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Xiaoying Hu, Hao Li, Shoubo Hu, Weiguo Liu, Mitsuo Takeda, Wei Wang, "Nonlinearity in polarization imaging system with partially polarized and partially coherent illumination," Proc. SPIE 11875, Computational Optics 2021, 118750P (14 September 2021); doi: 10.1117/12.2597114

Event: SPIE Optical Systems Design, 2021, Online Only

Nonlinearity in Polarization Imaging System with Partially Polarized and Partially Coherent Illumination

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ABSTRACT

As an advanced technology, polarization imaging has attracted widespread interests in recent years due to its unique ability to detect the polarization information of objects. With illumination of partially coherent and partially polarized light, the polarization imaging systems are in generally nonlinear systems because they do not possess a transfer matrix in the usual sense. Nonetheless, it is possible to apply a sinusoidal amplitude object at the input and to measure a periodic output. However, unlike the linear case, knowledge of the response of the polarization system to a sinusoidal input does not allow one to predict what the output will be for other types of inputs.

To address the fundamental nonlinearity in evaluation of the Stokes images, we provide a frequency- domain calculation of Stoke images of a sinusoidal amplitude grating under partially polarized and partially coherent illumination, and propose a new concept referred to as transmission cross coefficients to describe its frequency response of polarization imaging system. Some of the implications of such analysis are also given for polarization imaging evaluation.

Keywords: Polarization Imaging, Sinusoidal Amplitude Grating, Partially Coherent and Partially Polarized Light

1. INTRODUCTION

Before the birth of laser, people's research on the interaction of light and matter was limited to the category of linear optics[1]. Because of the short pulse duration, the laser has extremely high peak energy, which highlights many effects that are difficult to observe under conventional conditions, and greatly promotes the development of nonlinear optical technology[2]. Compared with conventional linear optical spectral imaging methods, nonlinear optical imaging has many advantages, which makes this technology attract extensive attention from researchers[3]. Many authors have studied objects with partially coherent illumination and demonstrated non-linear changes at the output[4]. For example, the authors of Becherer and Parrent, Swing and Clay have studied this problem from the perspective of checking the ambiguity of the transfer function of the object illuminated by partially coherent light[5,6]. Becherer and Parrent discussed the problem of objects with sinusoidal amplitude transmittance. After the work of Becherer and Parrent, Swing and Clay performed the same type of analysis on the more common sinusoidal targets with sinusoidal intensity transmittance.

At present, the research on objects with sine amplitude changes basically does not consider the polarization characteristics of light. Because of the various advantages of polarization imaging, there have been more and more researches on polarization imaging in recent years. In this paper, we have studied the nonlinearity of the polarization imaging of the sinusoidal amplitude grating under the illumination of partially polarized and partially coherent light and made relevant analysis. We have given the theory of the polarization image of sine wave with partially polarized and partially coherent light, and provided the calculation of the frequency domain of the Stokes image of a sinusoidal amplitude grating under partial polarization and partial coherent illumination is of great significance for the identification and detection of more information and details about the object.

2. NONLINEARITY IN IMAGE WITH PARTIALLY COHERENT LIGHT

The basic quantity in the vector theory of partial coherence is the mutual coherence function defined by

Computational Optics 2021, edited by Daniel G. Smith, Frank Wyrowski, Andreas Erdmann, Proc. of SPIE
Vol. 11875, 118750P · © 2021 SPIE · CCC code: 0277-786X/21/$21 · doi: 10.1117/12.2597114

