



Heriot-Watt University  
Research Gateway

# Deriving the weights for aggregating judgments in a multi-group problem: an application to curriculum development in entrepreneurship

## Citation for published version:

Wasim, J, Vyas, V, Amenta, P, Lucadamo, A, Marcarelli, G & Ishizaka, A 2022, 'Deriving the weights for aggregating judgments in a multi-group problem: an application to curriculum development in entrepreneurship', *Annals of Operations Research*. <https://doi.org/10.1007/s10479-022-04649-9>

## Digital Object Identifier (DOI):

[10.1007/s10479-022-04649-9](https://doi.org/10.1007/s10479-022-04649-9)

## Link:

[Link to publication record in Heriot-Watt Research Portal](#)

## Document Version:

Peer reviewed version

## Published In:

Annals of Operations Research

## Publisher Rights Statement:

This is a post-peer-review, pre-copyedit version of an article published in Annals of Operations Research. The final authenticated version is available online at: <https://doi.org/10.1007/s10479-022-04649-9>

## General rights

Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

## Take down policy

Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [open.access@hw.ac.uk](mailto:open.access@hw.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.

# Deriving the weights for aggregating judgments in a multi-group problem: an application to curriculum development in entrepreneurship.

Jahangir Wasim<sup>a</sup> and Vijay Vyas<sup>b</sup> and Pietro Amenta<sup>c</sup> and Antonio Lucadamo<sup>c</sup> and Gabriella Marcarelli<sup>c</sup> and Alessio Ishizaka<sup>d</sup>

<sup>a</sup>Edinburgh Business School, Heriot-Watt University, Edinburgh, UK; <sup>b</sup>University of Portsmouth, Portsmouth, UK; <sup>c</sup>University of Sannio, Benevento, Italy; <sup>d</sup>NEOMA Business School, Mont-Saint-Aignan, France

## ARTICLE HISTORY

Compiled February 2, 2022

## Abstract

In group decisions, two issues need to be tackled: weighting opinions of different decision-makers and aggregating their evaluations. Many group aggregation techniques analyse these issues. These approaches can be correctly applied only if the weights assigned to all decision-makers are available. Unfortunately, there are situations where such weights are unavailable or incomplete, the negotiation required to better define them is not possible or decision-makers are unwilling to revise their judgments. These situations could pose a critical problem for the correct application of aggregation procedures. This problem is exaggerated if there are more than one group of decision-makers.

In this paper, we present a new algorithm based on the Frobenius norm that considers the choice of the weights in aggregating judgments in a non-negotiable multi-group problem. This approach facilitates the computation of several sets of weights simultaneously, showing the roles played by each decision-maker and by each group in defining the global priority. To illustrate the method, we apply it in designing a new curriculum in entrepreneurship based on an entrepreneurial learning approach informed by perceptions of three stakeholders: entrepreneurship educators, entrepreneurs and entrepreneurship students. Data is collected by pairwise compar-

ison within the Analytic Hierarchy Process and is aggregated using our proposed approach.

## **KEYWORDS**

Multi-Criteria, Decision Analysis, Group decision Making, Entrepreneurship Education

## **1. Introduction**

In decision problems, pairwise comparison judgements of decision-makers (DMs) are often collected in a decision matrix and preferences are then calculated (Ho and Ma, 2018; Koczkodaj et al., 2016). It is well known that DMs express different judgements based on their interests, experiences and knowledge of the problem (Ishizaka and Siraj, 2018). These differences in their judgements imply the presence of multiple groups with different stakes. In order to obtain an overall preference vector, it is necessary to aggregate various judgements, which then becomes a difficult problem. The aggregation can be done in several ways mainly based on two approaches: the aggregation of individual judgments in a synthesis pairwise comparison matrix, and the aggregation of individual preferences. The most popular method for aggregating individual judgments in a synthesis (also called consensus or common) matrix is the weighted geometric mean method which preserves the reciprocal property and satisfies the homogeneity condition (Aczel and Saaty, 1983). Ramanathan and Ganesh (1994) highlighted, though, that Pareto's principle could not be satisfied in some cases using this approach. For the individual preferences, the weighted arithmetic mean (Forman and Peniwati, 1998) is often used. For a comprehensive review of literature on the Group Aggregation Techniques (GAT)s based on pairwise comparison matrices, see Ossadnik et al. (2016).

There are many proposed GATs. However, decision-making problems involving groups of decision-makers (hereafter multi-group decision-making problem) seem to be analysed only recently in the literature (Aguaron et al., 2016; Moreno-Jimenez et al., 2016). The most intuitive and straightforward solution for addressing multi-group decision-making problem is to apply sequentially a selected GAT to each group, obtain a single preference matrix, then derive a global synthesis for all these group-

matrices (using the same or a different GAT) and finally compute the global multi-groups priority vector (Grošelj et al., 2015). This strategy is not based on any property of optimality so it could lead to an unsound or inappropriate solution. Moreover, the synthesis of each group does not take into account the links between the groups and a lack of mathematical optimality property means that the global synthesis matrix will not be the most representative of all the groups.

A new approach, mainly based on the aggregation of individual preferences and dealing with multi-group decision-making problems has been recently proposed (Amenta et al., 2020a). The authors introduce the Common Priority Vector Procedure which accentuates the majority group preference and diminishes the influence of extreme individual opinions. The method has been further extended to deal with multi-actor, multi-criteria and multi-group decisions structured at several nested decision levels. This approach provides an optimal property for the solution at each decision level.

We point out that all the GATs and the multi-group approaches can be applied only if all the DM weights for each group and all the group weights for obtaining the synthesis are available.

Unfortunately, one may find situations where the DM weights are unavailable or incomplete and negotiation amongst the DMs is not possible (e.g. the process of reviewing the judgments expressed may not be feasible from a logistical or geographical standpoint or the DMs may be unwilling to revise their judgments). For example, stakeholders are not available when anonymous surveys are used. Moreover, in a large survey the weight of a DM is also difficult to determine (Tang and Liao, 2021). This is the case involving passengers and transport companies (Duleba and Blahota, 2021).

Such a situation could become a critical problem for the correct application of aggregation procedures. This problem is exaggerated if we consider a multi-group decision making problem.

Few solutions exist to resolve this issue in the Analytic Hierarchy Process (AHP) literature (Ishizaka and Labib, 2011b; Bernasconi et al., 2014; Dong and Cooper, 2016; Scala et al., 2016; Amenta et al., 2021). Weights are usually assigned by a manager (or supra DM) or by a researcher. The simplest solution is to assume that DMs have

equal importance. The main drawback of this arbitrary assumption is that it does not assure the property of robustness and/or does not apply different weights for DMs with different experiences and competence. Moreover, we point out that DMs may not necessarily agree with every arbitrary set of weights assigned to their decisions by the manager. Whichever way the solution is arrived at, not including a robust property, will always be questionable.

In the absence of a manager, the group-members could themselves determine the weights as in Ishizaka and Labib (2011a). This approach requires that the DMs have to interact with one-other for comparing their subjective opinions, leading to the non-anonymity of the DMs. Another approach is to have a supra-decision-maker who subjectively assigns weights (Aly and Vrana, 2008). This method is seldom adopted because it is difficult to find a supra decision-maker who wants to take the responsibility and also knows the DMs from the different groups very well to evaluate them (Ramanathan and Ganesh, 1994).

We point out that in many real situations, like the study in this paper, there is a panel of anonymous DMs and (often) no supra DM. In such cases, none of the existing methods can be used. To our knowledge, there are very few solutions in the literature that can effectively address this issue. One such solution is Scala et al. (2016) approach for aggregating judgments when the DMs are unwilling to revise their decisions. They propose a *practical strategy* applying Principal Component Analysis (PCA) to the logarithm of the entries of the upper triangular preference matrices. Entries of the lower triangular side are not used. They derive a synthesis matrix that provides the DM weights according to the first PCA component. The final priority vector is then obtained by applying the usual Eigenvalue Method (EM) (Saaty, 1980) to the synthesis matrix. Before applying PCA, they use a preliminary log-transformation of the judgments to let their approach be equivalent to using a weighted geometric mean with the weights coming from the PCA. However, as this pretreatment of the data does not assure all positive values for the DM weights, Scala et al. (2016) also arbitrarily square them, losing in the process, the optimality property (the basis for their computations) of coefficients. This makes it a random solution for the DM weights. This has the same drawback, which is caused by assuming equal importance to all DMs.

Recently, Amenta et al. (2021) proposed a method to derive DM weights for aggregating judgments in non-negotiable AHP group-decision making. This situation arises frequently in public decisions. Users want the best services but are reluctant to pay a high price. Service providers (e.g. transport companies) aim to lower the costs to increase the profits. The local authorities want satisfied population but also wish to spend as little as possible (Duleba and Blahota, 2021).

The solution obtained by Amenta et al. (2021) is based on the Frobenius norm and a congruence measure between DMs. The main property of the weights that they derive is that they show the roles played by each DM in forming the consensus matrix and they are computed by taking into account the majority effect and the congruence (compatibility) between the DMs based on their judgments as a whole. These authors also apply a preliminary log-transformation of all the entries of the preference matrices. However, they well clarify the meaning and the use of negative weights in their approach. Indeed, the consensus matrix is always computed with positive weights. The first normalized right eigenvector of the consensus matrix provides the final priority vector of the group, according to the usual EM approach (Saaty, 1980).

Taking a cue from these approaches (Scala et al., 2016; Amenta et al., 2021), we develop a new formal method to compute simultaneously: a) different sets of coefficients, one for each group, for aggregating individual judgments of anonymous DMs (with no supra DM) in several consensus group preference matrices; and b) a set of weights for the groups to obtain a common multi-group preference matrix. These coefficients show again the roles played by each DM in forming the consensus group matrix and by each group in the common multi-group preference matrix. The computation of all these sets of coefficients is equivalent to the achievement of a synthesis judgment matrix for each group and a single global synthesis groups matrix, simultaneously. All these matrices show interesting properties and the common multi-group preference matrix provides a unique synthesis vector of preferences for the groups as a whole. The main idea is to give greater weight to decision-makers who are not indifferent to all the alternatives. This means that in the proposed method we maximise the internal variability with respect to the condition of the indifference of a DM/group. It should be noted that our approach is not a strategy based on the sequential application of a

method for aggregating individual judgments to each group and successively aggregating these synthesis matrices using the same method. All the sets of weights and all the synthesis matrices are simultaneously computed, satisfying a suitable mathematical criterion in such a way that each consensus matrix is the most congruent with those of the group. At the same time, the common multi-group matrix is the most congruent with those of groups.

We also use a preliminary base two log-transformation of all judgments (Ishizaka and Labib, 2011b) to assure all positive values for the weights of all DMs. Moreover, we show that our approach is equivalent to using a weighted geometric mean for each group as well as for the group of the synthesis matrices, respectively, with the weights that are simultaneously computed.

To validate our new proposed aggregation method, we evaluate the current status of entrepreneurship education in the UK and - drawing from the scholarly insights on entrepreneurial learning - attempt to provide an entrepreneurship education framework that mimics the real-life learning of entrepreneurs. For this, we distributed a survey to three groups: the entrepreneurs, the entrepreneurship educators, and the entrepreneurship students. The collection of data has been done with the pairwise comparison method AHP. The advantage is to be able to check for inconsistencies and discard invalid responses. The data has then been aggregated with the newly proposed group decision method. The rest of the paper is organised as follows: Section 2 shows the notation and basic definitions; Sections 3 and 4 introduce the new method and its main properties; in Section 5 we apply the method to design a new curriculum in entrepreneurship, finally some concluding remarks are provided in Section 6.

