The effect of asymmetric predictability of conditional variances and covariances on asset allocation

THE EFFECT OF ASYMMETRIC PREDICTABILITY OF CONDITIONAL VARIANCES AND COVARIANCES ON ASSET ALLOCATION

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Abstract

Five multivariate-GARCH models and two less complicated benchmarks are used to analyse the effect on asset allocation of time variation, asymmetries and spill-over effects in the conditional covariance matrix of asset-class returns. The paper documents significant time variation, asymmetries and spill-over effects in the covariance matrix of UK stocks and government bonds. Volatility spills over from bonds to stocks but not vice versa. Comparison of portfolios created from the estimates of the various models show as much as 59 percent difference in the performance of efficient portfolios from taking these effects into account than from ignoring them.

Key words: GARCH; asymmetry; portfolio theory; asset allocation; performance measurement.

JEL classification: G11, G15, C32

1. Introduction

Investors usually have to diversify locally as well as internationally. For example, a typical aggregate allocation by UK pension funds puts more than 76% of funds in UK investments.[1] Asset allocation seeks to diversify among asset classes in the same manner as Markowitz portfolio theory diversifies among individual securities. Moreover, just as the covariances between individual securities are responsible for the reduction in total portfolio risk in diversified portfolios, so are the covariances between asset classes responsible for the 'optimal' asset allocation. The less is the covariation between asset classes (or the less is the correlation) the more is the reduction in total risk. Any misspecification in modelling this comovement between asset classes may eventually result in an inefficient or a sub-optimal allocation of funds.

There is a large body of literature showing evidence of certain predictability in the conditional second moments of asset returns.[2] Most of the literature, however, concentrates on modelling time variation in the conditional second moments using univariate techniques. These techniques usually use only the information in the asset's own return history, while multivariate techniques use the information embedded in the entire variance-covariance matrix of the returns of all assets considered. This ability to model the entire covariance matrix facilitates the investigation of various interactions between assets' conditional variances, covariances, and shocks. Some of these interactions have been useful in detecting certain predictabilities in stock returns, variances, and covariances. For example, Lo and MacKinlay (1990a) and Mech (1990) present evidence of an asymmetry in the ability to predict returns of stocks of different

[1] 49% in UK company securities, 9.2% in British government securities and the rest in other asset classes like cash (5.8%), UK land property and ground rents (8.7%), and unit trusts and short term assets (3.2%) - Her Majesty's Statistical Office, end 1991.

market value. They show that returns of large-capitalisation stocks can be used to predict reliably the returns of smaller stocks, but not vice versa. Lo and MacKinlay conclude that this asymmetry requires "further investigation of mechanisms by which aggregate shocks to the economy are transmitted from large capitalisation companies to small ones."

Conrad, Gultekin, and Kaul (1991) investigate further the predictable price behaviour of different securities. They examine empirically the differential predictability of the conditional variance of the returns of large versus small firms. Their motivation for analysing conditional volatilities follows directly from Ross (1989), who shows that the variance of price changes is related directly to the rate of flow of information. "Hence, studying the differential predictability of volatilities may shed light on the process by which information is assimilated across firms of different market value."

Using both univariate and multivariate specifications of the GARCH family of statistical processes on US stock returns from 1962 to 1988, Conrad, Gultekin, and Kaul model conditional variances and estimate the interaction between the conditional volatilities of different securities. They present evidence of "a distinct asymmetry in the predictability of the volatilities of large versus small firms: a volatility "surprise" to larger firms can be used to predict reliably the volatility of smaller market value firms, but not vice versa." They refer to this asymmetric predictability as volatility "spill-over effects" from larger to smaller firms.[3] However, Conrad, Gultekin, and Kaul (1991) do not report parameter estimates analysing any

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[2] Bollerslev, Chou, and Kroner (1992) provide and extensive survey of the literature up to and including 1991. Two subsequent surveys are Bera and Higgins (1992), and Diebold and Lopez (1994).

[3] Hamao, Masulis and Ng (1990), and Chan, Chan, and Karolyi (1992) used univariate models to report similar "spill-over effects." Through simulation experiments using the non-trading model of Lo and MacKinlay (1990b), Conrad, Gultekin, and Kaul (1991) show that the reported evidence of asymmetric spill-over effects are not due to infrequent/non-synchronous trading.

possible asymmetric effects in the conditional covariances, even though their model specification allows for such effects.

Kroner and Ng (1993) concentrate on analysing asymmetric effects in the conditional covariance matrix of large-firm versus small-firm portfolios. They formulate definitions for three types of possible asymmetries, namely (a) own-volatility (self-variance) asymmetry, (b) cross-volatility (cross-variance) asymmetry, and (c) covariance asymmetry. Own-variance asymmetry means that the conditional variance of an asset is affected by the sign of the asset's own return shock. For example, bad news to asset i in the last period might have a greater impact on the variance of asset i than good news. Cross-variance asymmetry means that the conditional variance of an asset is affected by the sign of the return shock of another asset. If cross-variance asymmetry exists, then unexpected bad (or good) news about asset i can affect the conditional volatility of asset j.[4] Finally, covariance asymmetry means that the covariance between two assets is affected by the sign of the return shock of at least one of the two assets. If covariance asymmetry exists between the returns of two asset classes, then unexpected bad (or good) news about at least one of the asset classes affect the degree of comovement (or flow of common information) between the two asset classes.

Kroner and Ng (1993) present results showing that the covariance between large-firm and small-firm returns is higher following a negative shock to the large-firm portfolio, while it is almost unaffected by shocks to the small-firm portfolio. This confirms the results of Conrad, Gultekin, and Kaul (1991) who concluded that small-firm news does not affect large-firm volatility. Furthermore, they report increases in the volatility of the large-firm portfolio following any news to the large firm, but especially following bad news. On the other hand, the

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[4] Some previously used spill-over models do not incorporate a cross-variance asymmetry term (e.g., Hamao, Masulis, and Ng (1990), and Chan, Chan, and Karolyi (1992)). Although the multivariate quadratic

variance of the small-firm portfolio is only "mildly affected" by news to the small firms while bad news to the large-firm portfolio has a "dominant impact" on small-firm variances. This also supports the findings of Conrad, Gultekin, and Kaul (1991), who show that large-firm news spill over to small-firm volatility. "But it provides the additional insight that it is only the bad news which spills over, and not the good news."

These studies show the existence of predictabilities in the variance-covariance matrix between US firms or US size-ranked portfolios. In this paper, I investigate the structural characteristics of the comovement between stocks and bonds, as well as the extent to which this structure and any predictabilities therein are responsible for the performance of efficient portfolios. Traditional asset-allocation models may ignore the time variability or any predictabilities that might exist in the conditional variances or covariances of asset returns. However, it is not yet clear whether models that consider these predictabilities lead to the formation of better portfolios. Furthermore, models that take such predictabilities into account do so in varying degrees and to different extents depending on the functional forms they assume and on how they parameterise the various elements of news and volatility. It is also not clear to what extent does this affect the performance of efficient portfolios. This paper aims to answer these questions by comparing the performance of portfolios formed from the inputs implied by various models of the conditional variance-covariance matrix of asset returns. The methodology adopted is outlined in Section 2. In Section 2.1, the procedure used in delinating the efficient frontier is presented. One of the inputs needed to this procedure is an estimate of the variance-covariance matrix of asset-class returns. The various models used to describe the dynamics of this matrix are presented in Section 2.2. The theoretical differences between these models are emphasised. The procedure of model comparison adopted to measure the effect of

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BEKK specification used by Conrad, Gultekin, and Kaul (1991) allows for all three forms of asymmetries, they do not report parameter estimates of asymmetric effects in the covariances.

time variation, asymmetries and spill-over effects on asset allocation is outlined in Section 2.3. The data and estimation are discussed in section 3. In Section 4 the results of assessing the impact on asset allocation are presented. A simple sub-sample robustness check is presented in Section 5. Finally, Section 6 concludes the paper.

2. The Methodology

2.1. The Portfolio Optimisation Procedure

In a portfolio optimisation exercise an investor would need estimates of expected returns, variances and covariances (or correlations) for the vector of assets considered as inputs to the optimisation procedure. Solving the optimisation procedure would then yield a vector of 'optimal' weights that represent a prescription to follow in order to achieve the highest return for a given level of risk (variance) or the lowest risk for a given level of return.