$$\begin{cases} S_0^{\mathrm{in}}(x_1,x_2) = \left\langle \tilde{E}_{\mathrm{x}}(x_1,t)\tilde{E}_{\mathrm{x}}^{\ *}(x_2,t) + \tilde{E}_{\mathrm{y}}(x_1,t)\tilde{E}_{\mathrm{y}}^{\ *}(x_2,t) \right\rangle \\ S_1^{\mathrm{in}}(x_1,x_2) = \left\langle \tilde{E}_{\mathrm{x}}(x_1,t)\tilde{E}_{\mathrm{x}}^{\ *}(x_2,t) - \tilde{E}_{\mathrm{y}}(x_1,t)\tilde{E}_{\mathrm{y}}^{\ *}(x_2,t) \right\rangle \\ S_2^{\mathrm{in}}(x_1,x_2) = \left\langle \tilde{E}_{\mathrm{x}}(x_1,t)\tilde{E}_{\mathrm{y}}^{\ *}(x_2,t) + \tilde{E}_{\mathrm{x}}^{\ *}(x_2,t)\tilde{E}_{\mathrm{y}}(x_1,t) \right\rangle \\ S_3^{\mathrm{in}}(x_1,x_2) = \left\langle \mathrm{i}[\tilde{E}_{\mathrm{x}}^{\ *}(x_2,t)\tilde{E}_{\mathrm{t}}(x_1,t) - \tilde{E}_{\mathrm{x}}(x_1,t)\tilde{E}_{\mathrm{y}}^{\ *}(x_2,t)] \right\rangle \end{cases} \tag{1}$$

$S_i^{\mathrm{in}}(x_1,x_2), i=0,1,2,3$ is the generalized Stokes parameters in the object plane of an optical imaging system in Eq. (1). The sharp brackets in Eq. (1) indicate a long time average and $E(x,t)$ is the analytic signal associated with the optical disturbance at the point x and time t.

For more complex polarization systems, the Stokes parameter and Jones matrix can be used to describe the polarization state of light and its superposition and transformation, the Jones matrix is

$$J_{(x,y)} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} \tag{2}$$

A more useful form for application to image analysis is obtained by considering the object to be transilluminated. For transilluminated objects, the expression for the mutual intensity in the image plane of an optical imaging system is

$$S_i^{\mathrm{im}}(x_1,x_2) = \int\int_{-\infty}^{\infty} S_i^{\mathrm{in}}(\xi_1,\xi_2)p(\xi_1)p^*(\xi_2)J_{mn}h(x_1-\xi_1)J_{mn}h^*(x_2-\xi_2)d\xi_1 d\xi_2 \tag{3}$$

In Eq. (3), $S_i^{\mathrm{im}}(x_1,x_2)$ are the generalized Stokes parameters in the image plane, $S_i^{\mathrm{in}}(\xi_1,\xi_2)$ is the mutual intensity in the object plane, $p(\xi)$ is the amplitude transmittance of the object , $J_{mn}\,(m=1,2;n=1,2)$ is an element of Jones matrix and $h(x-\xi)$ is essentially the spatially stationary amplitude impulse response of the imaging system.

When the detection process is included in the analysis, the image intensity is the quantity of interest. This image intensity is found from Eq. (3) and the definition (1) to be

$$S_i^{im}(x) = S_i^{im}(x,x) = \int\int_{-\infty}^{\infty} S_i^{in}(\xi_1,\xi_2)p(\xi_1)p^*(\xi_2)J_{mn}h(x-\xi_1)J_{mn}h^*(x-\xi_2)d\xi_1 d\xi_2 \quad i=0,1,2,3; m=1,2; n=1,2 \tag{4}$$

For the important practical case where $S_i^{in}(\xi_1,\xi_2), i=0,1,2,3$ is spatially stationary, i.e., for

$$S_i^{in}(\xi_1,\xi_2) = S_i^{in}(\xi_1-\xi_2) \tag{5}$$

Eq. (4) can be written

$$S_i^{im}(x) = \int\int_{-\infty}^{\infty} S_i^{in}(\xi_1-\xi_2)p(\xi_1)p^*(\xi_2)J_{mn}h(x-\xi_1)J_{mn}h^*(x-\xi_2)d\xi_1 d\xi_2 \tag{6}$$

From Eq. (6) it is clear that, for transilluminated objects, the transition from object intensity $p(\xi_1)p^*(\xi_2)$ to image intensity $S_i^{im}(x)$ is nonlinear. The significance of this conclusion is that the customary image-evaluation techniques and criteria are not, in general, applicable to such systems. Furthermore, the same optical system and object could be expected to yield different image intensities $S_i^{im}(x)$ if the coherence of the illumination represented by $S_i^{in}(\xi_1-\xi_2)$ were varied.

