Homogenization of biomechanical models of plant tissues with randomly distributed cells

Citation for published version:
Piatnitski, A & Ptashnyk, M 2020, 'Homogenization of biomechanical models of plant tissues with randomly distributed cells', *Nonlinearity*, vol. 33, no. 10, pp. 5510-5542. https://doi.org/10.1088/1361-6544/ab95ab

Digital Object Identifier (DOI):
10.1088/1361-6544/ab95ab

Published In:
Nonlinearity

Nonlinearity

PAPER • **OPEN ACCESS**

Homogenization of biomechanical models of plant tissues with randomly distributed cells

To cite this article: Andrey Piatnitski and Mariya Ptashnyk 2020 *Nonlinearity* **33** 5510

OPEN ACCESS
IOP Publishing | London Mathematical Society Nonlinearity

Nonlinearity **33** (2020) 5510–5542 https://doi.org/10.1088/1361-6544/ab95ab

Homogenization of biomechanical models of plant tissues with randomly distributed cells

Andrey Piatnitski[1] **and Mariya Ptashnyk**[2]

[1] The Arctic University of Norway, UiT, Campus Narvik, Norway and Institue for Information Transmission Problems RAS, Moscow, Russia
[2] Heriot-Watt University, Edinburgh, United Kingdom

E-mail: apiatnitski@gmail.com and m.ptashnyk@hw.ac.uk

Received 19 December 2019, revised 25 March 2020
Accepted for publication 22 May 2020
Published 15 September 2020

Abstract

In this paper homogenization of a mathematical model for biomechanics of a plant tissue with randomly distributed cells is considered. Mechanical properties of a plant tissue are modelled by a strongly coupled system of reaction-diffusion-convection equations for chemical processes in plant cells and cell walls, the equations of poroelasticity for elastic deformations of plant cell walls and middle lamella, and the Stokes equations for fluid flow inside the cells. The nonlinear coupling between the mechanics and chemistry is given by the dependence of elastic properties of plant tissue on densities of chemical substances as well as by the dependence of chemical reactions on mechanical stresses present in a tissue. Using techniques of stochastic homogenization we derive rigorously macroscopic model for plant tissue biomechanics with random distribution of cells. Strong stochastic two-scale convergence is shown to pass to the limit in the non-linear reaction terms. Appropriate meaning of the boundary terms is introduced to define the macroscopic equations with flux boundary conditions and transmission conditions on the microscopic scale.

Keywords: stochastic homogenization, poroelasticity, Stokes system, biomechanics of plant tissues, stochastic two-scale convergence

(Some figures may appear in colour only in the online journal)

1. Introduction

Formation of plant tissues and organs is a result of the coordinated expansion of hundreds of thousands of cells, different in size, shape, and composition. Plant organs are composed of several types of tissues, e.g. epidermis, cortex, endodermis, vascular tissue [55]. While the turgor pressure, the main force for cell expansion, acts isotropically, the anisotropic deformation and growth of plant cells and tissues rely on the mechanics of cell walls, surrounding plant cells, and the microstructure of cell walls and tissues. Plant tissues have complex hierarchical microstructures given by the size and arrangement of cells, connected by cross-linked pectin network of middle lamella, on one scale, and by the heterogeneous structure of cell walls on the other scale [39]. In some tissues, such as wood or cork, the geometric arrangement of cells is very regular and can be regarded as periodic [33], however many plant tissues exhibit random variations in their microstructure [28, 47, 48]. Plant cell walls mainly consist of cellulose microfibrils, pectin, hemicellulose, macromolecules, and water. The orientation of microfibrils, their length, high tensile strength and interaction with wall matrix macromolecules determine the wall stiffness. For irreversible deformation, the deposition of new wall materials and the loosening of the cell wall through the breaking of the load-bearing cross-links between microfibrils, pectin and hemicellulose by enzymes activity are required [62]. It is supposed that calcium-pectin cross-linking chemistry strongly influences elastic properties of plant cell walls [70]. Pectin is produced in Golgi apparatus inside the cells and is deposited to a cell wall in a methyl-esterified form, where it can be de-methylesterified by the enzyme pectin methylesterase (PME), which removes methyl groups by breaking ester bonds. The de-methyl-esterified pectin is able to form calcium-pectin cross-links, and so stiffen the cell wall and reduce its expansion, see e.g. [69], whereas mechanical stresses can break calcium-pectin cross-links and hence increase the extensibility of plant cell walls and middle lamella.

Considering the complex structure of plant tissues and organs, for a better understand and improvement of plant growth and development, it is important to model and analyse how microscopic structure and interactions between chemical processes and mechanical properties of individual cell walls and cells contribute to the properties of the plant tissues and organs [8, 39]. Different approaches were applied to analyse the interplay between micro- and macro-mechanics and transport processes in plant tissues [7]. Many results can be found for multiscale modelling and analysis of the periodic microstructure of wood [26, 46, 60]. Multiscale modelling and analysis of the impact of the microscopic structure of plant cell walls, especially orientation and distribution of microfibrils, on mechanical properties of cell walls were conducted in [58]. A vertex-element model and hybrid vertex-midline model for plant tissue deformation and growth, coupled with the cell-scale transport of plant hormone, were considered in [30, 31]. The impact of microfibrils on the mechanical properties of cell walls was accounted for by introducing an anisotropic viscous stress which depends on a pair of microfibril directions. A simple constitutive model at the cell scale which characterises cell walls via yield and extensibility parameters together with an appropriate averaging over a cross-section were used to derive the analogous tissue-level model describing elongation and bending of a plant root [27]. A mesh-free particle method was proposed in [43] to simulate the mechanics of both individual plant cells and cell aggregates in response to external stresses and to study how plant tissue mechanics is related to the micromechanics of cells. The interior of the cell is regarded as liquid phase and simulated using the smoothed particle hydrodynamics (SPH) method, where in the domain corresponding to the viscoelastic material of cell walls the particles are connected by pairwise interactions holding them together. A multiscale method for the simulation of large viscoelastic deformations of a plant tissue presented in [32] combines particle method on the microscopic level with standard finite elements methods on the macroscopic scale. The effect of non-periodic microstructure on effective (homogenized) elastic properties of two-dimensional cellular materials (honeycombs) was studied in [66] by considering non-periodic arrangement of cell walls in random Voronoi honeycombs and applying finite element analysis. The finite-edge centroidal Voronoi tessellation (FECVT) was introduced in

[28] to generate a realistic model of a non-periodic tissue microstructure and, combined with finite elements analysis, was used to determine the effective elastic properties of plant tissues, especially plant petioles and stems [29]. Smoothed particles hydrodynamics (SPH) framework was used in [52] to model plant tissue growth. The framework identifies the SPH particle with individual cells in a tissue, but the tissue growth is performed at the macroscopic level using SPH approximations and plant tissue is represented as an anisotropic poro-elastic material. A coarse-grained multiscale numerical model is proposed in [68] to predict macroscale deformations of food-plant tissues (e.g. apple tissues) during drying.

