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A Design Approach for Compact Wideband Transformer With Frequency-Dependent Complex Loads and Its Application to Wilkinson Power Divider

Liang Liu, Xianling Liang, Senior Member, IEEE, Ronghong Jin, Fellow, IEEE, Haijun Fan, Member, IEEE, Xudong Bai, Member, IEEE, Han Zhou, and Junping Geng, Senior Member, IEEE

Abstract—In this article, a new design approach for the wideband transformer with frequency-dependent complex loads is proposed, which is based on the least-square method and the generalized modified small reflection theory (MSRT) with proper electrical parameters. Compared with the wideband transformers based on other approaches such as the generalized MSRT and transmission line theory, the proposed ones feature easy design, compact size, and wide bandwidth. For validation, several wideband transformers with frequency-dependent complex loads are designed, simulated, and measured. The measured results are consistent with the calculated and simulated results, and a wideband transformer with a fractional bandwidth of 68.5% (defined by 25-dB isolation) and an overall size of 0.069λ is achieved. In addition, when applied to the design of wideband Wilkinson power divider, a fractional bandwidth of 110% (defined by 25-dB return loss) and an overall size of 0.025λ are attained.

Index Terms—Frequency-dependent complex loads, least-square method, modified small reflection theory (MSRT), wideband transformer, wideband Wilkinson power divider (WPD).

I. INTRODUCTION

NOWADAYS, with the rapid development of modern society, there is an increasing demand for high-date-rate wireless communication systems. To fulfill this requirement, much attention has been gained into the research of wideband transformers, which are widely used in various radio frequency components of wireless communication systems, such as Wilkinson power dividers (WPDs), couplers, transceivers, low-noise amplifiers, and power amplifiers [1]–[6].

In [7]–[10], the exact and approximate synthesis methods to realize the wideband transformers by using multisection quarter-wavelength transmission lines were proposed. However, they are only applicable to the transformers with fixed source and load. In [11], a dual-band transformer based on the conventional wideband Chebyshev response was presented. Although the source and load are fixed at real impedances, the results clearly indicate the feasibility of wideband transformer using the multifrequency transformer. In particular, the wideband transformer with frequency-dependent complex loads can be easily realized by multifrequency transformer with several adjacent matching frequencies as well as frequency-dependent complex loads. In [12] and [13], transformers consisting of lumped inductors and capacitors were proposed, which are based on the multiple low-pass–bandpass transformation sharing one topology for all operating bands. These transformers are characterized by wideband filtering responses, but they are limited to low-frequency applications due to the parasitic effect of lumped components. In order to achieve high-frequency applications, various transmission-line transformers were proposed, which are based on the methods such as the transmission line theory [14]–[17] and the modified small reflection theory (MSRT) [18]. Nevertheless, when these methods are applied to the wideband transformer with two adjacent matching frequencies, their dominated equations become a quartic equation [16], [17] and a single matrix equation [18]. Therefore, they are not convenient for practical engineering applications. Furthermore, their cascaded transmission lines are up to three and four sections, which results in bulk size. Also, their bandwidth of the single operating band is narrow since the adopted perfect matching method has a severe impact on the bandwidth of transformers.

In this article, a new design approach for the wideband transformer with frequency-dependent complex loads is presented. Its electrical parameters are no longer arbitrarily but properly selected, and two complex conjugated impedances are presented. Although the source and load are fixed at real impedances, the results clearly indicate the feasibility of wideband transformer using the multifrequency transformer. In particular, the wideband transformer with frequency-dependent complex loads can be easily realized by multifrequency transformer with several adjacent matching frequencies as well as frequency-dependent complex loads. In [12] and [13], transformers consisting of lumped inductors and capacitors were proposed, which are based on the multiple low-pass–bandpass transformation sharing one topology for all operating bands. These transformers are characterized by wideband filtering responses, but they are limited to low-frequency applications due to the parasitic effect of lumped components. In order to achieve high-frequency applications, various transmission-line transformers were proposed, which are based on the methods such as the transmission line theory [14]–[17] and the modified small reflection theory (MSRT) [18]. Nevertheless, when these methods are applied to the wideband transformer with two adjacent matching frequencies, their dominated equations become a quartic equation [16], [17] and a single matrix equation [18]. Therefore, they are not convenient for practical engineering applications. Furthermore, their cascaded transmission lines are up to three and four sections, which results in bulk size. Also, their bandwidth of the single operating band is narrow since the adopted perfect matching method has a severe impact on the bandwidth of transformers.

In this article, a new design approach for the wideband transformer with frequency-dependent complex loads is proposed, which is based on the least-square method and the generalized modified small reflection theory (MSRT) with proper electrical parameters. Compared with the wideband transformers based on other approaches such as the generalized MSRT and transmission line theory, the proposed ones feature easy design, compact size, and wide bandwidth. For validation, several wideband transformers with frequency-dependent complex loads are designed, simulated, and measured. The measured results are consistent with the calculated and simulated results, and a wideband transformer with a fractional bandwidth of 68.5% (defined by 25-dB isolation) and an overall size of 0.069λ is achieved. In addition, when applied to the design of wideband Wilkinson power divider, a fractional bandwidth of 110% (defined by 25-dB return loss) and an overall size of 0.025λ are attained.
the dominated equations of transformer are simplified to closed-form formulas. Meanwhile, the number of transmission lines for the realization of wideband transformer with dual adjacent matching frequencies is reduced to two or three sections, which leads to a compact size. Moreover, the bandwidth of the transformer is enhanced due to the wideband matching based on the least-square method instead of the conventional perfect matching. In addition, the proposed approach can be easily applied to the design of wideband WPDs with a compact size.

II. Theory and Design

In order to clearly distinguish the generalized MSRT from the proposed method, one first considers the wideband transformer based on the former method in [18], which is realized by a dual-frequency transformer with two adjacent matching frequencies of $f_l$ and $mf$. As shown in Fig. 1(a), the wideband transformer comprises quad-section transmission lines with the characteristic impedance of $Z_n$ ($n = 1, 2, 3, 4$) and electrical length of $\theta = \theta_n$ at $f_l$ (the transmission line number is four since four unknown characteristic impedances can be obtained from the four independent equations, namely, the real and imaginary parts of two arbitrary complex impedances at dual matching frequencies). The source and load of the transformer are assumed as a fixed real impedance of $Z_0$ and a frequency-dependent complex impedance of $Z_L(\theta)$. According to the perfect matching conditions at $f_l$ and $mf$ as well as the restricted condition at the zero frequency, the dominated equations of the transformer based on the generalized MSRT are given in [18]–[20]

$$\Gamma_4(\theta = 0) = \sum_{n=0}^{4} \frac{\tilde{\Gamma}_n}{Z_0+Z} = \frac{Z_0-Z}{Z_0+Z}$$ (1a) $$\Gamma_4(\theta) = \sum_{n=0}^{4} \frac{\tilde{\Gamma}_n e^{-j2\theta_n}}{Z_0+Z} = \frac{Z_L(\theta)-Z}{Z_L(\theta)+Z}$$ (1b)

where $\Gamma_4(\theta)$ and $\Gamma_4(\theta)$ denote the total reflection coefficients of transformer at $f_l$ and $mf$ respectively, and the number of total transmission lines is 4. Meanwhile, the asterisk of $\cdot$ denotes the complex conjugate operation and $Z$ is an arbitrarily selected virtual real load. $\tilde{\Gamma}_n$ is the modified small reflection coefficient represented in terms of the small reflection coefficient $\Gamma_n$, the characteristic impedance of $Z_n$, and $Z$ [19]

$$\Gamma_0 = \Gamma_n = \frac{Z_n-Z}{Z_n+Z}, \quad n = 0, 2, 3, 4$$ (2a) $$\Gamma_n = \begin{cases} \frac{Z_n-Z}{Z_n+Z}, & n = 0, 2, 3 \\ \frac{Z_n-Z}{Z_n+Z}, & n = 4. \end{cases}$$ (2b)

From (1), it is noted that for a dual-frequency transformer based on the generalized MSRT, by selecting any virtual real load of $Z$ and the reference frequency of $rf$, a total of five dominated equations are obtained by separating (1b) and (1c) into real- and imaginary-part equations. Although these dominated equations can be reduced to a single matrix equation with $5 \times 5$ coefficient matrix, its closed-form solution involves as many as 600 terms according to Cramer’s rule.

A. Proposed Wideband Transformer With Simplified Structure

In order to simplify the computation and structure of wideband transformer based on the generalized MSRT, one expects that part of the dominant equations in (1) can be eliminated by proper selection of the electrical parameters. Subsequently, the unknown parameters of dominated equations and the required transmission lines are reduced.

