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Citation for published version:
Nie, J, Xiao, S, Takeda, M & Wang, W 2021, Performance analysis for a polarization imaging system with rotational alignment errors from polarizing elements. in DG Smith, F Wyrowski & A Erdmann (eds), *Computational Optics 2021.*, 118750Q, Proceedings of SPIE, vol. 11875, SPIE, Computational Optics 2021, Virtual, Online, Spain, 13/09/21. https://doi.org/10.1117/12.2599863

Digital Object Identifier (DOI):
10.1117/12.2599863

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Computational Optics 2021

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Jianlin Nie, Shengzhu Xiao, Mitsuo Takeda, Wei Wang, "Performance analysis for a polarization imaging system with rotational alignment errors from polarizing elements," Proc. SPIE 11875, Computational Optics 2021, 118750Q (14 September 2021); doi: 10.1117/12.2599863

Event: SPIE Optical Systems Design, 2021, Online Only

Performance analysis for a polarization imaging system with rotational alignment errors from polarizing elements

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ABSTRACT

In this paper, we give a spatial frequency analysis of polarization imaging system and study the effects of rotational alignment errors of the polarizer from their ideal orientations. With the help of newly proposed Optical Transfer Matrix (OTM) for a diffraction-limited polarization imaging system, we investigate the effects of polarization-sensitive aberrations from the generalized pupil matrix at the exit pupil plane. Here, polarization aberrations stemming from the angular alignment errors of a linear polarizer has been demonstrated. The performance of a polarization imaging system with rotational alignment error has also been evaluated based on a cost function based on OTM.

Keywords: Optical Transfer Matrix, polarizing elements, rotational alignment errors

1. INTRODUCTION

As a new type of imaging technology, polarization imaging technology not only has great achievements in target detection, but also has unique applications in machine vision. Traditional light intensity imaging technology is generally affected by environmental factors. In harsh environment, due to the weak light intensity, imaging will have a certain degree of difficulty. But polarization imaging technology can be used for remote image acquisition in harsh environment. Meanwhile, it has absolute advantages in suppressing background noise, improving detection range, obtaining detailed features and detecting targets. Therefore, polarization imaging technology is widely used in biomedical diagnosis, remote sensing, mineralogy and so on [1-6]. However, few investigations have been made to the effects of rotational alignment errors of polarizing elements from their ideal orientations when they are used in any polarization imaging systems.

On the other hand, as a natural extension to the well-known Optical Transfer Function for scalar imaging system, we proposed a new concept referred to as Optical Transfer Matrix (OTM) to analyze the performance of polarization imaging systems [7]. The approach of OTM The OTM describing the frequency transfer characteristic of the system for Stokes parameters between the object plane and the image plane, enjoys some distinct advantages, such as more objective, more reliable, and it can be applied to both small and large aberration optical systems.

In this paper, we give a spatial frequency analysis of polarization imaging system by using a method referred to as Optical Transfer Matrix and mainly study the effects of rotational alignment errors of the polarizer from its ideal orientations. For a diffraction-limited polarization imaging system, we consider the effects of polarization-sensitive aberrations or departures of the exit-pupil wavefront embedded with polarization information from ideal spherical form. Here, polarization aberrations stem from the angular alignment errors of polarizing elements. This paper is structured as follows. In Section 2, after introduction of the general pupil matrix, we present the optical transfer matrix as the frequency transfer characteristics for the imaging system. Section 3 gives the details for the calculation of the generalized pupil matrix for evaluating the optical transfer matrix. In Section 4, based on some newly proposed concepts such as the generalized pupil matrix and the Optical Transfer Matrix, we assess the quality of the Stokes images with the help of a cost functions. Conclusions are presented in Section 5.