Since systems of this type are inherently nonlinear, it is impossible to characterize them by a transfer function. This point is easily established by taking the Fourier transform of both sides of Eq. (6). Where J_{mn} can be regarded as a constant. Then

$$\mathbf{S}_i^{im}(f_x) = \int \mathbf{P}(\gamma)\mathbf{P}^*(f_x-\gamma)\{\int \mathbf{S}_i^{in}[f_x-(\sigma+\gamma)]J_{mn}\mathbf{H}(f_x-\sigma)J_{mn}\mathbf{H}^*(\sigma)d\sigma\}d\gamma \tag{7}$$

In Eq. (7) the inner integral is characteristic of the instrument and the illumination while the factors $\boldsymbol{P}(\gamma)$ and to $\boldsymbol{P}^*(f_x-\gamma)$ are determined solely by the object. However, the right side of Eq. (7) is not in the form of a product of object spectrum and transfer function as it would be if the system were linear. The inner integral in Eq. (7) has been referred to as a generalized transfer function, but that nomenclature is rather misleading since the function is not used as a transfer function. A better terminology may be the more cumbersome one introduced by Wolf, i.e., the "transmission cross coefficient."

3.DESCRIPTION OF IMAGING SYSTEM

This section contains a description of an optical imaging system for which the transfer function is to be measured. The mutual intensity of the light incident on the object and the amplitude impulse response of the imaging system, and their Fourier transforms, are found in this section. These are used in the following sections to solve the imaging problem and to determine the apparent transfer functions for sinusoidal objects.

Figure 1 indicates a schematic diagram of polarization imaging of sine-wave object under illumination of a partially coherent and partially polarized light source.

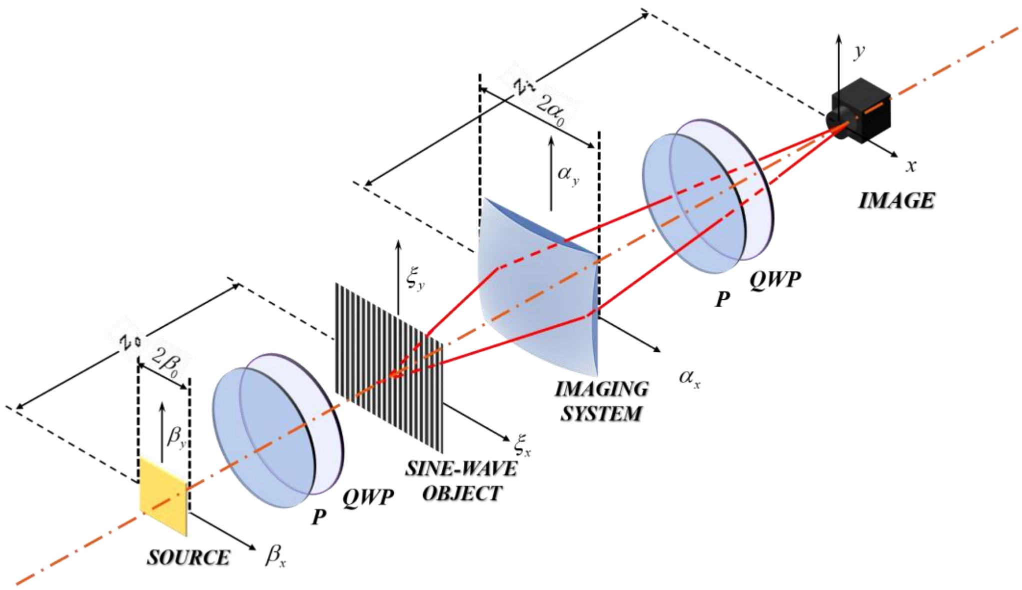

Fig.1. Schematic diagram for polarization imaging of sine-wave object illuminated by partially coherent and partially polarized light source.

For mathematical simplicity, all the treatments here are restricted to one-dimensional variations for light source, object, apertures and images. An incoherent source in the β plane illuminates an object in the ξ plane. The ξ plane is imaged onto the x, y plane by an imaging system with exit pupil in the α plane. The distance in image space is normalized by the lateral magnification of the imaging system and the positive direction in image space is opposite to that in object space. This is done to make the ordinate of a given object point equal to that of its corresponding image point.

