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Forecasts of Prices and Informed Sensitivity Analysis: Applications in Project Valuations

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Abstract

From corporate budgeting to public planning, we hear that commodity prices are uncertain and that when they vary, key investment measures sway with them. However, claiming that commodity prices are outside a firm's domain of control, corporate decision makers tend to disregard this uncertainty or at best reflect it in naïve sensitivity analyses. Yet, firms should take in the understanding about key uncertain factors to avoid inferior decisions and loss of value. In this paper, we show that the customary practice of analysis with arbitrary "high" and "low" forecasts of prices is inconsistent with the general understanding about commodity price dynamics and the financial theory. To alleviate this, we develop consistent and project-specific forecast of prices that support valuations and decision making.

1. Introduction

For most project appraisals, a change in the forecast of product prices could dramatically sway the investment decision measures. An upstream petroleum project that is highly appealing at, e.g., 70 US/bbl crude oil price could be marginal at 50 USD/bbl or disastrous at 30 USD/bbl. Price forecasts are central to commodity asset valuations, but with their inherent uncertainty, how should the investors decide? The answer has traditionally been along the lines: "Of course prices are uncertain, we evaluate projects at expected prices and run sensitivity analyses to see how the value measures change".

While it is common (and recommended) to examine changes in value by changing the parameters of the valuation model, this begs another, yet unanswered, question: "How much of a change in prices?" For most projects, an unrealistic range of prices could lead to confusion rather than insight.[1]

The uncertainty we perceive should be consistent with our information and understanding. Yet with unjustified estimates, the price ranges could be either too wide, too narrow, or inconsistent with the general economic understanding. In practice, commodity prices do not have an arbitrary range. They reflect the rates of production and consumption, constantly changing to keep the fine balance of supply and demand. This makes some price scenarios more likely, and others outright impossible. The scenarios we consider in sensitivity analyses should reflect this economically reasonable range of prices.

Traditional sensitivity analysis defines the range of prices as an arbitrary span between extremes. But extreme values of a variable do not necessarily show a useful range. They may even be mistaken as absolute minimum or maximum (Leamer, 1985). Perhaps if valuations were studies of *fragility*, then assessing the models with extreme prices would have made sense. We could for example see if a configuration in a project stands the test of extreme prices and is still robust across the conceivable extremes.

Project appraisals are not about robustness.[2] They are about making investment decisions. Sensitivity analyses should support the formulation of investment decisions using a reasonable range of value

[1] Often corporate analyses assume "high" and "low" prices as compared with an "expected" price forecast—also known as the corporate planning price curves. Often, the "high" and "low" forecasts are arbitrary deviations from the base case, say 40% lower or higher than the expected prices. It is unclear why such prices stand for a reasonable range and why this should lead to useful decision insights.

[2] Sensitivity analyses could serve a variety of goals, from applications in risk analysis, e.g., Frey and Patil (2002), to building econometric models, e.g., Leamer (1985). We believe valuations are integral to decision-making and should contribute at an early stage to the project design by finding or constructing value-maximizing alternatives.

drivers and then screening off any inferior courses of action. It should also show how project values (and managerial decisions) change by changing value drivers. With this description, the range of prices for sensitivity analyses cannot be arbitrary. They should be economically informed and project specific.

In this paper, we develop forecasts of prices for sensitivity analysis. Our model that is consistent with market dynamics and adjusts to a project's features. Like pieces of a puzzle that fit only in a certain way to show a picture, we believe the components of valuation—such as forecasts of prices and discounting method—should also fit together. This unified view of valuations (e.g., Myers and Turnbull, 1977, and recently Hahn et al, 2018, and Jafarizadeh and Bratvold, 2021) concludes that project risks depend on at least the project length. Then in sensitivity analyses, forecast ranges should also be a function of project length.

Using the valuation scheme in Jafarizadeh and Bratvold (2019, 2021), in this paper we formulate the effect of duration on the forecasts of prices for sensitivity analysis. We describe the market dynamics of prices using the economics of supply and demand and we use the price model in Schwartz and Smith (2000) to estimate ranges for forecasts of prices.[3]

In our numerical procedure, we first simulate future spot prices for the duration of a project. Then, we estimate the "optimistic" and "pessimistic" forecasts of prices (planning prices) so that they reflect the distribution of project values as if using simulated prices. We discuss using the "sum-discounted prices" as a measure of the effect of prices on value.

In our analysis, we generate forecasts of prices that are simple and informative. In addition, these forecasts are consistent with the valuation scheme and economic dynamics. We further compare our results with crude oil price forecasts available to the businesses (e.g., analysis from the U.S. Energy Information Administration, EIA) and discuss some misconceptions.

The next section discusses the economic dynamics and the general understanding about uncertain commodity prices. Then in section 2, we use a two-factor process to describe the price uncertainty. Based on this description, in section 3 we devise forecasts of prices for sensitivity analysis. In section 4 we discuss the applications, compare our results with other price forecasts, and apply our method to an example. In section 5, we summarize the discussions and conclude. The appendices discuss the use of single factor price processes and parameter estimation for calibration of the price model.

2. A Description of Price Uncertainty

2.1 General Understanding

Early attempts in modeling commodity investments used a random walk process to describe the dynamics of the prices. Commodity prices were thought to behave like the prices of financial stocks—random independent ticks building up to a Brownian motion. The assumption was easy and convenient. It also drew similarities to financial investments, allowing the spread of financial valuation techniques to real assets (e.g., Brennan and Schwartz, 1985).

Admittedly, commodity prices are random and unpredictable. Statistical studies of the history of the commodity prices, for example crude oil prices reveal randomness. The studies also show mean reversion in historical prices (Pindyck, 1999, Geman, 2007, Xu et al, 2012). Yet in general, the past is

However, recommending a course of action based on its robustness (rather than value creation potentials) is also destructive. In this paper, we focus on sensitivity analyses that support valuations.

[3] In any valuation, we estimate the present value of uncertain future cash flows. Through risk-return tradeoff, the present value already reflects the time value of money and risk. Sensitivity analysis in addition shows the effect of uncertain (or misestimated) forecasts of a variable. We study the effect of "optimistic and pessimistic values of that variable", as discussed by Brealey et al (2012), on project value. In this paper, we estimate economically consistent optimistic and pessimistic price forecasts.

not a predictor of the future. As Brealey et al (2012) put it, with uncertainty "more things can happen than will happen". The past is but one realization of the uncertain prices. Therefore, in addition to studying the past, we should also see whether our perceived behavior of prices agrees with the general belief in the commodity markets. Here, the random walk assumption is in odds with the general understanding about commodity markets (Lund, 1993).[4]

All market prices follow equilibrium considerations of supply and demand, but commodities are more tightly bound to the physical world. When prices are higher than the equilibrium level, we expect higher cost producers to come online and existing projects to produce longer. This results in more barrels of oil in the market, pushing the prices down towards the equilibrium. On the other hand, when prices are below the equilibrium level, then expensive producers go offline faster and existing production ceases earlier. This results in fewer barrels of oil in the market pushing the prices up towards the equilibrium.