In [57] we derived and analysed a mathematical model for plant tissue biomechanics, which describes the interactions between calcium–pectin dynamics and deformations of a plant tissue. The microscopic model, at the length scale of plant cells, comprises a strongly coupled system of the Stokes equations modelling water flow inside plant cells, the equations of poro-elasticity defining elastic deformations of plant cell walls and middle lamella, and reaction-diffusion-convection equations describing the dynamics of the methyl-esterfied pectin, de-methyl-esterfied pectin, calcium ions, and calcium–pectin cross-links. The interplay between the mechanics and the chemistry comes in by assuming that the elastic properties of cell walls and middle lamella depend on the density of the calcium–pectin cross-links and the stress within cell walls and middle lamella can break the cross-links. Assuming periodic distribution of cells in a plant tissue in [57] we derived rigorously macroscopic model for plant tissue biomechanics. The two-way coupling between chemical processes and mechanics is the main novelty of the model, which also induces some non-standard elements in the analysis of the model and in the rigorous derivation of macroscopic equations. In this paper we generalise the results obtain in [57] by considering random distribution of cells in a plant tissue, observed experimentally in many plant tissues and organs [28, 48]. The derivation of macroscopic equations from a continuum description of the microscopic processes on the cell level using stochastic homogenization techniques results into a continuum macroscopic two-scale model containing the information on the microscopic interactions. Our microscopic model incorporates microscopic properties of plant cell walls, essential for plant tissue mechanics. The macroscopic model takes into account the microscopic structure of a plant tissue via effective (macroscopic) elasticity and permeability tensors and includes the interplay between the fluid in cell inside and poroelastic nature of cell walls and middle lamella. The effect of the microstructure and heterogeneity of the processes is also reflected in the equations for calcium–pectin chemistry via effective (macroscopic) diffusion coefficients, reaction terms and advective velocity. In the relation to particle and vertex-elements methods, continuum modelling approach proposed here may be beneficial when consider large size plant tissues and organs.

To analyse macroscopic mechanical properties of plant tissues with a random distribution of cells, we derive rigorously a macroscopic model for plant biomechanics using techniques of stochastic homogenization. The stochastic two-scale convergence [74] is applied to obtain the macroscopic equations. The main mathematical difficulties in the derivation of the macroscopic equations arise from the strong coupling between the equations of poro-elasticity and the system of reaction-diffusion-convection equations, as well as due to transmission conditions between the free fluid and poro-elastic material. The strong stochastic two-scale convergence for the displacement gradient and flow velocity is proven to pass to the limit in the nonlinear reactions terms. Extension arguments and formulations of surface integrals as volume integrals are used to pass to the stochastic two-scale limit in the equations with non-homogeneous Neumann boundary conditions and transmission conditions. To pass to the limit in the flux boundary conditions defined on the surfaces of the microstructure, Palm measure and the proven here trace inequality for H^1-function in the probability space, see lemma 8.1, are used.

Some of the first results on the stochastic homogenization of linear second-order elliptic equations were obtained in [41, 56, 72]. The homogenization of quasi-linear elliptic and parabolic equations with stochastic coefficients and convex integral operators was considered in [9, 21, 24, 25]. Subadditive ergodic theory and the method of viscosity solutions were applied to homogenize Hamilton-Jacobi, viscous Hamilton–Jacobi equations, and fully nonlinear elliptic and parabolic equations in stationary ergodic media [4, 20, 40, 44, 45] (see also references therein). The stochastic two-scale convergence introduced in [74] has been extended to Riemannian manifolds and has been applied to analyze heat transfer through composite and polycrystalline materials with nonlinear conductivities [36, 37]. The two-scale convergence in the mean [15] has been applied to derive macroscopic equations for single- and two-phase fluid flows in randomly fissured media [13, 71].

The poro-elastic equations, modelling interactions between fluid flow and elastic deformations of a porous medium, has been first obtained by Biot using a phenomenological approach [10–12] and subsequently derived by applying formal asymptotic expansion [5, 18, 42, 61, 64] or the two-scale convergence method [22, 34, 38, 49, 50, 54]. Along many results for poroelastic equations, only few studies of interactions between a free fluid and a deformable porous medium can be found. In [65] nonlinear semigroup method was used for mathematical analysis of a system of poroelastic equations coupled with the Stokes equations for free fluid flow. A rigorous derivation of interface conditions between a poroelastic medium and an elastic body was considered in [51]. Numerical methods for coupled system of poroelastic and Navier–Stokes equations were studied in [6, 19].

One of the approaches commonly used in numerical homogenization to approximate the effective coefficients of a microscopic problem describing some processes in a random medium is the so-called periodization [14]. The key idea of this method is to choose a large enough sample of the random medium, to extend it periodically, and to take the effective coefficients of the obtained periodic problem as an approximation of the effective coefficients of the original random problem. Recent years an essential progress was achieved in this approach, see the work [35], and references therein. Justification of this method for the model studied in the present paper is an interesting problem. Mixed multiscale finite element method [1] or stochastic variational multiscale method [2] can also be used for numerical simulation of multiscale stochastic problems.

The paper is organised as follows. In section 2 we formulate the microscopic model for plant tissue biomechanics. The main results of the paper are summarised in section 3. The *a priori* estimates and convergence results are given in sections 4 and 5. In section 6 we derive macroscopic equations for the coupled poro-elastic and Stokes problem. The strong stochastic two-scale convergence for displacement gradient and flow velocity is proven in section 7. The macroscopic equations for the system of reaction-diffusion-convection equations are derived in section 8.

2. Microscopic model

We consider a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with probability measure $\mathcal{P}$. We define a 3-dimensional dynamical system $\mathcal{T}_x : \Omega \to \Omega$, i.e. a family $\{\mathcal{T}_x \, : \, x \in \mathbb{R}^3\}$ of invertible maps, such that for each $x \in \mathbb{R}^3$, $\mathcal{T}_x$ is measurable and satisfy the following conditions:

(a) $\mathcal{T}_0$ is the identity map on Ω, and for all $x_1,\ x_2 \in \mathbb{R}^3$ the semigroup property holds:

$$\mathcal{T}_{x_1+x_2} = \mathcal{T}_{x_1}\mathcal{T}_{x_2}.$$

(b) $\mathcal{P}$ is an invariant measure for $\mathcal{T}_x$, i.e. for each $x \in \mathbb{R}^3$ and $F \in \mathcal{F}$ we have that

$$\mathcal{P}(\mathcal{T}_x^{-1}F) = \mathcal{P}(F).$$

(c) For each $F \in \mathcal{F}$, the set $\{(x,\omega) \in \mathbb{R}^3 \times \Omega : \mathcal{T}_x\omega \in F\}$ is a $\mathcal{L} \times \mathcal{F}$-measurable subset of $\mathbb{R}^3 \times \Omega$, where $\mathcal{L}$ denotes the Lebesgue σ-algebra on $\mathbb{R}^3$.

We consider a fixed measurable set Ω_f such that $\mathcal{P}(\Omega_f) > 0$ and $\mathcal{P}(\Omega\backslash\Omega_f) > 0$ and denote $\Omega_e = \Omega\backslash\Omega_f$. We also consider $\Omega_\Gamma \subset \Omega$, with $\mathcal{P}(\Omega_\Gamma) > 0$ and $\mathcal{P}(\Omega_\Gamma \cap \Omega_j) > 0$, for $j = e, f$.