An incoherent source with side length $2\beta_0$ illuminates sine-wave object located far away with a distance of z_o. For such a light source with uniform irradiance, its Stokes parameters can be written as Eq. (8) with $\widehat{s}_i = S_i \Big/ \sqrt{\sum_i S_i^2}$ being the normalized Stokes parameters for the tensor wave of light source.

$$S_i(\beta) = \begin{cases} A\widehat{s}_i, & |\beta| \le \beta_0 \\ 0, & |\beta| > \beta_0 \end{cases} \qquad (i = 0 \sim 3) \tag{8}$$

The generalized Stokes parameters of the incident beam at the object plane is found from the well-known van Cittert-Zernike theorem to be given to a good approximation by

$$\mathbf{S}_i^{in}(\xi_1, \xi_2) = \int S_i(\beta) e^{2\pi i \beta(\xi_1 - \xi_2)/\lambda z_0} d\beta \qquad (i = 0 \sim 3) \tag{9}$$

For points ξ_1 and ξ_2 close to the optical axis, here z_o is the distance between the incoherent-source plane β and the object plane ξ. When Eq. (8) is used to evaluate Eq. (9), the mutual intensity in the object plane is found to be

$$\mathbf{S}_i^{in}(\xi_1,\xi_2) = const \sin c\left[2\pi i\beta_0\left(\xi_1-\xi_2\right)/\lambda z_o\right] \qquad \left(i=0\sim 3\right) \tag{10}$$

The distance from the center to the first zero of the mutual intensity function in Eq. (10) will be referred to as the coherence interval of the object illumination. The Fourier transform of the mutual intensity function Eq. (10) is

$$\mathbf{S}_i^{in}(f_x) = \int \mathbf{S}_i^{in}(\xi_1-\xi_2)e^{2\pi i f_x(\xi_1-\xi_2)}d(\xi_1-\xi_2) = A\begin{cases}\hat{s}_i, & |f_x|\le \beta_0/\lambda z_o = f_1\\ 0, & |f_x|>\beta_0/\lambda z_o = f_1\end{cases} \tag{11}$$

Equation (11) serves to define the parameter f_1.

To emphasize the coherence effects and to minimize the complications arising from aberrations, the restriction to diffraction-limited optics is imposed. The amplitude in the exit pupil of the imaging system due to a point object is taken to be

$$A(\alpha) = \begin{cases}1, & |\alpha|\le\alpha_0\\ 0, & |\alpha|>\alpha_0\end{cases} \tag{12}$$

Where α_0 is a constant. Under the usual approximations which characterize Fraunhofer diffraction, the amplitude impulse response corresponding to Eq. (12) is

$$h(x) = \int A(\alpha)e^{2\pi i\alpha x/\lambda z_i}d\alpha = const \sin c\left[2\pi i\alpha_0 x/\lambda z_i\right] \tag{13}$$

Where z_i is the distance from the plane α of the exit pupil to the image plane x. The distance from the center to the first zero of the impulse response function in Eq. (13) will be referred to as the size of the imaging system's diffraction pattern. The Fourier transform of Eq. (13) is

$$\mathbf{H}(f_x) = \int h(x)e^{2\pi i f_x x}dx = const\begin{cases}1, & |f_x|\le\alpha_0/\lambda z_i = f_0\\ 0, & |f_x|>\alpha_0/\lambda z_i = f_0\end{cases} \tag{14}$$

Equation (14) serves to define the parameter f_0.

4. POLARIZATION IMAGE FOURIER TRANSFORM FOR SINE WAVE OBJECT

The object to be considered is a sinusoidal object with amplitude transmittance

$$p(\xi) = 1+\cos 2\pi f_{x_0}\xi \tag{15}$$

Where f_{x_0} is the spatial frequency. The Fourier transform of Eq. (15) is

$$\boldsymbol{P}(r) = \delta(r)+\frac{1}{2}\delta\left(r-f_{x_0}\right)+\frac{1}{2}\delta\left(r+f_{x_0}\right) \tag{16}$$

The corresponding intensity transmittance of this object is

$$\left|p(\xi)\right|^2 = 3/2+2\cos 2\pi f_{x_0}\xi + 1/2\left(\cos 2\pi 2 f_{x_0}\xi\right) \tag{17}$$

The intensity transmittance of the sine-wave object often used in laboratory measurements of the optical transfer function has a single spatial-frequency component rather than the two- spatial-frequency components which appear in Eq. (17). When the imaging system is linear, there is no fundamental difference between these two objects. When the system becomes nonlinear, the sine-wave loses its value as a test object. The necessity of specifying the amplitude transmittance of the object for use in Eq. (7) when the illumination is partially coherent makes a simple object of the form shown by Eq. (15) particularly suitable for the present analysis. The object represented by Eqs. (15) and (17) is, of course, physically realizable.