In a single-period model, the portfolio choice problem was initially solved using constant unconditional estimates of the inputs (see Grubel (1968), Levy and Sarnat (1970) and Elton and Gruber (1976)). However, in a multi-period model and in light of the overwhelming evidence of time variable conditional second moments (and more recently, covariances) it might prove beneficial to produce estimates of inputs by models that take into consideration any predictable time variability, asymmetries and spill-over effects in the conditional variance-covariance matrix. In this analysis, the following asset allocation problem is considered:

$$\begin{aligned} &\underset{\{\omega_t\}}{\text{Minimize}} \quad \omega_t' H_t \omega_t \\ &\text{subject to:} \quad E_{t-1}(r_{p,t}) = \omega_t' E_{t-1}(r_t) = R, \\ &\qquad\qquad\quad \omega_t' \underline{1} = 1. \end{aligned} \tag{1}$$

where $r_{p,t}$ is the return on a portfolio formed with the vector of weights ω_t and the vector of individual asset-class returns r_t, $\underline{1}$ is a vector of ones, and H_t is the conditional variance-covariance matrix of the asset-classes considered. Notice the time subscript, t, which indicates that the conditional variance-covariance matrix and the vector of portfolio weights are allowed to vary over time. In this analysis the interest is not in expected returns, but instead concentrates on the effect of estimating the conditional variance-covariance matrix, H_t, on the asset allocation problem outlined above. As such, H_t is estimated by five multivariate-GARCH models, presented in section 2.2, and two less complicated benchmarks. Tests are conducted to see whether estimates from a particular model consistently lead to a more efficient set of portfolios.

2.2. The Conditional Covariance Matrix

To model the time variability and any predictabilities in the conditional covariances (as well as the conditional variances) a multivariate setting is called for. The multivariate-GARCH (MGARCH) family of models is well suited to such analysis.[5]

This family of models can be presented as follows. Let ε_t be an N-dimensional vector of conditional mean zero random variables with an N×N conditional variance-covariance matrix, H_t, that can change over time. Then

$$\varepsilon_t | \psi_{t-1} \sim MD(0, H_t), \qquad (2)$$

$$H_t = G(\Psi_{t-1}; Z). \qquad (3)$$

[5] In addition to the literature reviewed in the text see Kraft and Engle (1983), Bollerslev, Engle, and Wooldridge (1988), Engle and Kroner (1993), and Longin and Solnik (1992).

is defined as a multivariate-GARCH process, where MD is a multivariate distribution and *G* is a non-negative function of elements in the information set Ψ_{t-1} and the parameter space Z. Usually ε_t is taken as the unexpected component (or residuals) resulting from modelling the first moment of some other stochastic process Y_t, where

$$Y_t = f(X_{t-1}; C) + \varepsilon_t , \tag{4}$$

and $f(X_{t-1}; C)$ is the hypothesised mean of Y_t as a function of the parameter vector C and the set of endogenous and exogenous variables X_{t-1} observed at time t-1 and included in the information set Ψ_{t-1}.

Several suggestions for formulating and parameterising the function *G* have been brought forward. Bollerslev, Engle and Wooldridge (1988) proposed a natural multivariate extension to the univariate GARCH specification of Bollerslev (1986) and Taylor (1986). This has come to be known as the Diagonal-VECH (DVECH) specification. To simplify the exposition assume that the information set, Ψ_{t-1}, available at time t-1, contains only past variances and past squared innovations and no exogenous variables. The DVECH model is given by equation (2) and

$$\mathrm{vech}(H_t) = C + \sum_{i=1}^{q} A_i \mathrm{vech}(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j \mathrm{vech}(H_{t-j}), \tag{5}$$

where vech(·) is the vector-half operator that stacks the lower half of a matrix into a vector, C is a $\frac{1}{2}N(N+1)\times 1$ vector parameter of constants, A_i, i = 1,...,q and B_j, j = 1,...,p, are $\frac{1}{2}N(N+1)\times\frac{1}{2}N(N+1)$ diagonal matrices of parameters, and ε_{t-i} is the N×1 vector of innovations of the N assets at time t-i, i=1,...,q. This specification assumes that each element of the covariance matrix, $h_{ij,t}$ depends only on past values of itself and past values of the product $\varepsilon_{i,t}\varepsilon_{j,t}$. To illustrate, the DVECH model in the bivariate GARCH(1,1) case is

$$
\begin{aligned}
h_{11,t} &= c_1 + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1} \\
h_{12,t} &= c_2 + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{22}h_{12,t-1} \\
h_{22,t} &= c_3 + a_{33}\varepsilon_{2,t-1}^2 + b_{33}h_{22,t-1}
\end{aligned} \tag{6}
$$

It is evident from these equations that the model is a simultaneous system of simple univariate-GARCH(1,1) processes in the variances and in the covariance. Sufficient conditions for positive definiteness of H_t is that a_{ii} and b_{ii} be greater or equal to zero and stationarity is ensured if $a_{ii} + b_{ii}<1$, i=1,2,3.

Baba, Engle, Kraft and Kroner (1991) propose a parameterization that allows for more general dynamics in the variances and the covariance. This parametrization, known as the BEKK model, can be written as

$$
H_t = C'C + \sum_{i=1}^{q} A_i'\varepsilon_{t-i}\varepsilon_{t-i}'A_i + \sum_{j=1}^{p} B_j'H_{t-j}B_j, \tag{7}
$$

where C, A_i and B_j are N×N parameter matrices, with C triangular. This parameterization is quadratic in the parameters involved as the ARCH term, $\varepsilon_{t-i}\,\varepsilon'_{t-i}$, and the GARCH term, H_{t-j}, are post and pre-multiplied by the corresponding parameter matrices thus ensuring positive definiteness. To illustrate the BEKK model consider the simple bivariate GARCH(1,1) case,

$$
\begin{aligned}
h_{11,t} &= c_{11} + a_{11}^2\varepsilon_{1,t-1}^2 + 2\cdot a_{11}a_{21}\cdot\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 \\
&\quad + b_{11}^2h_{11,t-1} + 2\cdot b_{11}b_{21}\cdot h_{12,t-1} + b_{21}^2h_{22,t-1}\ , \\
h_{12,t} &= c_{12} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\cdot\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 \\
&\quad + b_{11}b_{12}h_{11,t-1} + (b_{21}b_{12} + b_{11}b_{22})\cdot h_{12,t-1} + b_{21}b_{22}h_{22,t-1}\ , \\
h_{22,t} &= c_{22} + a_{12}^2\varepsilon_{1,t-1}^2 + 2\cdot a_{12}a_{22}\cdot\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 \\
&\quad + b_{12}^2h_{11,t-1} + 2\cdot b_{12}b_{22}\cdot h_{12,t-1} + b_{22}^2h_{22,t-1}\ .
\end{aligned} \tag{8}
$$

It is evident that this system of equations allows for more interactions between the elements of news, ε_i, and volatility, h_{ij}, than the DVECH.

Another specification suggested by Bollerslev (1990) is the Constant Correlation model. This restricts the conditional covariance between two assets to be proportional to the product of the conditional standard deviations, while the conditional variances follow a GARCH(p,q) process. In this model the conditional correlation coefficient between two asset-returns is assumed to be constant, but the conditional covariance may still vary over time through the GARCH evolution of the variances. The Constant Correlation GARCH(p,q) for N assets can be written in matrix notation as,

$$H_t = D_t \Gamma D_t , \tag{9}$$

where D_t is a N×N diagonal matrix with elements $\{ h_{11,t}^{1/2}, \dots, h_{NN,t}^{1/2} \}$, and Γ is the time invariant correlation matrix. The variance of each asset follows a univariate GARCH(p,q) process. This representation is positive definite if Γ is positive definite and the requirements for positive definiteness of the univariate GARCH(1,1) processes of the conditional variances are held. Stationarity is also ensured if the variances are stationary and $-1 \leq \rho_{12} \leq +1$. To illustrate, a bivariate Constant Correlation GARCH(1,1) is written as,

$$\begin{aligned} h_{11,t} &= c_1 + a_1 \varepsilon_{1,t-1}^2 + b_1 h_{11,t-1} \\ h_{12,t} &= \rho_{12} \cdot \sqrt{h_{11,t}} \cdot \sqrt{h_{22,t}} \\ h_{22,t} &= c_2 + a_2 \varepsilon_{2,t-1}^2 + b_2 h_{22,t-1} \end{aligned} \tag{10}$$

This model assumes the same dynamics in the variances as the Diagonal-VECH, but imposes a proportionality restriction on the covariance. Bollerslev (1990) used this model on five nominal European versus dollar exchange rates and found evidence of higher co-movement between the currencies following the inception of the European Monetary System. Longin and Solnik (1992) used the model to test the extent of integration between international capital markets.