For discussion, the operating frequency range of wideband transformer is assumed to be $(f_l, f_h)$. Then, the term of “capacitive and inductive loads” can be defined in the two subranges of $(f_l, f_l/2+f_s/2)$ and $(f_h/2+f_s/2, f_h)$, whose reactance is almost negative across one of the above two subranges but positive across the other subrange. Fig. 1(b) shows the proposed wideband transformer with the simplified structure consisting of two parts. The first part, shown in the black dotted box, is the load transformation stage comprising only one transmission line with the characteristic impedance of $Z_1$ and electrical length of $\theta_1$, which is utilized to transform an arbitrary frequency-dependent load into the obtained capacitive and inductive loads to the source of $Z_0$. Obviously, the matching and load transformation stages are equivalent to the two transformers shown in Fig. 2(a) and 2(b), respectively.
Fig. 2. Equivalent transformers. (a) Matching stage. (b) Load transformation stage.

B. Matching Stage of Wideband Transformer

1) Closed-Form Algebraic Solution by Properly Selecting Electrical Parameters in Generalized MSRT: For the transformer shown in Fig. 2(a), its dominated equations are

\[
\Gamma_2(\theta = 0) = \sum_{n=0}^{2} \bar{\Gamma}_n = \frac{Z_0-Z}{Z_0+Z} (3a)
\]

\[
\Gamma_2(\theta) = \sum_{n=0}^{2} \bar{\Gamma}_n e^{-j2n\theta} = \frac{Z_L^{*}(\theta)-Z}{Z_L^{*}(\theta)+Z} (3b)
\]

\[
\Gamma_2(m\theta) = \sum_{n=0}^{2} \bar{\Gamma}_n e^{-j2n(m\theta)} = \frac{Z_L^{*}(m\theta)-Z}{Z_L^{*}(m\theta)+Z} (3c)
\]

where \( \Gamma_2(\theta) \) denotes the total reflection coefficient of the matching stage composed of dual-section transmission lines and \( \bar{\Gamma}_n(n=0, 1, 2) \) is redefined by \( Z, Z_2, Z_3, \) and \( Z_0. \)

Substituting \( \pi - \theta \) for \( \theta \) in (3b), one has

\[
\Gamma_2(\pi-\theta) = \sum_{n=0}^{2} \bar{\Gamma}_n e^{-j2n(\pi-\theta)} = \frac{Z_L^{*}(\pi-\theta)-Z}{Z_L^{*}(\pi-\theta)+Z} (4)
\]

Equalizing (3c) and (4), one obtains the following relationship \[21\]:

\[
m\theta = \pi - \theta \Leftrightarrow \theta = \frac{\pi}{1+m}, \quad Z_L^{*}(\theta) = Z_L^{*}(m\theta). (5)
\]

Thus, by properly selecting the electrical parameters of (5) in the generalized MSRT, the dominated equations are simplified to (3a) and (3b). In order to further deduce the closed-form solution to the simplified dominated equations, substitute \( Z_L^{*}(\theta) = R' + jX' \) into (3b), separate it into real- and imaginary-part equations, and then combine it with (3a), one has

\[
\sum_{n=0}^{2} \bar{\Gamma}_n \cos(2n\theta) = \frac{R'^2+X'^2-Z^2}{(R+Z)^2+X^2} (6a)
\]

\[
\sum_{n=0}^{2} \bar{\Gamma}_n \sin(2n\theta) = \frac{2X'Z}{(R+Z)^2+X^2} (6b)
\]

\[
\sum_{n=0}^{2} \bar{\Gamma}_n = \frac{Z_0-Z}{Z_0+Z} (6c)
\]

Generally, when \( Z \) is an arbitrarily selected virtual load similar to \[18\], the formula of (6a) is nonzero. Then, applying the proposed method to the simplified structure in Fig. 2(a), the entire dominated equations are reduced to 3 due to the selected electrical parameter condition of (5). The total terms of the solution to (6) are reduced from 600 to 18 according to Cramer’s rule. In particular, by selecting a virtual real load of \( Z = (R'^2+X'^2)^{1/2} \), the formula of (6a) is equal to 0. The total terms of the solution of (6) are then further reduced from 18 to 12 according to Cramer’s rule. After some algebraic computations, one obtains the following closed-form algebraic solution:

\[
k_0 = \frac{\sqrt{R'^2+X'^2-Z_0}}{\sqrt{R'^2+X'^2+Z_0}} (7a)
\]

\[
k_1 = \begin{cases} \frac{\sqrt{R'^2+X'^2-R'}}{X'}, & X' \neq 0 \\ 0, & X' = 0 \end{cases} (7b)
\]

\[
\tilde{\Gamma}_2 = -0.5(k_1+k_0 \cot \theta) \csc(2\theta) (7c)
\]

\[
\tilde{\Gamma}_1 = [k_1+k_0 \cot(2\theta)] \cot \theta, \quad \tilde{\Gamma}_0 = -(k_0+\bar{\Gamma}_1+\bar{\Gamma}_2). (7d)
\]

From (7), it is observed that when the source, the complex conjugated loads, and \( m \) or \( \theta \) are given, the closed-form algebraic solution of \( \tilde{\Gamma}_n \) can be obtained. Subsequently, the small reflection coefficient of \( \Gamma_n \) can be derived from (2a), and the characteristic impedances of two transmission lines are given by the following formulas according to (2b):

\[
Z_2 = \sqrt{R'^2+X'^2} \frac{1+\Gamma_0}{1-\Gamma_0}, \quad Z_3 = Z_2 \frac{1+\Gamma_1}{1-\Gamma_1}. (8)
\]

It should be pointed out that the electrical length at the center frequency of two matching frequencies \( f \) and \( mf \) is \( \pi/2 \) according to (5). Therefore, the center frequency of two matching frequencies is the reference frequency of \( rf \) in the generalized MSRT, and the condition of (5) is equivalent to

\[
r_f = \frac{1+m}{2}\sqrt{f}, \quad Z_L^{*}(\theta) = Z_L^{*}(m\theta). (9)
\]

From (9), the parameter of \( r = (1+m)/2 \) can be obtained from the two given adjacent matching frequencies, \( f \) and \( mf \), which is distinctly different from that in the generalized MSRT \[18\]–\[20\].

2) Realization of Two Complex Conjugated Loads and Wideband Matching Based on Least-Square Method: In Section II-B1, although a wideband transformer based on two adjacent matching frequencies can be realized by two-section transmission lines, its loads at the two matching frequencies should be conjugated according to (5) or (9). To fulfill this requirement, one widely used approach is the introduction of an additional impedance-transformation transmission line \[16\], \[17\]. However, this may lead to a considerable increase in size. In addition, it is not applicable to all dual-frequency transformers with frequency-dependent complex loads \[6\]. Therefore, it is preferred that the two complex conjugated loads can be constructed directly.

For analysis, the constructed two complex conjugated loads are assumed to be \( R' \pm jX' \) at \( f \) and \( mf \), and the load of the matching-stage equivalent transformer is assumed to be capacitive and inductive load of \( Z_L^{*}(\theta_0) = R'_n+jX'_n \). Considering that \( Z_L^{*}(\theta_0) \) is frequency-dependent and its reactance across the two half frequency-ranges has opposite signs, one expands it around the constructed two complex conjugated
loads. In addition, the mean impedance \( \overline{R} + j\overline{X} \) of loads is defined by a minimum error of \( Z_{\text{min}} \) between itself and the loads at \( N \) equal-interval frequencies. Namely

\[
Z_{L}(\theta_n) = R'_n + jX'_n
\]

\[
= \begin{cases} 
R' + jX' + \Delta R'_n + j \Delta X'_n, & 1 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor \\
R' - jX' + \Delta R'_n + j \Delta X'_n, & N + 1 - \left\lfloor \frac{N}{2} \right\rfloor \leq n \leq N
\end{cases}
\]

(10a)

\[
Z_{\text{min}} = \frac{1}{2\pi} \left\{ \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left[ (\overline{R} - R'_n)^2 + (\overline{X} - X'_n)^2 \right] + \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} \left[ (\overline{R} - R'_n)^2 + (\overline{X} - X'_n)^2 \right] \right\}
\]

(10b)