For $\mathcal{P}$-a.a. $\omega \in \Omega$ we define the following random subdomains in $\mathbb{R}^3$

$$G_j(\omega) = \{x \in \mathbb{R}^3 :\ \mathcal{T}_x\omega \in \Omega_j\}, \quad \text{for} \ \ j = e, f, \quad G_\Gamma(\omega) = \{x \in \mathbb{R}^3 :\ \mathcal{T}_x\omega \in \Omega_\Gamma\},$$

and surfaces

$$\Gamma(\omega) = \partial G_f(\omega), \qquad \widetilde{\Gamma}(\omega) = \Gamma(\omega) \cap G_\Gamma(\omega).$$

We shall consider the following assumptions on $G_f(\omega)$, $\Gamma(\omega)$, and $\widetilde{\Gamma}(\omega)$:

Assumption 1.

(a) $G_f(\omega)$ consists of countable number of disjoined Lipschitz domains for $\mathcal{P}$-a.a. $\omega \in \Omega$ with a uniform Lipschitz constant.

(b) The distance between two connected components of $G_f(\omega)$ is uniformly bounded from above and below.

(c) The diameter of connected components of $G_f(\omega)$ is bounded from below and above by some positive constants.

(d) The surface $\widetilde{\Gamma}(\omega) \subset \Gamma(\omega)$ is open on $\Gamma(\omega)$ and Lipschitz continuous.

Consider a bounded $C^{1,\alpha}$-domain $G \subset \mathbb{R}^3$, with $\alpha > 0$, representing a part of a plant tissue. In a plant tissue individual cells, consisting of cell inside and cell walls, are connected by the pectin network of middle lamella. Then the microscopic structure of a plant tissue with a random distribution of cells is defined as

$$G_f^\varepsilon = \{x \in \mathbb{R}^3 :\ \mathcal{T}_{x/\varepsilon}\omega \in \Omega_f\} \cap G, \quad G_\Gamma^\varepsilon = \{x \in \mathbb{R}^3 :\ \mathcal{T}_{x/\varepsilon}\omega \in \Omega_\Gamma\} \cap G, \quad G_e^\varepsilon = G\backslash G_f^\varepsilon,$$

$$\Gamma^\varepsilon = \partial G_f^\varepsilon, \quad \widetilde{\Gamma}^\varepsilon = \Gamma^\varepsilon \cap G_\Gamma^\varepsilon,$$

$\mathcal{P}$-a.s., where G_e^ε represent the subdomains occupied by cell walls and middle lamella, G_f^ε denotes the cell inside, and $\widetilde{\Gamma}^\varepsilon$ defines a part of cell membrane which is impermeable to calcium ions.

Assumption 1.2 states that the thickness of cell walls and middle lamella is uniformly bounded from above and below and assumption 1.3 postulates that the diameter of cells is bounded from above and below.

Due to assumed random distribution of cells in a plant tissue, the permeability and elastic properties of plant cell walls and middle lamella are characterised by the corresponding random variables. For this we define statistically homogeneous random fields $\mathbf{E}_1(x,\omega,\xi) = \widetilde{\mathbf{E}}_1(\mathcal{T}_x\omega,\xi)$ and $K_p(x,\omega) = \widetilde{K}_p(\mathcal{T}_x\omega)$, where $\widetilde{\mathbf{E}}_1(\cdot,\xi)$ and $\widetilde{K}_p(\cdot)$ are given measurable functions from Ω to $\mathbb{R}^{3^4}$ and $\mathbb{R}^{3\times 3}$, respectively, for $\xi \in \mathbb{R}$ representing the dependence of the elastic properties on the calcium–pectin cross-links density. It is observed experimentally that the load bearing calcium–pectin cross-links reduce cell wall expansion, see e.g. [70], and hence we assume that elastic properties of cell walls and middle lamella depend on the density of calcium–pectin cross-links.

Then for each $\omega \in \Omega$ and $\xi \in \mathbb{R}$ the microscopic elasticity tensor $\mathbf{E}_1^\varepsilon$ and permeability tensor K_p^ε are defined as

$$\mathbf{E}_1^\varepsilon(x,\xi) = \mathbf{E}_1(x/\varepsilon,\omega,\xi), \quad K_p^\varepsilon(x) = K_p(x/\varepsilon,\omega). \tag{1}$$

In the mathematical model for biomechanics of a plant tissue we consider concentration of calcium c_e^ε and c_f^ε in cell walls and middle lamella G_e^ε and in cell cytoplasm G_f^ε (cell inside), respectively. In addition, in the domain of cell walls and middle lamella G_e^ε densities of methylesterified and de-methylesterified pectins n_e^ε and n_d^ε and of calcium–pectin cross-links n_b^ε are considered. We shall use the notation $b_e^\varepsilon = (n_e^\varepsilon, n_d^\varepsilon, n_b^\varepsilon)$ and $D_b(b_{e,3}^\varepsilon) = \mathrm{diag}(D_{n_e}(n_b^\varepsilon), D_{n_d}(n_b^\varepsilon), D_{n_b}(n_b^\varepsilon))$ denotes the diagonal matrix of diffusion coefficients for n_e^ε, n_d^ε, and n_b^ε respectively. We assume that the inflow of new calcium is facilitated only on parts of the cell membrane $\Gamma^\varepsilon \backslash \widetilde{\Gamma}^\varepsilon$. Here we consider a passive flow of calcium between cell wall and cell inside. The regulatory mechanism for calcium inflow by mechanical properties of cell walls will be considered in further studies. For elastic deformations of plant cell walls and middle lamella we consider homogenized equations of poro-elasticity reflecting the microscopic structure of cell walls composed of elastic cellulose microfibrils and cell wall matrix permeable for the fluid flow. The differences in the elastic properties of cell walls and middle lamella are reflected in the elasticity tensor $\mathbf{E}_1^\varepsilon$, which depends on the microscopic variable x/ε. Here we consider diffusion coefficients depending on calcium–pectin cross-links density. The analysis in the case of diffusion coefficients depending additionally on microscopic and macroscopic variables will follow the same steps.

We shall use the notations $G_T = (0,T) \times G$, $(\partial G)_T = (0,T) \times \partial G$, $G_{j,T}^\varepsilon = (0,T) \times G_j^\varepsilon$ for $j = e, f$, $\Gamma_T^\varepsilon = (0,T) \times \Gamma^\varepsilon$, and $\widetilde{\Gamma}_T^\varepsilon = (0,T) \times \widetilde{\Gamma}^\varepsilon$. By $\Pi_\tau w$ we define the tangential projection of a vector w, i.e. $\Pi_\tau w = w - (w \cdot n)n$, where n is a normal vector and τ indicates the tangential subspace to the boundary.