Another specification is developed from the fact that many economic variables tend to react to the same information, whether it is micro or macro information. Accordingly, there may exist a few factors that are responsible for the dynamics in the conditional variances and covariances of all the assets considered. The possible existence of such factors implies that assets move together only as a response to the movement in the common factor. One model that captures such dynamics is the Factor ARCH model of Engle (1987). Let λ and ω be $N\times 1$ vectors of factor loadings and portfolio weights, and let α and φ be two scalar parameters. The specification for a one-factor GARCH (1,1) model is

$$H_t = C'C + \lambda\lambda' \cdot [\varphi.\omega' H_{t-1}\omega + \alpha.(\omega'\varepsilon_{t-1})^2], \tag{11}$$

where $\omega' H_{t-1}\omega$ is the conditional variance of the common factor and $\omega'\varepsilon_{t-1}$ is its innovation at time t-1. As is evident from this equation, the conditional variances and covariances of the assets at time t, H_t, depend solely on the conditional variance and the shock of the common factor at time t-1. In fact, the common factor in the above specification is a portfolio constructed from the N assets under consideration weighted by the vector ω. Moreover, this portfolio follows a GARCH(1,1) process. The one-Factor ARCH(1,1) can also be written as

$$\begin{aligned} h_{ij,t} &= \sigma_{ij} + \lambda_i\lambda_j \cdot h_{p,t}, \qquad \forall\, i,j = 1, \ldots ,N, \\ h_{p,t} &= w_p + \varphi \cdot h_{p,t-1} + \alpha \cdot \varepsilon^2_{p,t-1}, \end{aligned} \tag{12}$$

where $w_p \equiv \omega' C' C\omega$, $\sigma_{ij} = c_{ij} - \lambda_i\lambda_j w_p$ and c_{ij} is the $(i,j)^{th}$ element of C. It is interesting to notice that this model is a special case of the BEKK representation in which $A=\sqrt{\alpha}\omega\lambda'$ and $B=\sqrt{\varphi}\omega\lambda'$. As such, the one-factor GARCH model is positive definite as long as $C'C$ and $\omega\lambda'$ intersect only at the origin.[6]

Kroner and Ng (1993) propose a more general specification that nests the above four models together with an extension for capturing asymmetries. They call this specification the Asymmetric Dynamic Covariance (ADC) matrix model. Define a_i and b_i to be N×1 vectors of parameters equivalent to the i^{th} column of the parameter matrices A and B in the BEKK specification. Also define $\eta_{i,t} \equiv \max\{0, -\varepsilon_{i,t}\}$ and $\eta_t \equiv \{\eta_{1,t}, \ldots, \eta_{N,t}\}$. Finally, define

$$w_{ij,t} \equiv z_{ij} + b_i' H_{t-1} b_j + a_i' \varepsilon_{t-1} \varepsilon_{t-1}' a_j + \gamma_i' \eta_{t-1} \eta_{t-1}' \gamma_j . \tag{13}$$

The ADC is given by

$$\begin{aligned} h_{ii,t} &= w_{ii,t} \\ h_{ij,t} &= \rho_{ij} \cdot \sqrt{h_{ii,t} h_{jj,t}} + \phi_{ij} \cdot w_{ij,t} \end{aligned} \tag{14}$$

where the constant matrix with elements $z_{ij,t}$ is positive definite, $\rho_{ij} \in (-1,+1)$ for all $i \neq j$, and $|\phi_{ij}| < 1 - |\rho_{ij}|$. This model is reducible to either a positive version of the Diagonal-VECH model, the BEKK model, the Constant Correlation model or the Factor-GARCH model (c.f. Kroner and Ng (1993)). Unlike the other models, the asymmetric extension in the ADC captures such effects in an unrestricted manner.

The above models differ in the number of parameters required for estimation, the kind of cross-sectional structure they impose on the variance-covariance matrix, the functional form they assume, and in how they deal with asymmetries and spill-over effects. For example, the DVECH model does not allow for causality in variance (and covariance) as in Granger, Engle and Robins (1986). Volatility is not allowed to 'spill-over' from one asset to the other. This is evident in the system of equations (6) from the absence of $h_{jj,t-1}$ from the equation of $h_{ii,t}$, $i \neq j$ and the equation of $h_{ij.t}$. The Constant Correlation model does not allow for causality in variance but allows for causality in the covariance through the proportionality restriction that the model imposes. The BEKK and the Factor GARCH

[6] For a fuller description of the model and stationarity constraints see Engle(1987), Lin (1992) and Engle, Ng and Rothschild (1990).

models allow for all possible spill-over effects, however, the two models differ in how they parameterise these effects.

The models also differ in how they deal with various elements of news. Consider the bivariate illustrations above. In the DVECH and the Constant Correlation models the variance equations are simple univariate-GARCH processes where the variance of one asset, $h_{i,t}$, depends on the square of past innovation, $\varepsilon_{i,t-1}^2$, of the same asset. Since this relationship is quadratic and symmetric around $\varepsilon_{i,t-1} = 0$, no distinction is made between the effect of bad and good news (negative and positive shocks).[7] Only the effect of the magnitude of news is considered. Bad and good news of the same magnitude have the same impact on future volatility. Moreover, news about one asset is not allowed to affect the variance of the other asset. Therefore, the models do not allow self-variance or cross-variance asymmetry. In addition, the Constant Correlation model does not allow covariance asymmetry. The covariance is proportional to the product of the standard deviations. Since standard deviations are always positive, no distinction is made between the effects of negative and positive shocks on the covariance. The DVECH allows for covariance asymmetry but only when the shocks of the two assets have opposite signs. Although the BEKK and the Factor-GARCH models allow for all three types of asymmetry, they differ in the way they parameterise the elements of news.

The models also differ in the number of parameters needed for estimation. Table 1 provides a summary of these properties.

2.3. Measuring The Effect On Asset Allocation

The comparison between the performance of the various models is made against benchmarks of progressive complexity. The most 'naïve' benchmark is a Constant

[7] See Engle and Ng (1993) for a discussion on how various univariate-GARCH models deal with asymmetries.

Variance-Covariance Matrix model where the estimates of the variances and the covariances are taken as constant and equal to their unconditional sample values. This represents the most naïve of asset allocation models. Comparison of the five multivariate-GARCH models against this benchmark should reveal whether time variability and volatility clustering modelled by an autoregressive-moving average process have an impact on the efficiency of allocation. The second, Constant Covariance, benchmark assumes that the covariance between the asset-classes is constant and equal to its unconditional sample value but the variances are time varying and this time-variability is modelled by simple univariate and symmetric GARCH processes. This represents possible asset-allocation practices of modelling volatility using univariate techniques while assuming constant covariances. A comparison of more sophisticated models that allow time variability in both the variances and the covariances against the second benchmark should reveal whether covariances exhibit autoregressive conditional heteroscedasticity and the extent to which this may affect the efficiency of allocation. The third benchmark is the Constant Correlation model of Bollerslev (1990). As discussed above, the Diagonal-VECH model takes the same form in the variances as the Constant Correlation model but allows for a more general time variability in the covariance. A comparison between those two models should throw some light on the effect of the covariance restriction in the Constant Correlation model on the efficiency of allocation. Finally, if the presence of asymmetries has an effect on the portfolio problem, then the ADC matrix model might produce better estimates of the variance-covariance matrix than the other four multivariate-GARCH models. Notice that the ADC model is expected, by construction, to produce better estimates than the four multivariate-GARCH models it nests. If this is confirmed by the data in estimation, the model would provide a well-specified 'measuring rod' against which the performance of all other models can be compared. This allows the measurement of the effect on asset allocation of the shortcomings of the other models, especially that these shortcomings are outcomes of ignored asymmetries, ignored spill-over effects, assumed functional forms, or restrictive parameterisations.

To examine the relative performance of the various models, portfolios on the efficient frontier are examined. First, each model or benchmark is estimated, and the resulting series of estimated variances and covariance is used in place of H_t in the optimisation procedure. By fixing the target level of return, R, on a portfolio, the optimisation procedure will yield a set of 'optimal' weights for that portfolio for each period of time t-1 to t, t=2,...,T. Repeating this procedure for a few target returns one can trace the efficient frontier implied by the input, $\hat{H}_{k,t}$, estimated by model k, k=1,..,7. By repeating this process for the seven models the efficient frontier implied by each of the models at any particular period of time, t-1 to t, t = 2, ..., T is constructed. Therefore, for each interval of time there is a set of competing portfolios that yield the same return. Each of these portfolios, however, has a different amount of risk depending on which model is used to estimate the variance-covariance matrix. It is important to understand that these portfolios do not represent different choices available to an investor, but rather different estimates of the true efficient portfolio available at that time. The true efficient portfolio is not observable, however. The investor has to use some procedure to estimate the composition of that portfolio. The question being asked is whether an estimate obtained by using one of the models discussed above that is based on certain assumptions and restrictions regarding time variability, asymmetry and spill-over effects, fares better than other estimates based on different sets of assumptions and restrictions. The models discussed in section 2.2 capture these effects in different ways and to various extents, it is the aim of this paper to measure the impact this has on the formation of optimal portfolios.