In (10), \( \left\lfloor \frac{N}{2} \right\rfloor \) denotes the maximum integer equal or smaller than \( N/2 \), whereas \( \Delta R'_n \) and \( \Delta X'_n \) represent the resistance and reactance difference between the \( n \)th actual and constructed loads, respectively. Obviously, the minimum error of (10b) can be seen as a least-square equation with two unknowns of \( \overline{R} \) and \( \overline{X} \), whose solution is given by the least-square method [22]–[25].

\[
A_{av} = (B_{av}^T \cdot B_{av})^{-1} \cdot B_{av}^T \cdot Y_{av} \quad \text{and} \quad A_{av} = (\overline{R} \quad \overline{X})^T, \quad Y_{av} = (y_1 \ y_2 \ \cdots \ y_n \ \cdots \ y_{4\left\lfloor \frac{N}{2} \right\rfloor})^T
\]

(11a)

\[
y_n = \begin{cases} 
R'_{\text{odd}, n}, & \text{odd } n, 1 \leq n \leq 2 \left\lfloor \frac{N}{2} \right\rfloor \\
X'_{\text{odd}, n}, & \text{even } n, 1 \leq n \leq 2 \left\lfloor \frac{N}{2} \right\rfloor \\
R'_{\text{even}, n+\text{mod}(N,2)}, & \text{odd } n, 2 \left\lfloor \frac{N}{2} \right\rfloor + 1 \leq n \leq 4 \left\lfloor \frac{N}{2} \right\rfloor \\
X'_{\text{even}, n+\text{mod}(N,2)}, & \text{even } n, 2 \left\lfloor \frac{N}{2} \right\rfloor + 1 \leq n \leq 4 \left\lfloor \frac{N}{2} \right\rfloor
\end{cases}
\]

(11b)

\[
B_{av} = \begin{pmatrix} 
1 & 0 & \cdots & 1 & 0 & 1 & \cdots & 1 & 0 & \cdots & 1 & 0 \\
0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 & \cdots & 0 & 1
\end{pmatrix}^T
\]

(11c)

where the superscript of \( T \) denotes the transpose operation of a matrix and mod \( (N, 2) \) is equal to 1 or 0 for the odd or even numbers \( N \), respectively. After some algebraic computations, \( \overline{R} \) and \( \overline{X} \) in (11) can be simplified as

\[
\overline{R} = \frac{1}{2\pi} \left( \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} R'_n + \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} R'_n \right)
\]

(12a)

\[
\overline{X} = \frac{1}{2\pi} \left( \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} X'_n - \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} X'_n \right)
\]

(12b)

As the conventional perfect matching condition with zero reflection at the target frequency usually leads to narrow matching, in order to achieve a wideband matching, the condition of minimum average power reflection coefficient \( |\Gamma_p(\theta)| \) in a given operating frequency range of \( (f_i, f_h) \) should be satisfied, namely

\[
|\Gamma_p(\theta)| = \frac{1}{N} \sum_{n=1}^{N} |\Gamma_{p,n}(\theta_n)| = \frac{1}{N} \sum_{n=1}^{N} |\Gamma_{v,n}(\theta_n)|^2
\]

(13)

where \( \Gamma_{p,n}(\theta_n) \) and \( \Gamma_{v,n}(\theta_n) \) denote the power and voltage reflection coefficients at the \( n \)th operating frequency, respectively, and \( |\Gamma_{v,n}(\theta_n)| \) is given by [18], [26]

\[
|\Gamma_{v,n}(\theta_n)| = \left| \frac{Z_m(\theta_n) - Z'_L(\theta_n)}{Z_m(\theta_n) + Z'_L(\theta_n)} \right| = \left| \frac{Z_m(\theta) - R'_n + jX'_n}{Z_m(\theta) + R'_n + jX'_n} \right|.
\]

(14)

In (14), \( Z_m(\theta) \) is the input impedance of matching stage, which is complex conjugated to the optimum loads of \( Z'_L(\theta) = R' + jX' \) for a well-matched transformer. Then, one has

\[
Z_{m}(\theta) = \begin{cases} 
R' - jX', & 1 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor \\
R' + jX', & N + 1 - \left\lfloor \frac{N}{2} \right\rfloor \leq n \leq N
\end{cases}
\]

(15)

Substituting (10a), (14), and (15) into (13), after some algebraic computations, one obtains

\[
|\Gamma_p(\theta)| = \frac{1}{2\pi} \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left[ \frac{(R'_n - R'_n)^2 + (X'_n - X'_n)^2}{(R'_n + R'_n)^2 + (X'_n + X'_n)^2} \right] + \frac{1}{2\pi} \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} \left[ \frac{(R'_n - R'_n)^2 + (X'_n - X'_n)^2}{(R'_n + R'_n)^2 + (X'_n + X'_n)^2} \right]
\]

\[
\approx \frac{1}{2\pi} \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left[ \frac{(R'_n - R'_n)^2 + (X'_n - X'_n)^2}{(R'_n + R'_n)^2 + (X'_n + X'_n)^2} \right] + \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} \left[ \frac{(R'_n - R'_n)^2 + (X'_n - X'_n)^2}{(R'_n + R'_n)^2 + (X'_n + X'_n)^2} \right]
\]

\[
= \frac{1}{2\pi} \left\{ \sum_{n=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left[ (b_n R'_n - b_n R'_n)^2 + (b_n X'_n - b_n X'_n)^2 \right] \right\} + \sum_{n=N+1-\left\lfloor \frac{N}{2} \right\rfloor}^{N} \left[ (b_n R'_n - b_n R'_n)^2 + (b_n X'_n + b_n X'_n)^2 \right]
\]

(16a)

\[
b_n = \frac{1}{\sqrt{(R'_n + R'_n)^2 + (X'_n - X'_n)^2}} \quad \text{for } 1 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\]

\[
= \frac{1}{\sqrt{(R'_n + R'_n)^2 + (X'_n + X'_n)^2}} \quad \text{for } \left\lfloor \frac{N}{2} \right\rfloor + 1 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\]

(16b)
where, due to the relationship of \(|R' - R| \ll R + R'\), the expressions of \((R' + R_n^2)(X' - X_n^2)\) and \((R' + R_n^2 + X' + X_n^2)\) in the denominator are well approximated to \((R + R_n^2)^2 + (X' - X_n^2)^2\) and \((R + R_n^2 + X' + X_n^2)^2\), respectively. Similar to (10b), the two unknowns of \(R'\) and \(X'\) in (16b) can also be obtained by the least-square method

\[
A = (B^T B)^{-1} B^T \cdot Y \\
A = (R' X')^T, \quad Y = \begin{pmatrix} y_1 & y_2 & \cdots & y_n & \cdots & y_{L(\frac{1}{2})} \end{pmatrix}^T
\]

\[
y_n = \begin{cases} \frac{b_{n+1} R_{n}^{-1}}{a_{n}}, & \text{odd } n, 1 \leq n \leq 2 \cdot \left\lfloor \frac{N}{2} \right\rfloor \\
\frac{b_n X'}{a_n}, & \text{even } n, 1 \leq n \leq 2 \cdot \left\lfloor \frac{N}{2} \right\rfloor \\
\frac{b_{n+1} R_{n}^{-1} \cdot \text{mod}(N,2)}}{a_{n}}, & \text{odd } n, 2 \cdot \left\lfloor \frac{N}{2} \right\rfloor + 1 \leq n \leq 4 \cdot \left\lfloor \frac{N}{2} \right\rfloor \\
\frac{b_n X'}{a_n} \cdot \text{mod}(N,2)}, & \text{even } n, 2 \cdot \left\lfloor \frac{N}{2} \right\rfloor + 1 \leq n \leq 4 \cdot \left\lfloor \frac{N}{2} \right\rfloor
\end{cases}
\]

\[
B = \begin{pmatrix} b_1 & 0 & \cdots & b_n & 0 & b_{n+1} & 0 & \cdots & b_{L(\frac{1}{2})} & 0 \\
0 & b_1 & \cdots & b_n & 0 & b_{n+1} & \cdots & 0 & b_{L(\frac{1}{2})} \end{pmatrix}^T
\]

In summary, when the capacitive and inductive loads of \(Z'_L(\theta_n) = R'_n + jX'_n\) in the matching stage are given in an operating frequency range of \((f_1, f_n)\), the two complex conjugated loads, \(R' \pm jX'\), can be directly obtained from (17d). On the other hand, when a wideband transformer in the operating frequency range of \((f, f_n)\) is realized by a dual-frequency transformer with the matching frequencies of \(f'\) and \(mf\), it is usually well-matched for the frequencies adjacent to \(f\) and \(mf\) due to their accurate input impedances of \(Z_{0}(\theta) = R' \pm jX'\). Therefore, in order to obtain the small power reflection coefficient covering the entire operating frequency range of \((f_1, f_n)\), the entire operating frequency range is divided into the two subranges of \((f, f_1/2 + f_n/2)\) and \((f_1/2 + f_n/2, f_n)\), as shown in Fig. 3. Meanwhile, \(f\) and \(mf\) are usually selected to be the center frequencies of the above subranges. Thus, one has \(f = (3f_1 + f_n)/4\) and \(mf = (f_1 + 3f_n)/4\), and the frequency ratio of \(m\) in (5) is then given by

\[
m = \frac{f + f_1 + 3f_n}{3f_1 + f_n}.
\]  

\[
\text{C. Load Transformation Stage of Wideband Transformer}
\]

In Section II-B, two complex conjugated loads are directly constructed based on the least-square method. According to this, wideband transformer with two well-matched frequencies is realized by only two transmission lines. However, the load of matching-stage equivalent transformer should be capacitive and inductive. Therefore, similar to [27], the load transformation stage shown in Fig. 2(b) should be introduced to obtain the desired capacitive and inductive loads of matching stage. According to the reactance of loads, the discussion is classified into the following three cases.