Since the object illumination is partially coherent, it is necessary to describe the experiment very carefully. The resulting image is shown to have an intensity function of the form

$$S_i^{im}(x) = A_i + B_i \cos 2\pi f_{x_0} x + C_i \cos 2\pi 2 f_{x_0} x \tag{18}$$

Where, of course, A, B, and C are yet to be determined. The ratio of the image modulation in terms of intensity to the object modulation, in terms of intensity, will be calculated. This ratio will be calculated for both spatial frequency components separately, following the practice which would be used under the condition of incoherent object illumination for an object whose intensity contained more than one spatial-frequency component. If the system were linear in intensity, these modulation ratios for the components of frequency f_{x_0} and $2f_{x_0}$ should be the same except for the scale factor of 2 in spatial frequency.

Where Eq. (13) and Eq. (15) have been used to write $h = h^*$ and $p = p^*$. By substituting Eq. (16) into Eq. (7), using Eq. (14) to write $\boldsymbol{H}(-\sigma) = \boldsymbol{H}(\sigma)$, and taking the inverse Fourier transform of both sides of Eq. (7), we find that

$$\begin{aligned} S_i^{im}(x) = & \int \mathbf{S}_i^{in}(-\sigma) J_{mn} \mathbf{H}(-\sigma) J_{mn} \mathbf{H}(\sigma) d\sigma \\ & + \frac{1}{4} \int J_{mn} \mathbf{H}(\sigma) \mathbf{S}_i^{in}(-\sigma - f_{x_0}) J_{mn} \mathbf{H}(-\sigma) d\sigma \\ & + \frac{1}{4} \int J_{mn} \mathbf{H}(\sigma) \mathbf{S}_i^{in}(-\sigma + f_{x_0}) J_{mn} \mathbf{H}(-\sigma) d\sigma \\ & + \int \mathbf{S}_i^{in}(f_{x_0} - \sigma) J_{mn} \mathbf{H}(f_{x_0} - \sigma) J_{mn} \mathbf{H}(\sigma) d\sigma \cos 2\pi f_{x_0} x \\ & + \int \mathbf{S}_i^{in}(-\sigma) J_{mn} \mathbf{H}(f_{x_0} - \sigma) J_{mn} \mathbf{H}(\sigma) d\sigma \cos 2\pi f_{x_0} x \\ & + \frac{1}{2} \int \left[\mathbf{S}_i^{in}(f_{x_0} - \sigma) J_{mn} \mathbf{H}(2 f_{x_0} - \sigma) J_{mn} \mathbf{H}(\sigma) \right] d\sigma \cos 2\pi 2 f_{x_0} x \\ = & A + B + C \cos 2\pi f_{x_0} x + D \cos 2\pi 2 f_{x_0} x \end{aligned} \tag{19}$$

Where the coefficients A, B, C and D are determined by using Eq. (11) and (14) to evaluate the integrals in Eq. (19). The result is

$$A = 2 \times \widehat{s}_i \times J_{mn} \times J_{mn} \begin{bmatrix} f_0 \\ f_1 \end{bmatrix} \tag{20}$$

$$B = \frac{1}{2} \times \widehat{s}_i \times J_{mn} \times J_{mn} \left\{ \begin{matrix} 2f_0 & f_0 < f_1 & and & 0 \le f_{x_0} \le f_1 - f_0 \\ f_0 + f_1 - f_{x_0} & f_0 < f_1 & and & f_1 - f_0 \le f_{x_0} \le f_0 + f_1 \\ 2f_0 - f_{x_0} & f_0 = f_1 & and & 0 \le f_{x_0} \le 2f_0 \\ 0 & f_0 = f_1 & and & 2f_0 \le f_{x_0} \\ 2f_1 & f_0 > f_1 & and & 0 \le f_{x_0} \le f_0 - f_1 \\ f_0 + f_1 - f_{x_0} & f_0 > f_1 & and & f_0 - f_1 \le f_{x_0} \le f_0 + f_1 \\ 0 & f_0 + f_1 \le f_{x_0} & & \end{matrix} \right\} \tag{21}$$

