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Kink soliton behavior study for systems with power-law nonlinearity

Xiaoning Liu [a], Yubin Jiao [a], Ying Wang [a,*], Qingchun Zhou [a], Wei Wang [b,*]

[a] School of Science, Jiangsu University of Science and Technology, Zhenjiang 212100, China
[b] Institute of Photonics and Quantum Sciences, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom

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ABSTRACT

In this study, we investigate the kink soliton dynamics for power-law nonlinear systems. Based on the F-expansion method, we first derive the novel kink soliton solution of the nonlinear Schrödinger equation (NLSE) with third-order dispersion term, power-law dependent nonlinearity term, linear attenuation term, and self-steepness term under appropriate parameter settings. With pictorial demonstration, we show that the obtained kink soliton solution not only has the soliton features of the classical NLSE, but also has power-law features. The theoretical results presented in our work can be used to guide the observation of soliton behavior in power-law dependent media.

Introduction

In the physical world, there exist many nonlinear physical systems, which can be described by various nonlinear models [1–4]. Among them, nonlinear Schrödinger equation (NLSE) is such an equation model with a high degree of universality in nonlinear science, and soliton solutions are usually the ultimate goal of the analytical study of the NLSE [5,6]. Solitons arise from the balance between nonlinear interactions and dispersion effects in a nonlinear system as highly localized perturbation wave existing in a continuous medium, where their amplitude and shape are stable during transportation. In recent years, solitons have been discovered and studied in solid-state physics, plasma physics, nonlinear optics, nonlinear quantum field theory, and hydrodynamics [7–11]. Dai et al. [12] constructed the coupled cn–sn-type periodic wave solution and the corresponding photo refractive bright–dark soliton pair solution. Wang et al. [13] analyzed the dynamical properties of analytical fractional soliton solutions by using two fractional dual-function methods. In prior work, bright solitons and dark solitons have been the hot topic of nonlinear phenomena study. The study of certain category of solitons, kink solitons for example, is relatively rare for nonlinear systems with power-law nonlinearity. The progress in nonlinear phenomena experimental technology and the corresponding theoretical investigation call for the study of novel soliton phenomena in complex systems in reality [14,15].

Among the various soliton modes, the kink soliton is a semi-local nonlinear mode where the waveform exhibits a sharp turn between a fixed bottom height and many decaying oscillation tails [16]. Kink solitons produce self-steepness or nonlinear effects in nonlinear fibers, so the wavefront shape affects the high-intensity short-pulse properties. In practical applications, kink solitons can be used as polarization switches between two different domains or optical logic units [17,18]. There have been many methods employed to obtain soliton solutions of the various NLSE with nonlinear and dispersive terms, including the Bäcklund transformation method, variational iteration method, Exp expansion method, homotopy analysis method, and so on [19–21]. Among them, the F-expansion method solves the NLSE with typical traveling wave solutions by representing the solution of the NLSE as a Jacobi elliptic function in the form of a power expansion [22].

In this work, we study the dynamic behavior of kink solitons in systems with power-law nonlinearity. We utilize the novel parameterized power indexes $2m$ and $4m$ instead of the simpler cubic and quintic power indexes in the classic NLSE. The power index can be adjusted freely according to the experimental settings, reflecting the tunability of nonlinear interactions. We derive the kink soliton solution of the power-law nonlinear system through the F-expansion method and illustrate the dynamic evolution model of the derived solution. We find that there exists an adjustable group velocity dispersion term and a nonlinear term. In addition, there is a third-order dispersion term, linear attenuation term, and self-steepness term. As the power index m increases, the kink range of the soliton becomes smaller. By adjusting the power index parameter of the derived kink soliton, the shape of the kink soliton can be controlled. The theoretical results derived in this work can enrich the study related to kink soliton behavior, and the obtained analytical kink soliton results can be used to guide the detection and discovery of kink solitons in nonlinear systems such as liquid crystals and nonlinear optical fibers.

* Corresponding authors.
E-mail addresses: wangying@just.edu.cn (Y. Wang), zhouqc@just.edu.cn (Q. Zhou), w.wang@hw.ac.uk (W. Wang).