For $\mathcal{P}$-a.a. realizations $\omega \in \Omega$ the microscopic model for the concentration of calcium and densities of pectins and calcium–pectin cross-links reads

$$\begin{aligned}
&\partial_t b_e^\varepsilon = \mathrm{div}(D_b(b_{e,3}^\varepsilon)\nabla b_e^\varepsilon) + g_b(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)), && \text{in } G_{e,T}^\varepsilon,\\
&\partial_t c_e^\varepsilon = \mathrm{div}(D_e(b_{e,3}^\varepsilon)\nabla c_e^\varepsilon) + g_e(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)) && \text{in } G_{e,T}^\varepsilon,\\
&\partial_t c_f^\varepsilon = \mathrm{div}(D_f \nabla c_f^\varepsilon - \mathcal{G}(\partial_t u_f^\varepsilon)c_f^\varepsilon) + g_f(c_f^\varepsilon) && \text{in } G_{f,T}^\varepsilon,\\
&D_b(b_{e,3}^\varepsilon)\nabla b_e^\varepsilon \cdot n = \varepsilon R(b_e^\varepsilon) && \text{on } \Gamma_T^\varepsilon,\\
&c_f^\varepsilon = c_e^\varepsilon && \text{on } \Gamma_T^\varepsilon \backslash \widetilde{\Gamma}_T^\varepsilon,\\
&D_e(b_{e,3}^\varepsilon)\nabla c_e^\varepsilon \cdot n = (D_f \nabla c_f^\varepsilon - \mathcal{G}(\partial_t u_f^\varepsilon)c_f^\varepsilon) \cdot n && \text{on } \Gamma_T^\varepsilon \backslash \widetilde{\Gamma}_T^\varepsilon,\\
&D_e(b_{e,3}^\varepsilon)\nabla c_e^\varepsilon \cdot n = 0, \quad (D_f \nabla c_f^\varepsilon - \mathcal{G}(\partial_t u_f^\varepsilon)c_f^\varepsilon) \cdot n = 0 && \text{on } \widetilde{\Gamma}_T^\varepsilon.
\end{aligned} \tag{2}$$

Here u_e^ε stands for the displacement from the equilibrium position in poroelastic material of cell wall and middle lamella, $\mathbf{e}(u_e^\varepsilon) = (\mathbf{e}(u_e^\varepsilon)_{ij})_{i,j=1,2,3}$ for its symmetrized gradient, with $\mathbf{e}(u_e^\varepsilon)_{ij} = (\partial_{x_i} u_{ej}^\varepsilon + \partial_{x_j} u_{ei}^\varepsilon)/2$, and $\partial_t u_f^\varepsilon$ denotes the fluid velocity in the cell inside. The pressures in the poroelastic and fluid domains are denoted by p_e^ε and p_f^ε, respectively. The function $\mathcal{G}$ defines the velocity field in the convection term in cell inside and is a Lipschitz continuous bounded function of the intracellular flow velocity $\partial_t u_f^\varepsilon$. The condition that $\mathcal{G}$ is bounded is natural from the biological and physical point of view, because the flow velocity in plant tissues is bounded. This condition is also essential for the derivation of *a priori* estimates.

The water flow inside the cells and elastic deformations of plant cell walls and middle lamella are modelled by a coupled system of poro-elastic and Stokes equations

$$\begin{aligned}
&\rho_e \partial_t^2 u_e^\varepsilon - \operatorname{div}(\mathbf{E}^\varepsilon(b_{e,3}^\varepsilon)\mathbf{e}(u_e^\varepsilon)) + \nabla p_e^\varepsilon = 0 && \text{in } G_{e,T}^\varepsilon,\\
&\rho_p \partial_t p_e^\varepsilon - \operatorname{div}(K_p^\varepsilon \nabla p_e^\varepsilon - \partial_t u_e^\varepsilon) = 0 && \text{in } G_{e,T}^\varepsilon,\\
&\rho_f \partial_t^2 u_f^\varepsilon - \varepsilon^2 \mu \operatorname{div}(\mathbf{e}(\partial_t u_f^\varepsilon)) + \nabla p_f^\varepsilon = 0 && \text{in } G_{f,T}^\varepsilon,\\
&\operatorname{div} \partial_t u_f^\varepsilon = 0 && \text{in } G_{f,T}^\varepsilon,\\
&(\mathbf{E}^\varepsilon(b_{e,3}^\varepsilon)\, \mathbf{e}(u_e^\varepsilon) - p_e^\varepsilon I)n = (\varepsilon^2 \mu\, \mathbf{e}(\partial_t u_f^\varepsilon) - p_f^\varepsilon I)n && \text{on } \Gamma_T^\varepsilon,\\
&\Pi_\tau \partial_t u_e^\varepsilon = \Pi_\tau \partial_t u_f^\varepsilon, \quad n \cdot (\varepsilon^2 \mu\, \mathbf{e}(\partial_t u_f^\varepsilon) - p_f^\varepsilon I)n = -p_e^\varepsilon && \text{on } \Gamma_T^\varepsilon,\\
&(-K_p^\varepsilon \nabla p_e^\varepsilon + \partial_t u_e^\varepsilon) \cdot n = \partial_t u_f^\varepsilon \cdot n && \text{on } \Gamma_T^\varepsilon,\\
&u_e^\varepsilon(0,x) = u_{e0}^\varepsilon(x), \quad \partial_t u_e^\varepsilon(0,x) = u_{e0}^1(x), \quad p_e^\varepsilon(0,x) = p_{e0}^\varepsilon(x) && \text{in } G_e^\varepsilon,\\
&\partial_t u_f^\varepsilon(0,x) = u_{f0}^1(x) && \text{in } G_f^\varepsilon,
\end{aligned} \tag{3}$$

where ρ_e denotes the poroelastic wall density, ρ_p is the mass storativity coefficient, and ρ_f denotes the fluid density. We assume that ρ_e, ρ_p, and ρ_f are positive and constant. The dependence of the elastic properties of the cell wall matrix and middle lamella on calcium–pectin cross-links is reflected in the dependence of the elasticity tensor $\mathbf{E}^\varepsilon$ on $b_{e,3}^\varepsilon(\cdot)$. In what follows we assume that this dependence is non-local in temporal variable which reflects the time of reaction, i.e. the stretched cross-links have different impact (stress drive hardening) on the elastic properties of the cell wall matrix than newly-created cross-links, see e.g. [17, 59, 63]. More precisely, we assume in (1) that $\xi = \mathcal{K}\left(b_{e,3}^\varepsilon(\cdot)\right)(t,x) = \int_0^t \kappa(t-\tau) b_{e,3}^\varepsilon(\tau,x)\mathrm{d}\tau$, where $\kappa(\cdot)$ is a smooth non-negative kernel, and define

$$\widetilde{\mathbf{E}}\left(\omega, b_{e,3}^\varepsilon(\cdot)\right) = \widetilde{\mathbf{E}}_1\left(\omega, \int_0^t \kappa(t-\tau) b_{e,3}^\varepsilon(\tau,x) d\tau\right), \quad \mathbf{E}^\varepsilon\left(x, b_{e,3}^\varepsilon(\cdot)\right) = \widetilde{\mathbf{E}}\left(\mathcal{T}_{x/\varepsilon}\omega, b_{e,3}^\varepsilon(\cdot)\right).$$

Together with the profile of function $\mathbf{E}_1^\varepsilon$ this kernel specifies how the elastic properties of cell walls and middle lamella depend on calcium-pectin cross-links, see assumption A2 for further conditions on κ.