## 2. Notation and basic definitions

Let us consider a multi-group problem. Each single group  $g$  ( $g = 1, \dots, G$ ) is characterized by  $K_g$  judges ( $k = 1, \dots, K_g$ ) and by the same number  $n$  of attributes or alternatives. Consider, for every single group, the  $n \times n$  positive reciprocal matrix  $\mathbf{X}_k^g = ({}_k x_{ij}^g)$  where  ${}_k x_{ij}^g = 1/{}_k x_{ji}^g$  represents the judgment expressed by the  $k$ th decision-maker when comparing the  $i$ th with the  $j$ th alternative. The entries of the

matrix are expressed by using Saaty's fundamental scale (Saaty, 1980). Other scales are also available but the choice of the best judgement scale is not yet settled (Ishizaka et al., 2010; Ishizaka and Labib, 2011a).

It is well known that if  ${}_k x_{ij}^g = {}_k x_{is}^g {}_k x_{sj}^g$ , for each  $i, j, s \in \{1, \dots, n\}$ , then pairwise comparison matrix  $\mathbf{X}_k^g$  is perfectly consistent. To measure the consistency level of a judgment matrix, Saaty (1980) suggested the Consistency Ratio (CR), based on the difference between the maximum eigenvalue of the matrix and its order  $n$ . Moreover, he suggested the EM approach to derive the final priority values as well. Let us now consider the weighted geometric mean method (WGMM), which is the most common group preference aggregation method in the AHP. We write the WGMM common group preference aggregation matrix for the  $g$ th group  $\mathbf{X}_g^C = \odot_{k=1}^{K_g} (\mathbf{X}_k^g)^{\alpha_k} = \left( \prod_{k=1}^{K_g} {}_k x_{ij}^{g \alpha_k} \right)$ , where  $\odot$  is the Hadamard product and  $\alpha_k$  reflects the positive weight of the  $k$ th expert in forming the group opinion such that  $\sum_{k=1}^{K_g} \alpha_k = 1$ . The preference aggregation matrix of the (less performing) weighted arithmetic mean method (WAMM) is instead defined as  $\mathbf{X}^C = \sum_{k=1}^{K_g} \alpha_k \mathbf{X}_k^g = \left( \sum_{k=1}^{K_g} \alpha_k {}_k x_{ij}^g \right)$ .

We replace all the original comparison values with their base 2 logarithms (Ishizaka et al., 2010), that is  $z = \log_2(x+1)$  with  $x = \{1/9, 1/8, \dots, 1/2, 1, 2, \dots, 8, 9\}$ , avoiding negative values with the log transformation and facilitating the following calculations. Let us denote  ${}_k a_{ij} = {}_k x_{ij} + 1$  and  ${}_k z_{ij} = \log_2({}_k a_{ij})$ .  ${}_k a_{ij}$  values across all the DMs. We point out that the weighted arithmetic mean (noted  ${}_c z_{ij}$ ) of the log2-transformed data  ${}_k z_{ij}$  is equivalent to considering the base 2 logarithm of the weighted geometric mean of the  ${}_k a_{ij}$  values across all the DMs:

$${}_c z_{ij} = \sum_{k=1}^K \alpha_k {}_k z_{ij} = \sum_{k=1}^K \alpha_k \log_2({}_k a_{ij}) = \log_2 \left( \prod_{k=1}^K {}_k a_{ij}^{\alpha_k} \right) = \log_2(g_{ij})$$

where  $g_{ij} = \prod_{k=1}^K {}_k a_{ij}^{\alpha_k}$ . These identities imply that  $2^{{}_c z_{ij}} = g_{ij}$  and so the following optimization criteria can also be read in terms of a weighted geometric mean. The preliminary step of our approach is then to replace all the modified comparison values  ${}_k a_{ij}$  with their base-2 logarithms.

We denote by  $\|\mathbf{X}_k\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^n {}_k x_{ij}^2}$  the Frobenius norm, which is one of the



most used matrix norms in numerical analysis and has been already introduced in AHP literature. Indeed, this norm is at the heart of several approaches (Gass and Rapcsak, 2004; Bozóki and Lewis, 2003; Benítez et al., 2011, 2012; Lin et al., 2013; Benítez et al., 2015). The choice of the Frobenius norm was also supported by its reputation and properties (Gentle, 2007). To our knowledge, there is no mention of the use of any other matrix norms in AHP literature. Perhaps it is due to their lack of attractiveness caused by the computational and practical difficulties in AHP context. We use this norm in a different way taking advantage of a less recognized property: if we consider the judgments of the  $k$ th DM as centred with respect to global mean value then it could be considered as a measure (up to a constant) of the dispersion (variability) of the judgments based on the arithmetic mean (Saaty and Vargas, 2007). Without centring the judgments (as it happens in our approach), the squared Frobenius norm can also be viewed as a measure of the departure of the  $k$ th DM from the indifference condition in his evaluations ( ${}_k x_{ij} = 1, \forall i, j$ ) (Amenta et al., 2020a, 2021) implying that if a DM is indifferent then his/her dispersion is zero. So we can consider the squared Frobenius norm as a measure of the decisional power of the DM. We highlight that the basic idea of our approach is to compute a consensus preference matrix that has to be close to the underlying common judgment structure of the group, with the highest decisional power. It is then evident that it is possible to satisfy this purpose by maximizing the Frobenius norm of a linear combination of the preference DM matrices with suitable weights (Amenta et al., 2021). This result allows us to use this mathematical tool in the more complex case of multi-group decision-making problem as well.

### 3. A non-negotiable multi-group decision-making problem: The theory.

The computation of all sets of coefficients is equivalent to for  $G+1$  synthesis judgment matrices  $\mathbf{Z}_g^C = ({}_c z_{ij}^g)$  simultaneously. We have  $G$  matrices, each of which is representative of the  $K_g$  judgment matrices  $\mathbf{Z}_k^g = ({}_k z_{ij}^g)$  of the  $g$ th group. The last synthesis judgments matrix represents the  $G$  synthesis matrices  $\mathbf{Z}_g^C$ . This global matrix will provide a unique synthesis vector of preferences for the  $G$  groups as a whole, by taking into account the congruences between all pairs of judgment matrices within each group

(within-group congruence), likewise between the synthesis matrices of the  $G$  groups (between-groups congruence), simultaneously.

We name this approach "Multi-Group Decision-Making Problem" (hereafter MGDMP). This new procedure, based on judgment matrices, could be considered as a special case of a method named Stasis-4 (Sabatier and Vivien, 2008) and it is an extension of the method proposed by Amenta et al. (2021). This method analyses the multivariate structure of a four-way multi-block data set given by  $G$  sets ( $g = 1, \dots, G$ ) of  $p_g^k$  real variables ( $k = 1, \dots, K_g$ ) measured on  $n$  statistical units. These measures are all gathered into the matrices  $\mathbf{Y}_k^g$  of order  $n \times p_g^k$ , respectively. Stasis-4 is based on the proximity of scalar product operators  $\mathbf{W}_k^g = \mathbf{Y}_k^g \mathbf{Y}_k^{gT}$  computed for each matrix  $\mathbf{Y}_k^g$ . These scalar product operators reflect the proximities between the statistical units and they are used to compute the group similarities. We point out that this kind of operator cannot be used in the context of judgment matrices, due to the different characteristics and nature of  $\mathbf{Y}_k^g$  and  $\mathbf{Z}_k^g$ . For this reason, Stasis-4 cannot be so directly applied in the AHP context but its basic idea can be a seminal concept for facing our non negotiable multi-group decision-making problem.

Let  $\mathbf{Z}_g^C = \sum_{k=1}^{K_g} \alpha_k^g \mathbf{Z}_k^g$  be the synthesis judgment matrix of the  $g$ th group (g-SJM) ( $g = 1, \dots, G$ ) and  $\mathbf{Z}^C = \sum_{g=1}^G \mu_g \mathbf{Z}_g^C$  be the Global Synthesis Judgments Matrix (GSJM) of the  $G$  groups.  $\mathbf{Z}^C$  provides an underlying common judgment structure of all synthesis judgment matrices  $\mathbf{Z}_g^C$ .  $\mu_g$  reflects instead the weight of the  $g$ th synthesis judgment matrix and will assume the highest value when its synthesis structure is close to the underlying common judgment structure provided by  $\mathbf{Z}^C$ .

The  $G + 1$  weight vectors,  $\mathbf{a}_g = (\alpha_1^g, \dots, \alpha_{K_g}^g)^T$  and  $\mathbf{m} = (\mu_1, \dots, \mu_G)^T$ , are simultaneously obtained (see Fig. 1) subject to the constraint  $\mathbf{a}_g^T \mathbf{a}_g = \sum_{k=1}^{K_g} \alpha_k^{g2} = 1$  (with  $g = 1, \dots, G$ ) and  $\mathbf{m}^T \mathbf{m} = \sum_{g=1}^G \mu_g^2 = 1$ .

[Figure 1 about here.]

These weights have to maximise the squared Frobenius norm (Sabatier and Vivien, 2008) of  $\mathbf{Z}^C = \sum_{g=1}^G \sum_{k=1}^{K_g} \mu_g \alpha_k^g \mathbf{Z}_k^g$  and are solutions of the maximisation problem

attached to the following:

$$\begin{cases} \max_{\{\mathbf{a}, \mathbf{m}\}} \|\mathbf{Z}^C\|_F^2 = \max_{\{\alpha_k^g, \mu_g\}} \left\| \sum_{g=1}^G \sum_{k=1}^{K_g} \mu_g \alpha_k^g \mathbf{Z}_k^g \right\|_F^2 \\ \text{with} \quad \sum_{k=1}^{K_g} \alpha_k^g = 1 \\ \quad \quad \sum_{g=1}^G \mu_g^2 = 1 \end{cases} \quad (1)$$

where  $\|\mathbf{Z}^C\|_F^2$  can be now rewritten as

$$\begin{aligned} \|\mathbf{Z}^C\|_F^2 &= \text{tr}(\mathbf{Z}^C \mathbf{Z}^{C^T}) = \sum_{g=1}^G \sum_{g'=1}^G \mu_g \mu_{g'} \text{tr} \left( \sum_{k=1}^{K_g} \sum_{k'=1}^{K_{g'}} \alpha_k^g \alpha_{k'}^{g'} \mathbf{Z}_k^g \mathbf{Z}_{k'}^{g'T} \right) = \\ &= \sum_{g=1}^G \sum_{g'=1}^G \mu_g \mu_{g'} \text{tr}(\mathbf{Z}_g^C \mathbf{Z}_{g'}^{C^T}) = \sum_{g=1}^G \sum_{g'=1}^G \mu_g \mu_{g'} \check{c}_{gg'} = \mathbf{m}^T \check{\mathbf{C}} \mathbf{m} \end{aligned}$$

with  $\check{c}_{gg'} = \text{tr}(\mathbf{Z}_g^C \mathbf{Z}_{g'}^{C^T})$ . The solutions are then obtained by means of the Lagrangian function  $\mathfrak{F}$  given by

$$\mathfrak{F}(\mathbf{a}_g, \mathbf{m}) = \mathbf{m}^T \check{\mathbf{C}} \mathbf{m} - \frac{1}{2} \sum_{g=1}^G \lambda_g (\|\mathbf{a}_g\|^2 - 1) - \frac{1}{2} \eta (\|\mathbf{m}\|^2 - 1)$$

where  $\lambda_g$  and  $\eta$  are the Lagrange multipliers associated with the  $G$  constraints  $\mathbf{a}_g^T \mathbf{a}_g = \sum_{k=1}^{K_g} \alpha_k^g = 1$  and  $\mathbf{m}^T \mathbf{m} = \sum_{g=1}^G \mu_g^2 = 1$ , respectively.