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This paper is organized as follows: "Nonlinear Schrödinger equation model and F-expansion method" presents the formulation of the model of the (1 + 1)-dimensional NLSE with power-law dependent nonlinearity and introduces the F-expansion method. "Soliton solution for (1 + 1)-dimensional NLSE with power-law dependent nonlinear interaction" investigates the procedure for obtaining the kink soliton solution of the corresponding (1 + 1)-dimensional NLSE. "Kink soliton features and stability analysis" verifies the accuracy and stability of our analytical results and reaches the two-dimensional kink soliton solution based on the self-similar method. "Conclusion" gives the conclusive remarks.

Nonlinear Schrödinger equation model and *F*-expansion method

The (1 + 1)-dimensional NLSE model with power-law dependent nonlinearity

The (1 + 1)-dimensional NLSE model with power-law dependent nonlinearity is expressed as follows [23,24]

$$\begin{aligned} & i\frac{\partial\psi(x,t)}{\partial t}+\gamma_1|\psi(x,t)|^{2m}\psi(x,t)+\gamma_2|\psi(x,t)|^{4m}\psi(x,t)+g_0\frac{\partial^2\psi(x,t)}{\partial x^2} \\ & = ig_1\psi(x,t)+ig_2\frac{|\psi(x,t)|^{2m}\psi(x,t)}{\partial x}-ig_3\frac{\partial^3\psi(x,t)}{\partial x^3}, \end{aligned} \tag{1}$$

where the space and time coordinates x and t are the independent variables of the wave function $\psi(x,t)$. $i\frac{\partial\psi(x,t)}{\partial t}$ describes the evolution of the wave function, $\gamma_1|\psi(x,t)|^{2m}\psi(x,t)$ and $\gamma_2|\psi(x,t)|^{4m}\psi(x,t)$ are the nonlinear terms, m is the power index. The nonlinear interaction strength parametric constants $\gamma_1(t)$ and $\gamma_2(t)$ can be adjusted via system's inter-particle interaction modulation, by Feshbach resonance experimental technique [25] in ultracold atomic systems, for example. In our study, taking into consideration of the time duration when the nonlinear interaction modulation is not applied, so that we take γ_1 and γ_2 as constants in our theoretical treatment. $g_0\frac{\partial^2\psi(x,t)}{\partial x^2}$ represents the group velocity dispersion term, g_0 is the coefficient of the dispersion term. g_1 is the time-dependent coefficient of the linear attenuation term, g_2 is the time-dependent coefficient of the self-steepness term, and g_3 is the time-dependent coefficient of the third-order dispersion term. Here, we take consideration of the cases where the parametric coefficients in Eq. (1) can be adjusted according to the experimental setting and remain stable in certain time period under study.

The nonlinear terms $\gamma_1|\psi(x,t)|^{2m}\psi(x,t)$ and $\gamma_2|\psi(x,t)|^{4m}\psi(x,t)$ correspond to the two-body interaction and the three-body interaction in Bose–Einstein condensation (BEC). $m = 1$ corresponds to the BEC limit in ultracold atomic system for example, with the variation of parameterized power index m corresponding to the tunable nonlinear inter-particle interaction [26]. The particle number density n is proportional to the square of the wave function modulus $|\psi|$ as $n \propto |\psi|^2$ and the two-body interaction is proportional to the square of the particle number density n^2; the three-body interaction is proportional to n^4. Thus, the two-body and three-body interactions are proportional to $|\psi|^4$ and $|\psi|^6$ in the Hamiltonian respectively. In our work, the generalized power index m formulation is adopted.

We consider the cases where the parameter coefficient functions in Eq. (1) do not change with time within a certain period but can be freely adjusted according to the experimental settings. Some special sets of these parameters correspond to specific descriptions of physical phenomena. For example, $m = 1$ corresponds to the conventional scenario of the cubic–quintic dispersion effect in quantum optics system, and kink solitons are identified in nonlinear systems such as liquid crystals and nonlinear optical fibers [23]. We will then briefly describe the F-expansion method to be used for solving Eq. (1).

F-expansion method

The F-expansion method has been widely applied in solving problems of many physical settings since it was proposed. It reveals the relationship between complex nonlinear partial differential equations and appropriate solvable auxiliary ordinary differential equations. Precisely, the solution of the nonlinear partial differential equation can be expressed in polynomial form with respect to the base function F (function of variable $\xi = px + qt$), where the highest order of F in the polynomial is obtained by the homogeneous balance of the highest order derivative term and the highest order nonlinear term in the nonlinear partial differential equation, and F satisfies the auxiliary ordinary differential equation [22].