On the external boundaries we consider some given forces applied to plant tissues and flux conditions for pectins and calcium:

$$\begin{aligned}
&D_b \nabla b_e^\varepsilon \cdot n = F_b(b_e^\varepsilon), \quad D_e \nabla c_e^\varepsilon \cdot n = F_c(c_e^\varepsilon) && \text{on } (\partial G)_T,\\
&\mathbf{E}^\varepsilon(b_e^\varepsilon)\mathbf{e}(u_e^\varepsilon)\, n = F_u && \text{on } (\partial G)_T,\\
&(K_p^\varepsilon \nabla p_e^\varepsilon - \partial_t u_e^\varepsilon) \cdot n = F_p && \text{on } (\partial G)_T.
\end{aligned} \tag{4}$$

A detailed derivation of the model equations (2) and (3) can be found in [57].

System (2)–(4) is studied under the following assumption on the coefficients and nonlinear functions:

Assumption 2.

A1 $D_b^{jj}, D_e \in C(\mathbb{R})$ such that $d_j \leqslant D_b^{jj}(\xi) \leqslant \tilde{d}_j$ and $d_e \leqslant D_e(\xi) \leqslant \tilde{d}_e$ for all $\xi \in \mathbb{R}$, with some $d_j, d_e, \tilde{d}_j, \tilde{d}_e > 0$ and $j = 1, 2, 3$.

A2 Elasticity tensor $\widetilde{\mathbf{E}}(\omega,\xi) = (\widetilde{E}_{ijkl}(\omega,\xi))_{1\leqslant i,j,k,l\leqslant 3}$ satisfies $\widetilde{E}_{ijkl} = \widetilde{E}_{klij} = \widetilde{E}_{jikl} = \widetilde{E}_{ijlk}$ and $\alpha_1|A|^2 \leqslant \widetilde{\mathbf{E}}(\omega,\xi)A\cdot A \leqslant \alpha_2|A|^2$ for all symmetric $A\in\mathbb{R}^{3\times 3}$, $\xi\in\mathbb{R}_+$, $\mathcal{P}$-a.a. $\omega\in\Omega$, and $0<\alpha_1\leqslant\alpha_2<\infty$,
$\widetilde{\mathbf{E}}(\omega,\varsigma(\cdot)) = \widetilde{\mathbf{E}}_1(\omega,\mathcal{K}(\varsigma(\cdot)))$, where $\widetilde{\mathbf{E}}_1\in C(\Omega; C^2_b(\mathbb{R}))$ and $\mathcal{K}(\varsigma(\cdot)) = \int_0^t \kappa(t-\tau)\varsigma(\tau,x)d\tau$, with a smooth function $\kappa:\mathbb{R}_+\to\mathbb{R}_+$ such that $\kappa(0)=0$.

A3 $\widetilde{K}_p\in L^\infty(\Omega)$ and $\widetilde{K}_p(\omega)\eta\cdot\eta\geqslant k_1|\eta|^2$ for $\eta\in\mathbb{R}^3$, $\mathcal{P}$-a.a. $\omega\in\Omega$, and $k_1>0$.

A4 The convection function $\mathcal{G}$ is a Lipschitz continuous function on $\mathbb{R}^3$ such that $|\mathcal{G}(r)|\leqslant\rho$, for some $\rho>0$.

A5 For functions g_b, g_e, g_f, R, F_b, and F_c we assume

$$g_b\in C(\mathbb{R}\times\mathbb{R}^3\times\mathbb{R}^6;\mathbb{R}^3),\quad g_e\in C(\mathbb{R}\times\mathbb{R}^3\times\mathbb{R}^6),\quad F_b,\, R\in C(\mathbb{R}^3;\mathbb{R}^3),$$

and F_c and g_f are Lipschitz continuous. Moreover, the following estimates hold:

$$|g_b(s,r,A)|\leqslant C_1(1+|s|+|r|)+C_2|r||A|, \qquad |F_b(r)|+|R(r)|\leqslant C(1+|r|),$$
$$|g_e(s,r,A)|\leqslant C_1(1+|s|+|r|)+C_2(|s|+|r|)|A|, \qquad |F_c(s)|+|g_f(s)|\leqslant C(1+|s|),$$

where $s\in\mathbb{R}_+$, $r\in\mathbb{R}^3_+$, and A is a symmetric 3×3 matrix. Here and in what follows we identify the space of symmetric 3×3 matrices with $\mathbb{R}^6$.

It is also assumed that for any symmetric 3×3 matrix A we have that $g_{b,j}(s,r,A)$, $F_{b,j}(r)$, $R_j(r)$ are non-negative for $r_j=0$, $s\geqslant 0$, and $r_i\geqslant 0$, with $i=1,2,3$ and $j\neq i$, and $g_e(s,r,A)$, $g_f(s)$, and $F_c(s)$ are non-negative for $s=0$ and $r_j\geqslant 0$, with $j=1,2,3$.

We assume also that F_b and R are locally Lipschitz continuous, and

$$\begin{aligned}
|g_b(s_1,r_1,A_1)-g_b(s_2,r_2,A_2)| &\leqslant C_1(|r_1|+|r_2|)|s_1-s_2| + C_2(|s_1|+|s_2|+|A_1|+|A_2|)|r_1-r_2| \\
&\quad + C_3(|r_1|+|r_2|)|A_1-A_2|, \\
|g_e(s_1,r_1,A_1)-g_e(s_2,r_2,A_2)| &\leqslant C_1(|r_1|+|r_2|+|A_1|+|A_2|)|s_1-s_2| \\
&\quad + C_2(|s_1|+|s_2|+|A_1|+|A_2|)|r_1-r_2| \\
&\quad + C_3(|r_1|+|r_2|+|s_1|+|s_2|)|A_1-A_2|,
\end{aligned}$$

for $s_1,s_2\in(-\mu,+\infty)$, $r_1,r_2\in(-\mu,+\infty)^3$, for some $\mu>0$, and A_1,A_2 are symmetric 3×3 matrices.

A6 $b_{e0}\in L^\infty(G)^3$, $c_0\in L^\infty(G)$, and $b_{e0,j}\geqslant 0$, $c_0\geqslant 0$ a.e. in G, where $j=1,2,3$, $u^1_{e0}\in H^1(G)^3$, $u^1_{f0}\in H^2(G)^3$ and div $u^1_{f0}=0$ in G^ε_f for $\mathcal{P}$-a.a. realization $\omega\in\Omega$, $u^\varepsilon_{e0}\in H^1(G^\varepsilon_e)^3$, $p^\varepsilon_{e0}\in H^1(G)$, are defined as solutions of

$$\begin{aligned}
&\operatorname{div}(\mathbf{E}^\varepsilon(b_{e0,3})\mathbf{e}(u^\varepsilon_{e0})) = f_u && \text{in } G^\varepsilon_e,\\
&\Pi_\tau(\mathbf{E}^\varepsilon(b_{e0,3})\mathbf{e}(u^\varepsilon_{e0})\,n) = \varepsilon^2\mu\,\Pi_\tau(\mathbf{e}(u^1_{f0})n) && \text{on } \Gamma^\varepsilon,\\
&n\cdot\mathbf{E}^\varepsilon(b_{e0,3})\mathbf{e}(u^\varepsilon_{e0})\,n = 0 && \text{on } \Gamma^\varepsilon,\quad u^\varepsilon_{e0}=0 \text{ on } \partial G,\\
&\operatorname{div}(K^\varepsilon_p\nabla p^\varepsilon_{e0}) = f_p && \text{in } G,\quad p^\varepsilon_{e0}=0 \text{ on } \partial G,
\end{aligned}$$

