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Revisit Time-variant Beampatterns for Frequency Diverse Arrays

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***Abstract*—Recently, an argument on the possibility of achieving the time-invariant beampatterns of frequency diverse arrays (FDAs) was discussed. The conclusion that time-invariant FDA beampatterns are impossible in free space applications is indisputable. However, the analyses are not convincing as they tangle two issues, namely the time-range relationship and the pattern propagation as time elapses. In this paper, the time-variant FDA beampatterns with a single maximum at the specific spatial position have been revisited and carefully examined. Following rigorous illustrative simulations and analysis, the authors argue that at a fixed time instant, the previously reported time-variant FDA beampatterns with a single maximum at the target locations are possible when the time references are carefully selected.**

***Index Terms*—Frequency diverse arrays (FDAs), time-variant beampatterns, time references, time-range relationship.**

I. INTRODUCTION

Frequency diverse arrays (FDAs) were developed as a type of novel Radar transmit technique, which, in its original form, is able to provide a range-angle-time dependent beampattern by way of adding a fraction of radio frequency (RF) carrier frequency increment across the linear array [1]. This unique range-dependent beampattern can resist interference in the range domain therefore attracting much attention in recent years [1]–[11]. Two well known limiting issues, however, exist in an FDA Radar, i.e., inherent range-angle coupling and range periodicity [2]. In order to address these two issues, the FDAs which can produce a time-variant beampattern with only a single maximum in the range domain at a time instant have been intensively studied [3]–[8]. In [3] the logarithmically increasing frequency offset FDA, namely log-FDA, was first presented, claiming that an uncoupled range-non-periodic beampattern at the target location can be achieved by way of designing the complex weight associated the radiated signal. This log-FDA is further extended to the multiple-input-multiple-output (MIMO) scenarios in [4], [5], and to the multicarrier waveforms in [6]. In [7], a non-monotonically increasing frequency increment combined with logarithmic offset was described, aiming to obtain improved performance of range-angle localization and reduced sidelobe levels. In [8], the Hamming window-based non-uniform frequency offset for FDA was proposed and compared with the log-FDA, showing that a better signal-to-interference-plus-noise ratio (SINR) performance can be achieved at the desired range-angle location than that does the log-FDA. These reported works illustrate promising characteristics of the FDA beampatterns.

However, it was reported recently that the conclusions of some previous FDAs and their applications can be misleading [9]–[11]. Specifically, in [9], the conventional FDA and its variants for physical-layer wireless security was carefully examined, proving that the claimed security in the range domain is time-dependent, indicating that the FDA concept cannot be used to secure a free-space wireless transmission in the range domain. In [10] and [11], it was pointed out that one important factor was overlooked in the previously reported time-dependent frequency offset FDA investigations, which is the target distance r is uniquely determined by the time instant t, i.e., $r = c\cdot t$ where c is the speed of light. It concludes that the previous FDA schemes which were claimed to produce time-invariant beampatterns are impossible in free space applications. This conclusion is indisputable. However, the FDAs which produce time-variant beampatterns, e.g. log-FDA, were mistakenly included in the argument due to the incompleteness of the theoretical analysis [10]. This paper is dedicated to rectify this error to direct future research in this area.

We now pose the question '*Are the time-variant beampatterns of the FDAs claimed in [3]–[8] possible when considering the time-range relationship*?'. Note: The conclusion reached in [10] claimed that they were impossible. This is different to the other proposition in [10] that is 'the time-invariant FDA beampatterns are impossible in free space applications'. Here in this paper the time-variant beampatterns refer to those range-angle uncoupled beampatterns, at a time instant, which only have a single maximum in the range domain.

II. PREVIOUSLY REPORTED TIME-VARIANT FDA BEAMPATTERNS

In this section, we present the previously reported time-variant beampattern FDA transmitter modelling in its general form, from which its transmit beampattern expressions are derived. An example of the time-variant FDA beampattern, i.e. log-FDA, is produced, leaving their discussions revealing and rectifying the flaws presented in Section III.

