Visualizing probability density distribution of polarization speckle with electron cloud diagrams

Proceedings Volume 12126, Fifteenth International Conference on Correlation Optics; 121260N (2021)
https://doi.org/10.1117/12.2615510

PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Visualizing probability density distribution of polarization speckle with electron cloud diagrams

Zang, Mengmeng, Liu, Xin, Dai, Jun, Wang, Ying, Wang, Wei

Mengmeng Zang, Xin Liu, Jun Dai, Ying Wang, Wei Wang, "Visualizing probability density distribution of polarization speckle with electron cloud diagrams," Proc. SPIE 12126, Fifteenth International Conference on Correlation Optics, 121260N (20 December 2021); doi: 10.1117/12.2615510

SPIE.

Event: Fifteenth International Conference on Correlation Optics, 2021, Chernivtsi, Ukraine

Visualizing probability density distribution of polarization speckle with electron cloud diagrams

Mengmeng Zang, [a] Xin Liu, [a] Jun Dai, [a] Ying Wang, [a] and Wei Wang [b]
[a] Department of Physics, Jiangsu University of Science and Technology, Zhenjiang,212003, P.R. China
[b] School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, UK

ABSTRACT

In this paper, a three-dimensional representation of the probability density distribution of the Stokes parameters in polarization speckle was developed. Following the electron cloud model used widely to visualize the hydrogen atomic orbitals, we present the density of the dots sculptured in the Hilbert space shows the probability density of finding the Stokes parameters in polarization speckle. Advantages of the electron cloud presentation for the statistics of the Stokes parameters are also discussed.

Keywords: Polarization speckle, electron cloud, Von Neumann's rejection method, probability density distribution

1. INTRODUCTION

Since continuous-wave lasers became commercially available in the early 1960s, extensive studies have been made on their basic properties and applications of laser speckle. In the majority of studies on speckle phenomena, these random optical fields have been treated as scalar optical fields, and the main interest has been in the statistical properties and applications of the intensity distribution of the speckle patterns. Recently, statistical properties of random electric vector fields have come to attract new interest of researchers because of their importance in wide areas of practical applications such as biology and metrology [1–3].

The statistical properties and spatial structure of the Stokes parameters in speckle fields have been investigated by Fercher and co-workers [4–7]; the statistics of partially polarized light has been studied by Barakat [8,9]; the correlation properties of the stochastic electric fields have been studied [10–14]; the polarization analyses of speckle field with applications to the surface and bulk scattering have been widely investigated [15–17]. The studies on polarization of the light scattered by a rough-surfaced birefringent material have been conducted to reveal their isotropic and anisotropic statistics for the generated polarization speckle [18,19].

In a variety of applications related to polarization speckle, the resultant electric field consists of a sum of random polarization phasors with random states of polarization and a constant background with a known polarization state. Such is the case in polarization holography with a polar interferometer, where the constant polarization background field corresponds to the reference beams with known polarization states, and the random polarization phasor sum corresponds to the electric field scattered from the object[20].In addition, when the polarization speckle is generated by diffusion from a rough birefringent surface smaller than the wave length, the diffusion electric field should normally consist of a constant mirror with a transmittance of a constant polarization state [18,19]. The phenomenon of polarization speckle is an important

Fifteenth International Conference on Correlation Optics, edited by Oleg V. Angelsky, Proc. of SPIE Vol. 12126, 121260N
© 2021 SPIE · 0277-786X · doi: 10.1117/12.2615510

problem in some polarization imaging systems and polarization sensitive optical coherence tomography because the determination of image quality depends fundamentally on the coherence of the probe light used in the imaging process. After considering a series of cases, we conclude that knowledge of the statistical properties of the synthesized randomly polarized field is of theoretical and practical significance.

In this paper, the three-dimensional representation of Stokes parameter probability density distribution in polarized speckle is studied. Based on the known probability density of Stokes parameter in polarized speckle, we give the density of the points carved in Hilbert space by referring to the model of hydrogen atom electron cloud, and show the variation of Stokes parameter in polarized speckle.