Computational Optics 2021, edited by Daniel G. Smith, Frank Wyrowski, Andreas Erdmann, Proc. of SPIE Vol. 11875, 118750Q · © 2021 SPIE · CCC code: 0277-786X/21/$21 · doi: 10.1117/12.2599863

2. OPTICAL TRANSFER MATRIX FOR POLARIZATION IMAGING SYSTEM

Polarization imaging systems that use incoherent illumination have been seen to obey the matrix convolution integral[8]

$$
\begin{aligned}
{}_{im}\mathbf{S}(x,y) &= \kappa \iint_{-\infty}^{\infty} \mathbf{H}(x-\xi, y-\eta; x-\xi, y-\eta)\, {}_{ob}\mathbf{S}(\xi,\eta)\, d\xi d\eta \\
&= \kappa \mathbf{H}(x,y) * {}_{ob}\mathbf{S}(x,y),
\end{aligned}
\tag{1}
$$

where $\mathbf{S}$ is a Stokes vector with its left subscript of 'im' and 'ob' indicating the image plane and object plane, respectively, and $\mathbf{H}$ indicates an impulse point-spread matrix. The symbol '$*$' indicates a matrix convolution and can be understood as replacements of the multiplication operations in a matrix product by the corresponding convolution operations[8]. Such polarization imaging systems should therefore be frequency-analyzed a linear mapping of Stokes parameters distributions. To this end, let the Fourier frequency spectra of ${}_{ob}\mathbf{S}$ and ${}_{im}\mathbf{S}$ be defined

$$
{}_{ob}\boldsymbol{\mathcal{S}}(f_x, f_y) = \Im\left\{ {}_{ob}\mathbf{S}(x,y) \right\}, \tag{2.1}
$$

$$
{}_{im}\boldsymbol{\mathcal{S}}(f_x, f_y) = \Im\left\{ {}_{im}\mathbf{S}(x,y) \right\}. \tag{2.2}
$$

In a similar fashion, the transfer function of the polarization system can be defined by $\boldsymbol{\mathcal{M}}(f_x, f_y) = \Im\left\{ \mathbf{H}(x,y) \right\}$. Note from the definition for the impulse point-spread matrix

$$
\mathbf{H} = \mathbf{A}[\mathbf{h} \otimes \mathbf{h}^*]\mathbf{A}^{-1}, \tag{3}
$$

where $\mathbf{h}$ is the amplitude point-spread matrix, $\otimes$ indicates the Kronecker product, and $\mathbf{A}$ is the unitary transformation matrix given by

$$
\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{bmatrix}. \tag{4}
$$

Substitution of Eq. (3) and application of the correlation theorem yields the frequency-domain relation

$$
\begin{aligned}
\boldsymbol{\mathcal{M}}(f_x, f_y) &= \mathbf{A}[\Im\{\mathbf{h} \otimes \mathbf{h}^*\}]\mathbf{A}^{-1} \\
&= \iint_{-\infty}^{\infty} \mathbf{A}[\boldsymbol{\hbar}(p+f_x, q+f_y) \otimes \boldsymbol{\hbar}^*(p,q)]\mathbf{A}^{-1} dp dq,
\end{aligned}
\tag{5}
$$

where $\boldsymbol{\hbar}$ can be referred to as the amplitude transfer matrix given by

$$
\boldsymbol{\hbar}(f_x, f_y) = \Im\left\{ \mathbf{h}(x,y) \right\} = \mathbf{P}(\bar{\lambda} z_i f_x, \bar{\lambda} z_i f_y)\,. \tag{6}
$$

The amplitude transfer matrix $\boldsymbol{\hbar}$ is itself equal to the generalized pupil matrix $\mathbf{P}$, which is a product of the scalar pupil function (aperture function) of the conventional scalar imaging system and the Jones matrix indicating system's polarizing modulation at the exit pupil plane. Therefore, the relation in Eq. (6) is of the great importance since its supplies very revealing information about the behavior of diffraction-limited coherent polarization imaging systems in the frequency domain.