Considering a one-dimensional (1D) N-element linear FDA with uniform spacing d, see Fig. 1, the excitation carrier frequency injected into the n^{th} element can be expressed as

$$f_n = f_0 + \Delta f_n, \quad n = 1, 2, \ldots, N. \tag{1}$$

In (1), f_0 is the reference carrier frequency and for FDA $f_0 >> \Delta f_n$, where Δf_n denotes the small frequency offset for the n^{th} antenna branch. The radiated carrier by the n^{th} antenna element (assuming isotropic active element patterns) can be written as

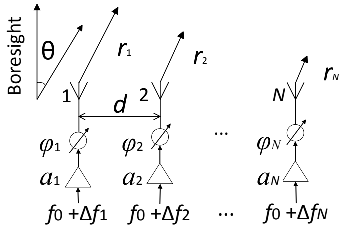


Fig. 1. Illustration of a 1D FDA.

$$S_n(t) = a_n \exp\{j(2\pi f_n t+\varphi_n), \quad 0 \le t \le T \tag{2}$$

where a_n and φ_n, respectively, denote the excitation amplitude and initial phase of the n^{th} antenna transmit signal at the time instant $t = 0$. Commonly, uniform amplitude is considered in the schemes, i.e., $a_n = 1$. T is the duration of the transmit pulse. The transmit beampatterns of the FDAs can be expressed as

$$\begin{aligned}&\boldsymbol{B}(t;r,\theta)\\&=\left|\sum_{n=1}^{N}\exp\left[j\left(2\pi\Delta f_n\left(t-\frac{r}{c}\right)+\frac{2\pi\left(f_0+\Delta f_n\right)(n-1)d\sin\theta}{c}+\varphi_n\right)\right]\right|^2,\end{aligned} \tag{3}$$

where θ is the spatial direction with the array boresight set as the angle reference, seen in Fig. 1. In order to steer the beampattern peak at the target location (r_x, θ_x), the excitation phase weightings in [3]–[5], [7], [8], were designed as

$$\varphi_n = 2\pi\Delta f_n\frac{r_x}{c}-\frac{2\pi\left(f_0+\Delta f_n\right)(n-1)d\sin\theta_x}{c}. \tag{4}$$

Substituting (4) into (3), the transmit beampattern can be rewritten as

$$\begin{aligned}&\boldsymbol{B}(t;r,\theta)\\&=\left|\sum_{n=1}^{N}\exp\left[j\left(2\pi\Delta f_n\left(t-\frac{r-r_x}{c}\right)+\frac{2\pi\left(f_0+\Delta f_n\right)(n-1)d\left(\sin\theta-\sin\theta_x\right)}{c}\right)\right]\right|^2.\end{aligned} \tag{5}$$

As an example to facilitate further discussion, the transmit beampatterns of the FDA scheme in [3] are replicated in Figs. 2(a) and (b). The parameters used in the simulations are as follows: $\Delta f_n = \log(n)$, = 2 kHz, f_0 = 5 GHz, d = 0.015 m, N = 10, and T = 2 ms.

III. ISSUES DISCUSSION AND RECTIFICATION

In this section, the authors firstly argue that the time-variant FDA beampatterns produced in previous works are erroneously claimed if the time instant when the signals are radiated from the antenna is selected as the time reference $t = 0$. Subsequently, the two cases with regard to the time reference selections are discussed.

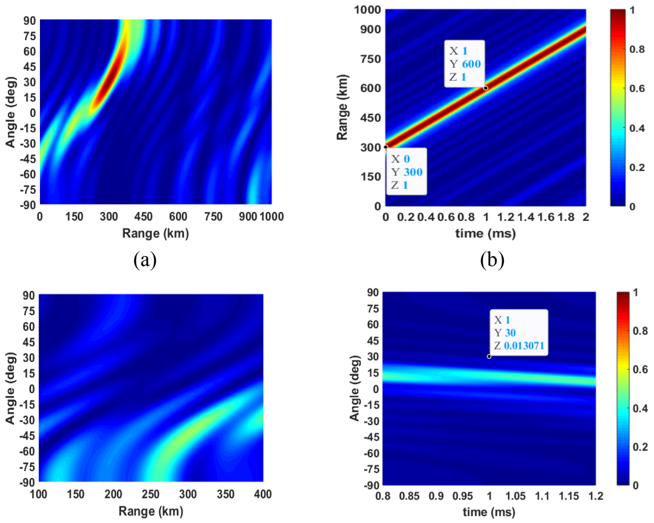
A. Issues in Previous Time-Variant Beampatterns