1) Wideband Transformer With Capacitive and Inductive Loads: In this case, there is no need for load transformation stage. In other words, the electrical length of the load transformation stage and the load of matching stage are given by

\[
\theta_{1} = 0, ~ Z_L(f) = Z_{L}(\theta).
\]

2) Wideband Transformer With Capacitive Loads: In this case, the reactance of the loads is almost negative in the operating frequency range of \((f, f_n)\). For analysis, the load of transformer is assumed to be \(Z_{L}(\theta) = R_n - jX_n(X_n > 0)\). Then, the transformed impedance of \(Z'_L(f)\), shown in Fig. 2(b), is given by

\[
Z'_L(f) = Z_L(\theta_1) + jZ_1 \tan \left( \frac{f_{\theta_1}}{f_0} \right) Z_1 + jZ_2 \tan \left( \frac{f_{\theta_2}}{f_0} \right) Z_2 + jX_n.
\]

where \(f_0\) is the lower matching frequency of wideband transformer with two matching frequencies and \(\theta_1\) is the electrical length at \(f_0\). When the characteristic impedance of \(Z_1\) is sufficiently large, one obtains the inequality of \(Z_1 \gg |Z_{L}(\theta) \tan(\theta_1 f/f_0)|\). Then, the input impedance of \(Z'_L(f)\) can be well approximated to

\[
Z'_L(f) \approx Z_L(\theta_1) + jZ_1 \tan \left( \frac{f_{\theta_1}}{f_0} \right) \approx R_n + \left[ Z_1 \tan \left( \frac{f_{\theta_1}}{f_0} \right) - X_n \right] .
\]

From (21), it is seen that \(Z'_L(f)\) is obtained by adding a positive reactance to \(Z_L(\theta_1)\). Ideally, one expects \(\text{Im}[Z'_L(f)] = 0\) in the entire operating frequency range. Note that the frequency number is often much larger than the only variable of \(\theta_1\), so the relationship of \(\text{Im}[Z'_L(f)] = 0\) generally is not valid at all frequencies. However, the solution of \(\theta_1\) can be obtained with the least-square method.

Considering that the expression of \(f_{\theta_1}/f_0\) is definitely positive, one expands \(\tan(f_{\theta_1}/f_0)\) around 0.5 rather than the normal value of 0 to obtain an approximate expression applicable to a wider frequency range

\[
\tan \left( \frac{f_{\theta_1}}{f_0} \right) \approx \left( \frac{f_{\theta_1}}{f_0} - 0.0793 \right) \sec^2 0.5 .
\]

Substituting (22) into (21) and solving for \(\theta_1\) based on the condition of \(\text{Im}[Z'_L(f)] = 0\) and the least-square method, one
has
\[
\theta_1 = \frac{1}{\sum_{n=1}^{N} f_n^2} \sum_{n=1}^{N} \left[ f_0 f_n \left( 0.0793 + 0.7702 \frac{X_n}{Z_1} \right) \right].
\] (23)

In (23), the imaginary part of loads \(-X_n\), the frequency of \(f_n\), and the lower matching frequency \(f_0\) are known for a given transformer. Meanwhile, \(Z_1\) is selected as a sufficiently large characteristic impedance such as 130 or 140 \(\Omega\) according to the used substrate. Therefore, the parameter of \(\theta_1\) can easily be obtained. Based on it, the matching-stage loads of \(Z_L'(\theta)\) are transformed into capacitive and inductive impedances, which can be derived by substituting \(\theta_1\) and \(Z_1\) into (20). Subsequently, the remaining electrical parameters of the wideband transformer can be solved according to Section II-B.

3) Wideband Transformer With Inductive Loads: In this case, the reactance of the loads is almost positive in the operating frequency range of \((f_1, f_2)\). For analysis, the load of transformer at the \(n\)th frequency is assumed to be \(Z_L(\theta_n) = R_n + jX_n\) \((X_n > 0, \theta_n > 0)\). Then, the discussion is classified into the following two cases according to whether the condition of \(\theta_n \ll 1\) is valid.

In the case of \(\theta_n \ll 1\), the transformation at a single frequency from \(Z_L(\theta_n) = R_n + jX_n\) to \(Z_0\) is first considered. In the Smith chart with the normalized characteristic impedance of \((Z_0, Z_L(\theta_n))^{1/2}\), the reflection coefficients of the source and load are
\[
\frac{Z_0 - \sqrt{Z_0 Z_L(\theta_n)} \left( \frac{Z_L(\theta_n)}{Z_0} \right) - \sqrt{Z_0 Z_L(\theta_n)} \right) / \sqrt{Z_0 Z_L(\theta_n)} \cdot Z_L(\theta_n) / \sqrt{Z_0 Z_L(\theta_n)} = \frac{Z_L(\theta_n)}{Z_L(\theta_n)} + \sqrt{Z_0 Z_L(\theta_n)} / \sqrt{Z_0 Z_L(\theta_n)}
\] (24)

where \(e^{j\theta_n} \approx 1 + j\theta_n \approx 1\) is used due to the condition of \(\theta_n \ll 1\). From (24), it is seen that the source and load are on a circle of almost equal reflection coefficient. Thus, the load of \(R_n + jX_n\) can be well matched to \(Z_0\) by load-transformation stage with the characteristic impedance \(Z_1 = (Z_0 Z_L(\theta_n))^{1/2}\).

Now, one considers the frequency-dependent complex load \(Z_L(\theta)\) in the frequency range of \((f_1, f_2)\), which can be well represented by its mean impedance of \(Z_L(\theta)\). Similarly, \(Z_L(\theta)\) also can be well matched to \(Z_0\) by a load transformation stage with the characteristic impedance of \(Z_1 = Z_1\), where \(Z_1\) is given by
\[
Z_1 = \sqrt{Z_0 |Z_L(\theta)|} = \sqrt{Z_0} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} (R_n + jX_n). \tag{25}
\]

In order to obtain \(\theta_1\) of the load transformation stage, substituting \(R_n + jX_n\) and \(Z_1\) for \(Z_L(\theta)\) and \(Z_1\) in (20) and separating it into the real and imaginary parts, one obtains
\[
Z_L'(f) = Z_1 R_n \sec^2 \left( \frac{f \theta_n}{f_0} \right) \left[ Z_1 - X_n \tan \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2}
\]
\[
+ j Z_1 X_n \sec \left( \frac{f \theta_n}{f_0} \right) \left[ Z_1 - X_n \tan \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2} + Z_1 X_n \tan \left( \frac{f \theta_n}{f_0} \right) \left[ Z_1 - X_n \tan \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2}
\] (26)

For the optimum parameter of \(\theta_1\), it is expected \(\text{Im}[Z_L'(f)] = 0\) across \((f_1, f_2)\). Thus, the least-square solution to \(\theta_1\) is
\[
k_n = \frac{\sum_{n=1}^{N} f_0 f_n \arctan(k_n)}{\sum_{n=1}^{N} f_0^2} / \sqrt{c_n}, \quad \zeta < Z_0 \tag{27a}
\]
\[
\theta_1 = \frac{\sum_{n=1}^{N} f_0 f_n \arctan(k_n)}{\sum_{n=1}^{N} f_0^2} / \sqrt{c_n}, \quad \zeta > Z_0 \tag{27b}
\]