Target return portfolios

In comparing the performance of the multivariate-GARCH models through their effect on portfolio formation the four models nested by the ADC are included even though they fit the data less well. To compare the models, the impact of different estimates of the conditional variance-covariance matrix on the reward-to-variability ratio of some pre-chosen portfolios is examined. The portfolios analysed are those with annual target excess returns of 5, 10 and 12 percent in addition to the global minimum-variance portfolio. The target-return portfolios are identified by setting a value for the excess return a portfolio should have and solving for the weights given the expected returns of the asset classes. The weights are then used in conjunction with the estimate of the variance-covariance matrix from a particular model to calculate the variance of that target-return portfolio. The reward-to-variability ratio is then calculated by dividing the target excess return by the standard deviation of that portfolio. Notice that under this set-up and during any particular period of time the asset-class weights are the same for all models of the variance-covariance matrix. Furthermore, these weights are solely determined by the set of expected excess-returns of the asset classes and the target excess-return. In fact, no risk minimisation is needed because with two asset-classes and the constraints on the weights (they sum up to unity) and the returns (being weighted and added to equal the target return) the portfolio weights are identifiable with only the set of excess returns and the target excess-return. However, the set of weights varies from period to period due to the variation in the excess returns of the two asset-classes. This variation is common for all the models of the variance-covariance matrix. Therefore, the proposed comparison between the models based on the relative performance of their reward-to-variability ratios is largely determined by the difference in their estimates of the variance-covariance matrix of the two asset-classes. In these circumstances short selling is allowed; otherwise, a portfolio with a pre-specified target excess-return may not be attainable during all periods.

The global minimum-variance portfolio

The construction of the global minimum variance portfolio is different, however. The weights are no longer determined by the set of excess returns and a target return. Portfolio variance is minimised (subject to the constraint that the weights add up to one), and the weights are obtained. These weights are then used in the usual manner to calculate the excess return and the reward-to-variability ratio of the global minimum-variance portfolio.

The constant covariance benchmark

It is also worth noting that the construction of the Constant Covariance benchmarks is theoretically incorrect. When the correlation between assets is not zero, assuming a constant covariance but time-varying variances does not impose the requirements for stationarity. In a time-varying context, the ratio of the conditional covariance to the product of the conditional standard deviations must be between -1 and +1 during all periods. The Constant Covariance benchmark does not impose this requirement. Indeed, in estimation this ratio has exceeded the upper boundary during two periods. The benchmark is included in the comparison, however, to represent possible asset allocation practices of modelling variances using univariate techniques while assuming constant covariances.

Performance of competing portfolios

One way to compare the performance of competing portfolios is to see which one achieves the lowest variance. This criteria is based on the fact that for a given level of expected return a portfolio on the true minimum-variance frontier will have a lower variance than any other portfolio with the same expected return. This is true regardless of what volatility this other portfolio is estimated to have by an incorrect model. To test this, the difference in the return-to-risk ratio of a particular return portfolio across the various models is calculated. This involves calculating the time series of the ratio of expected return-to-standard deviation for a portfolio with a fixed target return R for a particular model, and then testing the difference between this series and another calculated from a different model. A simple non-parametric t-test is used.

3. Data and Estimation Results

3.1. The Data

Two indices are used as diversified portfolios to represent two major UK asset classes. Daily return data on the Financial Times All Share (FTALL) index and the FTA all-government-bonds index are compounded from Wednesday close to Wednesday close to obtain weekly return series. Wednesdays are chosen for three reasons: a minimum number of holidays occur at this day, there is no problem of weekend effects and it avoids the Friday book settlement procedure every two weeks. The data covers the period from the first week in January 1976 to the first week in February 1995, giving a total of 995 observations. Weekly intervals are chosen as a compromise between the relatively few monthly observations and the noisy daily data. Two asset classes are considered for two reasons. First, considering the objectives of this study and some of the more elaborate models, a bivariate system is sufficient for the purposes of this analysis. Second, the number of parameters to be estimated rises geometrically with larger systems making convergence difficult to achieve and time consuming.

The returns from the two indices include dividend payments and are adjusted for events such as scrip issues, rights issues, and stock splits. The FTALL share index contains roughly 650 stocks listed on the London International Stock Exchange and the FTA all-government-bonds index is constructed to contain a selection of UK government bonds. Excess returns are calculated relative to the offer rate on seven-day lending and borrowing in the money market.[8]

3.2. The Conditional Mean

The focus of this analysis is on the conditional variance-covariance matrix rather than the conditional mean. The analysis concentrates on the unexpected part of the asset-class excess returns. An adjustment procedure similar to those used in Kroner and Ng (1993), Engle and Ng (1993), and Pagan and Schwert (1990) is used to remove the predictable component in asset-class excess returns. This procedure is a Vector Auto-Regression (VAR) of order q together with lags of a threshold term. Specifically, for two asset-classes, i = 1,2,

$$R_{i,t} = C_i + \sum_{j=1}^{2} \sum_{L=1}^{q} (\delta_{jL} R_{j.t-L} + \beta_{jL} D_{j,t-L}) + \varepsilon_{i,t} \qquad (15)$$

where C, δ and β are coefficients, $R_{i,t}$ is the excess return on asset-class i, and $D_{j,t-L}$ is a threshold term that takes the absolute value of the excess return, R_j, when it is negative. Throughout this analysis i = 1 refers to stocks and i = 2 refers to bonds. The presence of the lags in the excess returns of one asset in the return equation of the other asset, removes any predictable spill-over effects in the mean. This, and the presence of the lags of the excess returns of the same asset and the presence of the lags on the threshold term, reduces the

8 Bid and middle rates are also considered but no qualitative differences are observed in the results.

likelihood that any detectable auto-correlation, spill-over effects and asymmetries in the conditional variances and covariances are not caused by a misspecification in the mean.

The estimation results of the vector-autoregression are not reported but are available from the author. The lag order, q, is experimented with until no serial correlation is left in the residuals. Table 2 reports some diagnostics on the residuals. From the Ljung-Box test statistic for twelfth-order serial correlation for the levels, there is no significant serial correlation left in the excess-return series of stocks and bonds after the adjustment procedure. The coefficients of skewness and kurtosis indicate that the sample distribution of the excess returns of stocks is slightly skewed to the left and has slightly fatter tails than the standard normal, while the sample distribution of the excess returns of bonds is slightly skewed to the right and is less fat-tailed than that of stocks. It should be noted that ignored autoregressive dependencies in the conditional variances induce leptokurtosis (fat tails) in the unconditional distribution of the mean.[9] Therefore, the excess kurtosis reported could be an early indication of GARCH dependencies or due to some other misspecification in the mean equation. However, the significance of the Ljung-Box test statistic for twelfth-order serial correlation in the squares indicates a significant presence of time-varying volatility.

Table 2 also reports two sets of misspecification tests on any asymmetries found in the series of adjusted excess-returns. A brief description of these tests is provided in Appendix 1. The test statistics that are significant for stocks are also significant for bonds. Therefore, the following descriptions apply equally to both stocks and bonds. The first set of tests examine the different impacts of negative and positive shocks and the differential effects of their size. From the reported values of the sign-bias tests there is no preliminary evidence that past negative-return shocks affect volatility differently than past positive-return shocks. The significance of the negative-size-bias test suggests that large negative

9 See Bollerslev, Engle and Nelson (1993).

return shocks increase future volatility more than smaller ones. The positive-size-bias test is not significant, but when the size term is dropped it is - suggesting that positive return-shocks increase future volatility regardless of size.

The second set of tests examine similar aspects but in the extreme-size band of the shocks (ε_{t-1}^2). The tests suggest similar conclusions: past shocks increase volatility regardless of the sign, while large negative shocks have a significant impact on future volatility. These results, together with the joint tests, strongly indicate that the past return shock, ε_{t-1}, has an influence on current volatility.