The early time-variant FDA beampatterns were depicted based on (5). From (5), it can be concluded that the maximum of the beampattern is steered at (r_x, θ_x) when $r = r_x$, $\theta = \theta_x$, and $t = 0$. It was claimed that the beampattern peak can be focused at the target location, thus achieving superiority in Radar applications. However, the time-range relationship was overlooked. In (2), the 't' represents the time of the radiated signals and $t = 0$ indicates the signals are about to radiate. Therefore, the waveforms radiated by the antenna elements **CANNOT** reach far-field target location at the time instant $t = 0$. While in previous papers, e.g. [3], see Fig. 2(a), it can be observed that the beampattern peak is concentrated at the target location (300 km, 30°) at $t = 0$ and at $t = 1$ms, the peak arrives at 600 km, see Fig. 2(b). Obviously, these results contradict to the definition of t. Considering time-range relationship, the radiated waveform from the antennas can only reach the distance 300 km at the time instant $t = 1$ ms. However, at the time instant $t = 1$ ms, no maximum occurs at target location (300 km, 30°), shown in Fig. 2(c). This indicates that the beampattern peak cannot illuminate the target. This conclusion is further confirmed in Fig. 2(d).

These incorrect results are caused by the ambiguity of selected time references, i.e., for (5) it corresponds to selecting time reference $t = 0$ when the signals reach the target, while, for (2), which indicates that the time when the signals are radiated from antennas is selected as the time reference $t = 0$. This mismatch of the selected time references between (2) and (5) leads to essential misconceptions. Next, we would like to rectify this issue respectively associated with the two selected time reference cases.

B. Selecting Time Reference t = 0 When the Signals are Radiated from Antenna

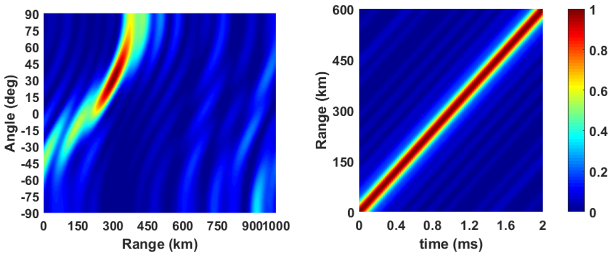
As defined in (2), the 't' refers to the time with the reference ($t = 0$) of when the pulse signal is starting to radiate. In this case, considering the time-range relationship, the excitation phase weightings should be designed as



(a) (b)

(c) (d)

Fig. 2. The normalized transmit beampatterns of the FDA scheme in [3] with (r_x = 300 km, θ_x = 30°) for (a), (c) at time instant t = 0 ms and t = 1 ms, respectively, in the range-angle domains; (b) along angle θ = 30° in the time-range domain; (d) at the distance r_x = 300 km in the time-angle domain.



(a) (b)

Fig. 3. The normalized transmit beampatterns of the FDA scheme in [3] using the re-designed excitation phase weightings in (6) with $\theta_{\mathrm{x}} = 30°$ for (a) at time instant $t = 1$ ms in the range-angle domain; (b) along angle $\theta = 30°$ in the time-range domain.

$$\varphi_n = \frac{-2\pi\left(f_0+\Delta f_n\right)(n-1)d\sin\theta_{\mathrm{x}}}{c}. \tag{6}$$

Substituting (6) into (5), the transmit beampattern is given as

$$\boldsymbol{B}(t;r,\theta) = \left|\sum_{n=1}^{N}\exp\left[j\left(2\pi\Delta f_n\left(t-\frac{r}{c}\right)+\frac{2\pi\left(f_0+\Delta f_n\right)(n-1)d\left(\sin\theta-\sin\theta_{\mathrm{x}}\right)}{c}\right)\right]\right|^2. \tag{7}$$

In (7), the beampattern peak can be steered at the target location (r_{x}, θ_{x}) at the time instant $t = r_{\mathrm{x}}/c$ which is proved in Fig. 3. In Fig. 3(a), it can be observed that the beampattern peak is steered at the target direction, i.e. 30°, and, at $t = 1$ ms, the peak arrives at the location (300 km, 30°). This beampattern peak propagates along the target direction as time elapses and the time-range relationship has been met, confirmed in Fig. 3(b). Therefore, when time reference $t = 0$ represents the time when the signals starting to radiate, through re-designing the excitation phase weightings, the beampattern peak can be focused in the desired spatial position.