Consider the following nonlinear partial differential equation with variable coefficients

$$N\left(u,u_t,u_x,u_{xx},\dots\right)=0, \tag{2}$$

where $u(x,t)$ is the unknown function to be solved, N is a polynomial in $u(x,t)$ and its partial derivatives of all orders.

$$u(x,t)=u(\xi),\quad \xi=px+qt, \tag{3}$$

where p, q are variable parameters. The system of ordinary differential equations about $u(\xi)$ with p, q can be obtained by substituting Eq. (3) into the nonlinear partial differential Eq. (2). Assuming that $u(\xi)$ can be expanded into a finite power series of $F(\xi)$ of the following form

$$u(\xi)=\sum_{i=0}^{m}h_i(t)F^i(\xi),\quad h_m(t)\neq 0, \tag{4}$$

where $h_i(t)$ are the parametric function of timing variable t, which are to be determined by the subsequent equation solution steps. Using the homogeneous balance principle which means that the highest-order nonlinear term in the ordinary differential equation is balanced with the highest-order derivative term, we can determine the power index m of the highest order term of Eq. (4). $F(\xi)$ is defined as

$$\left(\frac{dF(\xi)}{d\xi}\right)^2=a_nF^n(\xi)+a_{n-1}F^{n-1}(\xi)+\cdots+a_2F^2(\xi)+a_1F(\xi)+a_0,\quad n\geq 4. \tag{5}$$

We differentiate both sides of Eq. (5) with respect to ξ. The transformed equation for $F(\xi)$ is

$$2\frac{d^2}{d\xi^2}F(\xi)=na_nF^{n-1}(\xi)+(n-1)a_{n-1}F^{n-2}(\xi)+\cdots+2a_2F(\xi)+a_1, \tag{6}$$

where $n, n-1, \dots$ are integers, a_2, a_1, a_0 are parametric constants to be determined in ensuing steps. By substituting Eq. (4) into the system of ordinary differential equations (ODEs) and using Eq. (5) or Eq. (6) to simplify and combine similar items of F, a polynomial of F can be obtained. Furthermore, if we make the coefficients of all orders of F in the polynomial equal to zero, we can determine the power index m of the highest order term of Eq. (4). By solving the system of ODEs, we can determine $h_i(t)\,(i=1,2,\dots,m)$ and Eq. (2) is solved accordingly.

Soliton solution for (1 + 1)-dimensional NLSE with power-law dependent nonlinear interaction

We assume that the traveling wave solution format of Eq. (1) is as follows

$$\psi(x,t)=\varphi(\xi)e^{i(At+Bx)}, \tag{7}$$

where $\xi = px + qt$, $\varphi(\xi) = |\psi(x,t)|$, $\varphi(\xi)$ is the modulus of the wave function $\psi(x,t)$. p, q, A, and B are the constants to be determined in the following steps. Substituting Eq. (7) into Eq. (1), we obtain the expressions for each differential term

$$i\psi_t = \left(iq\varphi' - A\varphi\right)e^{i(At+Bx)}, \tag{8a}$$

$$\psi_x = \left(p\varphi' + iB\varphi\right) e^{i(At+Bx)}, \tag{8b}$$

$$\psi_{xx} = \left(p^2\varphi'' + 2iBp\varphi' - B^2\varphi\right) e^{i(At+Bx)}, \tag{8c}$$

$$\psi_{xxx} = (p^3\varphi''' + 3iBp^2\varphi'' - 3B^2p\varphi' - iB^3\varphi)e^{i(At+Bx)}, \tag{8d}$$

where $\varphi' = \frac{d\varphi}{d\xi}, \varphi'' = \frac{d^2\varphi}{d\xi^2}, \varphi''' = \frac{d^3\varphi}{d\xi^3}$. Substituting Eqs. (8a), (8b), (8c), (8d) into Eq. (1), we get

$$\begin{aligned}&\left(iq\varphi' - A\varphi\right) + \gamma_1\varphi^{2m+1} + \gamma_2\varphi^{4m+1} + g_0\left(p^2\varphi'' + 2ipB\varphi' - B^2\varphi\right)\\ &\quad = ig_1\varphi + ig_2\left[(2m+1)p\varphi^{2m}\varphi' + iB\varphi^{2m+1}\right]\\ &\quad - ig_3(p^3\varphi''' + 3iBp^2\varphi'' - 3B^2p\varphi' - iB^3\varphi).\end{aligned} \tag{9}$$