These dynamics support the "mean-reverting" assumption about prices. We mostly expect the deviations from the equilibrium to dissipate and disappear. Prices move randomly but tend to revert towards an average. Beyond these dynamics, changes in technology could still cause random but permanent changes in prices.

The collective market belief—the market expectation about prices' trend and uncertainty—supports mean reverting behavior of prices. The traded derivative contracts imply that the volatility of futures prices is different from that of spot prices, unlike how the Geometric Brownian motion behaves (e.g., Schwartz, 1997), and as if prices were mean reverting. But this is not all. The market also admits random and permanent price moves (caused by changes in technology), like those of the Geometric Brownian motion. Eventually, we expect to see both effects, yet with differing levels of importance, in the dynamics of prices.

The above discussion naturally leads to models that account for both random permanent moves and transitory deviations—at least two stochastic factors describing the dynamics of prices (e.g., Schwartz and Smith, 2000). We could model the random moves with a geometric Brownian motion process and assume the transitory deviations follow a mean-reverting stochastic process. Perhaps we could even refine the model further by considering a third type of dynamic that describes the movements of the mean-reverting long-term (e.g., model three in Schwartz, 1997, or Schwartz and Smith, 1998). Still, more intricate models are not necessarily better. As Carlson et al (2007) discuss, there are inherent problems about extrapolation of models to long-term price forecasts for project valuations.

This brings us to the main point—a price model cannot ignore the frame of the investment decision. The decision frame should inform the uncertainty model. For example, for the decision to invest in an upstream petroleum development project with a long lead-time, the short-term transitory dynamics may not be significant. However, for a tight oil project with a short lifespan and rapidly declining production, value creation is in the short-term price dynamics.

If we add features to our price model but still take the identical course of action that a simpler model suggests, then the added features would have been useless. A model should conform to the general market understanding and economic reasoning. Yet in addition, a price model should be relevant and useful in the context of decisions.

2.2 Model Description

The Schwartz and Smith (2000) price model assumes the price S_t at time t has a short- and long-term factor, so that $\ln S_t = \chi_t + \xi_t$. The long-term factor ξ_t follows a Brownian motion and reflects the

[4] Similarly, we build geological subsurface models with limited available information that shows just a glimpse of the subsurface. We must therefore consider the information within the context of geological understanding (like e.g., deeper depositions are older, or low-density fluids migrate upwards). The geological models that we build should be consistent with both the information and background knowledge.

persisting changes in equilibrium prices while the short-term factor χ_t follows a mean-reverting process and stands for the transitory deviations from the equilibrium.

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{1}$$

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \tag{2}$$

Here κ reflects the speed of mean-reversion in the short-term factor, μ_ξ is the drift of the long-term factor, and σ_χ and σ_ξ are the standard-deviations for, respectively, the short- and long-term factors. In addition, dz_χ and dz_ξ are correlated increments of the standard Brownian motion with $\rho_{\chi\xi}dt = dz_\chi dz_\xi$.

The log of prices in the future will be normally distributed. Depending on the current price factors χ_0 and ξ_0, they will have the following expectation and variance

$$\mathrm{E}(\ln S_t) = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \tag{3}$$

$$\mathrm{Var}(\ln S_t) = (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \tag{4}$$

In addition, because $\ln \mathrm{E}(S_t) = E(\ln S_t) + \frac{1}{2}\mathrm{Var}(\ln S_t)$, we can define the expectation of future prices as

$$\ln \mathrm{E}(S_t) = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t + \frac{1}{2}\left((1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right) \tag{5}$$

This expectation serves as a forecast of prices. Once we describe prices as a stochastic time series, the best estimate of their future values at time $t > 0$, would be their expectation.

For risk-neutral prices, we deduct the short-and long-term risk-premiums, λ_χ and λ_ξ respectively, from the expectation. Then the expected future spot prices would be equal to the futures prices at the same delivery. In other words, the current value of a futures contract for delivery at T, $F_{0,T}$ would be

$$\begin{aligned}\ln F_{0,T} = {} & e^{-\kappa T}\chi_0 + \xi_0 + (\mu_\xi - \lambda_\xi)T - (1 - e^{-\kappa T})\frac{\lambda_\chi}{\kappa} \\ & + \frac{1}{2}\left((1 - e^{-2\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\end{aligned} \tag{6}$$

The two-factor model can also easily reduce to its constituent single-factor price models if valuations call for more simple models at the expense of losing realism.

For a set of parameters shown in Table 1, Figure 1 shows the expectation of spot prices and their confidence bands. Note that these bands are associated with spot price percentiles in each period considered independently.

Table 1—Parameters of the two-factor price process

Parameter	Value
χ_0	-0.2
ξ_0	4.2
μ	0.005
σ_χ	0.3
κ	0.6
σ_ξ	0.1
$\rho_{\chi\xi}$	0.2

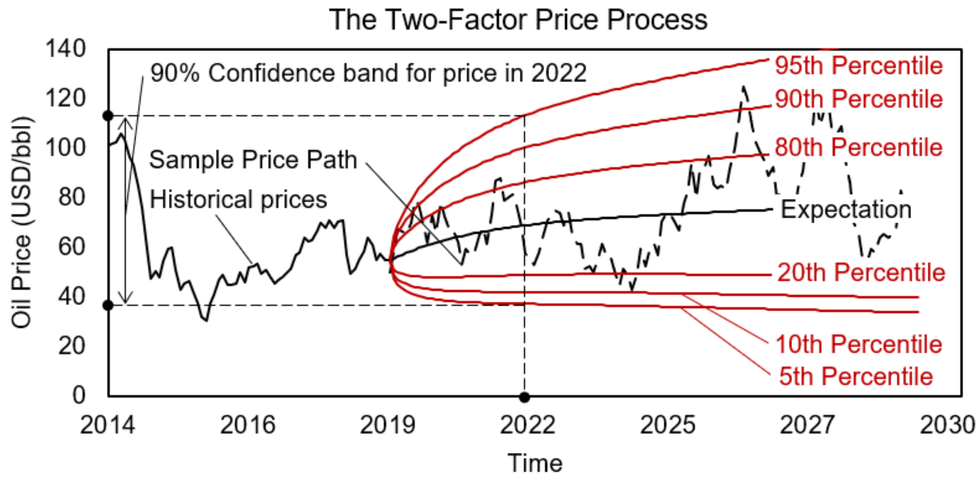

Figure 1—The two-factor price model and the uncertainty in the future spot prices (e.g., the dashed lines show the 90% confidence band for price in the beginning of 2022)

The two-factor price process is simple, versatile, and consistent with the economic dynamics. The model reflects both the permanent and the transitory dynamics of prices. In addition, it leads to an explicit formulation of expected future prices (as well as derivative contracts like futures and options) consistent with the market observations.