2. STOCHASTIC SIMULATION

The motion of points in Hilbert space is random, and electron cloud graph is usually used to intuitively describe the randomness of points, that is, the density of points is used to reflect the probability of electrons appearing somewhere. Monte Carlo method has excellent performance in dealing with stochastic problems, so this method is used for simulation. Using Monte Carlo method to deal with the problem in this paper, first of all, according to the law of the problem to be dealt with, the Stokes parameter probability density distribution in the polarization speckle must be random sampling test, so as to get a set of random number sequence according to the known distribution; Finally, according to this random number sequence, the probability density of finding stokes parameters in the polarized speckle is displayed by MATLAB.

To find the spherical expression for our own probability density [20]in Hilbert space, we start from

$$p(S_1,S_2,S_3)=\frac{\exp\left\{-(\sqrt{S_1^2+S_2^2+S_3^3}+E_0^2)/(2\sigma^2)\right\}}{16\pi\sigma^4\sqrt{S_1^2+S_2^2+S_3^2}}I_0(\sigma^{-2}E_0\sqrt{(\sqrt{S_1^2+S_2^2+S_3^2}+S_1)/2}). \quad (1)$$

Based on $\sqrt{S_1^2+S_2^2+S_3^2}=r$, $S_1=r\cos\varphi\sin\theta$ where $\theta\in[0,\pi]$, $\varphi\in[0,2\pi)$, we have

$$p(r,\theta,\varphi)=\frac{r\exp\left\{-(r+E_0^2)/(2\sigma^2)\right\}}{16\pi\sigma^4}I_0(\sigma^{-2}E_0\sqrt{(r+r\cos\varphi\sin\theta)/2})\sin\theta. \quad (2)$$

Let fix $E_0=[0, 2, 4, 8]$

Now, let's consider another case of electron cloud with asymmetric structure[20]

$$p(r,\theta,\varphi)=(16\pi\sigma_x^2\sigma_y^2r)^{-1}\exp\left\{-(\sigma_x^{-2}-\sigma_y^{-2})r\cos\varphi\sin\theta/4\right\}\exp\left\{-(\sigma_x^{-2}+\sigma_y^{-2})r/4\right\} \quad (3)$$

Let fix $\sigma_x=1$, $\sigma_y=[0.01, 0.1, 0.2, 0.5, 0.8, 1]$.

By random sampling according to the Stokes parameter probability density function in the polarization speckle provided, the random number sequence of probability density distribution is obtained, and the density of the carved points in Hilbert space is obtained. A prerequisite to any Monte Carlo simulation is the ability to produce in abundance random numbers distributed according to some probability density. In the Monte Carlo method, there are many methods to achieve random sampling according to the known distribution. According to the characteristics of the problem to be dealt with in this paper, Von Neumann's rejection method[21-24] is adopted, We wish to construct random samples from the density $p(r,\theta,\varphi)$.

It is known that the meaning of the probability density function is that the probability of taking a number is P, but the sum of the probabilities of all events is 1, so the global integral of the probability density function is 1. Therefore, we put

forward the following inference:

$$\int_0^{\infty} p dx = 1. \tag{4}$$

Let a goes to 0, $a \to 0$,
then

$$\int_0^{a} p dx \ll 1. \tag{5}$$

Similarly, let b goes to ∞, $b \to \infty$,
then

$$\int_b^{\infty} p dx \ll 1. \tag{6}$$

The effect of $\int_0^a p dx \ll 1$ and $\int_b^{\infty} p dx \ll 1$ on the overall probability density function is small enough to be negligible. Therefore, the probability density function with interval $[a,b]$ as boundary is used to replace the probability density function without boundary, so as to meet the preconditions of Von Neumann's rejection method.

The probability density function provided in this paper is a three-dimensional case, but when the multidimensional problem is encountered, it should be simplified to a series of one-dimensional sampling problems to solve. Consider randomly extracting vector points from the sphere in three dimensions, using polar coordinates, $\theta \in [0,\pi]$, $\varphi \in [0,2\pi)$.