The real part of Eq. (9) is

$$\begin{aligned}&\gamma_2\varphi^{4m+1} + \left(\gamma_1 + Bg_2\right)\varphi^{2m+1}\\ &\quad - \left(A + B^2g_0 - B^3g_3\right)\varphi\\ &\quad + \left(p^2g_0 - 3Bp^2g_2\right)\varphi'' = 0.\end{aligned} \tag{10}$$

The imaginary part of Eq. (9) is

$$\begin{aligned}&(2\,m+1)pg_2\varphi^{2m}\varphi' + g_1\varphi\\ &\quad - \left(q + 2Bpg_0 - 3B^2pg_3\right)\varphi' - p^3g_3\varphi''' = 0.\end{aligned} \tag{11}$$

According to Eq. (10), we assume that

$$\lambda_1 = \gamma_1 + Bg_2, \tag{12a}$$

$$\lambda_2 = A + B^2g_0 - B^3g_3, \tag{12b}$$

$$\lambda_3 = p^2g_0 - 3Bp^2g_2. \tag{12c}$$

We set the coefficients in Eqs. (10) and (11) to zeros, A, q are expressed by B, p as follow,

$$A = B^3g_3 - B^2g_0, \tag{13a}$$

$$q = 3B^2pg_3 - 2Bpg_0. \tag{13b}$$

From Eq. (13), we make B, p as free parameter constants so that $\lambda_{1,2,3}$, A, q can be expressed by B, p. To proceed with Eq. (10), we define

$$T(\varphi) = \left[\frac{d\varphi(\xi)}{d\xi}\right]^2 = \left(\varphi'\right)^2. \tag{14}$$

So

$$\frac{dT}{d\xi} = \frac{dT}{d\varphi}\frac{d\varphi}{d\xi} = \varphi'\frac{dT}{d\varphi}, \tag{15}$$

and

$$\begin{aligned}\varphi'' &= \frac{d\varphi'}{d\xi} = \frac{d\varphi}{d\xi}\frac{d\varphi'}{d\varphi} = \varphi'\frac{d\varphi'}{d\varphi}\\ &= \frac{1}{2}\frac{d(\varphi')^2}{d\varphi} = \frac{1}{2}\frac{dT}{d\varphi} = \frac{1}{2}T',\end{aligned} \tag{16}$$

where $T' = \frac{dT}{d\varphi}$. Substituting Eqs. (12a), (12b), (12c) into Eq. (10), we get

$$\gamma_2\varphi^{4m+1} + \lambda_1\varphi^{2m+1} - \lambda_2\varphi + \frac{1}{2}\lambda_3T' = 0, \tag{17}$$

where we apply boundary condition $T(\varphi = 0) = 0$ (imply $T'(\varphi = 0) = 0$). To proceed further with Eq. (17), we assume $\varphi(\xi) = F(\xi)$, and choose the F base function as follows

$$T = a_4\varphi^{4m+2} + a_2\varphi^{2m+2} + a_0\varphi^2, \tag{18}$$

where a_0, a_2, a_4 are all constants to be determined later. Substituting Eq. (18) into Eq. (17) and making the coefficient of each polynomial term of φ equal to zero

$$\varphi^{4m+1} : \gamma_2 + (2m+1)\lambda_3a_4 = 0, \tag{19a}$$

$$\varphi^{2m+1} : \lambda_1 + (m+1)\lambda_3a_2 = 0, \tag{19b}$$

$$\varphi : \lambda_3a_2 - \lambda_2 = 0. \tag{19c}$$

From Eqs. (19a), (19b) and (19c), we get

$$a_4 = -\frac{\gamma_2}{(2m+1)\lambda_3}, \tag{20a}$$

$$a_2 = -\frac{\lambda_1}{(m+1)\lambda_3}, \tag{20b}$$

$$a_0 = \frac{\lambda_2}{\lambda_3}. \tag{20c}$$

Substituting Eqs. (12a), (12b) and (12c) into Eqs. (20a), (20b) and (20c), we obtain

$$a_4 = -\frac{\gamma_2}{(2m+1)\left(p^2g_0 - 3Bp^2g_2\right)}, \tag{21a}$$

$$a_2 = -\frac{\gamma_1 + Bg_2}{(m+1)\left(p^2g_0 - 3Bp^2g_2\right)}, \tag{21b}$$

$$a_0 = \frac{A + B^2g_0 - B^3g_3}{p^2g_0 - 3Bp^2g_2}. \tag{21c}$$

From Eqs. (14) and (18), we have

$$T = a_4\varphi^{4m+2} + a_2\varphi^{2m+2} + a_0\varphi^2 = \left(\frac{d\varphi}{d\xi}\right)^2. \tag{22}$$