The estimation is done in two steps. First, the vector-autoregression in equation (9) is estimated by OLS and the residuals, $\varepsilon_{i,t}$, are taken as observable data in the estimation of the conditional variance-covariance matrix. This two-step procedure is necessary in order to isolate the effects of different models of the conditional variance-covariance matrix on asset allocation. The second estimation is done by maximising the log-likelihood at time t given by

$$L_t(\theta) \equiv \ln(2\pi) - 0.5[\ln|H_t| + (\varepsilon_t' H_t^{-1} \varepsilon_t)], \tag{16}$$

using the Brendt, Hall, Hall and Hausman (BHHH) (1974) numerical optimisation algorithm assuming a multivariate-normal distribution.[10] The information matrix under this two-step procedure is block diagonal, thus, in spite of a loss in efficiency, the estimated parameters are consistent (c.f., Bollerslev and Wooldridge (1992)).

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[10] The constant term in the log-likelihood function is ignored in estimation, as it does not affect the position of the sought global maxima in terms of the estimated parameter values. The second step could be estimated while assuming a more general distribution such as the multivariate student t-distribution. However, this is not necessary since the coefficients of kurtosis of the residuals from the first step are not large, especially before the dependence in the conditional variance-covariance matrix is accounted for. Neglected ARCH can induce leptokurtosis in the unconditional distribution (see, for example, Bollerslev, Engle and Nelson (1993).

3.3. The Conditional Covariance Matrix

As a first step a simple univariate GARCH process is fitted to the adjusted excess returns of stocks and bonds. These univariate estimations are necessary for the Constant Covariance benchmark, where the variances are allowed to vary over time but the covariance is assumed constant and equal to its unconditional sample value. The estimation of the univariate processes also offers an indication of the lag order, p, q, of the GARCH(p,q) process that characterises the data. Separate univariate GARCH(p,q) processes are estimated for the excess returns of stocks and bonds with different lag structures ranging from p=q=1 to p=q=3. Among the various lag structures, p=q=1 seems to be the most appropriate for the sample data. The value of the log-likelihood function for an estimated GARCH(1,3) process for stocks is 3290.988 and for a GARCH(1,1) process is 3290.890 giving a likelihood-ratio test statistic of 0.19526 which is not significant at the five percent level. In the case of bonds, the value of the log-likelihood function for an estimated GARCH(1,3) process is 4091.197 and for a GARCH(1,1) is 4089.100 giving a likelihood-ratio test-statistic of 4.1926, which, again, is not significant at the five percent level. The estimates of a GARCH(1,1) for the excess-returns of stocks and bonds are reported in Table 3, together with Lagrange-multiplier type tests on any misspecification in the order of the lags. The tests reported in the table are a variation of the Engle and Ng (1993) misspecification tests explained in Appendix 2, and are similar to those used in Ng, Engle and Rothschild (1992). These tests indicate that a lag structure of p=q=1 is adequate in describing the autoregressive nature of the conditional heteroscedasticity that characterises the excess returns of stocks and bonds. All the coefficients of ARCH lags p=2 to p=4 (coefficients on $\varepsilon^2_{i,t-j}$, $i = 1,2,\ j = 2,3,4$) are not significant at the five-percent level, and the F-test statistics for joint misspecification (testing if all coefficients are insignificantly different from zero) are also not significant at the same level for both stocks and bonds.

The estimated coefficients reported in the table indicate a medium persistence in the volatility of stocks ($b_1^2 + c_1^2 = 0.84$), and high persistence in the volatility of bonds ($b_2^2 + c_2^2 = 0.96$), suggesting that shocks in the past level of volatility do not die quickly but have a prolonged effect on the level of future volatility - a fact that indicates a high degree of time variability in the conditional second moments of these series. This fact is of importance to asset allocators as it highlights possible long-term risk characteristics of their funds. Periods of low market volatility are likely to be followed by more periods of low volatility, while periods of high volatility are often followed by yet more periods of high volatility. This pattern appears to be more persistent in the bond market than in the stock market.

Table 3 also reports a set of tests designed to capture any misspecification regarding asymmetries. These robust conditional moment tests are developed by Kroner and Ng (1993) using the framework of Wooldridge (1990), and are similar to the Lagrange-multiplier type tests of Engle and Ng (1993). A description of these tests is provided in Appendix 2 for ease of reference. In Table 3, the robust conditional moment tests in the estimated variance of stocks reveal significant values at the five percent level for the x_1 and x_6 misspecification indicators. These values suggest that there is a significant degree of asymmetry, present in the data, arising from the effect of negative shocks (bad news) in the stock market and from positive shocks (good news) in both the stock and the bond markets that is not accounted for by the symmetric univariate GARCH model. On-the-other-hand, the estimated variance of the excess returns of bonds shows no significant misspecification regarding asymmetries. None of the misspecification indicators are significant. The assumed constant covariance shows a significant misspecification (x_9) in capturing the effect of large news in the bond market when there is bad news in the stock market. Perhaps an initial indication of the presence of this type of asymmetry in the covariance between stocks and bonds.

Following the results of Table 3, the above multivariate-GARCH models are estimated assuming a (p=1, q=1) lag structure. Estimation results are reported for only the Constant Correlation and the ADC matrix models. These are in Table 5 and Table 6.[11] Significant results of the robust conditional moment tests are reported in Table 4 for all models. In this table no test results are reported for the variance of bonds because none are significant. This indicates that the models do not differ substantially in their description of bond volatility and that there are no significant asymmetric effects of news from either market on the future volatility of bonds.

The estimation results of the ADC confirms a well specified model. The coefficients γ_{11} and γ_{12} are significant, suggesting that the ADC captures certain asymmetries unaccounted for by the other models. The misspecification indicators support this fact by exhibiting no significant misspecification in the ADC with regard to asymmetries either in the variances or the covariance. Moreover, the log-likelihood function value of 7556.13 attained by the ADC is significantly higher than that of any of the four models it nests, as indicated by a likelihood ratio of 26.618 against the BEKK, 29.424 against the Diagonal-VECH, 34.444 against the Constant Correlation and 85.361 against the Factor-GARCH model. All multivariate-GARCH models confirm the presence of persistent time variability in the variances and in the covariance. The ADC also confirms the results of the other models regarding the persistent time variation and insignificant asymmetries in the variance of bonds.

[11] Estimation results of other models are available from the author.

From the estimation results of the Constant Correlation model in Table 5, all coefficients are significantly different from zero, and the time-invariant correlation coefficient between the excess returns of stocks and bonds takes a value of 0.5. The model shows no significant misspecification in the estimated conditional variance of bonds and the conditional covariance between stocks and bonds. The estimated conditional variance of stocks, however, exhibits significant misspecification with regard to the asymmetric indicators x_1 and x_6. This reflects the inadequacy of the model in capturing the impact of bad news in the stock market and good news in both markets on the volatility of stocks.

Note that, from the equations of the model in (10), the variances follow univariate GARCH(1,1) processes. Thus, there are no spill-over terms and information flow in one market is not allowed to affect the volatility in the other. This restriction is shared with the Diagonal-VECH model. However, unlike the Diagonal-VECH, the Constant Correlation model describes the conditional covariance to be dependent on the volatility in both markets. Information-flow in both markets are allowed to affect the degree of co-movement between their excess returns. In addition, the model restricts the correlation coefficient to be constant. Therefore, the time-variation in the covariance is restricted to be a consequence of the time-variation in the variances, and not due to changes in the correlation. On the other hand, the Diagonal-VECH model assumes a more general univariate GARCH process for the covariance. Comparing the results of these two models in Table 4, shows that the Diagonal-VECH (and indeed the BEKK) is misspecified with regard to the conditional covariance, while the Constant Correlation shows no such misspecification. This is a remarkable result, because although the Constant Correlation model restricts the correlation coefficient to be constant during the sample period of over nineteen years, it shows no significant misspecification in describing the dynamics of the covariance. This is perhaps an indication of the stability of the correlation coefficient between UK stock and bond excess returns, in spite of the fact that the conditional covariance exhibits significant time-variation. This is confirmed by the estimation results of the ADC reported in Table 6. The estimated value of the coefficient ϕ_{12} is not significant, while ρ_{12} has a significant estimated value of 0.5.

3.4. Information Flow Between The UK Stock and Bond Markets

A better way to assess how the above multivariate-GARCH models deal with asymmetries, than comparing their estimation results, is to look at their news impact surfaces. These are three dimensional graphs describing how news in the stock market, $\varepsilon_{1,t-1}$, and in the bond market, $\varepsilon_{2,t-1}$, affect the future volatility of stocks, $h_{11,t}$, the future volatility of bonds, $h_{22,t}$, and the future covariance between stocks and bonds, $h_{12,t}$. These surfaces are developed by Kroner and Ng (1993) as a bivariate extension of the news impact curves described in Pagan and Schwert (1990) and Engle and Ng (1993). If information at t-2 and earlier is held constant, one can evaluate the conditional variances and covariances at time t-1 at their unconditional sample values. Then the bivariate equations of the above models revert to functional relationships between news at time t-1 and variances and covariances at time t. The exact form of these functions are listed in Table 7.

