-1}$, corresponding to an oscillation frequency 0.7 MHz.

For linear oscillation, we choose the amplitude $d_x = 1\,\mu\text{m}$, which may be achievable by using ultrasound to accelerate the cavity. From (14) we then have $\frac{\text{d}}{\text{d}\tau}|\hat{A}_{\text{res}}| \approx 6 \times 10^2\,\text{s}^{-1}$, so that the mode mixing coefficient grows to order unity within a millisecond. For the squeezed states of section 2.4, the negativity (10) grows to order unity at the same timescale provided the squeezing parameter s is not much less than unity. Storing the quantum in the cavity for a millisecond could be challenging although recent achievements indicate that it may be feasible [43].

For circular motion, we choose the amplitude $d_x = d_y = 1\,\text{mm}$. At the threshold angular velocity $\omega_{\text{c}} \approx 4.2 \times 10^6\,\text{s}^{-1} \approx 4 \times 10^7$ rpm, the mode mixing coefficient then grows to order unity within a nanosecond, and for squeezed states similarly for the negativity (10) provided the squeezing parameter s is not much less than unity. The threshold angular velocity exceeds the angular velocity of medical ultracentrifuges by a factor of 200 [44], but this gap could possibly be bridged by a specifically designed system of sub-centimetre scale. We note that the centripetal acceleration at the threshold angular velocity equals $1.5 \times 10^6\,\text{m}\,\text{s}^{-2}$, which is already reached in ultracentrifuges that combine a smaller angular velocity with a larger radius [44].

As ω_{c} in these scenarios is much below the particle creation resonance (12*b*), particle creation in the cavity is not cumulative in the duration of the oscillation and is highly sensitive to the manner in which the acceleration is switched on and off. While this is a consequence of the idealized, fully confining character of our cavity, we may obtain an upper limit for the predicted particle creation by noting that in the extreme case of sharp switch-on and switch-off $\hat{B}_{mnp,m'np}$ (11*c*) has the order of magnitude

$$\frac{\pi^2 mm'(1-(-1)^{m+m'})|a_x|L_x}{L_x^4\left(\omega_{mnp}+\omega_{m'np}\right)^3\sqrt{\omega_{mnp}\omega_{m'np}}}. \tag{15}$$

The number of particles created in a mode with fixed m, n and p, each near their lowest value 1, can hence be given an upper bound by summing the square of (15) over m'. The result is a purely numerical factor times $(a_x L_x)^2$, which is of order 10^{-24} for our linear oscillation figures and of order 10^{-18} for our circular motion figures. At the mode-mixing resonance, the mixing hence overwhelmingly dominates over any particle creation effects. This is consistent with the usual estimates of 10^{-9} photons created per second [7] for experimentally less idealized cavities.

5. Conclusions

We have quantized a scalar field in a rectangular cavity that is accelerated arbitrarily in $(3+1)$-dimensional Minkowski spacetime, in the limit of small accelerations but arbitrary velocities and travel times. The Bogoliubov coefficients were expressed as explicit quadratures. For linear or circular periodic motions, we identified a configuration in which the mode-mixing resonance frequency is significantly below the frequencies of the cavity modes.

Our scalar field analysis adapts in a straightforward way to a Maxwell field with perfect conductor boundary conditions [45]. The mode mixing effects appear hence to be within the reach of a desktop experiment with photons, achievable with current technology in its mechanical aspects, if perhaps not yet in the storage capabilities required of a mechanically oscillating optical cavity.

We anticipate that the particle creation and mode mixing effects are not qualitatively sensitive to the detailed shape of the cavity, and this freedom could be utilized in the development of a concrete laboratory implementation. The experimental prospects could be further improved by filling the cavity with a medium that slows light down [46]. A laboratory implementation would also need to develop an experimental protocol for measuring the field within the cavity, and the data analysis would need to account for any experimental imperfections. A full detailed evaluation of these experimental issues would need to be carried out case by case for any proposed concrete implementation, but the frequency and lifetime estimates given in this paper do suggest the mode mixing effect to be at the threshold of current technology.

We underline that our experimental scenario does not involve significant particle creation. Nevertheless, it involves significant mode mixing. This mixing acts as a beam splitter quantum gate, creating or degrading entanglement in situations where particles are initially present. Finally, it is also worth underlining that although we have discussed the specific case of a mechanically oscillating cavity, the low-frequency resonance can be found whenever the quanta can be made highly transverse to the acceleration, and may therefore be similarly adopted to perform quantum gate operations also in other analogue systems, based e.g. on SQUID mirrors [8] or nonlinear optics [47, 48] that have been proposed to date. We anticipate that observations of entanglement will generally provide opportunities for experimental verification of both particle creation and mode mixing effects that are complementary to observations of fluxes or particle numbers [8].