The transformed equation for φ and ξ is

$$\frac{d\varphi}{\sqrt{a_4\varphi^{4m+2} + a_2\varphi^{2m+2} + a_0\varphi^2}} = d\xi. \tag{23}$$

If we denote $\phi = \varphi^{2m}$, and multiply the numerator and denominator of the left side of Eq. (23) by φ^{2m-1}, Eq. (23) then takes the following form

$$\begin{aligned}&\frac{\varphi^{2m-1}d\varphi}{\varphi^{2m-1}\sqrt{a_4\varphi^{4m+2} + a_2\varphi^{2m+2} + a_0\varphi^2}}\\ &= \frac{1}{2m}\frac{d\phi}{\phi\sqrt{a_4\phi^2 + a_2\phi + a_0}} = d\xi.\end{aligned} \tag{24}$$

With appropriate values setting of a_0, a_2 and a_4 and utilizing Eq. (20a), Eq. (20b), we will establish the following quadratic form of T (with the corresponding values of a_0, a_2 and a_4 set in the ensuing steps).

$$a_4\phi^2 + a_2\phi + a_0 = a_4\left(\phi - b\right)^2. \tag{25}$$

Here

$$b = -\frac{a_2}{2a_4} = -\frac{\left(\gamma_1 + Bg_2\right)(2m+1)}{2\gamma_2\left(m+1\right)}. \tag{26}$$

Substituting Eq. (25) into Eq. (24) and combining with Eq. (23), we get

$$\frac{d\phi}{\phi\left(b - \phi\right)} = 2m\sqrt{a_4}d\xi. \tag{27}$$

We require that the integration constant to be zero, since $\xi \to 0$, $\varphi \to 0$, and $\frac{d\varphi}{d\xi} \to 0$. After integration, Eq. (27) is transformed to the following form

$$\phi = \frac{be^{2mb\sqrt{a_4}\xi}}{1 + e^{2mb\sqrt{a_4}\xi}}. \tag{28}$$

Eq. (28) can be solved with the following resultant solution

$$\varphi = \phi^{\frac{1}{2m}} = \left(\frac{be^{2mb\sqrt{a_4}\xi}}{1 + e^{2mb\sqrt{a_4}\xi}}\right)^{\frac{1}{2m}}. \tag{29}$$

The modulus of wave function in Eq. (1) is

$$|\psi(x,t)| = \left(\frac{be^{2mb\sqrt{a_4}(px+qt)}}{1 + e^{2mb\sqrt{a_4}(px+qt)}}\right)^{\frac{1}{2m}}. \tag{30}$$

Since B, q are connected by Eq. (23), the number of free parameter constants is reduced to one, which can be chosen as p. Now, A, B, q, a_4 can be expressed by and $r_{1,2}$, $g_{0,1,2,3}$ (are constants within the timing range) and p. From Eq. (7) and (29), we will obtain the unknown wave function $\psi(x,t)$ in Eq. (1). It is not hard to see that solution Eq. (30) is in the classical form of kink soliton. That is, the (1 + 1)-dimensional quantum system with power-law dependent nonlinearity supports kink soliton behavior under appropriate experimental settings. Next, we will demonstrate higher-dimensional analog of the obtained one-dimensional kink soliton solution derived and analyzed the stability of the derived kink soliton solutions.