From Eqs. (2), (3) and (5), we note that polarization imaging systems with incoherent illumination have been seen to obey the relation: ${}_{im}\boldsymbol{\mathcal{S}}(f_x, f_y) = \kappa \boldsymbol{\mathcal{M}}(f_x, f_y)\, {}_{ob}\boldsymbol{\mathcal{S}}(f_x, f_y)$. Such systems should therefore be frequency analyzed as linear mappings of the distributions of the Stokes vectors. To this end, we can represent a normalized form

$$
{}_{im}\hat{\boldsymbol{\mathcal{S}}}(f_x, f_y) = \mathscr{M}(f_x, f_y)\, {}_{ob}\hat{\boldsymbol{\mathcal{S}}}(f_x, f_y)\,, \tag{7}
$$

where

$$
{}_{im}\hat{\boldsymbol{\mathcal{S}}}(f_x, f_y) = {}_{im}\boldsymbol{\mathcal{S}}(f_x, f_y) \big/ \left\| {}_{im}\boldsymbol{\mathcal{S}}(0,0) \right\|, \tag{8.1}
$$

$$ {}_{ob}\hat{\mathcal{S}}(f_x, f_y) = {}_{ob}\mathcal{S}(f_x, f_y) \big/ \left\| {}_{ob}\mathcal{S}(0,0) \right\|, \tag{8.2} $$

and

$$ \mathscr{M}(f_x, f_y) = \mathcal{M}(f_x, f_y) \big/ \left\| \mathcal{M}(0,0) \right\|, \tag{8.3} $$

where $\|\cdots\|$ indicates a Frobenius norm [9]. $\mathscr{M}$ can be referred to as the *Optical Transfer Matrix*, or the OTM. In the same way as the conventional OTF, this quantity of OTM represents the complex weighting matrix applied by the polarization imaging system to the Fourier spectra of object Stokes vector with spatial frequency relative to the weighting factor applied to the zero-frequency component.

3. ROTATIONAL ALIGNMENT ERRORS FOR AN POLARIZATION IMAGING SYSTEM

In the development of a generalized model of a polarization imaging system, it was specifically assumed that the presence of a point-source object yielded at the exit pupil a perfect specifical wave, converging toward the ideal geometrical image point with desired state-of-polarization. For such as diffraction-limited polarization imaging system, we consider now the effects of polarization aberrations, which are departures of the exit-pupil wavefront from ideal spherical form and from object's state-of-polarization. Polarization aberrations can arise in variety of ways, ranging from defects of imaging lens, polarization devices and their assembly errors between all the elements in the polarization imaging system. A complete treatment of polarization aberrations and their detailed effects on frequency response is beyond the scope of this development. In this paper, we concentrate on the rotational alignment error, which is a type of assembly error often encountered in the experiment, to demonstrate the effects on frequency response.

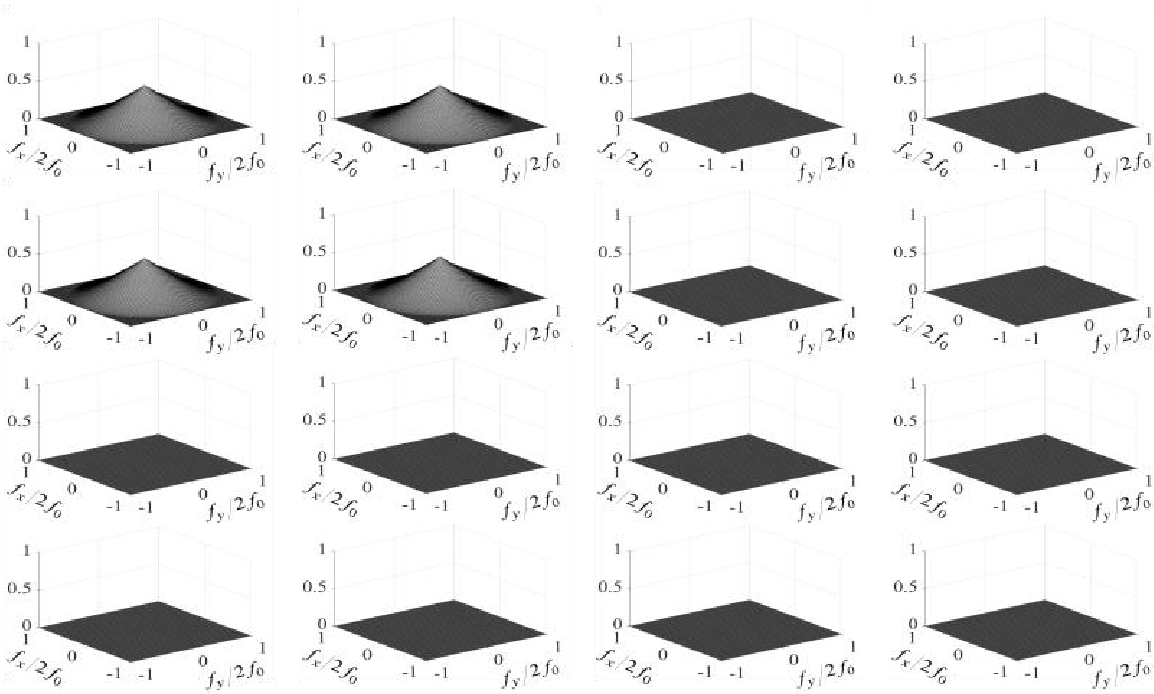
As we see from previous section, the key to evaluating the polarization imaging system using the proposed OTM approach is to get the expression of each element in the optical transfer matrix. The expression of the optical transfer matrix of a polarization imaging system can be obtained by correlation operations from the generalized pupil matrix at the exit pupil of the polarization imaging system. Note that the Jones matrix for a rotator is