It can be seen how a different parameterization or a functional specification governs the relationship between news and its impact on the conditional variance-covariance matrix. For example, in Table 7, the same elements of news ($\varepsilon_{1,t-1}, \varepsilon_{2,t-1}, \varepsilon^2_{1,t-1}, \varepsilon^2_{2,t-1}$) are present in the variances and covariances under both the BEKK(1,1) and the Factor-GARCH models. However, the BEKK model restricts the number of parameters which capture the effect of news to four: a_{11}, a_{12}, a_{21} and a_{22}, while the Factor-GARCH uses three: λ_1, λ_2, and ϕ. The difference in the parameter structure between the two models endows each corresponding element with a different intensity of impact.

Although, the difference in the parameter structure is what distinguishes the BEKK from the Factor-GARCH model, it is the functional specification that characterises the Constant Correlation model. The model restricts the covariance to be proportional to the product of the standard deviations. This restriction renders the covariance insensitive to differences between good and bad news, but sensitive to the magnitude of news.[12]

Figures 1 to 3 exhibit the news impact surfaces implied by the BEKK model, the Constant Correlation model, the Diagonal-VECH model and the Factor-GARCH model. Figure 4 exhibits the news impact surfaces implied by the ADC matrix model.

From Figure 1, the news impact surfaces implied by the Constant Correlation and the Diagonal VECH models are both symmetric in their response to news in the stock market and centred at $\varepsilon_{1,t-1}=0$. Good or bad news in the stock market increase the future volatility of stocks. Moreover, the increase in volatility is the same for good and bad news of the same magnitude. In fact, because of the symmetry stock volatility is indifferent to the type of news from the stock market. In addition, the surfaces are parallel to the axis representing news in the bond market, $\varepsilon_{2,t-1}$. This implies that the two models describe stock volatility to be indifferent to past news from the bond market. News of either type in the bond market has no effect on the future volatility of stocks. In summary, the two surfaces describe stock volatility to depend only on news from the stock market.

Apart from these apparent similarities there are considerable differences between any other pair of models. For example, the surface implied by the factor GARCH model describes an opposite relationship to the one described by the Diagonal VECH or the Constant Correlation model. The surface is almost symmetric at $\varepsilon_{2,t-1} = 0$, and almost

[12] The level of the return shock of either asset is not present in the covariance (or the variance) equation(s), however, the square of the shock is.

parallel to the axis representing shocks in the stock market, $\varepsilon_{1,t-1}$. This implies that the factor GARCH model describes the volatility of stocks to be largely dependent on news in the bond market, and almost indifferent to news in the stock market - a contradictory result to the descriptions of the above two models.

The surfaces implied by the above three models are, or almost, symmetric, thus it is easier to understand their implications. However, to study the implications of asymmetric surfaces, such as the one implied by the BEKK model, it might be helpful to divide the plane spanned by the two horizontal axes into four quadrants. For example, the quadrant spanned by positive shocks, $\varepsilon_{1,t-1}$ and $\varepsilon_{2,t-1} > 0$, represents good news in both markets, the quadrant spanned by negative shocks represents bad news in both markets, and the other two quadrants represent news of different types. The surface implied by the BEKK model depicts increases in stock volatility only when news in both markets are of the same type. When news from both markets are of different types the surface depicts a decrease in the level of stock volatility. Thus, the BEKK disagrees with the other three models in describing the impact of news on stock volatility.

Figure 2 exhibits the implied news impact surfaces for the conditional variance of bonds, $h_{22,t}$. All four surfaces have similar shapes suggesting that the four models do not vary substantially in their description of the impact of news on the future volatility of bonds. The surfaces depict an increase in bond volatility following news, of either type, in the bond market. In general, news in the stock market has little or no effect. When there is an effect, the models describe it slightly differently. The diagonal VECH and the Constant Correlation models agree that news in the stock market does not affect the volatility of bonds. The two surfaces are valley shaped and their troughs are parallel to the axis representing news in the stock market. On the other hand, the surfaces implied by the BEKK and the factor GARCH are slightly tilted, implying a slight impact from news in the stock market. Moreover, the surfaces are tilted in opposite directions to each other describing different effects. To clarify this, consider the two quadrants representing good news in the bond market. Both models describe an increase in the volatility of bonds regardless of the type, or the magnitude, of news in the stock market. However, the surface implied by the BEKK describes a higher level of bond volatility when news in the stock market is bad rather than good. The surface implied by the Factor GARCH describes the opposite; bond volatility is lower when news in the stock market is bad rather than good.

The differences between the four models are more prominent in the covariances. Figure 3 exhibits the news impact surfaces implied for the conditional covariance between stocks and bonds, $h_{12,t}$. All four surfaces have very different shapes. When there is good news in the stock market accompanied by bad news in the bond market, the Constant Correlation model describes an increase in the covariance between stocks and bonds. This increase is linear in its response to the magnitude of news. The Diagonal VECH model, however, describes a decrease. The BEKK and the Factor-GARCH describe an increase but in a quadratic form that is less extreme than the linear increase of the Constant Correlation model. These apparent differences between the models are caused by their assumed functional forms. For example, under the Constant Correlation model, the covariance, $h_{12,t}$, is proportional to the product of the standard deviations. Thus past return shocks in both markets, regardless of the sign, will increase the standard deviations, and hence the covariance. The inability of the Constant Correlation model to capture covariance asymmetry is evident from the symmetry in the star-shaped surface in Panel 2 of Figure 3. On the other hand, the covariance, $h_{12,t}$, under the Diagonal-VECH model is a function of the cross products of the past level of both shocks ($\varepsilon_{1,t-1}\varepsilon_{2,t-1}$). When the shocks are of opposite sign their product can decrease the covariance to negative values. However, when the shocks are of similar sign their product increases the covariance. This is evident from the saddle shape of the surface in panel 3 of Figure 3.

The differences between the four multivariate-GARCH models raise concerns about their adequacy in describing the dynamics of the covariance matrix. These differences may affect the efficiency of allocating funds between stocks and bonds. The ADC provides better estimates of variances and covariance and hence its implied news impact surfaces are better descriptions of the data. Figure 4 exhibits the three news impact surfaces, implied by the ADC. The surface for the variance of stocks has an irregular bowl shape indicating that news, of either type, in either the bond market or the stock market has an effect on the future volatility of stocks. In particular, when the arrived news is bad in both markets, the estimated increase in the future volatility of stocks is relatively large. Moreover, when news is bad in the bond market, the arrival of good news in the stock market decreases the volatility of stocks. The surface for the variance of bonds appears similar to those implied by the previous models and consistent with the notion that news, whether good or bad, in the stock market has minimal or no effect on the future volatility of bonds. The conditional variance of bonds seems to increase, almost symmetrically, in response to past shocks in the excess returns of bonds. The conditional covariance also seems to be affected by past shocks in excess returns. This is reflected in the irregular valley shape of the covariance surface shown in Panel 3 of Figure 4. News, of either type, in the bond market increases the covariance. However, news in the stock market can increase or decrease the covariance, depending on the type of news in the bond market. If news is good in the bond market ($\varepsilon_{2,t-1} > 0$), then good news arriving in the stock market increases the covariance, while bad news decreases it. The order is reversed when news is bad in the bond market ($\varepsilon_{2,t-1} < 0$); arrival of good news in the stock market decreases the covariance, while the arrival of bad news increases it. Good news in both markets seems to increase the future covariance between stocks and bonds but not as much as when news is bad in both markets. In general, the conditional covariance seems to be more responsive to past shocks in the excess returns of bonds.

4. The Effect on Asset Allocation

4.1. The Global Minimum Variance Portfolio

Descriptive statistics on the difference between the ratios estimated by the various models and those estimated by the ADC are listed in the first panel of Table 8. From the table, the BEKK seems to imply a set of ratios that are more different from those of the ADC than any of the other models. The difference has a mean of 0.01363, and represents around 59 percent of the scale of the corresponding reward-to-variability ratios. The next largest difference with a mean of 0.01275 is between the ADC and the Constant Variance-Covariance Matrix model. It represents around 53 percent of the scale of the corresponding reward-to-variability ratio. The rest of the differences have means ranging from 0.00106 to 0.00790 representing some 3 to 27 percent of the scale of their corresponding reward-to-variability ratios. However, the variation *over time* of these differences are quite large compared to their means, resulting in statistically insignificant t-statistics. Nevertheless, the scale of these differences relative to the scale of their corresponding average reward-to-variability ratios is quite large. At any particular week there could be more than 59 percent difference in the estimates of the reward-to-variability ratio between one model and the ADC.[13] Overall, the ADC produces larger estimates than any of the other models judging by the positive sign of all the mean differences.