In the case that \(\theta_n \ll 1\) is not valid, the approximate error in (24) will be introduced. However, \(Z_1\) can be expanded around \(\overline{Z}_1\) of (25) and then is deduced from the condition of minimum average power reflection coefficient \(\overline{T}_p(\theta)\) over \((f_1, f_2)\). After some algebraic computations, \(\overline{T}_p(\theta)\) is approximated to
\[
\overline{T}_p(\theta) = \frac{1}{N} \sum_{n=1}^{N} \left[ \frac{Z_1 \tan \left( \frac{f \theta_n}{f_0} \right)}{\sqrt{c_n}} \right. \left. \frac{Z_1 Z_0^2 \tan \left( \frac{f \theta_n}{f_0} \right) - X_n \left[ Z_1^2 + Z_0^2 \tan^2 \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2}}{\sqrt{c_n}} \right]
\]
\[
+ \frac{1}{N} \sum_{n=1}^{N} \left[ \frac{Z_0 Z_1^2 \sec^2 \left( \frac{f \theta_n}{f_0} \right) + R_n \left[ Z_1^2 + Z_0^2 \tan^2 \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2}}{\sqrt{c_n}} \right]
\] (28)

where, \(c_n\) is
\[
c_n = \left[ Z_0 Z_1^2 \sec^2 \left( \frac{f \theta_n}{f_0} \right) + R_n \left[ Z_1^2 + Z_0^2 \tan^2 \left( \frac{f \theta_n}{f_0} \right) \right]^{1/2} \right]
\]
\[
+ \left[ \overline{Z}_1 \left( Z_1^2 - Z_0^2 \right) \sec^2 \left( \frac{f \theta_n}{f_0} \right) + X_n \left[ Z_1^2 + Z_0^2 \tan^2 \left( \frac{f \theta_n}{f_0} \right) \right] \right]
\]

Similarly, the formula of (28) can be treated as a least-square equation with the only unknown of \(Z_1\). Then, the optimum characteristic impedance of \(Z_1\) corresponding to the minimum \(\overline{T}_p(\theta)\) is given by
\[
b_{1z} = \frac{Z_1 \tan \left( \frac{f \theta_n}{f_0} \right)}{\sqrt{c_n}}, \quad b_{2z} = \frac{Z_0 \sec \left( \frac{f \theta_n}{f_0} \right)}{\sqrt{c_n}} \tag{29a}
\]
\[
y_1z = \frac{Z_1 Z_0^2 \tan \left( \frac{f \theta_n}{f_0} \right) - X_n \left[ Z_1^2 + Z_0^2 \tan^2 \left( \frac{f \theta_n}{f_0} \right) \right]}{\sqrt{c_n}} \tag{29b}
\]


When $Z_1$ and $\theta_1$ of load transformation stage with inductive loads are derived, the capacitive and inductive loads of $Z'_1(\theta)$ in the matching stage can be obtained by (20). Subsequently, the electrical parameters of the matching stage can be attained according to Section II-B.

In summary, the load transformation stage can be absent for the wideband transformer with capacitive and inductive loads, while it can be realized by one-section transmission line for the transformer with capacitive or inductive loads. Considering that the matching stage comprises two-section transmission lines, the entire wideband transformer can be realized by only two- or three-section transmission lines.

D. Design Flowchart of Wideband Transformer

In order to clearly demonstrate the design process of a wideband transformer, Fig. 4 shows the design flowchart based on the proposed method, which includes eight steps in general. Alternately, in order to obtain a simple but less exact solution, a computer-aided design or optimization procedure with regard to the only two parameters of $Z_2$ and $Z_3$ can be used instead of those between steps 4 and 7.

E. Wideband Transformer in WPD Application

In Sections II-A–II-D, a generalized approach for the wideband transformer with frequency-dependent complex loads has been proposed. Based on it, a wideband WPD shown in Fig. 5(a) is developed, which comprises two stages. The first stage named “matching element” is composed of two-section coupled lines (CLs), whereas the second stage named “isolation element” comprises a one-section normal transmission line and an isolation resistor. Due to the symmetry of the WPD, its odd- and even-mode equivalent circuits are obtained and shown in Fig. 5(b) and (c), respectively. Obviously, the odd- and even-mode equivalent circuits of matching element act as the matching stage of a transformer discussed in Section II-B, while those of the isolation element work as the load transformation stage of a transformer. Therefore, the isolation element is used to obtain capacitive and inductive loads, which are desired for the matching element. Besides, it is also aimed to fulfill the isolation and load conditions of $Z_{\text{med}}(\theta_m) \approx Z_{\text{med}}(\theta_m) = 2Z_0$, and then, small and feasible coupling coefficients of matching-element CLs in practical applications can be obtained. The reason for this is that under the above condition, the odd- and even-mode equivalent circuits of WPD shown in Fig. 5(b) and (c) can be treated as two transformers with identical source of $Z_0$ and approximately equal loads of $Z_{\text{med}}(\theta_m)$ or $Z_{\text{med}}(\theta_m)$. Hence, the odd- and even-mode characteristic impedances of matching element are approximately equal. For analysis, the electrical length of isolation-element transmission line is set to $\theta_0$ at the center frequency of wideband WPD, and the characteristic impedance is assumed to be $Z_i$. Meanwhile, the electrical length of matching-element CLs defined as $\theta_m$ at the center frequency of wideband WPD, and the odd- and even-mode characteristic impedances are assumed to be $Z_{\text{men}}$ and $Z_{\text{men}}$ $(n = 1, 2)$, respectively.

1) Design of Isolation Element: Due to the additional isolation resistor $R_0$ and the zero termination impedance of odd-mode equivalent circuit in the isolation element, the odd-mode equivalent circuit of the isolation element is different from the load transformation stage in a wideband transformer. Thus, the parameters of the isolation element in WPD are not determined by (19), (23), (27), or (29) but can be derived from the load condition of $Z_{\text{med}}(\theta_m) \approx Z_{\text{med}}(\theta_m) = 2Z_0$.

In the even-mode equivalent circuit as shown in Fig. 5(c), in order to fulfill the load condition of $Z_{\text{med}}(\theta_m) = 2Z_0$, the termination impedance of $2Z_0$ should be transformed to
2\(Z_0\) by the transmission line of isolation element. Obviously, the characteristic impedance of isolation-element transmission line can be selected to be
\[
Z_i = 2Z_0. \tag{30}
\]

In the odd-mode equivalent circuit as shown in Fig. 5(b), the load of \(Z_{\text{med}}(\theta_m)\) is given by
\[
Z_{i\text{med}}(\theta) = jZ_i\tan\theta, \tag{31a}
\]
\[
Z_{\text{med}}(\theta_m) = \frac{Z_{i\text{med}}(\theta)R_0}{2Z_{i\text{med}}(\theta_i)+R_0} = \frac{R_0}{1+jZ_{i\text{med}}(\theta_i)R_0}. \tag{31b}
\]

From (31b), it is readily concluded that \(Z_{\text{med}}(\theta_m) \approx R_0/2\) under the condition of \(|Z_{i\text{med}}(\theta)| \gg R_0/2\). Therefore, in order to fulfill the load condition of \(Z_{\text{med}}(\theta_m) \approx 2Z_0\), \(Z_{i\text{med}}(\theta)\) should be chosen as large as possible, whereas \(R_0\) should be approximate to \(4Z_0\). Moreover, to obtain large \(|Z_{i\text{med}}(\theta)| = jZ_i\tan\theta|\) in the entire operating bandwidth of WPD, the frequency with maximum \(|Z_{i\text{med}}(\theta)|\) should be selected to be the center frequency. Obviously, \(|Z_{i\text{med}}(\theta)|\) increases with \(\theta\) in the range of \(0, \pi/2\) but decreases in the range of \((\pi/2, \pi)\). Thus, \(\theta_i\) should be selected as \(\pi/2\) at the center frequency of wideband WPD. Namely
\[
\theta_i = \frac{\pi}{2}. \tag{32}
\]

Now, let us consider the parameter of \(R_0\). As pointed out above, in the case of large \(|Z_{i\text{med}}(\theta)|\) in the entire operating frequency range of \((f_1, f_2)\), \(R_0\) should be approximate to \(4Z_0\) to fulfill the load condition of \(Z_{\text{med}}(\theta_m) \approx 2Z_0\). In the case of other \(|Z_{i\text{med}}(\theta)|\), although \(Z_{\text{med}}(\theta_m)\) is also approximate to the constant \(R_0/2\) by selecting \(R_0/2 \ll |Z_{i\text{med}}(\theta)|\), it results in \(Z_{\text{med}}(\theta_m) \approx 0\). Therefore, \(R_0\) should be sufficiently large to fulfill the load condition of \(Z_{\text{med}}(\theta_m) \approx 2Z_0\), and a compromise is made by selecting \(R_0\) as \(R_0 = \min[|Z_{i\text{med}}(\theta)|]\) for this case. Then, for any \(Z_{i\text{med}}(\theta)\), \(R_0\) can be given by
\[
R_0 = \min[|Z_{i\text{med}}(\theta)|, 4Z_0]. \tag{33}
\]