$$ \mathbf{J}_{rot}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \tag{9} $$

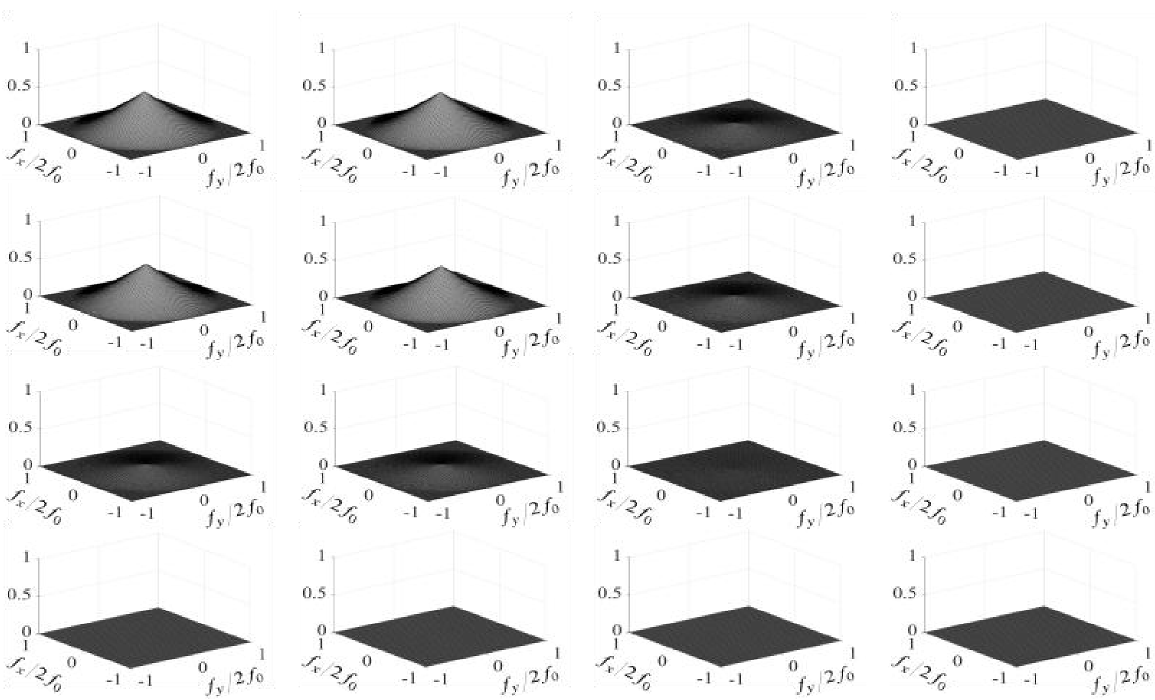
with θ being a rotational angle. For a rotated polarizing element, the Jones matrix is given by $\mathbf{J}(\theta) = \mathbf{J}_{rot}(-\theta)\mathbf{J}\mathbf{J}_{rot}(\theta)$ [10-16]. Therefore, we are able to write the generalized pupil matrix for a polarization imaging system as $\mathbf{P}(x, y) = P(x, y)\mathbf{J}(\theta)$ with P being a pupil function which is equal to one at every point within the pupil, and zero out with it. Given a linear polarizer with its axis of transmission horizontal as an example, we can express the generalized pupil matrix for a polarization imaging system with a circular aperture of diameter of w as

$$ \mathbf{P}(x, y) = circ(\frac{\sqrt{x^2 + y^2}}{w}) \begin{bmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{bmatrix}. \tag{10} $$