C. Selecting Time Reference t = 0 When the Signals Reach the Targets

In the case of when the signals reach the targets is selected as the time reference $t = 0$, the formula (2) needs to be redefined, which is modified as

$$S_n(t) = a_n \exp\{j(2\pi f_n t+\varphi_n)\}, \quad 0 \le \Delta t \le T. \tag{8}$$

In (8), 't' denotes the time with the reference ($t = 0$) of when the signals reach the target. Δt represents the time length of the transmitted signals, and, a_n and φ_n respectively represent the excitation amplitude and phase weighting for the n^{th} antenna branch when $\Delta t = 0$. Under this background, the distance-speed-time formula in free space is equivalent to $r = c\cdot\Delta t$. For instance, at time instant $t = 0$, the distance of the target from the reference antenna $r_{\mathrm{x}} = c\cdot\Delta t = c\cdot|0 - t_{\mathrm{s}}| = -ct_{\mathrm{s}}$, wherein t_{s} is the time instant when the signals starting to radiate, and it is negative. Hence, when $r_{\mathrm{x}} = 300$ km, we have $t_{\mathrm{s}} = -1$ ms. Therefore, when the signals reach the target is selected as the time reference $t = 0$, the signal model, i.e., (2), must be redefined in order to meet the time-range relationship of the beampattern propagation.

Based on the above simulation results and discussions, the conclusions on time-variant beampatterns of the FDA works [3]–[8] are reliable when the mismatched time reference issue is rectified.

IV. CONCLUSION

Here, the paper proved that the time-range relationship in time-variant FDA beampatterns can be met by way of re-designing the signal excitation phase weightings or re-defining the signal model in previous works. Furthermore, the misinterpretation of the time-variant beampatterns rooted in the mismatched time references was revealed. The illustration of the two cases of differently selected time references in this paper are helpful to clear up the confusions in the time-variant FDA research community.

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REFERENCES

[1] P. Antonik, M. Wicks, H. Griffiths, and C. J. Baker, "Frequency diverse array radars," in *Proc. IEEE Radar Conf.*, Verona, NY, USA, Apr. 2006, pp. 215–217.

[2] M. Secmen, S. Demir, A. Hizal, and T. Eker, "Frequency diverse array antenna with periodic time modulated pattern in range and angle," in *Proc. IEEE Radar Conf.*, Boston, MA, USA, 2007, pp. 427–430.

[3] W. Khan, I. M. Qureshi, and S. Saeed, "Frequency diverse array radar with logarithmically increasing frequency offset," *IEEE Antennas Wireless Propag. Lett.*, vol. 14, pp. 499–502, 2015.

[4] W. Khan, I. M. Qureshi, A. Basit and W. Khan, "Range-bins-based MIMO frequency diverse array radar with logarithmic frequency offset," *IEEE Antennas Wireless Propag. Lett.*, vol. 15, pp. 885–888, 2016.

[5] Y. Wang, G. Huang and W. Li, "Transmit beampattern design in range and angle domains for MIMO frequency diverse array radar," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 1003–1006, 2017.

[6] H. Shao, J. Dai, J. Xiong, H. Chen, and W. Wang, "Dot-shaped range-angle beampattern synthesis for frequency diverse array," *IEEE Antennas Wireless Propag. Lett.*, vol. 15, pp. 1703–1706, 2016.

[7] M. Mahmood and H. Mir, "Frequency diverse array beamforming using nonuniform logarithmic frequency increments," *IEEE Antennas Wireless Propag. Lett.*, vol. 17, no. 10, pp. 1817–1821, Oct. 2018.

[8] A. Basit, I. Qureshi, W. Khan, S. Rehman, and M. M. Khan, "Beampattern synthesis for an FDA radar with hamming window based nonuniform frequency offset," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 2283–2286, 2017.

[9] Y. Ding, A. Narbudowicz, and G. Goussetis, "Physical limitation of range-domain secrecy using frequency diverse arrays," *IEEE Access*, in press, 2020.

[10] B. Chen, X. Chen, Y. Huang, and J. Guan, "Transmit beampattern synthesis for the FDA radar," *IEEE Antennas Wireless Propag. Lett.*, vol. 17, no. 1, pp. 98–101, Jan. 2018.

[11] K. Chen, S. Yang, Y. Chen and S. Qu, "Accurate models of time-invariant beampatterns for frequency diverse arrays," *IEEE Trans. Antetennas Propag.*, vol. 67, no. 5, pp. 3022–3029, May. 2019.