$\mathcal{P}$-a.s., for given $f_u\in L^2(G)^3$ and $f_p\in L^2(G)$,

$$F_p\in H^1(0,T;L^2(\partial G)),\quad F_u\in H^2(0,T;L^2(\partial G))^3.$$

Remark 2.1. Under our assumptions on u^ε_{e0} and p^ε_{e0} by the standard stochastic homogenization arguments we obtain

$$\begin{aligned}&\tilde u^\varepsilon_{e0} \to u_{e0}, \quad p^\varepsilon_{e0}\to p_{e0} && \text{strongly in } L^2(G),\\ &\mathbf{e}(u^\varepsilon_{e0}) \to \mathbf{e}(u_{e0}) + U^0_{e,\mathrm{sym}} && \text{strongly stochastically two} - \text{scale}, \quad U^0_{e,\mathrm{sym}} \in L^2(G; L^2_{\mathrm{pot}}(\Omega))^3,\end{aligned}$$

for $\mathcal{P}$-a.a. $\omega\in\Omega$, where $\tilde u^\varepsilon_{e0}$ is an extension of u^ε_{e0}, and $u_{e0}\in H^1(G)^3$ and $p_{e0}\in H^1(G)$ are solutions of the corresponding macroscopic (homogenized) equations.

Here the subscript sym is used to emphasize that the corresponding matrix is symmetric.

Notice that in the equation for calcium c^ε_f inside plant cells we consider a bounded function of the water velocity u^ε_f. This technical assumption is biologically justified, since only bounded velocities are possible inside plant cells.

By $\langle\cdot,\cdot\rangle_{H^1(G)',H^1}$ we shall denote the duality product between $L^2(0,T;(H^1(G))')$ and $L^2(0,T;H^1(G))$, and

$$\langle\phi,\psi\rangle_{G_T} = \int_0^T\int_G \phi\,\psi\,\mathrm{d}x\mathrm{d}t \quad \text{for } \phi\in L^q(0,T;L^p(G)) \text{ and } \psi\in L^{q'}(0,T;L^{p'}(G)),$$

and

$$\langle\phi,\psi\rangle_{G_T,\Omega} = \int_0^T\int_G\int_\Omega \phi\,\psi\,d\mathcal{P}(\omega)\,\mathrm{d}x\mathrm{d}t \quad \text{for } \phi\in L^q(0,T;L^p(G\times\Omega)) \text{ and}$$
$$\psi\in L^{q'}(0,T;L^{p'}(G\times\Omega)),$$

where $1<p,q<+\infty$, $1/q+1/q'=1$ and $1/p+1/p'=1$.

Definition 2.2. Weak solution of (2)–(4) are functions

$$\begin{aligned}&u^\varepsilon_e \in [L^2(0,T;H^1(G^\varepsilon_e))\cap H^2(0,T;L^2(G^\varepsilon_e))]^3,\\ &p^\varepsilon_e\in L^2(0,T;H^1(G^\varepsilon_e))\cap H^1(0,T;L^2(G^\varepsilon_e)),\\ &u^\varepsilon_f\in [L^2(0,T;H^1(G^\varepsilon_f))\cap H^1(0,T;L^2(G^\varepsilon_f))]^3,\\ &\mathrm{div}\ u^\varepsilon_f = 0 \text{ in } G^\varepsilon_{f,T}, \quad \Pi_\tau u^\varepsilon_e = \Pi_\tau u^\varepsilon_f \text{ on } \Gamma^\varepsilon_T,\\ &p^\varepsilon_f\in L^2((0,T)\times G^\varepsilon_f)\end{aligned}$$

for $\mathcal{P}$-a.a. $\omega\in\Omega$, that satisfy the integral relation

$$\begin{aligned}&\langle\rho_e\partial_t^2 u^\varepsilon_e,\phi\rangle_{G^\varepsilon_{e,T}} + \langle\mathbf{E}^\varepsilon(b^\varepsilon_e)\mathbf{e}(u^\varepsilon_e),\mathbf{e}(\phi)\rangle_{G^\varepsilon_{e,T}} + \langle\nabla p^\varepsilon_e,\phi\rangle_{G^\varepsilon_{e,T}}\\ &\quad + \langle\rho_p\partial_t p^\varepsilon_e,\psi\rangle_{G^\varepsilon_{e,T}} + \langle K^\varepsilon_p\nabla p^\varepsilon_e - \partial_t u^\varepsilon_e,\nabla\psi\rangle_{G^\varepsilon_{e,T}} + \langle\partial_t u^\varepsilon_f\cdot n,\psi\rangle_{\Gamma^\varepsilon_T} - \langle p^\varepsilon_e,\eta\cdot n\rangle_{\Gamma^\varepsilon_T}\\ &\quad + \langle\rho_f\partial_t^2 u^\varepsilon_f,\eta\rangle_{G^\varepsilon_{f,T}} + \mu\,\varepsilon^2\langle\mathbf{e}(\partial_t u^\varepsilon_f),\mathbf{e}(\eta)\rangle_{G^\varepsilon_{f,T}} = \langle F_u,\phi\rangle_{(\partial G)_T} + \langle F_p,\psi\rangle_{(\partial G)_T}\end{aligned} \tag{5}$$

for all $\phi\in L^2(0,T;H^1(G^\varepsilon_e))^3$, $\psi\in L^2(0,T;H^1(G^\varepsilon_e))$, $\eta\in L^2(0,T;H^1(G^\varepsilon_f))^3$ such that $\Pi_\tau\phi = \Pi_\tau\eta$ on Γ^ε and $\mathrm{div}\,\eta = 0$ in $G^\varepsilon_{f,T}$, and functions

$$\begin{aligned}&b^\varepsilon_e\in[L^2(0,T;H^1(G^\varepsilon_e))\cap L^\infty(0,T;L^2(G^\varepsilon_e))]^3,\\ &c^\varepsilon = c^\varepsilon_e\,\chi_{G^\varepsilon_e} + c^\varepsilon_f\,\chi_{G^\varepsilon_f}\in L^2(0,T;H^1(G\backslash\widetilde\Gamma^\varepsilon))\cap L^\infty(0,T;L^2(G))\end{aligned}$$

that satisfy the integral relations

$$\begin{aligned}&\langle \partial_t b_e^\varepsilon, \varphi_1\rangle_{H^1(G_e^\varepsilon)', H^1} + \langle D_b(b_{e,3}^\varepsilon)\nabla b_e^\varepsilon, \nabla\varphi_1\rangle_{G_{e,T}^\varepsilon} - \langle g_b(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)), \varphi_1\rangle_{G_{e,T}^\varepsilon}\\ &\quad = \varepsilon\langle R(b_e^\varepsilon), \varphi_1\rangle_{\Gamma_T^\varepsilon} + \langle F_b(b_e^\varepsilon), \varphi_1\rangle_{(\partial G)_T}\end{aligned} \tag{6}$$