Let $p(r,\theta,\varphi)$ be a bounded density function and assume $p(r,\theta,\varphi)$ vanishes outside the interval (a,b). We wish to construct samples from the density $p(r,\theta,\varphi)$. Let $M=max_{p(r,\theta,\varphi)}$, M is the maximum Stokes parameter probability density function $p(r,\theta,\varphi)$ in polarized speckle and define

$$p_1(r,\theta,\varphi)=p(r,\theta,\varphi)/M\ . \tag{7}$$

so that $0 \le p_1(r,\theta,\varphi) \le 1$, generate a pair $(r_i, \theta_i, \varphi_i)$ of random numbers. And interpret $(a+\theta_i (b-a), \varphi_i)$ as a point in a rectangle with base $(b-a)$ and height 1. Use the Rand function to generate a random number H evenly distributed between 0 and 1. Compare $p_1(r,\theta,\varphi)$ with H, if $p_1(r,\theta,\varphi) \ge H$, then the point $(r_i, \theta_i, \varphi_i)$ is selected; if not, then the point $(r_i, \theta_i, \varphi_i)$ is omitted. In other words, If this point falls below the graph of $p_1(r,\theta,\varphi)$, then the point is accepted as the sample of $p(r,\theta,\varphi)$; if not, the previous process is repeated until sufficient sample points are successfully identified.

The efficiency of Von Neumann's rejection method, as measured by the fraction of pairs (p_1, p_2) which are not rejected, is just the ratio of the area under the curve $p_1(x)$ to the area of the enclosing rectangle. This ratio is clearly $1/[M(b-a)]$, and since this efficiency is properly ≤ 1.

A further point worth mentioning is that, in order to use the rejection method, it is necessary to find $M=max_{p(r,\theta,\varphi)}$, or at least an upper bound for $p(r,\theta,\varphi)$. If only a weak upper bound for $p(r,\theta,\varphi)$ can be found, the efficiency of the rejection method will suffer correspondingly [23].

The density of the points carved in Hilbert space can be obtained by using the random number sequence of $p(r_i, \theta_i, \varphi_i)$ distribution obtained and programming with MATLAB software.

3. RESULTS AND DISCUSSION

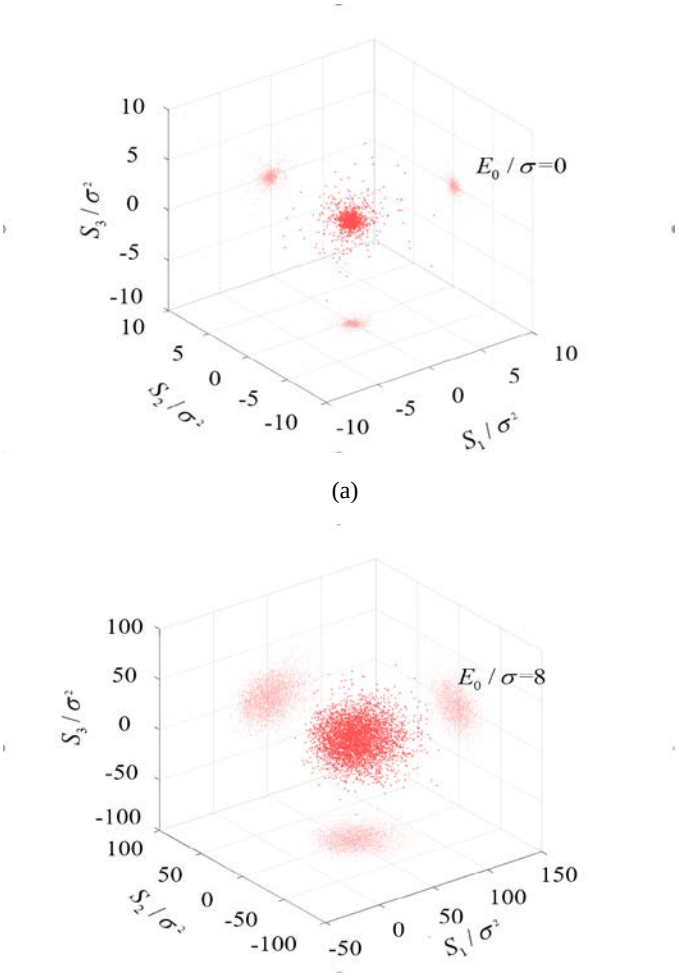
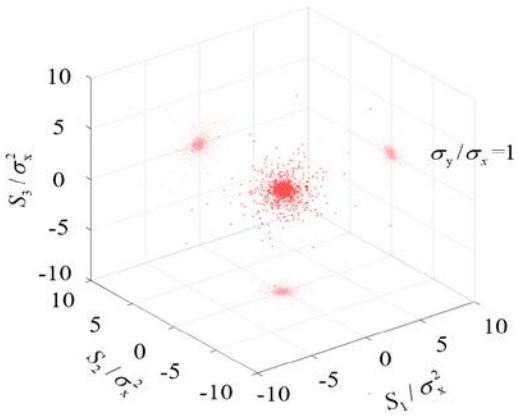
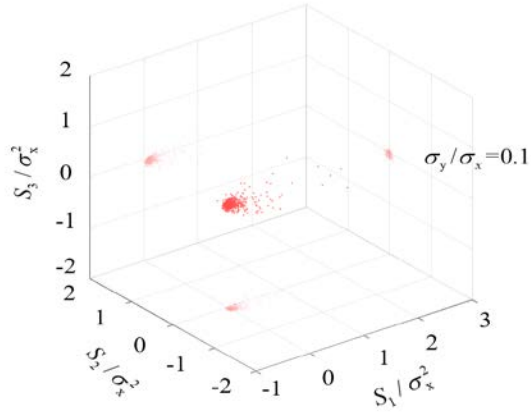