$$C = 2\times \widehat{s}_i \times J_{mn}\times J_{mn}\begin{cases} 2f_0 - f_{x_0} & f_0 < f_1 \quad and \quad 0\le f_{x_0}\le 2f_0 \\ 0 & f_0 < f_1 \quad and \quad 2f_0 \le f_{x_0} \\ 2f_0 - f_{x_0} & f_0 = f_1 \quad and \quad 0\le f_{x_0}\le 2f_0 \\ 0 & f_0 = f_1 \quad and \quad 2f_0\le f_{x_0} \\ 2f_1 & f_0 > f_1 \quad and \quad 0\le f_{x_0}\le f_0 - f_1 \\ f_0 + f_1 - f_{x_0} & f_0 > f_1 \quad and \quad f_0 - f_1 \le f_{x_0} \le f_0 + f_1 \\ 0 & f_0 > f_1 \quad and \quad f_0 + f_1 \le f_{x_0} \end{cases} \tag{22}$$

$$D = \frac{1}{2}\times \widehat{s}_i \times J_{mn}\times J_{mn}\begin{cases} 2f_0 - 2f_{x_0} & f_0 < f_1 \quad and \quad 0\le f_{x_0}\le f_0 \\ 0 & f_0 < f_1 \quad and \quad f_0 \le f_{x_0} \\ 2f_0 - 2f_{x_0} & f_0 = f_1 \quad and \quad 0\le f_{x_0}\le f_0 \\ 0 & f_0 = f_1 \quad and \quad f_0\le f_{x_0} \\ 2f_1 & f_0 > f_1 \quad and \quad 0\le f_{x_0}\le f_0 - f_1 \\ 2f_0 - 2f_{x_0} & f_0 > f_1 \quad and \quad f_0 - f_1 < f_{x_0} \le f_0 \\ 0 & f_0 > f_1 \quad and \quad f_0 < f_{x_0} \end{cases} \tag{23}$$

Here the symbol $\begin{bmatrix} f_0 \\ f_1 \end{bmatrix}$ is to be read " f_0 or f_1 , whichever is smaller." Obviously, A is a constant.

Since the optical imaging system has not been determined, Figures 2-4 in this article show that the Stokes parameter S_i and Jones matrix $J_{(x,y)}$ are not added. Without considering the Stokes parameter and Jones matrix, for the relevant parameters in formulas (20)-(23) are all taken as constant 1, where the y-axis is the coefficient corresponding to different parts of the image intensity formula (19). The x-axis represents the normalized ratio of the natural frequency of the grating to the cut-off frequency of the exit pupil. Observe the changes in the coefficients of different parts of formula (19) under three different sets of f0 and f1 ratios. The purpose of studying the coefficients of different parts is to better understand the various components of the nonlinear system, so as to add various differences. A better analysis method can be obtained after a complex optical system, which means that it can be identified and detect more information and details about the object.

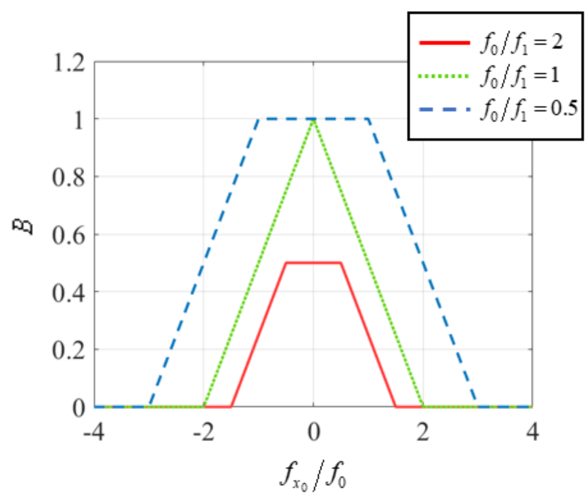

Figure 2. Schematic diagram of the normalized relationship between the coefficient B and the ratio of the natural frequency and the cut-off frequency under different ratios of f_0 and f_1