Despite the interesting analytics, we have only so far described the stochastic behavior of spot prices. We have not yet generated forecasts of prices. From figure 1, other than the expectation—a valid forecast—price forecasts for specific dates are meaningless in valuations. The confidence bands are not forecasts of prices. They merely show the range of price estimates at specific future dates.

By drawing a vertical line anywhere on the horizontal axis and finding its intersection with the confidence curves, we could find the confidence bands for price estimates at a specific date on the vertical axis. For example, (as shown in Figure 1) we would be 90% confident that price of crude oil in the beginning of 2022 would be between 37 USD/barrel and 110 USD/barrel. However, this single range is irrelevant in project decisions. Valuations depend on forecasts of prices over multiple periods.[5]

3. Informed Sensitivity Analysis

3.1 Sum-Discounted Prices

The stochastic models describe the uncertainty of spot prices in the future. They show the expectation and a probability distribution for future spot prices. Traditional sensitivity analyses do not fit such a description. They often define a range of price forecasts from the most optimistic to the most pessimistic—entirely detached from the stochastic models. Such analyses could not show, for example, if the value of a project at the pessimistic forecast is the lowest possible project value, the least likely value, or even a reasonable value at all. In this section, we discuss a method of generating optimistic and pessimistic price forecasts from stochastic price models.

[5] Consider the of random sequence $X_0, \dots, X_{n-1}, X_n$ that make up a stochastic series. If $Prob\{X_n \le x_n\} = p$, (i.e., x_n is the p-percentile of the random variable X_n) then in general, $\mathrm{Prob}\{X_0 \le x_0, \dots, X_{n-1} \le x_{n-1}, X_n \le x_n\} \neq p$. For example, we throw a die (dice) twice, and the series of variables X_0, X_1 show the number we get on the top face in each throw. The fiftieth percentile of the first or the second throw is number three, $\mathrm{Prob}\{X_0 \le 3\} = \mathrm{Prob}\{X_1 \le 3\} = 0.5$. However, the chance of getting a number less than or equal to three in the series of two throws is not fifty percent. The chance is twenty five percent. Using Bayes formula of conditional probabilities: $\mathrm{Prob}\{X_0 \le 3, X_1 \le 3\} = \mathrm{Prob}\{X_0 \le 3\} \times \mathrm{Prob}\{X_1 \le 3 \mid X_0 \le 3\} = 0.25$. The percentile of a series is different from the percentile of the variables.

Like the confidence bands of spot prices in the future, there should also be a range for forecasts of prices. Such "scenarios for prices" should be consistent with the stochastic properties of spot prices yet still useful in the detailed industrial cash flow models. We use the distribution of *sum of discounted prices* (e.g., Dixit, 1993) over the forecast horizon as a measure of effect of prices on project value.[6]

From the distribution of the sum-discounted prices, we could estimate forecasts of prices consistent with the stochastic process. For example, the expected forecast will be the price scenario we call S_t^*, $0 < t < T$, so that

$$\int_0^T S_t^* e^{-rt} dt = \mathrm{E}\left(\int_0^T S_t e^{-rt} dt\right) \tag{7}$$

Here, T is the forecast horizon, r is the discount rate, and e^{-rt} is the discounting factor.

Intuitively, the sum of discounted prices reflects both the aggregate effect of a series of prices over a specific period and the decision maker's time preference. In projects with multiple time periods, no single price is relevant. Instead, the decision maker considers a series of prices over time. To compare price series, it would be easier to consider the average, or the summation, of the prices within the series. In addition, the decision maker may weight prices of the near future as more important than those in far future—showing such a preference with the discount factor. Hence, the sum-discounted prices could be a natural measure of the effect of a series of prices on value.

The intuition behind using the sum-discounted prices as a proxy for the economic appeal of a series draws on the no-arbitrage arguments in valuation (e.g., Nau and McCardle, 1991, Smith and Nau, 1995). If prices are the only source of uncertainty in a project, then based on no-arbitrage arguments, the project's value should be the expected present value of its cash flows—which are uncertain merely because the prices are uncertain. A change in the forecast of prices would directly lead to a change in the net present value of the project. In the absence of non-linearities (like asymmetries and managerial flexibilities), the distribution of the project's present value would reflect the distribution of the sum-discounted prices.

To estimate the expectation (or any of the *n*th percentiles) of the forecasts of prices, we use a numerical approximation of equation (5). We simulate the spot prices within the forecast horizon and estimate the distribution of the sum of the discounted values. Then we ask: what multi-period forecast of prices shows the effect of the *n*th percentile of such a distribution? Using an optimization model, we will estimate the forecast scenarios that reflect the distribution of the sum-discounted prices.

3.2 Numerical Procedure

We use a discrete version of equations (1) and (2) to simulate prices.[7] We calculate the log spot price as

$$\ln S_{t+\Delta t} = \xi_{t+\Delta t} + \chi_{t+\Delta t} \tag{8}$$

$$\xi_{t+\Delta t} = \xi_t + \mu_\xi \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t} \tag{9}$$

[6] Using the earlier dice throwing example, we can further explain the point. Assume we receive payments based on the number we get on the top face of the dice. The variables X_0, X_1 show the payments as a series. We prefer money now to money later and r is our discount rate. Note that, for example, the value of the outcome $\{X_0 = 1, X_1 = 6\}$ is different from the value of $\{X_0 = 6, X_1 = 1\}$. When $r > 0$ we always prefer the latter. The sum-discounted value of the top faces is a measure of value for the dice throwing game.

[7] In using the discrete versions of the continuous processes, a too long time-step may be unrealistic, and a too short time-step increases computational details. We assumed monthly time-steps.

$$\chi_{t+\Delta t} = e^{-\kappa\Delta t}\chi_t - \left(1 - e^{-\kappa\Delta t}\right)\frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}} \tag{10}$$

In addition, we account for the correlation between the simulated factors by assuming ε_ξ is a standard normal distribution and ε_χ is a function of ε_ξ and ε (another independent normal distribution)

$$\varepsilon_\chi = \varepsilon_\xi \rho_{\chi\xi} + \varepsilon\sqrt{1 - \rho_{\chi\xi}^2} \tag{11}$$

Using these equations, we simulate price paths for the forecast period $0 < t < T$, and then use the following Riemann approximation to the sum-discounted prices

$$\int_0^T S_t e^{-rt} dt \cong \sum_{t=0}^{T} S_t e^{-rt}\, \Delta t \tag{12}$$

Note that in the discrete approximation, $t \in \{0, \Delta t, 2\Delta t, \ldots, (n-1)\Delta t, T\}$ and n is the number of time steps.

In our implementation, we assumed one-month time steps. In other words, we assumed that the price for one barrel of crude oil is the average on a specific month. We simulated these monthly prices for ten years (the forecast horizon) and calculated the sum-discounted value of each price path. Figure 2 shows the distribution of sum-discounted prices.