By differentiating  $\mathfrak{F}(\mathbf{a}_g, \mathbf{m})$  with respect to  $\mathbf{m}$  and  $\mathbf{a}_g$ , and setting the results equal to zero, we obtain the following normal equations

$$\frac{\partial \mathfrak{F}(\mathbf{a}_g, \mathbf{m})}{\partial \mathbf{m}} = \sum_{g'=1}^G \sum_{k=1}^{K_g} \sum_{k'=1}^{K_{g'}} \mu_{g'} \alpha_k^g \alpha_{k'}^{g'} \text{tr}(\mathbf{Z}_k^g \mathbf{Z}_{k'}^{g'T}) - \eta \mu_g = 0 \quad (2)$$

$$\frac{\partial \mathfrak{F}(\mathbf{a}_g, \mathbf{m})}{\partial \mathbf{a}_g} = \sum_{g'=1}^G \sum_{k=1}^{K_g} \mu_g \mu_{g'} \alpha_{k'}^{g'} \text{tr}(\mathbf{Z}_k^g \mathbf{Z}_{k'}^{g'T}) - \lambda_g \alpha_k^g = 0 \quad (3)$$

$$\frac{\partial \mathfrak{F}(\mathbf{a}_g, \mathbf{m})}{\partial \eta} = \mathbf{m}^T \mathbf{m} - 1$$

$$\frac{\partial \mathfrak{F}(\mathbf{a}_g, \mathbf{m})}{\partial \lambda_g} = \mathbf{a}_g^T \mathbf{a}_g - 1$$

with  $g = 1, \dots, G$ . It is easy to verify that Equation (2) can be rewritten as  $\check{\mathbf{C}} \mathbf{m} = \eta \mathbf{m}$ ,

showing that  $\mathbf{m}$  is an eigenvector of  $\check{\mathbf{C}}$  such that  $\mathbf{m}^T \check{\mathbf{C}} \mathbf{m} = \eta$ . This result implies that if we consider the eigenvector  $\mathbf{m}$  of  $\check{\mathbf{C}}$  associated with the maximum eigenvalue  $\mu$ , then the global synthesis matrix  $\mathbf{Z}^C$  has maximal norm. Moreover, the property of the non-negativity of judgment matrices implies that  $\check{\mathbf{C}}$  is a semi-definite matrix. According to the Perron–Frobenius theorem (Rencher, 2002), all the weights  $\mu_g$  have then the same sign and so, for convenience, can always be considered positive.

By using a straightforward reformulation, Equations (2) and (3) allow obtaining the following four relations

$$\mu_g = [\text{vec}(\mathbf{Z}_g^C)^T \text{vec}(\mathbf{Z}^C)]/\eta = \text{tr}(\mathbf{Z}_g^C \mathbf{Z}^{C^T})/\eta \quad (4)$$

$$\lambda_g = \eta \times \mu_g^2 \quad (5)$$

$$\eta = \sum_{g=1}^G \lambda_g \quad (6)$$

$$\alpha_k^g = \text{tr}(\mathbf{Z}_k^g \mathbf{Z}^{C^T})/(\eta \mu_g) \quad (7)$$

for  $g = 1, \dots, G$  and  $k = 1, \dots, K_g$ .

The weight  $\mu_g$  of the  $g$ th synthesis judgment matrix  $\mathbf{Z}_g^C$  and the congruence between this matrix and the underlying common judgment structure are both given by (4); relations (5) and (6) show how  $\lambda_g$  provides the contribution of the  $g$ th group of decision-makers in forming the optimum value of  $\|\mathbf{Z}^C\|_F^2$ ; relation (7) provides the weight of the  $k$ th decision-maker within the  $g$ th group.

We highlight that it is not possible to obtain a solution for the maximisation problem (1) by a single diagonalisation of a positive semi-definite matrix. To obtain a numerical solution, we have then to implement an iterative algorithm to solve (1). This is developed by initialising all the  $\alpha_k^g$  coefficients to fixed values and computing every other quantity iteratively until a convergence criterion or for a fixed number of steps. The vector of optimal coefficients  $\mathbf{m}$  will be the first eigenvector of  $\check{\mathbf{C}}$  computed in each iteration.

According to Sabatier and Vivien (2008), the flow of the algorithm of the entire procedure can be listed as follows.

---

**Algorithm 1** Multi-Groups Decision Making Problem (MGDMP) procedure

---

**Input:** A multi-group set of  $G$  groups ( $g = 1, \dots, G$ ) of matrices  $\mathbf{Z}_k^g$  characterised by the same  $K$  judges ( $k = 1, \dots, K$ ) and by the same number  $n$  of attributes or alternatives.

**Output:** The final priority vector of the Multi-Group and those of each group.

---

▷ **Step 1: Starting step**

- 1: For each group, compute all the  $\alpha_k^g$  coefficients given by the first eigenvector of  $\mathbf{C}^g = (c_{kk'}^g)$ , where  $c_{kk'}^g = \text{tr}(\mathbf{Z}_k^g \mathbf{Z}_{k'}^{gT}) = \text{vec}(\mathbf{Z}_k^g)^T \text{vec}(\mathbf{Z}_{k'}^g) > 0$  with  $k, k' = 1, \dots, K_g$ , defining in this way  $G$  judgment matrices  $\mathbf{Z}_g^C = \sum_{k=1}^K \alpha_k^g \mathbf{Z}_k^g$ ; the matrix  $\check{\mathbf{C}}$  is computed too;
- 2: Compute the first eigenvector  $\mathbf{m}$  of  $\check{\mathbf{C}}$  associated to the first eigenvalue  $\eta$ ;
- 3: The global synthesis matrix for the  $G$  groups  $\mathbf{Z}^C = \sum_{g=1}^G \mu_g \mathbf{Z}_g^C$  is then defined;
- 4: Compute the contribution of the  $g$ th group of decision-makers in forming the optimum value  $\eta$  from relation (5);
- 5: Set the iterative counter  $\xi$  to 1 and the convergence criterion variable  $\varepsilon(\xi)$  to  $\eta$ ;

▷ **Step 2: Iterative step**

- 6: Increase the iterative counter  $\xi = \xi + 1$ ;
- 7: Compute the weight of the  $g$ th synthesis judgment matrix  $\mathbf{Z}_g^C$  from relation (7) and such that  $\sum_{k=1}^{K_g} (\alpha_k^g)^2 = 1$  for  $g = 1, \dots, G$  and  $k = 1, \dots, K_g$ ;
- 8: Define the  $G$  judgment matrices  $\mathbf{Z}_g^C = \sum_{k=1}^{K_g} \alpha_k^g \mathbf{Z}_k^g$  and all the entries  $c_{kk'}^g = \text{tr}(\mathbf{Z}_k^g \mathbf{Z}_{k'}^{gT})$  of matrix  $\check{\mathbf{C}}$  are computed;
- 9: Compute the first eigenvector  $\mathbf{m}$  of  $\check{\mathbf{C}}$  associated to the first eigenvalue  $\eta$ ;
- 10: Compute the global synthesis matrix for the  $G$  groups  $\mathbf{Z}^C = \sum_{g=1}^G \mu_g \mathbf{Z}_g^C$ ;
- 11: Compute the contribution of  $g$ th group of decision-makers in forming the optimum value  $\eta$  from relation (5);
- 12: Set the value of the convergence criterion variable  $\varepsilon(\xi)$  to  $\eta$ ;

▷ **Step 3: Convergence step**

- 13: **if**  $\{|\varepsilon(\xi) - \varepsilon(\xi-1)|/\varepsilon(\xi) \leq 10^{-6}$  **or**  $\xi = 50\}$  **then** {Go to Step 4}
- 14: **else** {Set  $\varepsilon(\xi) = \varepsilon(\xi-1)$  and Go to Step 2}

▷ **Step 4: Exit**

- 15: Return the final priority vector of the Multi-Group by computing the first normalised right eigenvector of global synthesis matrix for the  $G$  groups  $2^{\mathbf{Z}^C}$  according to the usual EM approach (Saaty, 1980);
- 16: Return also the priority vector of each group by computing the first normalised right eigenvector of synthesis matrix  $2^{\mathbf{Z}_g^C}$

▷ **Step 5: End**

---

#### 4. Main properties

We point out that applying the EM approach to matrix  $2^{\mathbf{Z}^C}$  is equivalent to computing the priority vector of the final geometric synthesis judgment matrix  $\mathbf{Z}_G^C =$

$\prod_{g=1}^G (\mathbf{A}_G^g)^{\mu_g}$  which is weighted geometric matrix of the same matrix of each group  $\mathbf{A}_G^g = \prod_{k=1}^{K_g} (\mathbf{A}_k^g)^{\alpha_k^g} = [{}_g a_{ij}^G]$  ( $g = 1, \dots, G$ ). Indeed, let  $2^{\mathbf{Z}^C} = [2^{z_{ij}^C}]$  be a matrix whose generic element is  $2^{z_{ij}^C}$  with  $\mathbf{A}_k^g = [{}_k a_{ij}^g]$ ,  $\mathbf{Z}_k^g = [{}_k z_{ij}^g]$ ,  $\mathbf{Z}_g^C = [{}_g z_{ij}^C]$ ,  $\mathbf{Z}^C = [z_{ij}^C]$  and  ${}_k z_{ij}^g = \log_2({}_k a_{ij}^g)$ ,  ${}_g z_{ij}^C = \sum_{k=1}^{K_g} \alpha_{k,k}^g z_{ij}^g$  and  $z_{ij}^C = \sum_{g=1}^G \mu_g ({}_g z_{ij}^C)$ . We obtain then the following identity

$$\begin{aligned} 2^{\mathbf{Z}^C} &= 2^{\sum_{g=1}^G \mu_g \mathbf{Z}_g^C} = 2^{\sum_{g=1}^G \mu_g \sum_{k=1}^{K_g} \alpha_k^g \mathbf{Z}_k^g} = 2^{\sum_{g=1}^G \mu_g \sum_{k=1}^{K_g} \alpha_k^g \log_2(\mathbf{A}_k^g)} \\ &= 2^{\sum_{g=1}^G \mu_g \log_2 \left[ \prod_{k=1}^{K_g} (\mathbf{A}_k^g)^{\alpha_k^g} \right]} = \prod_{g=1}^G 2^{\{\mu_g \log_2 \left[ \prod_{k=1}^{K_g} (\mathbf{A}_k^g)^{\alpha_k^g} \right]\}} \\ &= \prod_{g=1}^G \{2^{\log_2 \left[ \prod_{k=1}^{K_g} (\mathbf{A}_k^g)^{\alpha_k^g} \right]}\}^{\mu_g} = \prod_{g=1}^G \left[ \prod_{k=1}^{K_g} (\mathbf{A}_k^g)^{\alpha_k^g} \right]^{\mu_g} = \prod_{g=1}^G (\mathbf{A}_G^g)^{\mu_g}. \end{aligned}$$

An additional property of this approach is that the suitable  $G + 1$  coefficients  $\mathbf{a}_g = (\alpha_1^g, \dots, \alpha_{K_g}^g)^T$  and  $\mathbf{m} = (\mu_1, \dots, \mu_G)^T$  are such that GSJM  $\mathbf{Z}^C$  and each g-SJM  $\mathbf{Z}_g^C$  are the most congruent matrices with every  $\mathbf{Z}_g^C$  and every PCM  $\mathbf{Z}_k^g$ , respectively. Coefficients  $\alpha_k^g$  and  $\mu_g$  will then assume a high value when the  $ktm$  judgment structure  $\mathbf{Z}_k^g$  or the  $g$ th g-SJM  $\mathbf{Z}_g^C$  are close to the respective underlying common judgment structures and a value close to zero when they are near to the indifference matrix. This property, therefore, guarantees the high robustness of the algorithm.