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References

[1] Unruh W G 1976 *Phys. Rev.* D **14** 870
[2] Crispino L C B, Higuchi A and Matsas G E A 2008 *Rev. Mod. Phys.* **80** 787
[3] Dodonov V V 2001 *Adv. Chem. Phys.* **119** 309
[4] Dodonov V V 2010 *Phys. Scr.* **82** 038105
[5] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
Hawking S W 1976 *Commun. Math. Phys.* **46** 206 (erratum)
[6] Davies P C W and Fulling S A 1976 *Proc. R. Soc. Lond.* A **348** 393

[7] Lambrecht A, Jaekel M-T and Reynaud S 1996 *Phys. Rev. Lett.* **77** 615
[8] Wilson C M *et al* 2011 *Nature* **479** 376
[9] Jaskula J-C *et al* 2012 *Phys. Rev. Lett.* **109** 220401
[10] Mundarain D F and Maia Neto P A 1998 *Phys. Rev.* A **57** 1379
[11] Bruschi D E, Fuentes I and Louko J 2012 *Phys. Rev.* D **85** 061701
[12] Birrell N D and Davies P C W 1982 *Quantum Fields in Curved Space* (Cambridge: Cambridge University Press)
[13] Fabbri A and Navarro-Salas J 2005 *Modeling Black Hole Evaporation* (London: Imperial College Press)
[14] Schützhold R and Unruh W G 2005 arXiv:quant-ph/0506028
[15] Unruh W G 2012 private communication
[16] Friis N, Lee A R, Bruschi D E and Louko J 2012 *Phys. Rev.* D **85** 025012
[17] Friis N, Bruschi D E, Louko J and Fuentes I 2012 *Phys. Rev.* D **85** 081701
[18] Friis N, Lee A R, Truong K, Sabín C, Solano E, Johansson G and Fuentes I 2013 *Phys. Rev. Lett.* **110** 113602
[19] Crocce M, Dalvit D A R and Mazzitelli F D 2001 *Phys. Rev.* A **64** 013808
[20] Crocce M, Dalvit D A R and Mazzitelli F D 2002 *Phys. Rev.* A **66** 033811
[21] Ruser M 2006 *Phys. Rev.* A **73** 043811
[22] Yuce C and Ozcakmakli Z 2008 *J. Phys. A: Math. Theor.* **41** 265401
[23] Dodonov A V and Dodonov V V 2001 *Phys. Lett.* A **289** 291
[24] Yuce C and Ozcakmakli Z 2009 *J. Phys. A: Math. Theor.* **42** 035403
[25] Schützhold R, Plunien G and Soff G 1998 *Phys. Rev.* A **57** 2311
[26] Dodonov V V and Andreata M A 1999 *J. Phys. A: Math. Gen.* **32** 6711
[27] Dodonov A V, Dodonov V V and Mizrahi S S 2005 *J. Phys. A: Math. Gen.* **38** 683
[28] Simon R, Mukunda N and Dutta B 1994 *Phys. Rev.* A **49** 1567
[29] Wolf M M, Eisert J and Plenio M B 2003 *Phys. Rev. Lett.* **90** 047904
[30] Friis N and Fuentes I 2013 *J. Mod. Opt.* **60** 22
[31] Alsing P M and Fuentes I 2012 *Class. Quantum Grav.* **29** 224001 (arXiv:1210.2223 [quant-ph])
[32] Briegel H-J, Englert B-G, Sterpi N and Walther H 1994 *Phys. Rev.* A **49** 2962
[33] Lutterbach L G and Davidovich L 1997 *Phys. Rev. Lett.* **78** 2547
[34] Lougovski P *et al* 2003 *Phys. Rev. Lett.* **91** 010401
[35] Pielawa S, Morigi G, Vitali D and Davidovich L 2012 *Phys. Rev. Lett.* **98** 240401
[36] Bruschi D E, Dragan A, Lee A R, Fuentes I and Louko J 2012 arXiv:1201.0663 [quant-ph]
[37] Friis N, Huber M, Fuentes I and Bruschi D E 2012 *Phys. Rev.* D **86** 105003
[38] Weedbrook C, Pirandola S, García-Patrón R, Cerf N J, Ralph T C, Shapiro J H and Lloyd S 2012 *Rev. Mod. Phys.* **84** 621
[39] Laurat J, Keller G, Oliveira-Huguenin J-A, Fabre C, Coudreau T, Serafini A, Adesso G and Illuminati F 2005 *J. Opt. B: Quantum Semiclass. Opt.* **7** S577
[40] Wu L-A, Xiao M and Kimble H J 1987 *J. Opt. Soc. Am.* B **4** 1465
[41] Vidal G and Werner R F 2002 *Phys. Rev.* A **65** 032314
[42] Epp R J, Mann R B and McGrath P L 2009 *Class. Quantum Grav.* **26** 035015 (arXiv:0810.0072 [gr-qc])
[43] Kuhr S *et al* 2007 *Appl. Phys. Lett.* **90** 164101
[44] Thermoscientific 2013 www.thermoscientific.com, visited 3 February
[45] Friis N, Lee A R and Louko J 2013 arXiv:1307.1631 [quant-ph]
[46] Lauprêtre T *et al* 2011 *Opt. Lett.* **36** 1551
[47] Dezael F X and Lambrecht A 2010 *Eur. Phys. Lett.* **89** 14001
[48] Faccio D and Carusotto I 2011 *Eur. Phys. Lett.* **96** 24006