and

$$\begin{aligned}&\langle \partial_t c_e^\varepsilon, \varphi_2\rangle_{H^1(G_e^\varepsilon)', H^1} + \langle D_e(b_{e,3}^\varepsilon)\nabla c_e^\varepsilon, \nabla\varphi_2\rangle_{G_{e,T}^\varepsilon} - \langle g_e(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)), \varphi_2\rangle_{G_{e,T}^\varepsilon} - \langle F_c(c_e^\varepsilon), \varphi_2\rangle_{(\partial G)_T}\\ &\quad + \langle \partial_t c_f^\varepsilon, \varphi_2\rangle_{H^1(G_f^\varepsilon)', H^1} + \langle D_f\nabla c_f^\varepsilon, \nabla\varphi_2\rangle_{G_{f,T}^\varepsilon} - \langle \mathcal{G}(\partial_t u_f^\varepsilon)c_f^\varepsilon, \nabla\varphi_2\rangle_{G_{f,T}^\varepsilon} - \langle g_f(c_f^\varepsilon), \varphi_2\rangle_{G_{f,T}^\varepsilon} = 0\end{aligned} \tag{7}$$

for all $\varphi_1 \in L^2(0, T; H^1(G_e^\varepsilon))^3$ and $\varphi_2 \in L^2(0, T; H^1(G\backslash\widetilde{\Gamma}^\varepsilon))$, and for $\mathcal{P}$-a.a. $\omega \in \Omega$. Moreover the initial conditions are satisfied in L^2-sense, i.e. $u_e^\varepsilon(t) \to u_{e0}^\varepsilon$, $\partial_t u_e^\varepsilon(t) \to u_{e0}^1$, $p_e^\varepsilon(0) \to p_{e0}^\varepsilon$, $b_e^\varepsilon(t) \to b_{e0}$, $c_e^\varepsilon(t) \to c_0$ in $L^2(G_e^\varepsilon)$ as $t \to 0$, $\partial_t u_f^\varepsilon(t) \to u_{f0}^1$, $c_f^\varepsilon(t) \to c_0$ in $L^2(G_f^\varepsilon)$ as $t \to 0$, $\mathcal{P}$-almost sure.

Examples of random geomerties

- Let $\mathcal{Q}$ be a smooth domain, $\mathcal{Q} \subset (0, 1)^3$, and assume that $\gamma = \text{dist}(\mathcal{Q}, \partial(0, 1)^3) > 0$. Let ξ_j be i.i.d. random vectors in $\mathbb{R}^3$ such that $|\xi_j| \leqslant \gamma/4$, and η_j, $j \in \mathbb{Z}^3$, be random variables with values in the interval $[1/2, 1]$. Letting $\mathcal{Q}_j = j + \xi_j + \eta_j\mathcal{Q}$ we define

$$G_f(\omega) = \bigcup_{j\in\mathbb{Z}^3} \mathcal{Q}_j(\omega).$$

- Let $\mathcal{P}$ be a stationary ergodic point process in $\mathbb{R}^3$ such that
* almost surely for any two points x_j and x_k from $\mathcal{P}(\omega)$ the inequality $|x_j - x_k| \geqslant c > 0$ holds with a deterministic constant c;
* There exists $r > 0$ such that the intersection of the process with any ball of radius r is a.s. non-empty. We then set $\mathcal{Q}_j = \{x \in \mathbb{R}^3 \,:\, \text{dist}(x, x_j) < \frac{1}{2}\text{dist}(x, \mathcal{P}(\omega)\backslash x_j)\}$ and define

$$G_f(\omega) = \bigcup_{j\in\mathbb{Z}^3} \mathcal{Q}_j(\omega).$$

- The last example admits the following modifications: for the same stationary point process $\mathcal{P}$ we consider the Voronoi tessellation

$$\mathcal{Q}_j(\omega) = \{x \in \mathbb{R}^3 \,:\, \text{dist}(x, x_j) < \text{dist}(x, \mathcal{P}(\omega)\backslash x_j)\}.$$

  Then $\bigcup_j \overline{\mathcal{Q}}_j = \mathbb{R}^3$ and, under the assumptions on $\mathcal{P}$, the diameters of the polyhedrons $\mathcal{Q}_j$ are uniformly bounded and their boundaries are uniformly Lipschitz continuous.

Given $\delta > 0$ we then set

$$G_e(\omega) = \{x \in \mathbb{R}^3 \,:\, \text{dist}(x, \bigcup_j \partial\mathcal{Q}_j) < \delta\}.$$

Notice that in this case the volume fraction of G_e is of order δ, if δ is sufficiently small. This allows to model cell structures with relatively small volume fraction of cell walls and middle lamella.

3. Main results

The main result of the paper is the derivation of the macroscopic equations for the microscopic problem (2)–(4) using methods of stochastic homogenization.

First we shall introduce the following notations. Denote by ∂_ω^j the generator of a strongly continuous group of unitary operators in $L^2(\Omega)$ associated with $\mathcal{T}_x$ along e_j-direction, i.e.

$$\partial_\omega^j u(\omega) = \lim_{\delta\to 0} \frac{u(\mathcal{T}_{\delta e_j}\omega) - u(\omega)}{\delta}.$$

The domains of ∂_ω^j, with $j = 1, 2, 3$, are dense in $L^2(\Omega)$. We denote $\nabla_\omega u = (\partial_\omega^1 u, \partial_\omega^2 u, \partial_\omega^3 u)^T$ and $H^1_\mathcal{T}(\Omega) = \{v : v, \nabla_\omega v \in L^2(\Omega)\}$. By $C_\mathcal{T}(\Omega)$ we denote the space of functions with continuous realizations and $C^1_\mathcal{T}(\Omega)$ defines the set of functions from $C_\mathcal{T}(\Omega)$ such that $(\partial_\omega^j u) \in C_\mathcal{T}(\Omega)$, for $j = 1, 2, 3$.