## 5. Examples

It has been shown in literature that aggregation methods behave differently in cases of small and large-scale evaluators (Tang and Liao, 2021) and in small and high-dimensional pairwise comparison matrices/priority vectors (Duleba and Blahota, 2021). The method proposed in this paper is not a new aggregation approach. We essentially use the WGMM technique as basic method of aggregation, but our aim is to derive the weights (to apply the WGMM method) in a very complex problem involving many grouped pairwise comparison matrices. See also Scala et al. (2016) and Amenta et al. (2021) on this topic, likewise Duleba and Blahota (2021) for aggregating individual preference vectors of three conflicting stakeholder groups.

In the same way as when a new aggregation method is proposed, it is essential to examine the applicability of these chosen weights for few (e.g. two to ten) participants or stakeholder groups and for large (from ten up to even thousand) evaluators. Moreover, the method for deriving the weights might be well-applicable to small PCM-s but for as large as 9x9, maybe another method is better (examining the dimensions extends from two to nine in AHP). The ideal verification of the proposed approach would probably be a very large simulation, however, it would be beyond the objective of the current paper.

In this section we provide some examples in which small and large or high-dimensional (four and nine) matrices are presented and the global consensus is computed. Since the problem how to choose suitable weights, for aggregating non-negotiable multi-groups pairwise comparison matrices, seems to be new in the literature, as far as we know, the outcomes of our approach and those of a WGMM-based strategy are then compared. In this comparative strategy, WGMM is applied sequentially and independently to each group, a WGMM global synthesis for all these group-preference matrices is then achieved and finally the global multi-groups priority vector is computed using the EM method. Uniform weights are always assigned in each phase of this strategy.

For a comparison of both approaches, we use two indicators. The former investigates how the final synthesis matrix is correlated with the single preference matrices of DMs (hereafter  $RV_{PM}$ ). In the meantime, this indicator is also used to derive the correlation between the priority vectors of DMs of each group and the final aggregated priority vector (hereafter  $RV_{PV}$ ). The latter computes the compatibility or closeness between the preference vectors of the (individual) priority vectors of DMs in each group and the final aggregated priority vector (hereafter  $G_{PV}$ ). The averages of each indicator are used to find the best techniques, according to several choices of alternatives, number of DMs and groups.

The choice of the first indicator is based on the fact that, in our approach, we use the Hilbert-Schmidt scalar product between the preference matrices  $\mathbf{X}_k^g$ . These scalar products define the element of matrix  $\mathbf{C}^g = (c_{kk'}^g)$  that provides the weights of DMs of group  $g$  (see Step 1 of Algorithm 1). As a distance can be derived from this scalar

product,  $\mathbf{C}^g$  contains the similarities between the preference matrices  $\mathbf{X}_k^g$ . The same information is achieved if we choose also to use the  $RV$  coefficient (Escoufier, 1973)

$$RV(\mathbf{X}_k^g, \mathbf{X}_{k'}^g) = \frac{\text{tr}(\mathbf{X}_k^g \mathbf{X}_{k'}^{gT})}{\sqrt{\text{tr}(\mathbf{X}_k^g \mathbf{X}_k^{gT}) \text{tr}(\mathbf{X}_{k'}^g \mathbf{X}_{k'}^{gT})}} \quad (8)$$

which is equivalent to the cosine of the angle between  $\mathbf{X}_k^g$  and  $\mathbf{X}_{k'}^g$  and it is a kind of correlation coefficient between these matrices ( $0 \leq RV \leq 1$ ). The closer it is to 1, the closer are the configurations of elements of the matrices. It also allows to compute the correlation between a matrix and a vector. The  $RV$  coefficient applied to two vectors is equivalent to the usual Bravais-Pearson correlation coefficient.

To measure the compatibility or closeness between two preference vectors  $\mathbf{x}$  and  $\mathbf{y}$  we refer to the Garuti's index  $G$  (Garuti, 1973)

$$G(\mathbf{x}, \mathbf{y}) = \frac{1}{2} \sum_{i=1}^n \frac{\min(\mathbf{x}_i, \mathbf{y}_i)}{\max(\mathbf{x}_i, \mathbf{y}_i)} (\mathbf{x}_i + \mathbf{y}_i) \quad (9)$$

with ( $0 \leq G(\mathbf{x}, \mathbf{y}) \leq 1$ ). Also in this case, the closer it is to 1, the closer is the compatibility between the vectors  $\mathbf{x}$  and  $\mathbf{y}$ . Other coefficients can be computed (e.g. Saaty's compatibility index (1996), Hilbert's index). See Garuti (1973) for a wide comparison.

We consider eight scenarios listed in Table 1. Scenarios vary depending on the number of alternatives and/or the number of groups and/or the number of decision-makers for each group. For each scenario, we randomly generate transitive matrices (Gass, 1998; Amenta et al., 2020b), using the Saaty's scale. For the sake of simplicity, we show only the matrices considered in the first scenario (4 alternatives, 3 groups with 3 to 5 DMs, respectively).



Group 1

$$DM_1 = \begin{pmatrix} 1 & 1/3 & 1/4 & 3 \\ 3 & 1 & 1 & 2 \\ 4 & 1 & 1 & 5 \\ 1/3 & 1/2 & 1/5 & 1 \end{pmatrix} \quad DM_2 = \begin{pmatrix} 1 & 7 & 1 & 4 \\ 1/7 & 1 & 1/4 & 1 \\ 1 & 4 & 1 & 9 \\ 1/4 & 1 & 1/9 & 1 \end{pmatrix} \quad DM_3 = \begin{pmatrix} 1 & 3 & 2 & 9 \\ 1/3 & 1 & 1 & 2 \\ 1/2 & 1 & 1 & 7 \\ 1/9 & 1/2 & 1/7 & 1 \end{pmatrix}$$

Group 2

$$DM_1 = \begin{pmatrix} 1 & 1/6 & 2 & 1/4 \\ 6 & 1 & 7 & 2 \\ 1/2 & 1/7 & 1 & 1/5 \\ 4 & 1/2 & 5 & 1 \end{pmatrix} \quad DM_2 = \begin{pmatrix} 1 & 7 & 3 & 2 \\ 1/7 & 1 & 1/5 & 1/9 \\ 1/3 & 5 & 1 & 1/5 \\ 1/2 & 1/2 & 5 & 1 \end{pmatrix} \quad DM_3 = \begin{pmatrix} 1 & 3 & 9 & 7 \\ 1/3 & 1 & 6 & 1 \\ 1/9 & 1/6 & 1 & 1/2 \\ 1/7 & 1 & 2 & 1 \end{pmatrix}$$

$$DM_4 = \begin{pmatrix} 1 & 3 & 5 & 1/4 \\ 1/3 & 1 & 1/2 & 1/7 \\ 1/5 & 2 & 1 & 1/6 \\ 4 & 7 & 6 & 1 \end{pmatrix}$$

Group 3

$$DM_1 = \begin{pmatrix} 1 & 1/5 & 1/9 & 1/6 \\ 5 & 1 & 1/2 & 1/2 \\ 9 & 2 & 1 & 3 \\ 6 & 2 & 1/3 & 1 \end{pmatrix} \quad DM_2 = \begin{pmatrix} 1 & 5 & 8 & 1/2 \\ 1/5 & 1 & 5 & 1/5 \\ 1/8 & 1/5 & 1 & 1/8 \\ 2 & 5 & 8 & 1 \end{pmatrix} \quad DM_3 = \begin{pmatrix} 1 & 8 & 6 & 9 \\ 1/8 & 1 & 1/2 & 4 \\ 1/6 & 2 & 1 & 5 \\ 1/9 & 1/4 & 1/5 & 1 \end{pmatrix}$$

$$DM_4 = \begin{pmatrix} 1 & 4 & 1/5 & 2 \\ 1/4 & 1 & 1/2 & 1 \\ 5 & 5 & 1 & 6 \\ 1/2 & 1 & 1/6 & 1 \end{pmatrix} \quad DM_5 = \begin{pmatrix} 1 & 9 & 1/3 & 2 \\ 1/9 & 1 & 1/9 & 1 \\ 3 & 9 & 1 & 5 \\ 1/2 & 1 & 1/5 & 1 \end{pmatrix}$$

Tables 2-4 illustrate the main results of the  $RV$  and  $G(\mathbf{x}, \mathbf{y})$  coefficients by comparing our approach with the classical WGMM.

[Table 1 about here.]

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

Our method is very promising and it seems not to be influenced by the number of alternatives, decision makers and groups. This drawback appears instead in other well-proven existing approaches. Indeed, in all scenarios and for each index, our approach works best providing the most suitable compromise vectors: on varying the number

of the alternatives and/or DMs and/or groups, all indices calculated for our approach are closer to 1 than those obtained using the classical WGMM. The results are then very encouraging and a wider simulation and comparison will be the focus of further studies.

## **6. Case study**

### ***6.1. Background***

Entrepreneurship, a sub-domain of management science, deals with the venture creation process. The scholars in this field, however, do recognise that the entrepreneurial process is somewhat chaotic and complex, with no one linear explanation (Neck and Greene, 2011). It constitutes much more than just starting up a venture. There are two issues at the core of entrepreneurship education. The first is whether it can be taught (Fiet, 2001; Henry et al., 2005; Ronstadt, 1987). The current consensus on this is that though it is possible to teach entrepreneurship, due to its uniqueness and complexity, it is not easy to do so (Kuratko, 2005). The second is that social networks play a crucial role in the entrepreneurial process, and entrepreneurship cannot be separated from its social context, nor can its process be holistically studied without it (Birley, 1985; Drakopoulou Dodd and Anderson, 2007; Huggins et al., 2015; Korsgaard and Anderson, 2011; Thornton et al., 2011). It is also suggested that entrepreneurial learning is a socially created experiential process (Funken et al., 2018; Pittaway and Cope, 2007; Pittaway and Thorpe, 2012; Wang and Chugh, 2014). Based on this, we argue that entrepreneurship education should follow an entrepreneurial learning trajectory. However, a notable feature of the current teaching of entrepreneurship in higher education is that it is widely based on business plan development exercises (Carrier, 2007; Nabi et al., 2018; Solomon et al., 2002), even when this approach has been shown to be inflexible (Honig, 2004) and excludes the true nature of entrepreneurial learning. As a result, there is an obvious disconnect between how entrepreneurs learn their craft in real-life and how students learn entrepreneurship in universities. We argue that entrepreneurship education requires more than just a knowledge transfer and should be based on the processes that mirror real-life entrepreneurial learning that lead to the

development of entrepreneurship skills, behaviours and mindsets. In the light of the current entrepreneurial learning discourse, this cannot be done without the incorporation of social networks in entrepreneurship education. The entrepreneurship learning framework that we propose here draws from the findings of recent research on how entrepreneurship education is perceived by entrepreneurs, entrepreneurship educators and entrepreneurship students (Wasim, 2019).