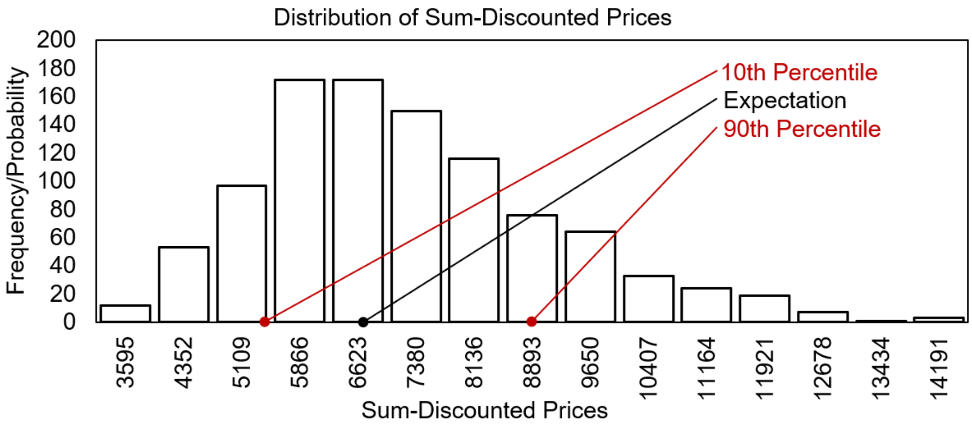

Figure 2—Distribution of sum-discounted prices for ten thousand simulated price paths

Next, from the set of sum-discounted prices, we picked the tenth and ninetieth percentiles of the distribution as the "low" and "high" scenarios. We then devised "pessimistic" and "optimistic" forecast of prices so that they have the same sum-discounted values as the "low" and "high" scenarios. For example, to generate the "pessimistic" forecast, we used an optimization model that varies a factor of the price curve until it has the same discounted value as the tenth percentile of the sum-discounted simulated prices.

To calculate forecast of prices S_t^* corresponding to the pth percentile of the discounted prices, we use the optimization procedure below

$$
\begin{aligned}
&\min_{\lambda_\xi}\left(\sum_{t=0}^{T} S_t^* e^{-rt} - X\right)^2 \\
&\text{s.t.} \\
&P\left(\sum_{t=0}^{T} S_t^* e^{-rt} < X\right) = \frac{p}{100}
\end{aligned}
\tag{13}
$$

Here, $t \in \{0, \Delta t, 2\Delta t, \ldots, (n-1)\Delta t, T\}$ for n time steps. In addition, X is an internal variable we defined to calculate the percentiles.

The solution to this optimization is not necessarily complex. For example, through a spreadsheet implementation, we used MS Excel and Solver to calculate the 10th and 90th percentile forecasts. We set the optimization by changing the long-term premium λ_ξ in equation (6) so that its sum-discounted monthly prices converge to respectively the 10th and 90th percentiles of the sum-discounted simulated prices. The results would be a "pessimistic" and an "optimistic" forecast. These are percentiles for forecasts of prices, as opposed to percentiles of prices. This distinction is important. Figure 3 shows the results. We have also implemented the procedure in the accompanying spreadsheet model.[8]

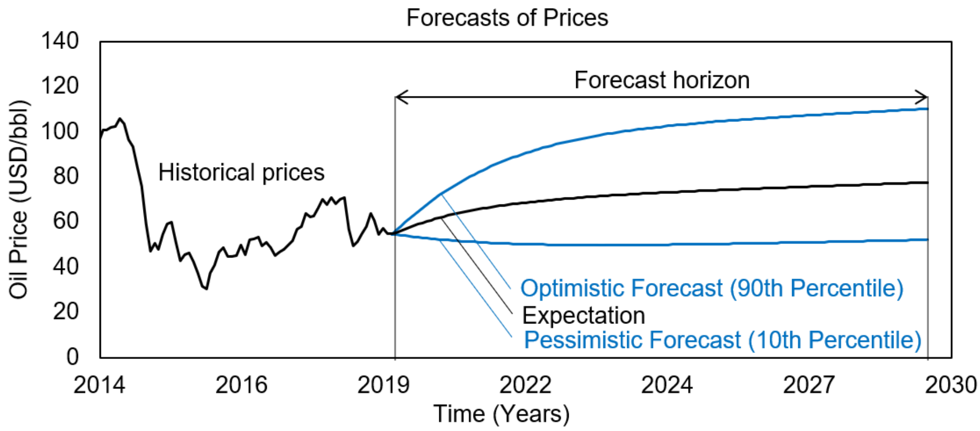

Figure 3—the "optimistic" and "pessimistic" forecasts of prices corresponding to respectively the ninetieth and the tenth percentiles of the sum-discounted simulated prices

The underlying procedure is versatile and applies to any stochastic model that we can discretize and simulate. We first need to simulate price paths, and then estimate the forecasts of prices from the distribution of sum-discounted simulated prices. In this paper, we used the Schwartz and Smith (2000) two-factor price process because it is simple, realistic, and useful in valuations.

We customize the sensitivity analysis to valuations. Projects of different duration would have different sensitivities to the uncertain prices. For a short-term project, say five years, only the variations in the immediate future matter. The uncertain prices beyond five years would be irrelevant. But a longer project of (say ten years) would be sensitive to price changes over a longer horizon. Its uncertainty

[8] We have also implemented the numerical procedure to generate optimistic and pessimistic forecasts for our dice throwing example. We receive payments based on the number on top face of the dice. We simulated the outcomes of throwing the dice twice and then calculated the sum-discounted value for each iteration. Then, from the distribution of sum discounted prices we calculated the tenth and ninetieth percentiles and used an optimization process to generate optimistic and pessimistic forecasts that have the same sum-discounted values. The link to the spreadsheet implementation is: https://www.dropbox.com/s/5uobir50d8y7wpe/Dice%20Prediction.xlsx?dl=0

profile would naturally be different. Our procedure tailors the optimistic and pessimistic price scenarios to the projects' forecast horizons. As we show in Figure 4, this leads to different sets of forecasts for different projects.

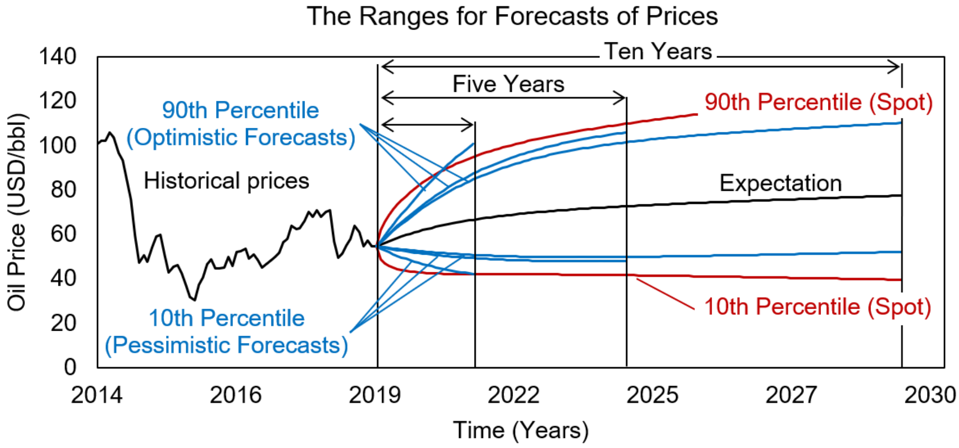

Figure 4—Ranges of forecasts for two-, five-, and ten-year durations shown against the confidence bands for spot prices

The optimistic and pessimistic forecasts spread wider for short-term projects than those for the longer projects. This means that under an identical price model, short-term forecasts would be more fanned out for sensitivity analysis. Why? Longer projects are riskier. But this pattern is due to an "averaging effect" over a longer horizon. A one dollar change in the forecast of prices will more intensely affect the cash flows of a longer project. In our analysis, the sweep between high and low forecasts (the area trapped between the curves, as measure of forecast uncertainty) is proportional to the length—larger for longer projects. Our optimistic and pessimistic forecasts reflect the uncertainty that matters to a project.