After substitution of Eq. (10) to the expressions for each element in OTM, we are able to evaluate the rotational alignment error. Figure 1(a) shows the Optical Transfer Matrix for diffraction-limited polarization imaging system with a linear polarizer of its transmission horizontal, where a cut-off frequency $f_0 = w/(\bar{\lambda} z_i)$ is the same as the conventional one for OTF of the scalar imaging system. As expected, each element of OTM in Fig.1(a) is seen to extend to a frequency that is twice the coherent cutoff frequency and the DC components of the OTM have the same as the conventional Mueller matrix. Figure 1(b) shows the OTM for the same polarization imaging system with a rotational alignment error of $\theta = 5^o$. From the height changes for each element in the optical transfer matrix, we can see the effect of rotational alignment errors of a linear polarizer to the polarization imaging system.



(a)



(b)

Figure 1. Optical transfer matrix for a polarization imaging system without (a) and with rotational alignment error (b) when a linear polarizer of its axis of transmission horizontal inserted

4. PERFORMANCE ANALYSIS FOR ROTATIONAL ALIGNMENT ERRORS

When any imaging system has been designed and made, a question will be asked, "How well does the system perform the imaging task in its intended use?''[17] If the polarization-imaging system performs well, than a second question could still be asked, "How should the degree of success be graded?'' When these questions are raised, we realize that we are now in the process of evaluating the Stokes images produced by the polarization imaging system, that is the degree of success to be rated by quantified information extracted from the actual system. Just as the OTF for a scalar imaging system, the OTM is the most complete presentation of the performance for a polarization imaging system and we could propose a merit function involving OTM for the assessment of Stokes imaging quality.

In general, to provide a comprehensive evaluation of the performance for a polarization imaging system, one should perform the analysis of the entire OTM for multiple object points and at various spatial frequency orientation. However, when manufacturing and assembling procedures are under control, acceptance testing can be safely reduced to establishing a single object point. Here, we propose a cost function to be set up for the volumes bounded two surfaces defined by OTMs for a perfect/ideal polarization imaging system and one with aberration. That is

$$\begin{aligned}\sigma^2 &= \sum_{p=0}^{3}\sum_{q=0}^{3}\iint_{-\infty}^{\infty}[\mathscr{M}_{pq}^{abb}(f_x,f_y)-\mathscr{M}_{pq}^{ide}(f_x,f_y)]^*[\mathscr{M}_{pq}^{abb}(f_x,f_y)-\mathscr{M}_{pq}^{ide}(f_x,f_y)]df_x df_y \\ &= \sum_{p=0}^{3}\sum_{q=0}^{3}\iint_{-\infty}^{\infty}\left|\mathscr{M}_{pq}^{abb}(f_x,f_y)-\mathscr{M}_{pq}^{ide}(f_x,f_y)\right|^2 df_x df_y,\end{aligned} \tag{11}$$

where the superscripts 'ide' and 'abb' indicate the OTMs for an ideal diffraction-limited polarization imaging system and a system with aberration, respectively.

Figure 2 gives an example of the merit function of a polarization imaging system for various rotational alignment errors when a linear polarizer with its axis of transmission horizontal is inserted. By changing the value of θ, we can calculate the corresponding OTM for a polarization imaging system with rotational alignment error. Then, the quality of the Stokes images can be evaluated with the help of the cost function. As expected for a polarization imaging system without any rotational alignment error when $\theta = 0^o$, the merit function is zero since the linear polarizer in the system has been adjusted well with its transmission axis along its designed orientation. The cost function increases when the rotational alignment angle for the linear polarizer becomes larger.