The literature on entrepreneurship education and entrepreneurial learning have evolved independently of each other following a 'Kiosk' approach which is typical of scholarly discourse in entrepreneurship and elsewhere. The existing insights on this issue are based on empirical evidence that has come from the investigations involving stand-alone samples of either entrepreneurs, or educators or students. In a clear departure from this unhelpful trend, we triangulate our findings by collecting data from all three stakeholders of entrepreneurship education and entrepreneurial learning. This study aims to understand how entrepreneurship education in the UK is currently delivered to provide an enhanced framework of entrepreneurship education in which students learn in the universities as entrepreneurs do in the actual world. To achieve this aim, the following objectives are proposed:

- To understand the current models of entrepreneurship education in the UK and highlight their shortcomings.
- To understand the viewpoints of entrepreneurs, entrepreneurship educators and entrepreneurship students on entrepreneurial learning and entrepreneurship education.
- To propose a framework for entrepreneurship education which reflects the learning process of entrepreneurs.
- To use a new multi-group decision method based on pairwise comparison matrices to achieve the above objectives.

## ***6.2. Data collection***

A six-part questionnaire, based on the following issues was designed:

- What is important for an entrepreneur?

- What is the purpose of entrepreneurship education?
- What are the elements of entrepreneurship education?
- In your view, how entrepreneurship is being taught?
- How do entrepreneurs learn?
- How entrepreneurial learning can be enhanced in entrepreneurship education?

The data was collected in the period March-April 2019, by asking pairwise comparison questions, see extract in figure 2.

[Figure 2 about here.]

The survey was completed in a structured interview setting, where participants were explained the objectives, process and details of the research. There were three stakeholder groups: 6 entrepreneurs, 10 entrepreneurship educators and 19 entrepreneurship students. Purposive sampling was used for the entrepreneurs and entrepreneurship educators with the help of gatekeepers. Entrepreneurship students were recruited using entrepreneurship educators as gatekeepers. Selected entrepreneurs were at various stages of the development of their businesses and from a diverse range of age and industry backgrounds. Entrepreneurship educators were selected from 7 different universities in the UK. Entrepreneurship students were recruited through the entrepreneurship educators in the same universities so their responses relate to the same context as that of the chosen entrepreneurship educators.

### **6.3. Results**

The proposed method has two results (see section 3). First, it provides weights to the decision-makers (Table 1: Weights of the decision-makers) and second, determines the importance of each alternative. Table 6 shows the aggregated results. Detailed results can be seen in figures 3 to 8. From Table 5 it can be seen that students receive the highest weight because they have the highest stake in what they learn. The entrepreneurs have the lowest weight because their answers are close to being indifferent to all alternatives.

[Table 5 about here.]

The results are very similar across the three groups. We find some differences in funding, which seems important for students but not for the two other groups (Figures 3 and 5). Probably their inexperience in finding funding gives them the impression that it is an important factor.

[Figure 3 about here.]

Creativity and innovation are by far the most important criteria for educators (Fig. 4). Although ranked high by the other two groups too, it is not the most important criterion from their perspective. Probably because not in all sectors of entrepreneurship (i.e., IT company VS hairdresser) creativity and innovation are of paramount importance.

[Figure 4 about here.]

It is interesting to see that the environment is an important element of education for entrepreneurship for educators (5). This is not the case for students and to a lesser extent for entrepreneurs.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

Entrepreneurs believe that resilience development should be taught more in courses (8). This is also true for the two other groups but not to the same extent. Probably entrepreneurs went through difficult times and were not well equipped to overcome the crises.

[Figure 8 about here.]

[Table 6 about here.]

#### **6.4. Discussion**

Adaptability and networks are highlighted as the most valuable elements for entrepreneurial learning (Table 6) but are not well reflected in the practice of en-

trepreneurship education. Contextual awareness and adapting actions emerge as the elements critical for the entrepreneurial process and learning. Context and networks are strongly tied together. People have a more positive perception of other people that have commonalities with them, such as geographical and organisational context (Welter, 2011). Different contexts equip people with adaptability skills and shape their various attributes. Literature highlights that attributes of students may play a larger role in success as compared to the entrepreneurship curriculum (Turner and Gianiodis, 2018).

All groups have recognised the importance of networks. However, the current literature and empirical evidence reflect the presence of social networks quite low in the current framework of entrepreneurship education. This is a critical gap between the two streams of literature on entrepreneurship education and entrepreneurial learning. Literature does indicate that there is no one best way to teach entrepreneurship (Huq et al., 2017). Hence the inclusion of context in entrepreneurship education can be an important element.

There is an element of teamwork in current entrepreneurship teaching. However, the literature shows us that in most cases, the students do not fully immerse themselves in such teamworking networked settings and participate in such activities for the assessment purpose only (Pfaff and Huddleston, 2003; Wilson et al., 2018). Scrutinising the student teamwork process in the light of the seminal findings of Birley (1985) (Birley, 1985), it is obvious that such teamwork does not meet the requirements of being an entrepreneurial network because of the lack of student immersion in such a networked process.

Many entrepreneurship education courses claim to be following the concept of effectuation (Perry et al., 2012). Effectuation involves, starting with the resources and means a person holds, without knowing what the outcome of such a venture would be (Sarasvathy and Venkatamaran, 2011). However, in reality, without the incorporation of contextual understanding and networks, it is hard to see how effectuation can be applied.

This vindicates our above argument that the current delivery of entrepreneurship education does not include or reflect the true nature of entrepreneurial learning. This

confirmation is the most striking in the comparison between the last two columns of table 9 which contrasts the current entrepreneurship education in UK universities with what an entrepreneurship curriculum informed by entrepreneurial learning should have.

## 7. Conclusion

In this paper, we analyse the problem of giving weights for aggregating individual judgments of DMs or groups of DMs when a negotiation is not possible. This happens, for example, when there is a panel of anonymous DMs that cannot interact and there is not a supraDM or a researcher who assigns the weights. In these cases, the simplest solution is to assume that DMs have equal importance, but this arbitrary assumption does not assure the property of robustness and does not apply different weights for DMs with different experiences and competence. Furthermore, DMs may not necessarily agree with the [manager](#).

Our new proposal is based on the maximisation of the variability expressed by the Frobenius norm. The approach allows us to compute several sets of weights simultaneously, showing the role played by each decision-maker in forming the consensus group matrices and by each group in defining the global priority. Greater importance is given to the group (or DM) that has a clear preference. The objective is to take a decision and, therefore, to avoid indifference amongst the proposed alternatives. As we want to take a decision, it would not be useful to have decision-makers who are indifferent to all the alternatives.

In the proposed method the weights and the synthesis matrices, computed simultaneously, satisfy a suitable mathematical criterion such that each consensus matrix is the most congruent with those of the group and, at the same time, the common multi-group matrix is the most congruent with those of the groups.

This new method is then applied to a real problem of designing an entrepreneurship curriculum. Decisions such as those analysed here are complex, especially if they are based on multiple criteria and involve several decision-makers from diverse groups. For the situations, in which decision-makers are unwilling or unable to revise their

judgments to achieve a consensus, various techniques have been proposed in the extant literature to aggregate diverse views. However, in this problem, the weights of the decision-makers and the groups are not available. During our exercise, we used a modified AHP as explained in this paper. As the proposed technique presented here is generic, we believe, it can be applied, with appropriate adjustments, to other similar cases in future research. More importantly, it can be used in a wide variety of group decision making in business as well as non-business settings. Our study also vindicates our core argument that the current delivery of entrepreneurship education does not include or reflect the true nature of entrepreneurial learning. There is an obvious need for a reconceptualisation of entrepreneurship education in a way that better reflects the elements of entrepreneurial learning and entrepreneurial values away from a developing business plan approach.

The contribution of this research is in its use of a recently proposed conceptual framework that aligns the literature on entrepreneurship education and entrepreneurial learning and empirically tests it through an AHP evaluation of perceptions of different stakeholders on entrepreneurial teaching and learning. The findings, thus generated using a new group decision based on AHP, a robust analytical method, rarely used in entrepreneurship research contributes to curriculum building in entrepreneurship both at novel conceptual as well as methodological levels and leads to a more grounded curriculum for entrepreneurship education.

With regard to the real problem, we highlight three limitations of our study that can be used as a basis for future research. First, as mentioned above, entrepreneurship is context-dependent. Therefore, there is a need for replicating this research in other countries to highlight the role that different geographical and cultural contexts might play. Secondly, a longitudinal study can provide better insights into the evolution of entrepreneurship education over time. Finally, we encourage researchers to conduct more multi-group research involving different stakeholders in entrepreneurship education to provide a more holistic and robust conceptualisation of the field.

As far as the methodology is concerned, our approach proposes a new algorithm based on the Frobenius norm to derive the weights in aggregating judgments: it is complex and the computational complexity increases by increasing the complexity of the problem



(in terms of number of alternatives, DMs and groups). In a further work, we will focus on a wide simulation study and analyze how the weights may vary when using some other methods, based on judgments matrices.

## References

- Aczel, J. and Saaty, T. (1983). Procedures for synthesizing ratio judgements. *Journal of Mathematical Psychology*, 7:93–102.
- Aguaron, J., Escobar, M., and Moreno-Jiménez, J. (2016). The precise consistency consensus matrix in a local ahp-group decision making context. *Annals of Operations Research*, 245:245–259.
- Aly, S. and Vrana, I. (2008). Evaluating the knowledge, relevance and experience of expert decision makers utilizing the fuzzy-ahp. *Agricultural Economics-Czech*, 54:529–535.
- Amenta, P., Ishizaka, A., Lucadamo, A., Marcarelli, A., and Vyas, V. (2020a). Computing a common preference vector in a complex multi-actor and multi-group decision system in analytic hierarchy process context. *Annals of Operations Research*, (284):33–62.
- Amenta, P., Lucadamo, A., and Marcarelli, A. (2021). On the choice of weights for aggregating judgments in non-negotiable ahp group decision making. *European Journal of Operational Research*, 288(1):294–301.
- Amenta, P., Lucadamo, A., and Marcarelli, G. (2020b). On the transitivity and consistency approximated thresholds of some consistency indices for pairwise comparison matrices. *Information sciences*, 507:274–287.
- Benítez, J., Delgado-Galván, X., Izquierdo, J., and Pérez-García, R. (2011). Achieving matrix consistency in ahp through linearization. *Applied Mathematical Modelling*, (35):4449–4457.
- Benítez, J., Delgado-Galván, X., Izquierdo, J., and Pérez-García, R. (2012). An approach to ahp decision in a dynamic context. *Decision Support Systems*, (53):499–506.
- Benítez, J., Delgado-Galván, X., Izquierdo, J., and Pérez-García, R. (2015). Consistent completion of incomplete judgements in decision making using ahp. *Journal of Computational and Applied Mathematics*, (290):412–422.
- Bernasconi, M., Choirat, C., and Seri, R. (2014). Empirical properties of group preference aggregation methods employed in ahp: Theory and evidence. *European Journal of Operational Research*, 232(3):584–592.
- Birley, S. (1985). The role of networks in the entrepreneurial process. *Journal of Business*