2) Design of Matching Element: In the odd-mode equivalent circuit, from (31b), it is easily concluded that the imaginary parts of \(Z_{\text{med}}(\theta_m)\) in the ranges of \((0, \pi/2)\) and \((\pi/2, \pi)\) are positive and negative, respectively. Thus, \(Z_{\text{med}}(\theta_m)\) is a capacitive and inductive load in the operating frequency range of WPD. Subsequently, the design of the odd-mode equivalent circuit is reduced to a wideband real-to-complex transformer with capacitive and inductive loads discussed in Section II-B, whose source of \(Z_S\) and load of \(Z'_L(\theta)\) are given by
\[
Z_S = Z_0, \quad Z'_L(\theta) = Z_{\text{med}}(\theta_m) = \frac{jZ_iR_0\tan\theta}{j2Z_i\tan\theta_i+R_0}. \tag{34}
\]

Thus, the electrical length of the matching element and two optimum complex conjugated loads of \(R' \pm jX'\) are derived from (5) and (17d), respectively. Then, the odd-mode characteristic impedances of matching-element CLs are easily obtained by (2a), (7), and (8).

In the even-mode equivalent circuit, obviously, the design is reduced to a real-to-real transformer, whose source of \(Z_S\) and load of \(Z'_L(\theta)\) are given by
\[
Z_S = Z_0, \quad Z'_L(\theta) = R' = 2Z_0. \tag{35}
\]

Based on (2a), (7), and (8), all the even-mode characteristic impedances of matching-element CLs can be easily obtained.

3) Design Flowchart of Wideband WPD: In order to clearly demonstrate the design process of a wideband WPD, Fig. 6 shows its design flowchart. Similarly, in order to obtain a simple but less exact solution, a computer-aided design or optimization procedure with regard to the only two parameters of \(Z_{\text{med}}\) and \(Z_{\text{med}2}\) can be used instead of those between steps 5 and 6.

III. SIMULATION, COMPARISONS, AND DISCUSSION

To validate the proposed method, several wideband transformers with various loads and a wideband WPD are taken as examples.

A. Wideband Transformers With Various Loads

1) Capacitive and Inductive Loads: The first example is a transformer with source impedance of \(Z_0 = 50\ \Omega\) and a frequency-dependent complex load of \(Z_L(\theta)\), whose equivalent circuit consists of an inductor of \(L = 25\ \text{nH}\), a capacitor of \(C = 3\ \text{pF}\), and a resistor of \(R = 180\ \Omega\), as shown in Fig 7. Based on the exact method [16], [17], the generalized MSRT [18], and the proposed method, their electrical parameters listed in Table I can be readily obtained, where all electrical lengths are referred to 0.71 GHz [for the generalized MSRT, the structure and parameter definition are shown...
TABLE I
ELECTRICAL PARAMETERS OF WIDEBAND TRANSFORMER WITH CAPACITIVE AND INDUCTIVE LOADS

<table>
<thead>
<tr>
<th>Methods</th>
<th>$\theta_1$</th>
<th>$\theta$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>64.3°</td>
<td>85.2°</td>
<td>63.0 Ω</td>
<td>89.5 Ω</td>
<td>83.8 Ω</td>
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<td>MSRT</td>
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<td>53.3°</td>
<td>45.4 Ω</td>
<td>56.6 Ω</td>
<td>48.7 Ω</td>
<td>67.2 Ω</td>
</tr>
<tr>
<td>Proposed</td>
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<td>77.9°</td>
<td>46.2 Ω</td>
<td>40.9 Ω</td>
<td>–</td>
<td>–</td>
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</table>

<table>
<thead>
<tr>
<th>Methods</th>
<th>$\theta_{total}$</th>
<th>Size Increase</th>
<th>Fractional Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>234.9°</td>
<td>50.8%</td>
<td>38.4%</td>
</tr>
<tr>
<td>MSRT</td>
<td>213.2°</td>
<td>36.7%</td>
<td>42.7%</td>
</tr>
<tr>
<td>Proposed</td>
<td>155.8°</td>
<td>–</td>
<td>48.7%</td>
</tr>
</tbody>
</table>

Fig. 8. Frequency responses of wideband transformer with capacitive and inductive loads.

in Fig. 1(a), whereas for the exact and proposed methods, they are shown in Fig. 1(b). From Table I, the transformer based on the proposed method comprises only two-section transmission lines, while those based on the exact method and the generalized MSRT comprise three- and four-section transmission lines, respectively. Accordingly, the size increase of the latter compared to the former is up to 50.8% and 36.7%.

Fig. 8 shows the frequency responses of transformers based on various methods. On the one hand, the return loss of transformer based on the exact method and generalized MSRT at two aimed frequencies is larger than 35 dB due to their adopted perfect matching method, while that based on the proposed method is limited to 15–20 dB in the entire operating band. On the other hand, as shown in Table I, the bandwidth (defined by 15-dB return loss) of transformer based on the proposed method is larger than those based on the exact method and generalized MSRT as the least-square method attributes to the wideband matching.

2) Inductive Loads: The equivalent circuit in [13], with electrical parameters of $L = 12 \, \text{nH}$, $C = 0.5 \, \text{pF}$, and $R = 200 \, \Omega$, is used to mimic the inductive frequency-dependent loads in the frequency range of 1.6–2.6 GHz, which covers the applications of digital cellular system (DCS), personal communications service (PCS), long-term evolution (LTE), as well as wireless local area network (WLAN). On the other hand, as shown in Table I, the bandwidth (defined by 15-dB return loss) of transformer based on the proposed method is larger than those based on the exact method and generalized MSRT as the least-square method attributes to the wideband matching.

![Fig. 8](image)

![Fig. 9](image)

![Fig. 10](image)

inductive loads of $Z_L'(\theta)$ as shown in Fig. 9 can be achieved by introducing a load transformation stage. Following the design flowchart shown in Fig. 4, all electrical parameters of the transformer can be obtained and listed in Table II ($\theta_1$, $\theta$, and $\theta_{total}$ are electrical lengths at 1.76 GHz).

Fig. 10 shows the frequency responses of wideband transformer with inductive loads. It is noted that the transformer is well-matched between 1.8 and 2.4 GHz, which corresponds to a fractional bandwidth of 30%.

3) Capacitive Loads: In the case of capacitive loads, the transformer in [13] is taken as an example, whose source and load are $Z_0 = 50 \, \Omega$ and $Z_L'(\theta)$ with the equivalent circuit parameters of $L = 2 \, \text{nH}$, $C = 2 \, \text{pF}$, and $R = 200 \, \Omega$ shown in Fig. 11. On the one hand, based on the exact method [16], [17] and generalized MSRT [18], the electrical parameters of wideband transformers are readily obtained and listed in the second and third rows of Table III, respectively ($\theta_1$, $\theta$, and $\theta_{total}$ are electrical lengths at 0.6 GHz). On the other hand, considering the capacitive load of transformer in the frequency range of 0.3–1.2 GHz, an additional transmission line with the characteristic impedance of $Z_1 = 140 \, \Omega$ is introduced for the transformer based on the proposed method.
Then, following the design flowchart shown in Fig. 4, its electrical parameters are obtained and listed in the fourth row of Table III. Although it comprises three-section transmission lines, its size is still smaller than those based on the exact method and the generalized MSRT.

Fig. 12 shows the frequency responses of transformers based on various design methods. It is observed that the bandwidths of transformers based on the exact method [16], [17] and the generalized MSRT [18] are smaller than the one based on the proposed method. The reason for this is that the latter is based on the wideband-matching method.

Another example with capacitive loads is the transformer with source of $Z_0 = 50 \, \Omega$ and frequency-dependent load in [18], which mimics the load of a power amplifier. The transformer operates in the frequency range of 0.9–3.0 GHz, which covers various applications such as global positioning system (GPS), DCS, PCS, LTE, as well as WLAN. In order to realize the frequency-dependent loads shown in Fig. 13, its equivalent circuit parameters are selected as the ones listed in Table IV. Following the design flowchart shown in Fig. 4, the electrical parameters of wideband transformer based on the proposed method are easily obtained and listed in Table IV (all electrical lengths are referred to 1.627 GHz). Then, the transformer is simulated and fabricated on a Rogers 4350B substrate with the relative dielectric constant of 3.48 and the thickness of 0.508 mm, whose structure and physical dimensions are shown in Fig. 14(a) and Table IV, respectively. Fig. 14(b) and (c) further shows the photographs of fabricated transformer with an overall size of 0.025 $\lambda_g$. 