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[13] Since 59 percent is an average figure, then during any particular period there could be more (or less) than 59 percent difference.

4.2. The Target Return Portfolios

Similar analysis is carried out on the target return portfolios. Descriptive statistics on the difference in reward-to-variability ratios are in panels 2 to 4 in Table 8. From the second panel the only mean difference with a negative sign is that between the ADC and the Constant Covariance benchmark. All other mean differences are positive. On average, the ADC produces higher reward-to-variability ratios than any other model except the Constant Covariance benchmark. The values of the mean differences, however, represent between 1 to 9 percent of the scale of the average reward-to-variability ratios. That is still high when converted to annual excess-returns per unit of risk. The values of the t-test show that all differences have significant means at any reasonable statistical level. The results for the other two target-return portfolios are similar. All values of the t-test are statistically significant, and the mean differences have the same ranking as those of the 5-percent portfolio.

4.3. An Implication On Performance Measurement

At first glance the above results may indicate that the Constant Covariance benchmark outperforms all the other models including the ADC. However, this is not true. First, as mentioned above, the benchmark is wrongly constructed. In a multivariate context, the assumptions of separate univariate GARCH processes governing the variances while the covariance is kept constant (except zero) are not valid in theory and, as seen above, may lead to inadmissible values for the correlation coefficient in empirical applications. Second, the estimation results of the ADC, and indeed of the other multivariate-GARCH models, indicate that the covariance between stocks and bonds is not constant, but rather exhibits a high degree of variation over time. Therefore, the estimates of the variance-covariance matrix by the Constant Covariance benchmark may identify portfolios that, in reality, do not achieve their target returns. The implication is that multivariate modelling may well make a difference for performance measurement.

Performance measurement in a conditional time-varying framework is an issue better addressed by a multivariate formulation of a conditional version of a pricing model like the CAPM. Such a formulation would consider the trade-off between risk and return that characterises asset pricing models. The analysis presented above does not take such a trade-off into account. Estimating the returns and the covariances are kept separate in order to isolate the effect of different ways of modelling the covariance matrix on asset allocation. The ARCH in Mean (ARCH-M) model of Engle, Lilien and Robins (1987) is better suited to investigating the relationship between the conditional mean and the conditional variance (c.f. Bollerslev, Chou and Kroner (1992)). For example, Bollerslev, Engle and Wooldridge (1988) use a multivariate formulation of a GARCH-M model to implement the CAPM for a market portfolio consisting of three assets (stocks, bonds and bills). They present evidence showing that the trivariate model used seems superior to the corresponding three univariate GARCH(1,1)-M. However, this just emphasises the notion presented above that multivariate modelling may well make a difference.

4.4. The Magnitude Of The Effect On Asset Allocation

On discounting the Constant Covariance benchmark, the ADC outperforms all the other models. In estimation it attains a statistically higher log-likelihood function value than any of the models it nests. Furthermore, the results of the robust conditional moment tests indicate that the ADC exhibits minimal or no misspecification regarding asymmetries. In portfolio formation, the ADC helps to identify portfolios that, on average, have a higher reward-to-variability ratio or have a lower variance for a given level of target return. Note that the ADC is expected, by construction, to outperform the other models because of its nesting property. This is true for all models except the non-nested Constant Covariance benchmark. However, the importance of the ADC is in providing a well specified benchmark against which the performance of the other models are compared. This allows the measurement of the effect, on asset allocation, of assuming a particular misspecified model than another. Comparing the scale of the differences in the performance ratios implied by the various models with the ADC, shows the extent of 'damage' on asset allocation that may result from assuming a model with a particular misspecification or a restrictive functional form.

Still discounting the Constant Covariance benchmark, the difference between the performance of the ADC and that of the other multivariate-GARCH models varies in scale. For the target-return portfolios the scale of the differences represents 1 to 9 percent of the scale of their corresponding reward-to-variability ratios. For the global minimum-variance portfolio the scale of the differences represents 5 to 59 percent of the scale of their corresponding reward-to-variability ratios. These are sizeable differences indicating the magnitude of the effect on asset allocation, portfolio management and performance measurement. These results strongly suggest that great care should be taken in choosing an adequate model to describe the time varying dynamics, and predictabilities therein, of the conditional variance-covariance matrix of asset-class returns.

4.5. A Note On The Impact Of Time Variation Versus Functional Form

It is worth noting that for the target return portfolios the largest difference from the ADC is that of the Constant Variance-Covariance Matrix model. For the global minimum variance portfolio the difference between the ADC and the Constant Variance-Covariance Matrix model is the second largest. These results may indicate that the impact of modelling time variability rather than not modelling it is larger than the impact of assuming a particular functional form than another among multivariate-GARCH models.

The main reason for including the Constant Variance-Covariance Matrix model as a benchmark is that it may represent traditional allocation models that assume constant variances and covariances. The large difference between the performance of this benchmark and the ADC reflects the magnitude of 'damage' on asset allocation that may result from the constancy assumption underlying traditional models. This emphasises the point made above that time variation in the variances, as well as in the covariance, is important to asset allocation. The 'efficiency' in allocation gained by taking time variation into account seems to be more than that gained from assuming different functional forms or taking other, albeit important, asymmetric predictabilities into consideration.

5. Subsample Robustness Check

In this section a simple analysis is conducted to check whether the predictabilities in the variance-covariance matrix discussed above are robust to the choice of sample period. The observations for 1994 and 1995 (a particularly volatile period) are dropped and the returns are adjusted by a vector autoregression as above. Diagnostics on the residuals from the adjustment procedure are reported in Table 9. The Ljung-Box (12) statistic for the squared residuals suggests the existence of autocorrelation in the squared residuals of stocks and bonds. The negative size bias test, as well as the negative sign bias, the positive sign bias and the negative extreme size bias tests are significant for stock residuals. The negative sign bias test and the positive sign bias tests are significant for bond residuals. The joint tests for both residuals are significant. These results indicate a size effect of news, especially in stocks, where the effect is stronger for bad news than for good news. Given the superiority of the ADC over the other models, its estimation is repeated for the subsample period. The results are reported in Table 10. Several of these results are worth special notice. First, the significance of the ARCH and GARCH parameters on both stocks and bonds confirms the presence of time variation in the variances and in the covariance. This time variation is persistent for both stocks and bonds - $b_{11}^2 + a_{11}^2 = 0.724$ and $b_{22}^2 + a_{22}^2 = 0.932$ - more so for bonds than stocks. This is consistent with the results for the whole sample period. Second, the asymmetric-extension coefficients, γ_{11} and γ_{12}, are significant, suggesting that the ADC captures certain asymmetries that would not be captured by the models it nests. This is true because the other models lack these asymmetric extensions. Third the estimated value of the correlation coefficient is equal to 0.5, and is significant, while the estimated value for the coefficient ϕ_{12} is small and not significant. These results are consistent with the notion discussed earlier, that the correlation coefficient between UK stocks and bonds seems to be stable over time, while the covariance exhibits significant time-variation due to the time variability in the variances. Fourth, the misspecification indicators are all insignificant, confirming that the ADC is well specified. All of the above results are fully consistent with the conclusions drawn for the whole sample period.

6. Conclusion

Multivariate models of the variance-covariance matrix of asset returns differ in how they describe time variation, asymmetries and spill-over effects. Surprisingly, however, very few studies investigate the impact of these effects on portfolio management, asset allocation and performance measurement. This paper fills this gap by comparing portfolios on the efficient frontier formed with the inputs obtained from estimating five multivariate-GARCH models and two less complicated benchmarks. The models are also used to describe the interaction in volatility and correlation between UK stocks and government bonds.

Results show that volatility spills over from bonds to stocks but not vice-versa. The covariance exhibits significant time variation but it is unlikely to be a result of time varying correlation. The correlation coefficient seems to be stable over time and has a value of 0.5. There is also a marked differential impact of bad over good news especially on the volatility of stocks and the covariance between stocks and bonds.

Portfolio comparison across the models shows that these predictable properties have a significant effect on the formation of optimal portfolios, performance measurement and asset allocation. On average there can be as much as 59 percent difference in the performance of efficient portfolios from taking these effects into account than from ignoring them. Although asymmetries and spill-over effects are significant, the largest impact on asset allocation seems to be a result of the time variation present in the variances and the covariance. These results suggest that great care should be exercised in choosing a multivariate model that adequately captures these effects. These effects could well make a difference.