- Venturing*, 1(1):107–117.
- Bozóki, S. and Lewis, R. (2003). Solving the least squares method problem in the ahp for  $3 \times 3$  and  $4 \times 4$  matrices. *Central European Journal of Operations Research*, (13):255–270.
- Carrier, C. (2007). Strategies for teaching entrepreneurship: What else beyond lectures, case studies and business plan? In Fayolle, A., editor, *Handbook of Research in Entrepreneurship Education*. Cheltenham, Massachusetts: Edward Elgar Publishing.
- Dong, Q. and Cooper, O. (2016). A peer-to-peer dynamic adaptive consensus reaching model for the group ahp decision making. *European Journal of Operational Research*, 250(2):521–530.
- Drakopoulou Dodd, S. and Anderson, A. R. (2007). Mumpsimus and the mything of the individualistic entrepreneur. *International Small Business Journal*, 25(4):341–360.
- Duleba, S. and Blahota, I. (2021). Determining optimal group weights for consensus creation in ahp for three conflicting stakeholder groups by vector distance minimization. *Journal of the Operational Research Society*.
- Escoufier, Y. (1973). Le traitement des variables vectorielles. *Biometrics*, 29:750–760.
- Fiet, J. O. (2001). The pedagogical side of entrepreneurship theory. *Journal of Business Venturing*, 16(2):101–117.
- Forman, E. and Peniwati, K. (1998). Aggregating individual judgments and priorities with the analytic hierarchy process. *European Journal of Operational Research*, pages 165–169.
- Funken, R., Gielnik, M. M., and Foo, M. D. (2018). How can problems be turned into something good? the role of entrepreneurial learning and error mastery orientation. *Entrepreneurship Theory and Practice*, pages 1–24.
- Garuti, C. (1973). A set theory justification of garuti’s compatibility index. *Journal of Multi-criteria Decision Analysis*, 27:50–60.
- Gass, S. (1998). Tournaments, transitivity and pairwise comparison matrices. *The Journal of the Operational Research Society*, 49(6):616–624.
- Gass, S. and Rapcsak, T. (2004). Singular value decomposition in ahp. *European Journal of Operational Research*, 154:573–584.
- Gentle, J. E. (2007). *Matrix algebra. Theory, computations, and applications in statistics*. New York: Springer.
- Grošelj, P., Zadnik Stirn, L., Ayrilmis, N., and Kuzman, M. (2015). Comparison of some aggregation techniques using group analytic hierarchy process. *Expert Systems with Applications*, 42(4):2198–2204.

- Henry, C., Hill, F., and Leitch, C. (2005). Entrepreneurship education and training: can entrepreneurship be taught? part i. *Education + Training*, 47(2):98–111.
- Ho, W. and Ma, X. (2018). The state-of-the-art integrations and applications of the analytic hierarchy process. *European Journal of Operational Research*, 267(21):399–414.
- Honig, B. (2004). Entrepreneurship education: Toward a model of contingency-based business planning. *Academy of Management Learning & Education*, 3(3):258–273.
- Huggins, R., Izushi, H., Prokop, D., and Thompson, P. (2015). Network evolution and the spatiotemporal dynamics of knowledge sourcing. *Entrepreneurship & Regional Development*, 27(7–8):474–499.
- Huq, A., Gilbert, D., Huq, A., and Gilbert, D. (2017). All the world’s a stage: transforming entrepreneurship education through design thinking. *Education + Training*, 50(2):155–170.
- Ishizaka, A., Balkenborg, D., and Kaplan, T. (2010). Influence of aggregation and measurement scale on ranking a compromise alternative in ahp. *Journal of the Operational Research Society*, 62(4):700–710.
- Ishizaka, A. and Labib, A. (2011a). Review of the main developments in the analytic hierarchy process. *Expert Systems with Applications*, 38(11):14336–14345.
- Ishizaka, A. and Labib, A. (2011b). Selection of new production facilities with the group analytic hierarchy process ordering method. *Expert Systems with Applications*, 38(6):7317–7325.
- Ishizaka, A. and Siraj, S. (2018). Are multi-criteria decision-making tools useful? an experimental comparative study of three methods. *European Journal of Operational Research*, 264(216):462–471.
- Koczkodaj, W., Mikhailov, L., Redlarski, G., Soltys, M., Szybowski, J., Tamazian, G., Wajch, E., and Yuen, K. (2016). Important facts and observations about pairwise comparisons. *Fundamenta Informaticae*, 144(3–4):291–307.
- Korsgaard, S. and Anderson, A. R. (2011). Enacting entrepreneurship as social value creation. *International Small Business Journal*, 29(2):135–151.
- Kuratko, D. F. (2005). The emergence of entrepreneurship education: development, trends, and challenges. *Entrepreneurship Theory and Practice*, 29(5):577–598.
- Lin, C., Kou, G., and Ergu, D. (2013). A heuristic approach for deriving the priority vector in ahp. *Applied Mathematical Modelling*, (37):5828–5836.
- Moreno-Jimenez, J., Salvador, M., Gargallo, P., and Altuzarra, A. (2016). Systemic decision making in ahp: A bayesian approach. *Annals of Operations Research*, 245:261–284.

- Nabi, G., Walmsley, A., Liñán, F., Akhtar, I., and Neame, C. (2018). Does entrepreneurship education in the first year of higher education develop entrepreneurial intentions? the role of learning and inspiration. *Studies in Higher Education*, 43(3):452–467.
- Neck, H. M. and Greene, P. G. (2011). Entrepreneurship education: Known worlds and new frontiers. *Journal of Small Business Management*, 49(1):55–70.
- Ossadnik, W., Schinkel, S., and Kaspar, R. (2016). Group aggregation techniques for analytic hierarchy process and analytic network process: A comparative analysis. *Group Decision and Negotiation*, 25:421–457.
- Perry, J. T., Chandler, G. N., and Markova, G. (2012). Entrepreneurial effectuation: A review and suggestions for future research. *Entrepreneurship Theory and Practice*, 36(4):837–861.
- Pfaff, E. and Huddleston, P. (2003). Does it matter if i hate teamwork? what impacts student attitudes toward teamwork. *Journal of Marketing Education*, 25(1):37–45.
- Pittaway, L. and Cope, J. (2007). Simulating entrepreneurial learning: Integrating experiential and collaborative approaches to learning. *Management Learning*, 38(2):211–233.
- Pittaway, L. and Thorpe, R. (2012). A framework for entrepreneurial learning: A tribute to jason cope. *Entrepreneurship & Regional Development*, 24(9/10):837–859.
- Ramanathan, R. and Ganesh, L. (1994). Group preference aggregation methods employed in ahp: An evaluation and an intrinsic process for deriving members’ weightages. *European Journal of Operational Research*, 79:249–265.
- Rencher, A. (2002). *Methods of Multivariate Analysis*. New York, John Wiley & Sons.
- Ronstadt, R. (1987). The educated entrepreneurs: A new era of entrepreneurial education is beginning. *American Journal of Small Business*, 11(4):37–53.
- Saaty, T. (1980). *The Analytic Hierarchy Process*. McGraw Hill, New York.
- Saaty, T. (1996). *The Analytic Network Process*. Pittsburgh, PA: RWS Publications.
- Saaty, T. and Vargas, L. (2007). Dispersion of group judgments. *Mathematical and Computer Modelling*, 46:918–925.
- Sabatier, R. and Vivien, M. (2008). A new linear method for analyzing four-way multiblock tables: Stasis-4. *Journal of Chemometrics*, 22(6):399–407.
- Sarasvathy, S. and Venkatamaran, S. (2011). Entrepreneurship as method: Open questions for an entrepreneurial future. *Entrepreneurship: Theory and Practice*, 35(1):113–135.
- Scala, N., Rajgopal, J., Vargas, L., and Needy, K. (2016). Group decision making with dispersion in the analytic hierarchy process. *Group Decision and Negotiation*, 25:355–372.
- Solomon, G. T., Duffy, S., and Tarabishy, A. (2002). The state of entrepreneurship edu-

- cation in the united states: A nationwide survey and analysis. *International Journal of Entrepreneurship Education*, 1(1):1–22.
- Tang, M. and Liao, H. (2021). From conventional group decision making to large-scale group decision making: What are the challenges and how to meet them in big data era? a state-of-the-art survey. *Omega*, 100:102–141.
- Thornton, P. H., Ribeiro-Soriano, D., and Urbano, D. (2011). Socio-cultural factors and entrepreneurial activity: An overview. 29(2), 105-118. *International Small Business Journal*, 29(2):105–118.
- Turner, T. and Gianiodis, P. (2018). Entrepreneurship unleashed?: Understanding entrepreneurial education outside of the business school. *Journal Of Small Business Management*, 56(1):131–149.
- Wang, C. L. and Chugh, H. (2014). Entrepreneurial learning: Past research and future challenges. *International Journal of Management Review*, 16(1):24–61.
- Wasim, J. (2019). *Closing the gap between university curriculum in Entrepreneurship and Entrepreneurial Learning in Networks: An interpretivist constructivist approach*. PhD thesis, University of Portsmouth.
- Welter, F. (2011). Contextualizing entrepreneurship-conceptual challenges and ways forward. *Entrepreneurship: Theory and Practice*, 35(1):165–184.
- Wilson, L., Ho, S., and Brookes, R. H. (2018). Student perceptions of teamwork within assessment tasks in undergraduate science degrees. *Assessment and Evaluation in Higher Education*, 43(5):786–799.

**Table 1.** Description of different scenarios analyzed in the study

| Scenario | Alternatives<br>( $n$ ) | Groups<br>( $G$ ) | Decision Makers<br>( $K_g$ ), $g = 1, \dots, G$ |       |       |       |       |       |
|----------|-------------------------|-------------------|---|-------|-------|-------|-------|-------|
|          |                         |                   | $K_1$   | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ |
| 1        | 4                       | 3                 | 3   | 4     | 5     | -     | -     | -     |
| 2        | 4                       | 3                 | 10  | 15    | 20    | -     | -     | -     |
| 3        | 9                       | 3                 | 3   | 4     | 5     | -     | -     | -     |
| 4        | 9                       | 3                 | 10  | 15    | 20    | -     | -     | -     |
| 5        | 4                       | 6                 | 3   | 3     | 4     | 4     | 5     | 5     |
| 6        | 4                       | 6                 | 10  | 10    | 15    | 15    | 20    | 20    |
| 7        | 9                       | 6                 | 3   | 3     | 4     | 4     | 5     | 5     |
| 8        | 9                       | 6                 | 10  | 10    | 15    | 15    | 20    | 20    |

Legend -  $n$ ,  $G$  and  $K_g$ : number of alternatives, groups and decision makers in the group  $g$ , respectively.

**Table 2.**  $RV$  and  $G(\mathbf{x}, \mathbf{y})$  coefficients for scenarios 1-4.

| Scenario | Index                       | g | MGDMP | WGMM |
|----------|-----------------------------|---|-------|------|
| 1        | $RV(PM)$                    | 1 | 0.88  | 0.76 |
|          |                             | 2 | 0.79  | 0.63 |
|          |                             | 3 | 0.83  | 0.70 |
|          | $RV(PV)$                    | 1 | 0.85  | 0.81 |
|          |                             | 2 | 0.71  | 0.60 |
|          |                             | 3 | 0.75  | 0.65 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.69  | 0.66 |
|          |                             | 2 | 0.58  | 0.50 |
|          |                             | 3 | 0.63  | 0.54 |
| 2        | $RV(PM)$                    | 1 | 0.82  | 0.68 |
|          |                             | 2 | 0.81  | 0.66 |
|          |                             | 3 | 0.81  | 0.67 |
|          | $Rv(PV)$                    | 1 | 0.76  | 0.68 |
|          |                             | 2 | 0.75  | 0.63 |
|          |                             | 3 | 0.75  | 0.66 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.61  | 0.55 |
|          |                             | 2 | 0.61  | 0.52 |
|          |                             | 3 | 0.62  | 0.55 |
| 3        | $RV(PM)$                    | 1 | 0.79  | 0.71 |
|          |                             | 2 | 0.79  | 0.70 |
|          |                             | 3 | 0.79  | 0.65 |
|          | $RV(PV)$                    | 1 | 0.71  | 0.60 |
|          |                             | 2 | 0.73  | 0.61 |
|          |                             | 3 | 0.72  | 0.52 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.60  | 0.52 |
|          |                             | 2 | 0.61  | 0.53 |
|          |                             | 3 | 0.61  | 0.48 |
| 4        | $RV(PM)$                    | 1 | 0.78  | 0.69 |
|          |                             | 2 | 0.78  | 0.67 |
|          |                             | 3 | 0.78  | 0.67 |
|          | $RV(PV)$                    | 1 | 0.73  | 0.60 |
|          |                             | 2 | 0.73  | 0.59 |
|          |                             | 3 | 0.73  | 0.60 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.61  | 0.52 |
|          |                             | 2 | 0.61  | 0.51 |
|          |                             | 3 | 0.61  | 0.52 |

$RV(PM)$ = Average of the  $RV$  coefficients between  $\mathbf{X}_k^g$  and  $\mathbf{Z}^C$  ;

$RV(PV)$ = Average of the  $RV$  coefficients between priority vectors of DMs of each group and the final aggregated priority vector;

$G(\mathbf{x}, \mathbf{y})$ = Average of Garuti's coefficients between (individual) priority vectors of DMs in each group and the final aggregated priority vector.