### Table III

<table>
<thead>
<tr>
<th>Methods</th>
<th>$\theta_1$</th>
<th>$\theta$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
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<tr>
<td>Exact</td>
<td>57.0°</td>
<td>68.6°</td>
<td>42.4 $\Omega$</td>
<td>13.5 $\Omega$</td>
<td>6.8 $\Omega$</td>
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<tr>
<td>MSRT</td>
<td>–</td>
<td>46.1°</td>
<td>69.6 $\Omega$</td>
<td>11.5 $\Omega$</td>
<td>18.2 $\Omega$</td>
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<tr>
<td>Proposed</td>
<td>23.1°</td>
<td>73.3°</td>
<td>140 $\Omega$</td>
<td>31.20 $\Omega$</td>
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### Table IV

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<th>$\theta_{L2}$</th>
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<th>$Z_{L4}$</th>
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<tr>
<td>30.4°</td>
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<td>120 $\Omega$</td>
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<th>$R$</th>
<th>$\theta_1$</th>
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<th>$\theta_{total}$</th>
<th>$Z_1$</th>
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<tr>
<td>60 $\Omega$</td>
<td>12.3°</td>
<td>73.2°</td>
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<td>130 $\Omega$</td>
<td>75.6 $\Omega$</td>
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<table>
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<th>$W_1$</th>
<th>$W_2$</th>
<th>$W_3$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 mm</td>
<td>0.51 mm</td>
<td>0.73 mm</td>
<td>4.2 mm</td>
<td>22.8 mm</td>
<td>22.2 mm</td>
</tr>
</tbody>
</table>
Fig. 15. Frequency responses of fabricated wideband transformer with frequency-dependent complex loads.

TABLE V
COMPARISONS OF FREQUENCY-DEPENDENT TRANSFORMERS BASED ON VARIOUS METHODS

<table>
<thead>
<tr>
<th>Refs</th>
<th>Transmission Lines</th>
<th>Dominated Equations</th>
<th>Based Methods</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>[13]</td>
<td>–</td>
<td>Closed-Form</td>
<td>FP</td>
<td>Low Frequency</td>
</tr>
<tr>
<td>[16]</td>
<td>3</td>
<td>Quartic</td>
<td>TLT</td>
<td>Narrow Matching</td>
</tr>
<tr>
<td>[17]</td>
<td>3</td>
<td>Quartic</td>
<td>TLT</td>
<td>Narrow Matching</td>
</tr>
<tr>
<td>[18]</td>
<td>4</td>
<td>S×5 Matrix</td>
<td>MSRT</td>
<td>Narrow Matching</td>
</tr>
<tr>
<td>This Work</td>
<td>2 or 3</td>
<td>Closed-Form and Algebraic Formulas</td>
<td>MSRT and LSM</td>
<td>–</td>
</tr>
</tbody>
</table>


Fig. 16. Physical dimension of wideband WPD.

TABLE VI
ELECTRICAL AND DIMENSION PARAMETERS OF WIDEBAND WPD

<table>
<thead>
<tr>
<th>$R_0$</th>
<th>$Z_m$</th>
<th>$Z_{m1}$</th>
<th>$Z_{m2}$</th>
<th>$Z_{m2}$</th>
<th>$\theta_m$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$L_4$</th>
<th>$L_5$</th>
<th>$g_2$</th>
<th>$g_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 $\Omega$</td>
<td>100 $\Omega$</td>
<td>75.49 $\Omega$</td>
<td>38.84 $\Omega$</td>
<td>65.05 $\Omega$</td>
<td>48.39 $\Omega$</td>
<td>90$^\circ$</td>
<td>1.1 mm</td>
<td>0.25 mm</td>
<td>0.75 mm</td>
<td>0.81 mm</td>
<td>1.2 mm</td>
<td>8.3 mm</td>
</tr>
<tr>
<td>90$^\circ$</td>
<td>$W_1$</td>
<td>$W_2$</td>
<td>$W_3$</td>
<td>$L_1$</td>
<td>$L_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 mm</td>
<td>11.2 mm</td>
<td>10.3 mm</td>
<td>0.12 mm</td>
<td>0.48 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

its thru-reflect-line (TRL) calibration fixture, respectively ($\lambda_g$ is the guided wavelength at 1.627 GHz).

Fig. 15 shows the calculated (MATLAB 2016a), simulated (HFSS 15.0), and measured (Agilent E8361C) frequency responses of the fabricated transformer, which are obtained on the basis of postprocessing similar to [18]. It is noted that the transformer is well-matched between 1.2 and 2.45 GHz with a return loss larger than 15 dB, which corresponds to a fractional bandwidth of 68.5%. Meanwhile, the insertion loss is smaller than 0.3 dB. In addition, good agreements among the calculated, simulated, and measured results are observed, and the slight differences are mainly attributed to the meandering transmission lines and the manufacture tolerance.
TABLE VII
COMPARISONS OF VARIOUS WIDEBAND POWER DIVIDERS

<table>
<thead>
<tr>
<th>Refs</th>
<th>Frequency Range (GHz)</th>
<th>FBW (%)</th>
<th>Return Loss (dB)</th>
<th>Insertion Loss (dB)</th>
<th>Isolation (dB)</th>
<th>Amplitude Imbalance (dB)</th>
<th>Phase Imbalance(°)</th>
<th>Lumped Elements</th>
<th>Size (λg²)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>[28]</td>
<td>3.1 - 10.6</td>
<td>109</td>
<td>10</td>
<td>3.5</td>
<td>25</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>Multilayer Structure</td>
</tr>
<tr>
<td>[29]</td>
<td>3.1 - 10.6</td>
<td>109</td>
<td>15</td>
<td>3.6</td>
<td>13</td>
<td>0.1</td>
<td>2</td>
<td>1</td>
<td>0.021</td>
<td>Multilayer Structure</td>
</tr>
<tr>
<td>[30]</td>
<td>1.45 - 4.61</td>
<td>105</td>
<td>15</td>
<td>3.7</td>
<td>15</td>
<td>0.15</td>
<td>6.8</td>
<td>2</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>[31]</td>
<td>1.25 - 2.5</td>
<td>62</td>
<td>15</td>
<td>-</td>
<td>20</td>
<td>0.2</td>
<td>0.5</td>
<td>1</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>[32]</td>
<td>2.05 - 3.95</td>
<td>62</td>
<td>16</td>
<td>3.6</td>
<td>16.5</td>
<td>0.11</td>
<td>2.65</td>
<td>3</td>
<td>0.132</td>
<td></td>
</tr>
<tr>
<td>[33]</td>
<td>2.8 - 9.6</td>
<td>110</td>
<td>11</td>
<td>3.8 - 5</td>
<td>13</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>0.093</td>
<td>Multilayer Structure</td>
</tr>
<tr>
<td>This Work</td>
<td>1.9 - 6.7</td>
<td>110</td>
<td>13</td>
<td>3.6</td>
<td>25</td>
<td>0.05</td>
<td>0.7</td>
<td>1</td>
<td>0.069</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 19. Imbalance of wideband WPD. (a) Amplitude. (b) Phase.

Table V summarizes various wideband transformers realized by dual-frequency transformers. It is noted that the wideband transformer based on the proposed method features easy design as well as compact size.

B. Wideband Transformer in WPD Application

For validation, a wideband WPD with a center frequency of 4 GHz is designed, simulated, and fabricated on a Rogers 4350B substrate with the relative dielectric constant of 3.48 and the thickness of 0.508 mm. Following the wideband-WPD design flowchart shown in Fig. 6, its electrical and physical dimension parameters in the structure in Fig. 16 are easily obtained and listed in Table VI ($R_0 = 72 \text{ Ω}$ is realized by two Yageo’s shunt resistors with resistances of 75 and 1800 Ω, and their part numbers are RE0402DRE0775RL and RE0402DRE071K8L, respectively). Fig. 17 shows the photograph of the fabricated WPD. It is seen that its overall size, excluding the connectors, is 0.069 $\lambda_g^2$ ($\lambda_g$ is the guided wavelength at the center frequency of 4 GHz).