3.3 The Choice of the Discount Rate

The discount factors are like preference weights showing the relative importance of individual prices within a price strip. By discounting and summing prices, we conveniently convert a price path to a single value. This simplifies comparisons between various price paths and forecasts. On the other hand, an inappropriate discount rate could lead to untuned forecasts and confused analysis. The proper rate for analysis should be consistent with the features of the valuation.

The discount rate is for comparing items in different points in time and varying levels of risk. Therefore, for risk-neutral prices (i.e., in the two-factor model $\lambda_\chi = 0$ and $\lambda_\xi = 0$) we should merely account for the differences in timing and discount them with r, the risk-free rate. On the other hand, if our prices include risk premiums (i.e., $\lambda_\chi \neq 0$ or $\lambda_\xi \neq 0$) then we use $r + \varepsilon$, a rate higher than the risk-free rate to compensate for the riskiness. Once our discounting is consistent with the perceived risk, the approach we take should not make a difference.[9] The results will be consistent.

In this paper, we estimate the parameters of our price model by fitting the model's futures and option volatility curves to the market observations (Appendix A). This implied method of parameter estimation, discussed in Jafarizadeh and Bratvold (2012, 2021), calibrates the parameters so that the price model yields the same estimate of futures and options prices as those contracts in the market. Because futures (and options on futures) are risk-free quotes designed to hedge price risk, our model

[9] This natural relationship between the riskiness of cash flows and the risk-adjusted discount rate is a recurring theme in finance literature. Jafarizadeh and Bratvold (2021) use this relationship to estimate the price risk premiums. In this paper, we discount prices not cash flows. Still, we can think of these prices as cash flows of an imaginary project that has no cost and produces one barrel of crude oil per period. The risk premium in the discount rate ε and the price risk premiums λ_χ and λ_ξ should be consistent so that they yield the same sum discounted prices as discounting risk-neutral prices with the risk-free rate.

will also be a risk-neutral description of prices. We use the risk-free rate for discounting. Most industrial valuations work with physical prices—having risk premiums, as in the real world. They should then use the comparable risk-adjusted rate for discounting.

Finally, it may appear that we are double discounting project's cash flows: first when we generate optimistic and pessimistic forecasts of prices and once again when we discount cash flows that we estimate using these forecasts. We do not double discount cash flows. Our process of generating forecasts uses the sum-discounted prices as a measure of importance. It simply means we select our optimistic and pessimistic forecasts by comparing their sum discounted values with the rest of admissible forecasts. It is however crucial to use a discount rate consistent with the risk assumptions of the price process.

4. Applications and Discussion

4.1 Comparison with Other Market Analytics

Other than their internal resources, corporate decision makers may also use publicly available price analytics. Such resources help cross-firm comparisons, but if misunderstood, could also lead to confusion and to loss of value. In this section we discuss the analytics from the U.S. Energy Information Administration (EIA, 2010) and compare them with the analytics and forecasts from our method.

In their Short-Term Energy Outlook, EIA (2010) show "confidence intervals around futures prices" and explain that they are based on "the implied volatilities of option contracts". In other words, EIA estimates the uncertainty in the futures prices using the volatility information embedded in the options contracts. The price of an option contract depends on various parameters, including the volatility of the futures price. With price and all other parameters known, we can use an option pricing formula in reverse and estimate the volatility—showing what the market participants think of the uncertainty in prices. This forward-looking approach (as opposed to statistical analysis of past price behavior) considers the most current information to draw insights about prices in the future.

Because the prices constantly change, and in the absence of information about their specific dataset, we cannot exactly reproduce EIA's analysis. However, we can still estimate confidence bands using their method on our dataset. In this section, we compare the results from their approach with our model and show similarities and slight differences.

We use the prices of light sweet crude oil's European options and futures in the NYMEX market and estimate the confidence bands for futures prices. The two-factor process's option pricing formula estimates the value of a European call option on a futures contract (delivering at T), with strike price K, and expiry at t as

$$\begin{gathered} e^{-rt}\left(F_{0,T}N(d) - KN\big(d - \sigma(t,T)\big)\right) \\ d = \frac{\ln\left({}^{F}/_{K}\right)}{\sigma(t,T)} + \frac{\sigma(t,T)}{2} \end{gathered} \tag{14}$$

Here, r is the risk-free rate, and $N(d)$ is the cumulative probability for the standard normal distribution. Using this equation in reverse, we can calculate $\sigma(t,T)$ the total volatility. Note that this is the volatility for the duration of the options contract and the annualized volatility would be $\sigma(t,T)/\sqrt{t}$.

We used a simple computer code that calculates the implied volatility of European options on futures by iteratively using equation (14) until it matches the observed option prices in the market. We then used these volatility estimates to draw confidence bands around the observed futures prices. The dots in Figure 5 show the results. Here, we show the futures prices for delivery in increasingly further maturities and their uncertainty. With eighty percent confidence, the futures prices would be between their corresponding 10th and 90th percentiles. Note that option contracts are available for limited dates.

We do not have options on long-maturity futures contracts and could not estimate their implied volatility. To compare the results with our analysis, in this figure we have also shown the confidence bands for future spot prices along with our model's optimistic and pessimistic forecasts of prices.