Appendix 1

These tests are fully described in Engle and Ng (1993). They are similar to the robust conditional moment tests of Kroner and Ng (1993). Engle and Ng (1993) use the following auxiliary regression:

$$v_t^2 = a + b_1 S_{t-1}^- + b_2 S_{t-1}^- \varepsilon_{t-1} + b_3 S_{t-1}^+ \varepsilon_{t-1} + \underline{\beta}' \underline{z}_{0t}^* + e_t \tag{17}$$

where $v_t^2 = \varepsilon_t^2 / h_{0t}$, h_{0t} is the conditional variance under the null, S_{t-1}^- is a dummy variable that takes the value of one when the return shock, ε_{t-1}, is negative and zero otherwise, S_{t-1}^+ is $1 - S_{t-1}^-$, and $\underline{z}_{0t}^*$ is a vector of scores. Here, I am testing the null that ε_t^2 is constant, therefore, $\underline{z}_{0t}^*$ is a vector of ones and the term is incorporated into the constant coefficient a. The sign-bias, the Negative-size bias, and the Positive-size bias tests are the t-ratios of the coefficients b_1, b_2 and b_3 in the regression, while the joint test is the lagrange-multiplier statistic $T \times R^2$, where T is the number of observations in the sample and R^2 is the squared multiple correlation of the regression. The coefficient b_1 picks up any asymmetry due to negative shocks in excess of the effect of positive shocks, while the coefficients b_2 and b_3 focus on the different effects that large and small negative and positive return shocks have on volatility. For the Negative-sign, Positive-sign, Negative-extreme-size and Positive-extreme-size bias tests reported in Table 2, I dropped the constant coefficient a, from the regression, and considered the following variables: S_{t-1}^-, S_{t-1}^+, $S_{t-1}^- \varepsilon_{t-1}^2$, and $S_{t-1}^+ \varepsilon_{t-1}^2$. The coefficients on the last two variables focus on the different effects that extreme negative and positive values of the return shock have on volatility. These are comparable to the Size-sign misspecification indicators in Kroner and Ng (1993).

Appendix 2

Kroner and Ng (1993) Robust Conditional Moment Tests

Kroner and Ng (1993) propose to test for systematic overstatement or understatement of the estimated variance-covariance matrix in relation to the sign and magnitude of the return shocks. The tests they develop use the fact that the models are correctly specified if and only if $E(\varepsilon_t\varepsilon_t'|\varepsilon_{t-1}) = H_t$, where ε_t is the vector of shocks of the N assets at time t. Therefore, they propose to test the null that $E(\varepsilon_t\varepsilon_t' - H_t|\varepsilon_{t-1}) = 0$ against the alternative that $E(\varepsilon_t\varepsilon_t' - H_t|\varepsilon_{t-1}) \neq 0$.

Define a "generalised residual", $u_{ij,t}$, to be the ijth element of the matrix of innovations $\varepsilon_t\varepsilon_t' - H_t$, so that $u_{ij,t} \equiv \varepsilon_{it}\varepsilon_{jt} - h_{ij,t}$. Also define $x_{1,t-1}, \ldots, x_{n,t-1}$ to be "misspecification indicators", which are variables known at time t-1. If an estimated model has adequately captured the predictability in the variance-covariance matrix, then $u_{ij,t}$ should have no relationship to any variables known at time t-1 including the misspecification indicators. Formally, $E(u_{ij,t}|x_{\ell,t-1}) = 0$, $\ell = 1, \ldots, n$. Alternatively, $u_{ij,t}$ should be uncorrelated with all elements in the information set, including the misspecification indicators $x_{1,t-1}, \ldots, x_{n,t-1}$. Therefore, one would expect

$$C_{cm} \equiv \frac{1}{T} \cdot \sum_{t=1}^{T} u_{ij,t} x_{\ell,t-1} \tag{18}$$

to be "close" to zero for all x_ℓ if the ijth covariance equation is correctly specified. C_{cm} is the unconditional sample covariance between the generalised residual and the misspecification indicator. To test if it is close to zero, Kroner and Ng (1993) apply the framework of Wooldridge (1990) which is not sensitive to the conditional distribution used when estimating the GARCH model. Wooldridge's robust conditional moment test statistic is

$$C_{rcm} \equiv \left(\frac{1}{T}\cdot\sum_{t=1}^{T} u_{ij,t}\lambda_{\ell,t-1}\right)^2 \cdot \left(\frac{1}{T}\cdot\sum_{t=1}^{T} u_{ij,t}^2\lambda_{\ell,t-1}^2\right)^{-1} \tag{19}$$

where $\lambda_{\ell,t-1}$ is the residual from a regression of $x_{\ell,t-1}$ on the derivatives of $h_{ij,t}$ with respect to each parameter in the model. Under certain conditions, Wooldridge (1990) shows that C_{rcm} has an asymptotic χ_1^2 distribution. These robust conditional moment tests can be computed from two auxiliary regressions. The first regression is $x_{\ell,t-1}$ on the derivatives of $h_{ij,t}$ with respect to all the parameters of the null model. The second regression is a vector of ones on the product $u_{ij,t}.\lambda_{\ell,t-1}$, where $\lambda_{\ell,t-1}$ is the residual from the first regression. The test statistic is T times the uncentred R-square from the second regression.

To complete the construction of the set of tests Kroner and Ng (1993) select a set of misspecification indicators which are likely to signal that the news impact surfaces are misspecified. Let I(·) be an indicator function that takes the value of one if the argument is true and zero otherwise. The "sign" misspecification indicators are:

$$x_{1t} = I(\varepsilon_{1,t-1} < 0),$$

$$x_{2t} = I(\varepsilon_{2,t-1} < 0).$$

If own-variance asymmetry exists in (for example) the first equation, then $E(u_{11,t}|x_{1,t-1}) \neq 0$, so the conditional moment test which examines the correlation between $u_{11,t}$ and $x_{1,t-1}$ should be significant. Similarly, if cross-variance asymmetry exists between (say) the variance of the first asset and news to the second asset, then $E(u_{11,t}|x_{2,t-1}) \neq 0$, so the conditional moment test which examines the correlation between $u_{11,t}$ and $x_{2,t-1}$ should be significant.

In a multivariate application, different combinations of the signs of the return shocks might be important, therefore, a set of "quadrant" misspecification indicators could also be useful. These are

$$x_{3t} = I(\varepsilon_{1,t-1} < 0; \varepsilon_{2,t-1} < 0)$$

$$x_{4t} = I(\varepsilon_{1,t-1} < 0; \varepsilon_{2,t-1} > 0)$$

$$x_{5t} = I(\varepsilon_{1,t-1} > 0; \varepsilon_{2,t-1} < 0)$$

$$x_{6t} = I(\varepsilon_{1,t-1} > 0; \varepsilon_{2,t-1} > 0).$$

Furthermore, since the null model might over-predict or under-predict the covariance following unexpectedly large or small return shocks of different signs, one can also use indicators of a combination of size and sign of the shocks. These are called the extreme size-sign misspecification indicators:

$$x_{7t} = \varepsilon^2_{1,t-1} I(\varepsilon_{1,t-1} < 0)$$

$$x_{8t} = \varepsilon^2_{1,t-1} I(\varepsilon_{2,t-1} < 0)$$

$$x_{9t} = \varepsilon^2_{2,t-1} I(\varepsilon_{1,t-1} < 0)$$

$$x_{10t} = \varepsilon^2_{2,t-1} I(\varepsilon_{2,t-1} < 0).$$

If large amounts of bad news to the first asset affects the variance of the second asset, then $E(u_{22,t} | x_{7,t-1}) \neq 0$, so the conditional moment test which examines the correlation between $u_{22,t}$ and $x_{7,t-1}$ should be significant.

In fact, as Engle and Ng (1993) suggest, one could test how well the model is specified by considering other sets of misspecification indicators. For example, one could test how well the lag structure of the GARCH(p,q) model is specified by considering the variables $\varepsilon^2_{t-2}, \varepsilon^2_{t-3}, \varepsilon^2_{t-4}$, ... etc. If the lag structure in (say) the first equation is correct, then the conditional moment test which examines the correlation between $u_{11,t}$ and

ε^2_{t-i}, $i = 2, 3, \ldots,$ should be insignificant. These extensions are also considered in estimation.

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Table 1

Model Summary For The Bivariate GARCH(1,1) Case

Model	Number of parameters	Asymmetries			Spill-overs
		Own Variance	Cross Variance	Covariance	
ADC	17	YES	YES	YES	YES
BEKK	11	YES	YES	YES	YES
D-VECH	9	NO	NO	YES	NO
CCORR	7	NO	NO	NO	NO
FGARCH	6	YES	YES	YES	YES