Fig. 18 shows the frequency responses of the wideband WPD. It is noted that the measured results are almost consistent with the calculated and simulated results, and the slight disagreements are mainly attributed to the meandering transmission lines, manufacture tolerance, impact of soldering, and parasitic effect of isolation resistor. Furthermore, in the entire operating frequency range of 1.94–6.68 GHz, the input and output return losses are better than 13 dB, and the insertion loss is less than 3.6 dB. In addition, a fractional bandwidth of 110% defined by 25-dB isolation is achieved due to the proposed wideband-matching method in the design of odd-mode equivalent circuit.

Fig. 19 shows the imbalances of simulated and measured WPD. It is observed that the amplitude and phase imbalances between the two output ports are less than 0.05 dB and 0.7°, respectively.

Table VII compares various reported wideband WPDs. It is noted that the proposed WPD based on the design of wideband transformer is characterized by only one lumped element, compact size, wide bandwidth, high isolation, and small amplitude imbalance.

IV. CONCLUSION

In this article, a design approach for the wideband transformer with frequency-dependent complex loads is presented. By properly selecting the electrical parameters in the generalized MSRT and directly constructing two complex conjugated loads instead of the actual impedances of transformer, closed-form design formulas of wideband transformer are deduced. Also, the wideband transformer is realized by only two- or three-section cascaded transmission lines. Furthermore, the bandwidth is enhanced by introducing the least-square method instead of the conventional perfect matching. In addition, a wideband transformer is readily applied to the design of WPD with compact size and wide bandwidth.

REFERENCES


Liang Liu received the B.S. degree in electronics science and technology from Southwest Jiaotong University, Chengdu, China, in 2005, and the M.S. degree in electronics science and technology from Zhejiang University, Hangzhou, China, in 2008. He is currently pursuing the Ph.D. degree at Shanghai Jiao Tong University, Shanghai, China.

From 2008 to 2009, he was an RF Engineer with Winhap Communications Inc., Shenzhen, China. From 2009 to 2010, he was a Research Assistant with the City University of Hong Kong, Hong Kong.

From 2010 to 2013, he was an RF Engineer with SED Wireless Communications Inc., Shenzhen. His current research interests include multifrequency/wideband power dividers, filters, and power amplifiers.

Xianling Liang (Senior Member, IEEE) received the B.S. degree in electronic engineering from Xidian University, Xi’an, China, in 2002, and the Ph.D. degree in electric engineering from Shanghai University, Shanghai, China, in 2007.

From 2007 to 2008, he was a Post-Doctoral Research Fellow with the Institute National de la Recherche Scientifique, University of Quebec, Montreal, QC, Canada. In 2008, he joined the Department of Electronic Engineering, Shanghai Jiao Tong University (SJTU), Shanghai, as a Lecturer where he became an Associate Professor in 2012. He has authored or coauthored over 250 papers, including 141 journal articles and 113 conference papers, and coauthored one book and three chapters in microwave and antenna fields. He holds 15 patents in antenna and wireless technologies. His current research interests include OAM-EM wave propagation and antenna design, time-modulated-4-D array and applications, anti-interference antenna and array, integrated active antenna and array, and ultrawideband wide-angle scanning phased array.

Dr. Liang was a recipient of the Award of Shanghai Municipal Excellent Doctoral Dissertation in 2008, the Nomination of the National Excellent Doctoral Dissertation in 2009, the Best Paper Award presented at the International Workshop on Antenna Technology: Small Antennas, Innovative Structures, and Materials in 2010, the SMC Excellent Young Faculty and the Excellent Teacher Award of SJTU in 2012, the Shanghai Natural Science Award in 2013, the Best Paper Award presented at the IEEE International Symposium on Microwave, Antenna, Propagation, and EMC Technologies in 2015, the 4th China Publishing Government Book Award, and the Okawa Foundation Research Grant in 2017.
Ronghong Jin (Fellow, IEEE) received the B.S. degree in electronic engineering, the M.S. degree in electromagnetic and microwave technology, and the Ph.D. degree in communication and electronic systems from Shanghai Jiao Tong University, Shanghai, China, in 1983, 1986, and 1993, respectively. In 1986, he joined the Department of Electronic Engineering, Shanghai Jiao Tong University, as a Faculty Member, where he has been an Assistant, a Lecturer, and an Associate Professor and is currently a Professor. From 1997 to 1999, he was a Visiting Scholar with the Department of Electrical and Electronic Engineering, Tokyo Institute of Technology, Meguro, Japan. From 2001 to 2002, he was a Special Invited Research Fellow with the Communication Research Laboratory, Tokyo, Japan. From 2006 to 2009, he was a Guest Professor with the University of Wollongong, Wollongong, NSW, Australia. He is also a Distinguished Guest Scientist with the Commonwealth Scientific and Industrial Research Organization, Sydney, NSW, Australia. He has authored or coauthored over 300 papers in refereed journals and conference proceedings and coauthored four books. He holds approximately 60 patents in antenna and wireless technologies. His current research interests include antennas, electromagnetic theory, numerical techniques for solving field problems, and wireless communication.

Dr. Jin is a Committee Member of the Antenna Branch of the Chinese Institute of Electronics, Beijing, China. He was a recipient of the National Technology Innovation Award, the National Nature Science Award, the 2012 Nomination of the National Excellent Doctoral Dissertation (Supervisor), the 2017 Excellent Doctoral Dissertation (Supervisor) of the China Institute of Communications, the Shanghai Nature Science Award, and the Shanghai Science and Technology Progress Award.

Haijun Fan (Member, IEEE) received the B.S. degree in applied physics from the University of Electronic Science and Technology of China, Chengdu, China, in 2009, and the M.S. degree in electromagnetic and microwave technology and the Ph.D. degree in electronic science and technology from Shanghai Jiao Tong University, Shanghai, China, in 2011 and 2016, respectively. From 2016 to 2017, he was a Post-Doctoral Fellow with the Department of Electronic Engineering, The Chinese University of Hong Kong, Hong Kong, SAR, China. From 2018 to 2019, he was a Research Associate with the Institute of Sensors, Signals and Systems, Heriot-Watt University, Edinburgh, U.K. From 2019 to 2020, he was a Senior Design Engineer with Ampleon, Nijmegen, The Netherlands. He is currently an Assistant Professor with the Institute of Sensors, Signals, and Systems, Heriot-Watt University, Edinburgh, U.K. His current research interests include monolithic microwave integrated circuit (MMIC) power amplifier, filter synthesis, antenna array, and 5G/6G related areas.

Dr. Fan was a recipient of the Outstanding Ph.D. Thesis Award of China Institute of Communications in 2017.

Junping Geng (Senior Member, IEEE) received the B.S. degree in plastic working of metals, the M.S. degree in corrosion and protection of equipment, and the Ph.D. degree in circuit and system from Northwestern Polytechnic University, Xi’an, China, in 1996, 1999, and 2003, respectively. From 2003 to 2005, he was a Post-Doctoral Researcher with Shanghai Jiao Tong University (SJTU), Shanghai, China. In 2005, he joined the Department of Electronic Engineering, SJTU, as a Faculty Member, where he is currently an Associate Professor. He has been involved in electromagnetic compatibility for high-altitude platform stations, multiantennas for terminals, smart antennas, and nanoantennas. He has authored or coauthored over 260 refereed journal and conference papers, two book chapters, and one book. He holds 50 patents with over 50 pending. His current research interests include antennas, electromagnetic theory, and computational techniques of electromagnetic and nanoantennas.

Dr. Geng is a member of the Chinese Institute of Electronics, Beijing, China. He was a recipient of the Technology Innovation Award of the Chinese Ministry of Education in 2007 and the Technology Innovation Award of the Chinese Government in 2008.

Han Zhou received the B.S. degree from Shanghai Jiao Tong University, Shanghai, China, in 2014, where he is currently pursuing the Ph.D. degree in electromagnetic and microwave technology. His current research interests include multi-input–multi-output antenna, metamaterials, microwave devices, and phased arrays.

Xudong Bai (Member, IEEE) received the B.S. degree in electronic engineering, the M.S. degree in electromagnetic and microwave technology, and the Ph.D. degree in electronic science and technology from Shanghai Jiao Tong University (SJTU), Shanghai, China, in 2009, 2012, and 2016, respectively. In 2016, he joined the Research and Development Center, Shanghai Aerospace Electronics Company Ltd., Shanghai, where he is currently an Advanced Engineer. His current research interests include phased arrays, electromagnetic metamaterials, and OAM-EM wave propagation and antenna design.

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