Fig. 1. Electron cloud map of Stokes parameter probability density function $p(r,\theta,\varphi)$ in homogeneous polarized speckles for (a) $E_0/\sigma=0$, (b) $E_0/\sigma=8$.

Figure 1 is the electron cloud map obtained by changing E_0/σ in the probability density function of Stokes parameter in the polarization speckle with the same dimension coordinate axis. The results show that the method of Von Neumann's rejection method is feasible. It can be seen that with the increase of E_0/σ, the electron cloud of Stokes parameter probability density function gradually migrates to the positive half axis of the S_1-axis, that is, the center point of the electron cloud gradually moves away from the origin. In addition, the electron cloud will become more and more expansive. At the same time, Figure 1 shows a three-dimensional projection to give the reader a better idea of the shape of the electron cloud.



(c)



(d)

Fig. 2. Electron cloud map of Stokes parameter probability density function $p(r,\theta,\varphi)$ in homogeneous polarized speckles for (c) $\sigma_x/\sigma_y=1$, (d) $\sigma_y/\sigma_x=0.1$.

Figure 2 is a further study of Figure 1 (a). Figure 2 is the electron cloud obtained by changing the Stokes parameter probability density function $p(r,\theta,\varphi)$ in the polarization speckle with the same dimension coordinate axis. It can be seen that with the decrease of σ_x/σ_y, the electron cloud of Stokes parameter probability density function will be inclined to the positive axis direction of S_1-axis, and then gradually change from spherical to ellipsoid, and then approximate to a line.

4. CONCLUSION

In this paper, we use the known Stokes parameter probability density function in the polarization speckle, and physical visualization of the Stokes parameter probability density with the help of MATLAB software. Above to adopt ways of intuitive see under the random field of gaussian assumptions of polarization of the electron clouds of the Stokes parameters of speckle briefly shape, but also simplifies the complexity of the problem, these results can be used as the theoretical inference further suppose that also can be regarded as the traditional laser speckle further development and extension of

figure reasoning. In addition, Von Neumann's selection method adopted in this paper can be applied to other probability density functions of the same type, and the electronic cloud graphs of other probability density functions can be constructed according to the ideas provided in this paper.

We hope that this visualization will give readers a deeper understanding of the evolution of the polarization states of the random electric field, and interested readers will use the methods provided in this article to build their own electron clouds.

REFERENCES

[1] J. C. Dainty, Laser Speckle and Related Phenomena (Springer,1975).

[2] J.W. Goodman, Statistical Optics, 2nd ed. (Wiley, 2015).

[3] J.W. Goodman, Speckle Phenomena in Optics, 2nd ed. (SPIE, 2020).