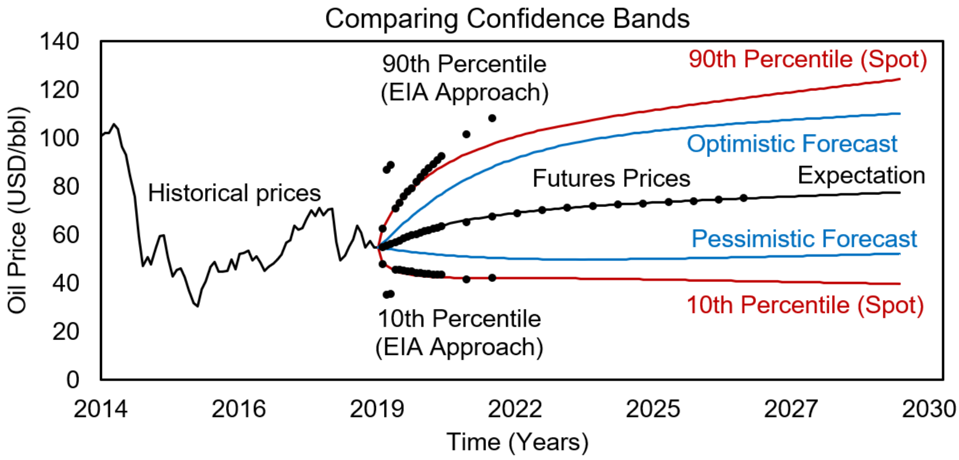

Figure 5—Comparing confidence bands from EIA approach (shown in dots) with the confidence bands of spot prices from our model. We also show the optimistic and pessimistic forecasts from our method.

The simulation result of our two-factor model is comparable to the EIA's analysis. First, the risk-neutral expectation in our model (calibrated to the information in the market, as shown in Appendix A) is identical to the futures curve in EIA's analysis. Second, the confidence bands for spot prices from our simulation are reasonably consistent with the confidence bands of futures prices in EIA approach. Because our calibrated two-factor model is an abstraction of the real-world price behavior, few confidence bands from the EIA's approach (real-world data) are inevitably different from our simulation results. When calibrating our price parameters, such outliers have negligible effect on parameter estimates. In effect, we have simplified the real world to build a useful model. Overall, the EIA's approach and our simulation of the two-factor price model are consistent.

Neither of the EIA's nor our simulation percentiles offer insights on forecasts for valuations. The confidence bands are not forecasts of prices. They simply show a range of values for price at a specific date in the future. Instead, we could use the method in this paper and generate optimistic or pessimistic forecasts of prices. For a project with a ten-year duration, we made forecasts corresponding to the tenth and ninetieth percentiles of sum-discounted simulated prices. Figure 5 also shows these forecasts.

4.2 Example: An Asymmetric Fiscal Regime

Governments use tax systems to generate value from their natural resources. In addition, they use taxes to encourage (or discourage) exploration and development activities. Yet, an ideal all-round tax system is hard to achieve. Business uncertainties are so diverse that no tax system could account for everything.

Therefore, authorities devise tax regimes specific to their economic resource conditions. For example, the Norwegian tax encourages exploration of even marginal opportunities by alleviating their downsides. A resource-rich country in the middle east, less concerned with minor assets, could devise a tax regime that increases government take on oil price spikes. For the latter, a consistent appreciation of oil price uncertainty is key to sound investment decisions.

The government would describe the price dynamics to predict the industry's investment decisions. The firms, on the other hand, would like to increase shareholder value. A disparity in the appreciation of price uncertainty could lead to losses. For example, if a firm uses an inconsistent price model and value metrics, then they could make poor investment decisions and lose value.

In this example, if annual oil prices go beyond a ceiling threshold, then the government absorbs all the extra income. The highest price that the oil company should expect is this threshold level. For an upstream petroleum development project, the government sets the threshold to 80 USD/bbl. In Table 2 we show that six different price scenarios. The first three scenarios are corporate prices and arbitrary "high" and "low" scenarios. The next three scenarios show ten-year price forecasts from our model.

To simplify the discussions, we only show the effect of asymmetric deductions. Assuming the project discount rate[10] is 5%, we compare the results from applying these sets of price scenarios and discuss their usefulness for the investment decisions.

Table 2—Cash flow element of the project and six price scenarios

Year	0	1	2	3	4	5	6	7	8	9	10
Corporate Prices (USD/bbl)	70	70	70	70	70	70	70	70	70	70	70
Corporate Prices -40%	42	42	42	42	42	42	42	42	42	42	42
Corporate Prices +40%	98	98	98	98	98	98	98	98	98	98	98
Expected Forecast (USD/bbl)	55	59	63	65	67	68	68	69	69	70	70
Pessimistic Forecast (P10)	54	53	51	50	50	50	50	50	51	51	52
Optimistic Forecast (P90)	57	70	83	92	97	101	104	105	107	108	109
Production (MMbbl)	0	0.5	0.4	0.3	0.3	0.2	0.2	0.2	0.1	0.1	0.1
Costs (Million USD)	50	10	5	5	5	5	5	5	5	5	5

Managers make investment decisions using a variety of measures and analytics. For example, the break-even oil price for this project is 47 USD/bbl. At this flat price—simply estimated by trial and error, or using a goal seek function—the net present value of the project is zero. In addition, using corporate price scenarios the net present value of the project could be as low as -10 USD million or as high as 67 USD million. These results may well lead to the conclusion that "with high break-even price, there is a reasonable chance of negative net present value, while the upside potentials of the project are taken by the government".

However, the analytics in this paper depict a different picture. First, within our optimistic and pessimistic forecasts of prices (a range showing 80% of the forecasts) the net present value is positive. Second, the break-even flat price of 47 USD/bbl for the duration of the project, corresponds to just the sixth percentile of the sum-discounted prices. We can estimate this percentile by finding the sum-discounted value of the break-even price on the distribution of sum-discounted values from our simulation. Any flat price assumption above 47 USD/bbl leads to a positive net present value and the sum-discounted price distribution (figure 2) shows that such price scenarios make up 94% of the outcomes. Third, the government threshold limit of 80 USD/bbl corresponds to the 73rd percentile of the sum-discounted prices—again by finding the sum discounted value of threshold limit on the distribution of sum-discounted prices. The government admittedly cuts the project's upside potentials, but it also leaves a sizable part to the oil company.

If the annual price surpasses the threshold, then the government exercises its option and receives the excess revenue. This happens again next year with the next year's price. Effectively the government holds a compound call option on the project's annual revenue. Our earlier analyses simplified this flexible pattern.

A dynamic price model would better reveal the value of the project and its embedded option. In Figure 6, we show the distribution of project value based on cash flow estimates from the simulated price paths.

[10] Based on the capital asset pricing models, the discount rate should account for the time value of money and the (systematic) risk of a project. Because each project is unique, this calls for project-specific discount rates. In this example, we have assumed the systematic risk of the projects justifies using this discount rate, following the discussions in Titman and Martin (2015).

We also compare the percentiles of the value distribution with our forecast model as well as the corporate prices. The figure shows that values from corporate prices may be misleading, but our model leads to value estimates close to the results of Monte Carlo simulation.