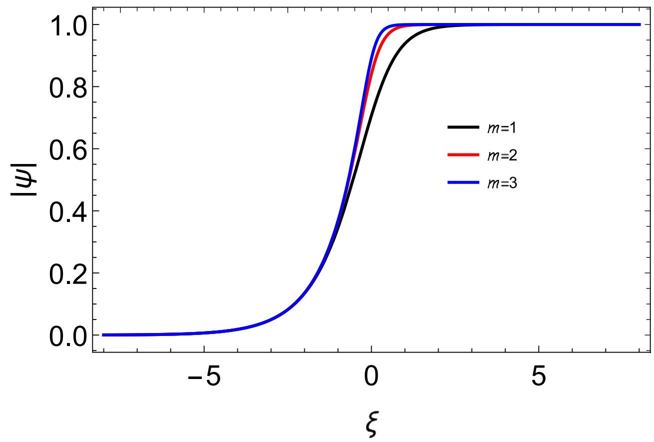

Fig. 1. Plot of one-dimensional kink soliton solution $|\psi|$ for various power indexes m vs. ξ for $b_1 = 1.0$, $a_4 = 1.0$, $p = 1.0$ and $q = 1.0$.

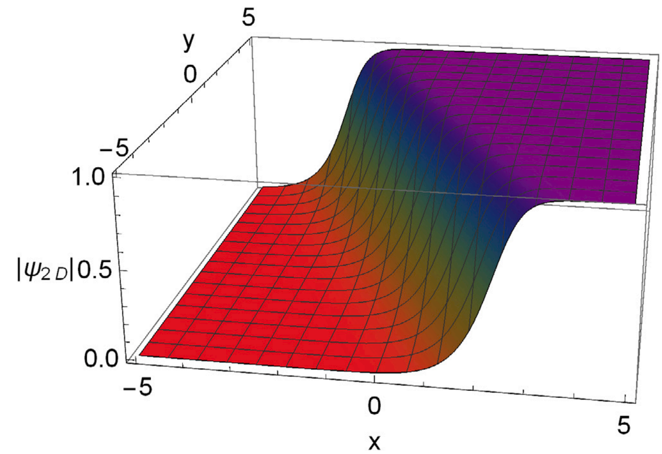

Fig. 2. Plot of two-dimensional kink soliton solution $|\psi|$ vs. ξ for $m = 1.0$, $b_2 = 1.0$, $a_4 = 1.0$, $\sigma_1 = 1.0$, $\sigma_2 = 1.0$ and $\sigma_3 = 1.0$.

Kink soliton features and stability analysis

Fig. 1 shows the form of the one-dimensional kink soliton at different power indexes, and it can be seen that the kink range of the soliton becomes smaller as the power index m increases. The obtained results can be used to observe kink solitons in nonlinear systems such as liquid crystals and nonlinear optical fibers. Compared with the previous work, we innovatively use the power indexes $2m+1$ and $4m+1$ to replace the cubic and quintic power indexes in the conventional NLSE and obtain the kink soliton solution. For example, in liquid crystals, the power index m corresponds to the kink elasticity coefficient of the liquid crystal. The larger the kink elasticity coefficient of the liquid crystal, the stronger the recovery force of the liquid crystal molecules after kinking, and the smaller the range of kink solitons affected. Therefore, by adjusting the power index m, we can control the morphological distribution of kink solitons in liquid crystals and change the properties of liquid crystals [23]. Next we use the $\frac{G'}{G}$ method [27,28] to verify that the results of the F-expansion method are correct.

Assuming that $U = \frac{G'}{G} = -2\frac{\psi'}{\psi}$, we get

$$U' = \frac{GG'' - (G')^2}{G^2} = -2\frac{\psi\psi'' - (\psi')^2}{\psi^2}. \tag{31}$$

When the Taylor series is in the neighborhood of $\xi = 0$, $\psi'' = 0$, we expand the above equation to obtain

$$U' = \frac{GG'' - (G')^2}{G^2} \simeq \psi^4(c_g + C_n(\psi - c_v)^2 + \cdots) \simeq c_g\psi^4. \tag{32}$$

Then

$$\psi = G^{-1/2} = \sqrt{\frac{d_1 e^{-d_2\xi}}{1 + e^{-d_3\xi}}}, \tag{33}$$

where $d_{1,2,3}$ are the integration constants, Eq. (33) is equivalent to Eq. (30). Thus, comparing Eq. (30) with the results from the F-expansion method, it can be seen that the $\frac{G'}{G}$ method yields the same result, indicating the accuracy of the derived soliton solution.