[4] F. Fercher and P. F. Steeger, “First-order statistics of Stokes parameters in speckle fields,” Opt. Acta 28, 443–448 (1981).

[5] P. F. Steeger, “Probability density function of the intensity in partially polarized speckle fields,” Opt. Lett. 8, 528–530(1983).

[6] P. F. Steeger, T. Asakura, K. Zocha, and A. F. Fercher, “Statistics of the Stokes parameters in speckle fields,” J. Opt. Soc. Am. A 1, 677–682(1984).

[7] S. Zhang, M. Takeda, and W. Wang, “Statistics of spatial derivatives of Stokes parameters for isotropic random polarization fields,” J. Opt.Soc. Am. A 27, 1999–2003 (2010).

[8] R. Barakat, “The statistical properties of partially polarized light,” Opt.Acta 32, 295–312 (1985).

[9] R. Barakat, “Statistics of the Stokes parameters,” J. Opt. Soc. Am. A 4, 1256–1263 (1987).

[10] I. Freund, M. Kaveh, R. Berkovits, and M. Rosenbluh, “Universal polarization correlations and microstatistics of optical waves in random media,” Phys. Rev. B 42, 2613–2616 (1990).

[11] D. Eliyahu, “Vector statistics of correlated Gaussian fields,” Phys.Rev. E 47, 2881–2892 (1993).

[12] D. Eliyahu, “Statistics of Stokes variables for correlated Gaussian fields,” Phys. Rev. E 50, 2381–2384 (1994).

[13] S. M. Cohen, D. Eliyahu, I. Freund, and M. Kaveh, “Vector statistics of multiply scattered waves in random systems,” Phys. Rev. A 43, 5748–5751 (1991).

[14] J. Dupont and X. Orlik, “Simulation of polarized optical speckle fields: effects of the observation scale on polarimetry,” Opt. Express 24,11151–11163 (2016).

[15] A. Ghabbach, M. Zerrad, G. Soriano, S. Liukaityte, and C. Amra, “Depolarization and enpolari-zation DOP histograms measured for surface and bulk speckle patterns,” Opt. Express 22, 21427–21440(2014).

[16] J. Dupont, X. Zerrad, G. Soriano, S. Liukaityte, and C. Amra, “Polarization analysis of speckle field below its transverse correlation width: application to surface and bulk scattering,” Opt. Express 22, 24133–24141 (2014).

[17] M. K. Schmidt, J. Aizpurua, X. Zambrana-Puyalto, X. Vidal, G. Molina-Terriza, and J. J. Sáenz, “Isotropically polarized speckle patterns,” Phys. Rev. Lett. 114, 113902 (2015).

[18] N. Ma, S. G. Hanson, M. Takeda, and W. Wang, “Coherence and polarization of polarization speckle generated by a rough-surfaced retardation plate depolarizer,” J. Opt. Soc. Am. A 32, 2346–2352 (2015).

[19] J. Ritter, N. Ma, W. Osten, M. Takeda, and W. Wang, “Depolarizing surface scattering by a birefrin-gent material with rough surface,” Opt. Comm. 430, 456–460 (2019).

[20] W. Wang, S. Zhang, and J. Grimble, "Statistics of polarization speckle produced by a constant polarization phasor plus a random polarization phasor sum," J. Opt. Soc. Am. A 37, 1888-1894 (2020)
[21] T. e. Hull and A. R. “Dobell, Random number generators,” SIAM Rev. 4, pp. 230-254(1962).
[22] J. V. Neumann, “Various Techniques Used in Connection with Random Digits, Applied Mathematics Series 12,” National Bureau of Standards, Washington, DC, pp. 36-38(1951).
[23] J. Spanier and E. M. Gelbard. “Monte Carlo Principles and Neutron Transport Problems,” Addison-Wesley, Reading, pp. 26-28(1969).
[24] G. Goertzel and M. H. Kalos, “Monte Carlo methods in transport problems, in Progress in Nuclear Energy, Series I: Physics and Mathematics,” D. J. Hughes et al., eds., Pergamon Press, New York, pp. 315-369(1958).