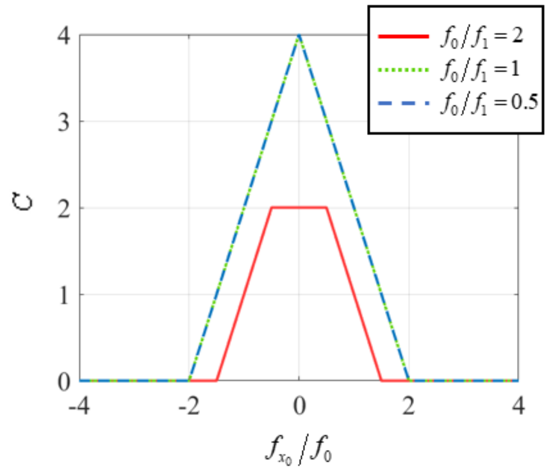

Figure 3. Schematic diagram of the normalized relationship between the coefficient C and the ratio of the natural frequency and the cut-off frequency under different ratios of f_0 and f_1

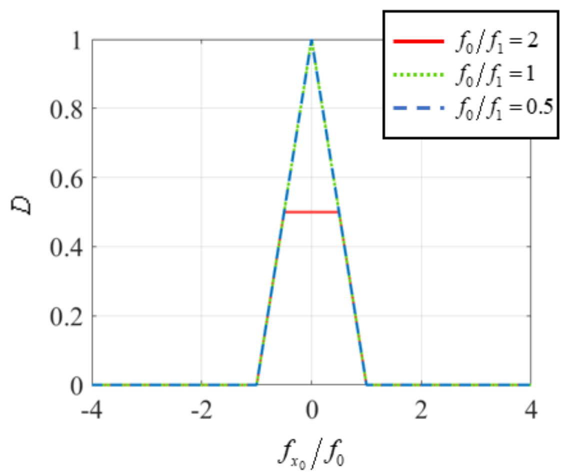

Figure 4. Schematic diagram of the normalized relationship between the coefficient D and the ratio of the natural frequency and the cut-off frequency under different ratios of f_0 and f_1

Figure 2 shows that as f_0/f_1 becomes smaller, the non-zero part of the coefficient B becomes more and more, which means that as f_0/f_1 becomes smaller, the non-zero part of B in the DC component becomes more and more; Figure 3 When f_0/f_1 is 0.5 and 1, the value of the coefficient C is the same, and when f_0/f_1 is 2, the non-zero part of the one-multiplier coefficient C becomes less, which means that f_0/f_1 is 2 than f0 /f1 is 0.5 and 1 octave the frequency component is lower; Figure 4 shows that the non-zero part of the double frequency coefficient D will not change with the change of the f_0/f_1 ratio; Figure 3 and Figure 4 can be compared The non-zero part of the one-time frequency coefficient C is more than the non-zero part of the two-time frequency coefficient D. This feature can be used to select that under a special fx0/f0 value, there can only be one frequency and the second frequency disappears. In the same way, only the DC component can be selected instead of the AC component.

6. CONCLUSIONS AND RESULTS

In conclusion, as an advanced sensing technology, polarization imaging has attracted more and more attention due to its unique ability to detect object polarization information. This paper studies the polarization imaging of a sinusoidal amplitude grating imaging system under the illumination of partially polarized and partially coherent light. First, it is explained that the polarization system is different from the traditional linear system, it is non-linear, and the Jones matrix and Stokes method are used to provide the frequency domain calculation of the sinusoidal amplitude grating image plane under partially polarized and partially coherent illumination. Secondly, under different f_0/f_1 ratios, the relationship between the natural frequency of the grating and the coefficients in the different parts of the Stokes image intensity formula of the sine wave object after being normalized by the cut-off frequency is obtained and the correlation analysis is carried out. These studies on identifying the amount of polarization information under different conditions are very meaningful and will be beneficial to many applications such as biomedical diagnosis, target detection, recognition and recognition.

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