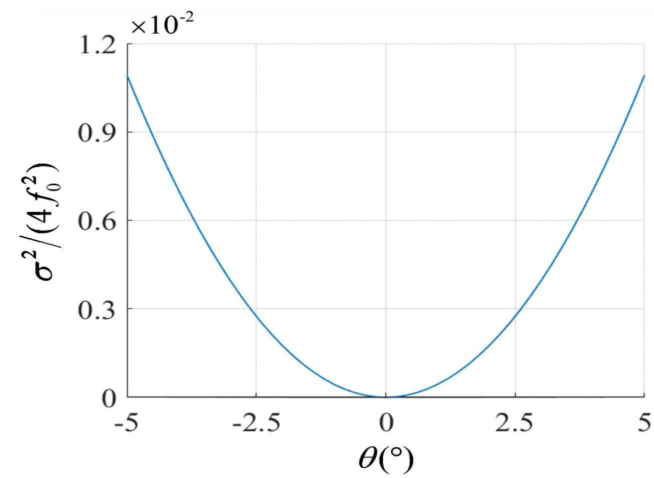


Figure 2. A cost function for a polarization imaging system with various rotational alignment errors

5. CONCLUSION

With the help of the newly proposed concept of OTM as the frequency response of a diffraction-limited polarization imaging system, we investigated the effect of rotational alignment error. Based on OTM, the cost function has also been proposed to evaluate the performance of a polarization imaging system with polarization aberration. The results presented in this paper can be regarded as a further development and extension of previous study on frequency response of a polarization imaging system and will provide a deeper insight into effects of alignment errors for polarizing elements deviating from their designed/ideal orientations when used in any polarization imaging systems.

REFERENCES

[1] V. V. Tuchin and L. Wang, [Optical Polarization in biomedical application], Springer (2006).
[2] Y. Zhao, C. Yi, S. G. Kong, Q. Pan and Y. Chang, [Multi-band polarization imaging and application], Springer, (2016).
[3] Y. Yamaguchi, [Polarimetric SAR imaging: Theory and Application], CRC Press, (2020).
[4] M. Born, E. Wolf, [Principles of Optics], Cambridge University Press, Chap.10 (1980).
[5] M. Gao, M. Hao. "Application of polarization imaging technology in various fields" Technology Innovation and Application, 2014(25):86.
[6] D. Goldstein D. [Polarized Light. Marcel Dekker, Inc, (2003).
[7] W. Wang and M. Takeda, "Effects of partial polarization and partial coherence in polarization imaging systems" JOSA.A (to be submitted).
[8] J. P. McGuire, Jr., and R. A. Chipman, "Diffraction image formation in optical systems with polarization aberrations. I. Formulation and examples," J. Opt. Soc. Am. A., 7(9), 1614-1626 (1990).
[9] R. A. Horn and C. R. Johnson, [Matrix Analysis] Cambridge Univ. Press, (1990).
[10] R. C. Jones, "A new calculus for the treatment of optical systems. I. Description and discussion of the calculus," J. Opt. Soc. Am., 31(7), 488-493 (1941).
[11] H. Hurwitz and R. C. Jones, "A new calculus for the treatment of optical systems. II. Proof of three general equivalence theorems," J. Opt. Soc. Am., 31(7), 493-499 (1941).
[12] R. C. Jones, "A new calculus for the treatment of optical systems. III. The Sohncke theory of optical activity," J. Opt. Soc. Am., 31(7), 500-503 (1941).
[13] R. C. Jones, "A new calculus for the treatment of optical systems. IV. Description and discussion of the calculus," J. Opt. Soc. Am., 32(8), 486-493 (1942).
[14] R. C. Jones, "A new calculus for the treatment of optical systems. V. A more general formulation and description of another calculus," J. Opt. Soc. Am., 37(2), 107-110 (1947).
[15] R. C. Jones, "A new calculus for the treatment of optical systems. VI. Experimental determination of the matrix," J. Opt. Soc. Am., 37(2), 110-112 (1947).
[16] R. C. Jones, "A new calculus for the treatment of optical systems. VII. Properties of the N-Matrices," J. Opt. Soc. Am., 38(8), 671-685 (1948).
[17] C. S. Williams and O. A. Becklund, [Introduction to the Optical Transfer Function], SPIE Press, (2002).