As shown in the previous work [29], we can obtain the two-dimensional NLSE self-similar analytical solution with power-law nonlinearity based on the one-dimensional solution of Eq. (30) as follows,

$$\left|\psi_{2D}(x,y,t)\right| = \left(\frac{b_2 e^{2mb_2\sqrt{a_4}(\sigma_1 x + \sigma_2 y + \sigma_3 t)}}{1 + e^{2mb_2\sqrt{a_4}(\sigma_1 x + \sigma_2 y + \sigma_3 t)}}\right)^{\frac{1}{2m}}, \tag{34}$$

where the parameter constants b_2, σ_1, σ_2, σ_3 are set according to the procedure shown in prior work [29]. Fig. 2 shows the kink soliton solution Eq. (34) of the two-dimensional NLSE. We now perform stability analysis on the derived results. We consider adding a perturbation term $\tilde{U}(x,t)$ to the solution $\psi(x,t)$ in Eq. (30) as follows,

$$\tilde{\psi}(x,t) = [\psi(x,t) + \tilde{U}(x,t)]e^{i\mu t}. \tag{35}$$

Substituting Eq. (35) into Eq. (1), we obtain the equation for $\tilde{U}(x,t)$ as follows,

$$\begin{aligned} &i\frac{\partial}{\partial t}\tilde{U} + \gamma_1\left(\psi^*\tilde{U} + \psi\tilde{U}^*\right)^m + \gamma_2|\psi|^{2m}\left(\psi^*\tilde{U} + \psi\tilde{U}^*\right)^m + g_0\frac{\partial^2}{\partial x^2}\tilde{U} \\ &= ig_1\tilde{U} + ig_2\frac{\partial}{\partial x}\left(\psi^*\tilde{U} + \psi\tilde{U}^*\right)^m - ig_3\frac{\partial^3}{\partial x^3}\tilde{U}. \end{aligned} \tag{36}$$

Eq. (36) can be simplified to

$$\begin{aligned} &i\eta_1\left(\psi^{2m}\frac{\partial}{\partial x}\tilde{U} + \psi^{2m}\frac{\partial}{\partial x}\tilde{U}^*\right) + i\eta_2\left(\psi^{4m}\frac{\partial}{\partial x}\tilde{U} + \psi^{4m}\frac{\partial}{\partial x}\tilde{U}^*\right) \\ &+ i\frac{\partial}{\partial t}\tilde{U} + g_0\frac{\partial^2}{\partial x^2}\tilde{U} - ig_1\tilde{U} + ig_3\frac{\partial^3}{\partial x^3}\tilde{U} = 0, \end{aligned} \tag{37}$$

where $\eta_{1,2}$ is a polynomial with respect to $\gamma_{1,2}$ and g_2. As shown in the stability analysis in prior work [29], the solution of Eq. (37) can be decomposed into the following normal modes,

$$\tilde{U}(x,t) = g(x)e^{\eta t} + s^*(x)e^{\eta^* t}. \tag{38}$$

As shown in the prior work [29], $\eta_{1,2}$ is a pure imaginary number, which guarantees that our kink soliton solution Eq. (30) is stable.

Conclusion

In this work, based on the NLSE with third-order dispersion, power-law dependent nonlinearity, linear attenuation, and self-steepness terms, we investigated the kink soliton behavior of power-law nonlinear systems. Via the F-expansion method, we derived the kink soliton solution of the NLSE of the one-dimensional and two-dimensional scenarios with appropriate parameter settings and illustrated the key kink soliton features graphically. The stability analysis is given for the derived kink soliton solutions of the power-law nonlinear system, and the accuracy of our theoretical results is verified by comparing it with the result from the $\frac{G'}{G}$ method. The obtained kink soliton solution not only has the key soliton features as shown in classical NLSE, but also has the typical power-law features. Our theoretical treatment demonstrates that systems with power-law nonlinear interaction support kink soliton behavior, showing the applicability of the theoretical treatment presented in this work.

CRediT authorship contribution statement

Xiaoning Liu: Formal analysis, Visulization, Writing – original draft. **Yubin Jiao:** Investigation, Visualization. **Ying Wang:** Conceptualization, Formal analysis, Methodology, Funding acquisition, Project administration, Writing – original draft. **Qingchun Zhou:** Supervision, Project administration. **Wei Wang:** Supervision, Project administration, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data sharing does not apply to this paper, as no new data were created or analyzed in this study. Our work is related to the establishment and analysis of the corresponding theoretical model, so we do not use the previously published data.

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