**Table 3.**  $RV$  and  $G(\mathbf{x}, \mathbf{y})$  coefficients for scenarios 5-6.

| Scenario | Index                       | g | MGDMP | WGMM |
|----------|-----------------------------|---|-------|------|
| 5        | $RV(PM)$                    | 1 | 0.79  | 0.64 |
|          |                             | 2 | 0.79  | 0.66 |
|          |                             | 3 | 0.79  | 0.64 |
|          |                             | 4 | 0.86  | 0.75 |
|          |                             | 5 | 0.80  | 0.66 |
|          |                             | 6 | 0.85  | 0.72 |
|          | $RV(PV)$                    | 1 | 0.71  | 0.57 |
|          |                             | 2 | 0.69  | 0.58 |
|          |                             | 3 | 0.72  | 0.63 |
|          |                             | 4 | 0.80  | 0.74 |
|          |                             | 5 | 0.72  | 0.62 |
|          |                             | 6 | 0.78  | 0.66 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.58  | 0.47 |
|          |                             | 2 | 0.59  | 0.49 |
|          |                             | 3 | 0.58  | 0.52 |
|          |                             | 4 | 0.63  | 0.57 |
|          |                             | 5 | 0.59  | 0.52 |
|          |                             | 6 | 0.65  | 0.55 |
| 6        | $RV(PM)$                    | 1 | 0.81  | 0.66 |
|          |                             | 2 | 0.81  | 0.64 |
|          |                             | 3 | 0.82  | 0.68 |
|          |                             | 4 | 0.81  | 0.67 |
|          |                             | 5 | 0.80  | 0.66 |
|          |                             | 6 | 0.80  | 0.65 |
|          | $RV(PV)$                    | 1 | 0.74  | 0.62 |
|          |                             | 2 | 0.76  | 0.64 |
|          |                             | 3 | 0.75  | 0.66 |
|          |                             | 4 | 0.75  | 0.65 |
|          |                             | 5 | 0.74  | 0.64 |
|          |                             | 6 | 0.73  | 0.63 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.61  | 0.52 |
|          |                             | 2 | 0.63  | 0.53 |
|          |                             | 3 | 0.60  | 0.53 |
|          |                             | 4 | 0.62  | 0.54 |
|          |                             | 5 | 0.59  | 0.52 |
|          |                             | 6 | 0.60  | 0.51 |

$RV(PM)$ = Average of the  $RV$  coefficients between  $\mathbf{X}_k^g$  and  $\mathbf{Z}^C$  ;

$RV(PV)$ = Average of the  $RV$  coefficients between priority vectors of DMs of each group and the final aggregated priority vector;

$G(\mathbf{x}, \mathbf{y})$ = Average of Garuti's coefficients between (individual) priority vectors of DMs in each group and the final aggregated priority vector.



**Table 4.**  $RV$  and  $G(\mathbf{x}, \mathbf{y})$  coefficients for scenarios 7-8.

| Scenario | Index                       | g | MGDMP | WGMM |
|----------|-----------------------------|---|-------|------|
| 7        | $RV(PM)$                    | 1 | 0.78  | 0.70 |
|          |                             | 2 | 0.78  | 0.71 |
|          |                             | 3 | 0.76  | 0.64 |
|          |                             | 4 | 0.80  | 0.69 |
|          |                             | 5 | 0.79  | 0.70 |
|          |                             | 6 | 0.77  | 0.64 |
|          | $RV(PV)$                    | 1 | 0.72  | 0.61 |
|          |                             | 2 | 0.71  | 0.59 |
|          |                             | 3 | 0.70  | 0.53 |
|          |                             | 4 | 0.74  | 0.60 |
|          |                             | 5 | 0.74  | 0.61 |
|          |                             | 6 | 0.72  | 0.56 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.61  | 0.53 |
|          |                             | 2 | 0.61  | 0.52 |
|          |                             | 3 | 0.60  | 0.48 |
|          |                             | 4 | 0.62  | 0.53 |
|          |                             | 5 | 0.62  | 0.53 |
|          |                             | 6 | 0.61  | 0.51 |
| 8        | $RV(PM)$                    | 1 | 0.77  | 0.67 |
|          |                             | 2 | 0.77  | 0.68 |
|          |                             | 3 | 0.77  | 0.66 |
|          |                             | 4 | 0.77  | 0.67 |
|          |                             | 5 | 0.78  | 0.67 |
|          |                             | 6 | 0.77  | 0.67 |
|          | $RV(PV)$                    | 1 | 0.71  | 0.56 |
|          |                             | 2 | 0.73  | 0.59 |
|          |                             | 3 | 0.73  | 0.58 |
|          |                             | 4 | 0.72  | 0.58 |
|          |                             | 5 | 0.73  | 0.60 |
|          |                             | 6 | 0.72  | 0.58 |
|          | $G(\mathbf{x}, \mathbf{y})$ | 1 | 0.60  | 0.50 |
|          |                             | 2 | 0.61  | 0.52 |
|          |                             | 3 | 0.62  | 0.53 |
|          |                             | 4 | 0.61  | 0.52 |
|          |                             | 5 | 0.61  | 0.52 |
|          |                             | 6 | 0.61  | 0.51 |

$RV(PM)$ = Average of the  $RV$  coefficients between  $\mathbf{X}_k^g$  and  $\mathbf{Z}^C$  ;

$RV(PV)$ = Average of the  $RV$  coefficients between priority vectors of DMs of each group and the final aggregated priority vector;

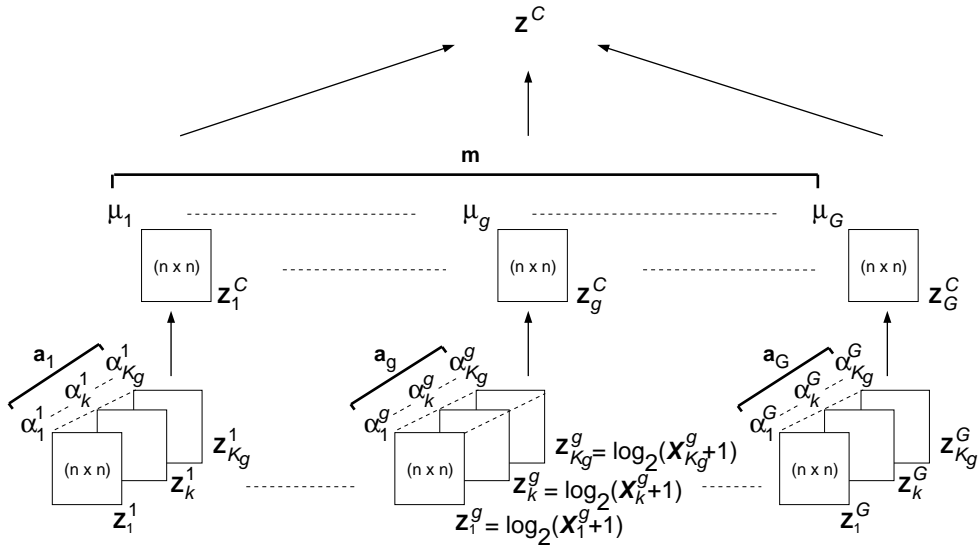
$G(\mathbf{x}, \mathbf{y})$ = Average of Garuti's coefficients between (individual) priority vectors of DMs in each group and the final aggregated priority vector.

**Table 5.** Weights of the decision-makers

|  | Entrepreneurs | Students | Entrepreneurship<br>educators |
|--|---------------|----------|-------------------------------|
| Entrepreneurial value  | 0.176         | 0.543    | 0.281                         |
| Purpose of entrepreneurship<br>education                       | 0.180         | 0.534    | 0.286                         |
| Elements of entrepreneurship<br>education                      | 0.186         | 0.528    | 0.286                         |
| Source of entrepreneurship<br>education                        | 0.174         | 0.528    | 0.298                         |
| Current entrepreneurship ed-<br>ucation                        | 0.176         | 0.520    | 0.304                         |
| From entrepreneurial learning<br>to entrepreneurship education | 0.194         | 0.527    | 0.279                         |

**Table 6.** Aggregated results

| Rank | Entrepreneurial Values    | Purpose of EE                       | Elements                            | Sources of EL                       | Current EE                        | From EL to EE                       |
|------|---------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-----------------------------------|-------------------------------------|
| 1    | Adaptability (0.167)      | Creativity & Innovation (0.195)     | Experience building (0.195)         | Mistakes (0.218)                    | Teamwork (0.182)                  | Resilience development (0.188)      |
| 2    | Networks (0.161)          | Opportunity recognition (0.190)     | Cross-disciplinary teamwork (0.175) | Network/social interactions (0.187) | Business Plan development (0.178) | Business simulations (0.167)        |
| 3    | Experience (0.143)        | Resource understanding (0.173)      | Venture knowledge (0.164)           | Experience (0.179)                  | Brainstorming activities (0.150)  | Cross-disciplinary teamwork (0.159) |
| 4    | Resilience (0.140)        | Employability (0.165)               | Context understanding (0.162)       | Environment / context (0.154)       | Resilience development (0.138)    | Brainstorming activities (0.150)    |
| 5    | Venture knowledge (0.136) | Preparing for uncertainty (0.164)   | Funding opportunity (0.155)         | Role Models (0.150)                 | Consultancy projects (0.129)      | Teamwork-same discipline (0.131)    |
| 6    | Funding (0.135)           | Increasing start-up numbers (0.113) | Resilience building (0.149)         | Academic qualification (0.112)      | Network exploitation (0.123)      | Role playing (0.108)                |
| 7    | Environment (0.118)       |                                     |                                     |                                     | Role playing (0.100)              | Case-studies (0.097)                |



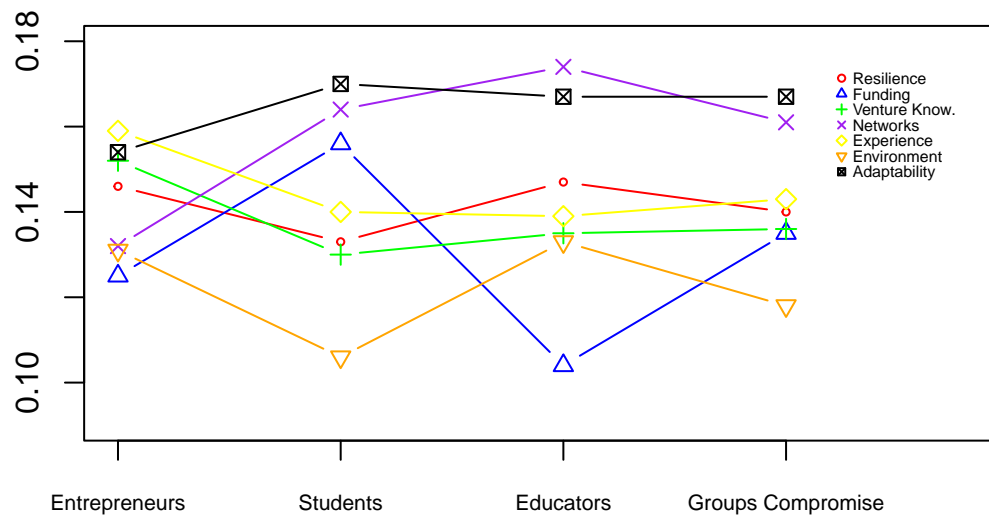
**Figure 1.** The  $G + 1$  weight vectors  $\mathbf{a}_g = (\alpha_1^g, \dots, \alpha_{K_g}^g)^T$  and  $\mathbf{m} = (\mu_1, \dots, \mu_G)^T$ , and the  $G + 1$  synthesis matrices  $\mathbf{Z}_g^C$  and  $\mathbf{Z}^C$  that have to be all computed simultaneously and not sequentially (with  $g = 1, \dots, G$ ).