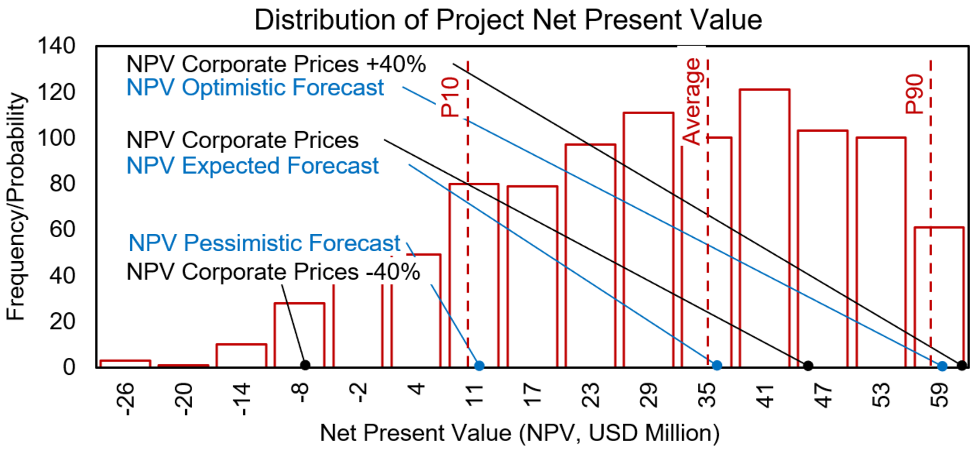

Figure 6—Distribution of project net present value (using simulation) and the net present values from six price scenarios

Overall, a probabilistic description of oil prices and their effect on decisions would normally be more informative than any arbitrary set of prices. While in an industrial setting a Monte Carlo simulation of project cash flows for every project may be impractical, we believe an informed sensitivity analysis could go a long way in supporting investment decisions.

In this paper we discuss two steps of analysis: 1) the sum-discounted distribution of prices for the forecast horizon, and 2) the "optimistic" and "pessimistic" forecasts, calculated from this distribution. Each step generates decision insights. For example, using the sum-discounted distribution, we can find the percentile for any price scenario. The optimistic and pessimistic forecasts also show an informed range for sensitivity analysis. Our method builds on an economically informed uncertainty model and generates forecasts that conform to this understanding. It is a major step forward in consistency.

Our method could further set up cross-project decision insight. We can apply the same price understanding (and uncertainty reasoning) to projects of various lengths. We generate tailor-made optimistic and pessimistic forecasts from a common price model that lead to comparable results.

5. Conclusions

Most projects appraisal models are intricate and complex. It would be hard to intuitively draw insights about the effect of uncertain factors on value. While no analysis can guarantee superior results, we can, however, assess the methods based on how they use the available information and conform to our understanding. Compared to other sensitivity analysis methods that ignore financial principles and theories of price behavior, our method is more consistent and more useful.

We develop percentiles on forecasts of prices rather than percentiles on prices. These are two different notions. For project appraisals we should use the former. Compared to the simplistic price descriptions and arbitrary scenarios common in practice, our model would be more informed and more useful in decision making.

In addition, compared to the burdens of a comprehensive simulation analysis, our method is simple to use and easy to communicate. By adjusting for project length, we can consistently apply our model to a variety of projects. This also makes our model suitable for comparative analyses across organizations, leading to consistency and transparency in valuations.

We focused on the effect of uncertain prices on investments. However, our method is general and applies to changes in value by any uncertain factor described as a stochastic process. For example, we could study the effect of fluctuating exchange rates on the value of international projects. Or study the effect of uncertain interest rates on value of coupon bonds. In any application, the analysis framework is the same: first, we use a stochastic model that conforms to our understanding about the behavior of the uncertain factor. Then, we estimate optimistic and pessimistic forecasts using the method in this paper. [11]

Supplementary Material

We implemented the analysis method of this paper in an MS Excel spreadsheet. It is available in the link below:

https://www.dropbox.com/s/89r9zvgzose9kw4/Data%20and%20Models_Simplified.xlsx?dl=0

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[11] I am grateful for the helpful comments and suggestions provided by Jim Dyer. I also appreciate the comments from the anonymous reviewers and the associate editor.

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Appendix A: Parameter Estimation for the Price Model

In this paper, we generate forecasts of a variable that are consistent with its stochastic model. We discuss that for informed sensitivity analyses, the stochastic model should first conform to our general understanding about the behavior of the variable. A crucial intermediate step would be to adjust the parameters of the stochastic process so that it reflects our understanding. For commodity prices, this usually means calibrating the parameters of the stochastic process so that it agrees with market observations and the perceived behavior of prices.

There are no ultimate solutions to parameter calibration. We could adjust a process to the historical data—assuming studying the past behavior of a variable helps predicting its future. Or we could calibrate a process to the market expectations (as implied by the prices of derivative contracts like futures and options). However, a good model always requires experts' judgement to be useful in decision making. In this paper, we estimate the parameters of our oil price model by fitting the futures and option volatility curves to the observed market information. We rely on the information from the derivative markets. We understand that this approach to calibration may not apply to all commodities or financial variables.

We use the following curve fitting procedure to estimate the parameters of the two-factor price process

$$\begin{aligned}\{\chi_t, \xi_t\} \in \underset{\chi_t, \xi_t}{\operatorname{argmin}} \sum_{i=1}^{N} & w_i\big(\ln \hat{F}_{0,i}(\chi_t, \xi_t, T_i, \mathbf{\Omega}) - \ln F_{0,i}\big)^2 \\ & + w_i \Big(Var\big(\ln \hat{F}_{0,i}(\chi_t, \xi_t, T_i, \mathbf{\Omega})\big) - Var\big(\ln F_{0,i}\big)\Big)^2 \\ \text{with } \mathbf{\Omega}&: \{\mu, \kappa, \sigma_\chi, \sigma_\xi, \rho\}\end{aligned} \qquad \text{B-1}$$

By minimizing the error between the market observations and the results of the model, we fit the futures and options volatility curves to the market observations. Here, $\hat{F}_{0,i}$ is the model's price for futures contract with maturity i, and $F_{0,i}$ is the observed market price at this same maturity. In addition, $Var\big(\ln \hat{F}_{0,i}\big)$ is the model's variance of the futures contract and $Var\big(\ln \hat{F}_{0,i}\big)$ is the implied variance of the options on the futures as seen in the market.

We assume there are N market observations across varying maturities. Fitting the curves of the model to the market observation leads to calibrating the parameters so that the model behaves like the market. We have also defined importance weights w_i to the error terms so that the user of the model could adjust the procedure: e.g., remove any outliers, or focus on fitting a specific part of the curves. Note that we do not estimate the price risk premiums, λ_χ and λ_ξ. Hedging instruments generally do not inform estimation of risk premiums.

A software model could estimate the parameters of the two-factor model by solving the following minimization problem.

$$\min_{\{\chi_0,\xi_0,\mathbf{\Omega}\}} \sum_{i=1}^{N} w_i\left(\ln \hat{F}_{0,i}(\chi_0,\xi_0,T_i,\mathbf{\Omega}) - \ln F_{0,i}\right)^2 + w_i\left(Var\left(\ln \hat{F}_{0,i}(\chi_0,\xi_0,T_i,\mathbf{\Omega})\right) - Var\left(\ln F_{0,i}\right)\right)^2 \qquad \text{B-2}$$

Here, we calculate model's futures prices $\hat{F}_{0,i}(\chi_0,\xi_0,T_i,\mathbf{\Omega})$ and their variance $Var\left(\ln \hat{F}_{0,i}(\chi_0,\xi_0,T_i,\mathbf{\Omega})\right)$. In addition, we calculate the implied variance of the futures prices, $Var\left(\ln F_{0,i}\right)$ by varying $\sigma_\varphi(T)$ so that the price of the option's contract converges to the observed price. The accompanying spreadsheet performs this parameter estimation process.