### What is important for an entrepreneur?

Please consider the following comparisons and assess the level of difference in traits, characteristics or outcome scale below. Please circle one number per row.

|                         | Extremely  | Very | Strongly | Moderately                                       | Equally | Moderately | Strongly  | Very | Extremely |   |   |   |   |   |   |   |   |                         |
|-------------------------|--|------|----------|--|---------|------------|---|------|-----------|---|---|---|---|---|---|---|---|-------------------------|
|                         | The trait X is <b>strongly</b> (or five times) higher than the trait Y |      |          | The trait X is <b>equal</b> than that of trait Y |         |            | The trait Y is <b>moderately</b> (or three times) higher than the trait X |      |           |   |   |   |   |   |   |   |   |                         |
| Product X               | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Product Y               |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (B) Funding             |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (C) Venture Knowledge   |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (D) Networks            |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (E) Experience          |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (F) Environment/context |
| (A) Resilience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |
| (B) Funding             | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (C) Venture Knowledge   |
| (B) Funding             | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (D) Networks            |
| (B) Funding             | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (E) Experience          |
| (B) Funding             | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (F) Environment/context |
| (B) Funding             | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |
| (C) Venture Knowledge   | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (D) Networks            |
| (C) Venture Knowledge   | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (E) Experience          |
| (C) Venture Knowledge   | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (F) Environment/context |
| (C) Venture Knowledge   | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |
| (D) Networks            | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (E) Experience          |
| (D) Networks            | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (F) Environment/context |
| (D) Networks            | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |
| (E) Experience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (F) Environment/context |
| (E) Experience          | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |
| (F) Environment/context | 9  | 8    | 7        | 6  | 5       | 4          | 3   | 2    | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | (G) Adaptability        |

Figure 2. Extract of the questionnaire



**Figure 3.** Results on what is important for an entrepreneur?

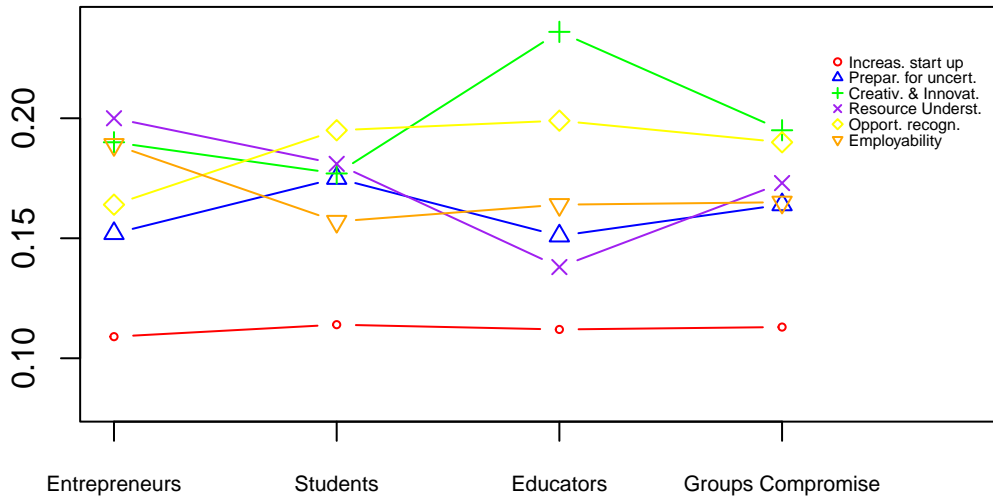


Figure 4. Results on what is the purpose of entrepreneurship education?

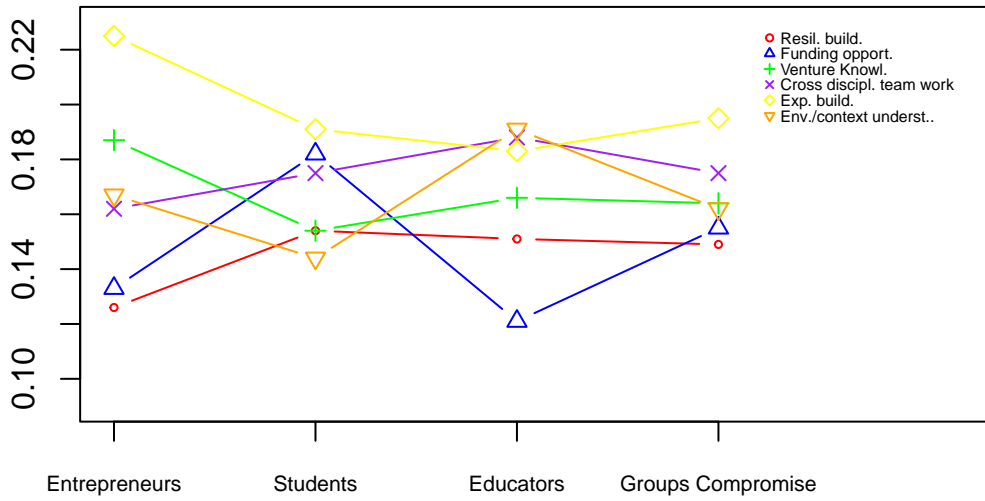
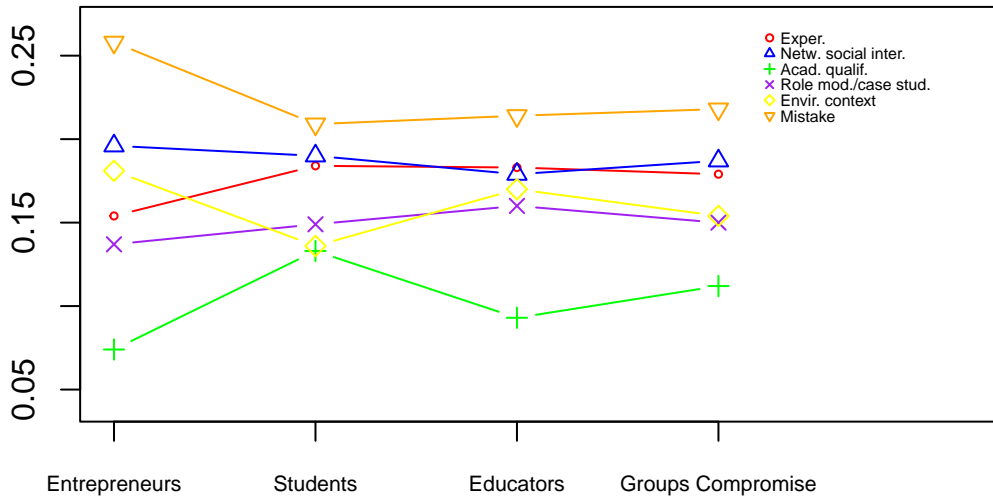


Figure 5. Results on what are the elements of entrepreneurship education?





**Figure 6.** Results on how entrepreneurship is being taught

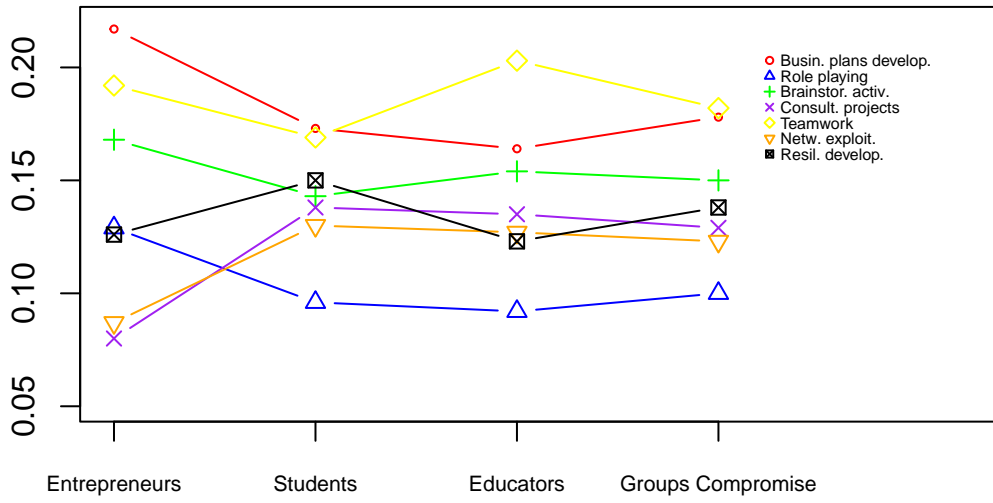


Figure 7. Results on how do entrepreneurs learn?

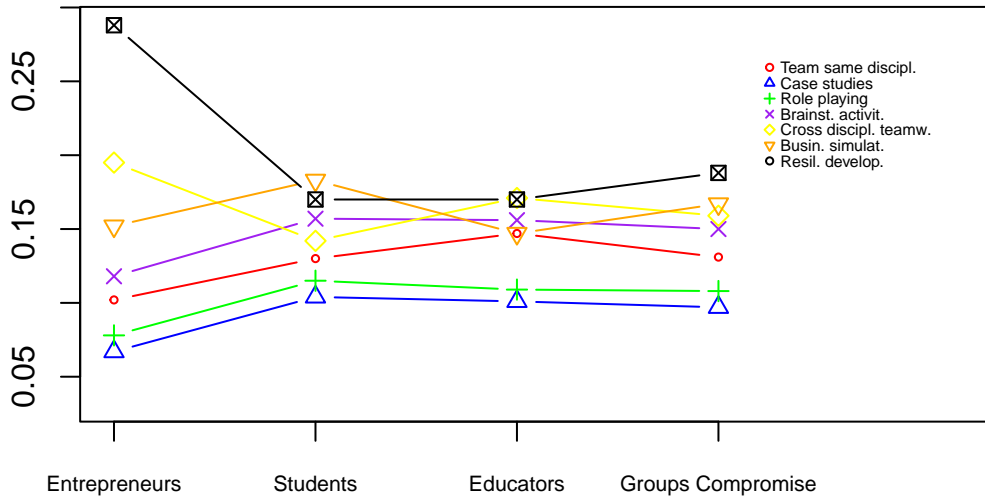


Figure 8. Results on how entrepreneurial learning can be enhanced in entrepreneurship education?