Note that parameter calibration is not an exact science and incompatible observations are common. Our models simplify and approximate the real world. The two-factor model is a narrow description of the crude oil prices, we may need even more factors to fully describe the behavior of prices. In addition, the assumptions of lognormal price distribution may not be proper. Commodity price distributions commonly have "fat tails"—i.e., the probability of experiencing extremely high or low prices is larger than what the normal distribution suggests. However, the goal in parameter calibration (and modeling in general) should not be to accurately describe price behavior but to support decisions. We believe using the two-factor model and implied parameter calibration in valuations generate useful decision insights.

Appendix B: Geometric Brownian Motion Process

Our approach is versatile and applies to a variety of underlying stochastic processes. In this appendix, we show the steps for generating optimistic and pessimistic forecasts if we assume that the spot prices follow a geometric Brownian motion process. Assume price S evolves in time t following the equation

$$dS = \mu S dt + \sigma_1 S dz \qquad \text{(B-1)}$$

Here μ is the drift rate, σ_1 is the volatility, and dz is the increment of the Brownian motion. The log of prices $x = \ln S$ follows a simple Brownian motion.

$$dx = \left(\mu - \frac{\sigma_1^2}{2}\right)dt + \sigma_1 dz \qquad \text{(B-2)}$$

If the current price is S_0, then the expected value $\mathrm{E}(\ln S_t)$ and variance $\mathrm{Var}(\ln S_t)$ will be

$$\mathrm{E}(\ln S_t) = \log S_0 + \left(\mu - \frac{\sigma^2}{2}\right)t \qquad \text{(B-3)}$$

$$\mathrm{Var}(\ln S_t) = \sigma^2 t \qquad \text{(B-4)}$$

For values $x_0 = 4$, $\mu = 0.5\%$, and $\sigma = 20\%$, Figure 7 shows the confidence bands for spot prices. We selected these parameters so that they describe price uncertainty comparable with our earlier analysis.

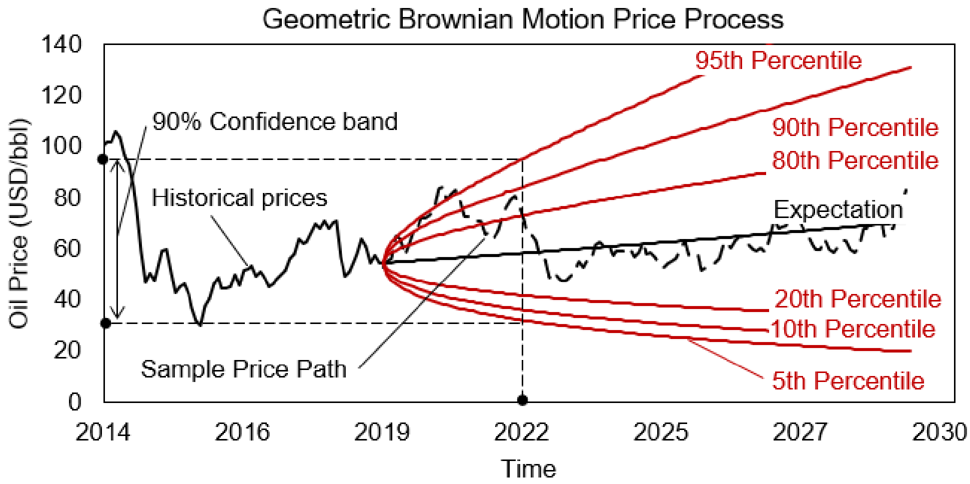

Figure 7—Confidence bands for spot prices assuming they follow a geometric Brownian motion process

The risk-neutral forecast (price of forward contracts) would be

$$\ln F_{0,T} = x_0 + (\mu - \lambda)T + \frac{1}{2}\sigma^2 T \tag{B-5}$$

In this equation, λ is the risk premium.

We follow our method to generate pessimistic and optimistic forecasts. We first simulate price paths using the equation

$$x_{t+\Delta t} = x_t + \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t} \tag{B-6}$$

Here, ε is the random variable that follows standard normal distribution and Δt is the time step.

Then, we estimate the distribution of sum-discounted prices and find its tenth and ninetieth percentiles. Finally, we adjust the variable λ in the equation B-5 so that its sum-discounted value matches the desired percentile of the sum-discounted simulated prices. Figure 8 shows the forecasts of prices matching the tenth and ninetieth percentiles.

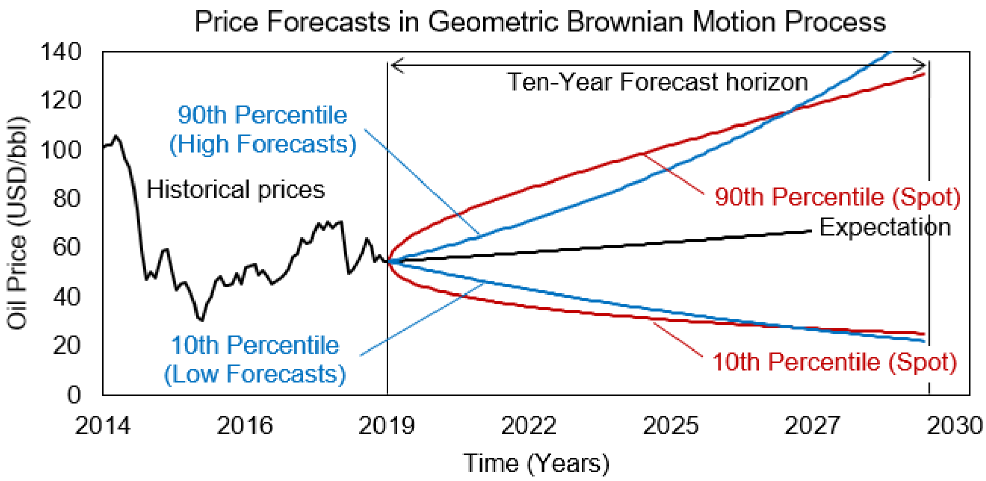

Figure 8—Pessimistic and optimistic price forecasts assuming spot prices follow a geometric Brownian motion

Note that the price dynamics are different. Forecasts in a geometric Brownian motion framework would be inevitably different from our earlier forecasts. Specifically, lack of mean-reversion would lead to the convex shape of the forecasts.[12] The expected price equation in the geometric Brownian motion framework (equation B-5), has fewer levers to work with. We adjusted λ to achieve the desired results.

[12] Studies (e.g., Lund, 1993) discuss the inconsistencies of the geometric Brownian motion in describing the commodities' price behavior.