First we introduce the spaces of potential and solenoidal vector fields:

$$L^2_{\rm pot}(\Omega) = \overline{\{\nabla_\omega u : u \in C^1_\mathcal{T}(\Omega)\}} \quad \text{and} \quad L^2_{\rm sol}(\Omega) = \left(L^2_{\rm pot}(\Omega)\right)^\perp,$$

where the closure in the definition of $L^2_{\rm pot}(\Omega)$ is with respect to the $L^2(\Omega)$-norm, see [73]. To introduce correctors we also need the space of functions whose realizations are discontinuous along the surface $\widetilde{\Gamma}(\omega)$. We define

$$L^2_{\mathrm{pot},\Gamma}(\Omega) = \overline{\{\nabla_x u(\mathcal{T}_x\omega)|_{x=0}, \quad u(\mathcal{T}_x\omega) \in H^1_{\rm loc}(\mathbb{R}^3\backslash\widetilde{\Gamma}(\omega)) \cap C^1(\mathbb{R}^3\backslash\widetilde{\Gamma}(\omega))\}}$$

with the norm

$$\|u\|^2 = \int_\Omega \int_{[0,1]^3\backslash\widetilde{\Gamma}(\omega)} |\nabla_x u(\mathcal{T}_x\omega)|^2 \mathrm{d}x\mathrm{d}\mathcal{P},$$

and

$$L^2_{\mathrm{sol},\Gamma}(\Omega) = \left(L^2_{\mathrm{pot},\Gamma}(\Omega)\right)^\perp.$$

We also denote

$$C_{\mathcal{T},\Gamma}(\Omega) = \{u : u(\mathcal{T}_x\omega) \in C(\mathbb{R}^3\backslash\widetilde{\Gamma}(\omega))\}.$$

We start with the definition of effective coefficients for macrosocpic poro-elastic equations, which are obtained by deriving the macroscopic equations for the microscopic problem (3) and (4). The macroscopic elasticity tensor $\mathbf{E}^{\rm hom} = (E^{\rm hom}_{ijkl})$ and permeability tensor $K_p^{\rm hom} = (K^{\rm hom}_{p,ij})$, along with $K_u = (K_{u,ij})$, are defined by

$$\begin{aligned}
E^{\rm hom}_{ijkl}(b_{e,3}) &= \int_\Omega \left[\widetilde{E}_{ijkl}(\omega, b_{e,3}) + \left(\widetilde{E}(\omega, b_{e,3}) W^{kl}_{e,\mathrm{sym}}\right)_{ij}\right] \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega), \\
K^{\rm hom}_{p,ij} &= \int_\Omega \left[\widetilde{K}_{p,ij}(\omega) + \left(\widetilde{K}_p(\omega) W_p^j\right)_i\right] \chi_{\Omega_e}\, \mathrm{d}\mathcal{P}(\omega), \\
K_{u,ij} &= \int_\Omega \left[\delta_{ij} - \left(\widetilde{K}_p(\omega) W_u^j\right)_i\right] \chi_{\Omega_e}\, \mathrm{d}\mathcal{P}(\omega),
\end{aligned} \tag{8}$$

where χ_{Ω_e} stands for the characteristic function of the set Ω_e, $W^{kl}_{e,\mathrm{sym}}$ denotes the symmetric part of the matrix W_e^{kl}, and $W_e^{kl} \in L^\infty(G_T; L^2_{\rm pot}(\Omega)^3)$ together with $W_p^k, W_u^k \in L^2_{\rm pot}(\Omega)$ are solutions of cell problems

$$\int_\Omega \widetilde{\mathbf{E}}\left(\omega, b_{e,3}\right)\left(W^{kl}_{e,\mathrm{sym}} + \mathbf{b}_{kl}\right)\, \Phi\, \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) = 0 \qquad \text{for all } \Phi \in L^2_{\mathrm{pot}}(\Omega)^3, \text{ a.a. } (t,x) \in G_T,$$
$$\int_\Omega \widetilde{K}_p(\omega)\left(W^k_p + e_k\right)\, \zeta\, \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) = 0 \qquad \text{for all } \zeta \in L^2_{\mathrm{pot}}(\Omega),$$
$$\int_\Omega \left(\widetilde{K}_p(\omega) W^k_u - e_k\right)\, \zeta\, \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) = 0 \qquad \text{for all } \zeta \in L^2_{\mathrm{pot}}(\Omega), \tag{9}$$

for $k, l = 1, 2, 3$, with $\mathbf{b}_{kl} = \frac{1}{2}(e_k \otimes e_l + e_l \otimes e_k)$ and $\{e_j\}_{j=1}^3$ is the canonical basis of $\mathbb{R}^3$.

We also define $Q(\partial_t u_f)$ as

$$Q(\partial_t u_f) = \int_\Omega \partial_t u_f\, \chi_{\Omega_f} \mathrm{d}\mathcal{P}(\omega) - \int_\Omega \widetilde{K}_p(\omega) Q_f(\omega, \partial_t u_f)\, \chi_{\Omega_e}\, \mathrm{d}\mathcal{P}(\omega), \tag{10}$$

where $Q_f(\cdot, \partial_t u_f) \in L^2_{\mathrm{pot}}(\Omega)$ is a solution of the problem

$$\int_\Omega \left(\widetilde{K}_p(\omega)\, Q_f\, \chi_{\Omega_e} + \partial_t u_f\, \chi_{\Omega_f}\right) \zeta\, \mathrm{d}\mathcal{P}(\omega) = 0 \quad \text{for } \zeta \in L^2_{\mathrm{pot}}(\Omega). \tag{11}$$

Then the macroscopic equations for the microscopic problem (3) and (4) are formulated as follows.

Theorem 3.1. *A sequence of solutions* $\{u^\varepsilon_e, p^\varepsilon_e, \partial_t u^\varepsilon_f, p^\varepsilon_f\}$ *of the microscopic problem* (3) *and* (4) *converges to a solution* $u_e \in H^2(0,T; L^2(G)) \cap L^2(0,T; H^1(G))$, $p_e \in L^2(0,T; H^1(G)) \cap H^1(0,T; L^2(G))$, $\partial_t u_f \in L^2(G_T; H^1(\Omega)) \cap H^1(0,T; L^2(G \times \Omega))$, $p_f \in L^2(G_T \times \Omega)$ *of the macroscopic equations*

$$\vartheta_e \rho_e \partial_t^2 u_e - \mathrm{div}(\mathbf{E}^{\mathrm{hom}}(b_{e,3}) \mathbf{e}(u_e)) + \nabla p_e + \int_\Omega \partial_t^2 u_f\, \chi_{\Omega_f} \mathrm{d}\mathcal{P}(\omega) = 0 \quad \text{in } G_T,$$
$$\vartheta_e \rho_p \partial_t p_e - \mathrm{div}(K_p^{\mathrm{hom}} \nabla p_e - K_u \partial_t u_e - Q(\partial_t u_f)) = 0 \quad \text{in } G_T, \tag{12}$$

with boundary and initial conditions

$$\mathbf{E}^{\mathrm{hom}}(b_{e,3}) \mathbf{e}(u_e)\, n = F_u \qquad \text{on } (0,T) \times \partial G,$$
$$(K_p^{\mathrm{hom}} \nabla p_e - K_u \partial_t u_e - Q(\partial_t u_f)) \cdot n = F_p \qquad \text{on } (0,T) \times \partial G, \tag{13}$$
$$u_e(0) = u_{e0}, \quad \partial_t u_e(0) = u^1_{e0}, \quad p_e(0) = p_{e0} \qquad \text{in } G,$$

and the equations for the flow velocity

$$\int_\Omega \left[\rho_f \partial_t^2 u_f\, \varphi + \mu\, \mathbf{e}_\omega(\partial_t u_f) \mathbf{e}_\omega(\varphi) + \nabla p_e\, \varphi\right] \chi_{\Omega_f} \mathrm{d}\mathcal{P}(\omega) - \int_\Omega P^1_e\, \chi_{\Omega_e}\, \varphi\, \mathrm{d}\mathcal{P}(\omega) = 0,$$
$$\mathrm{div}_\omega \partial_t u_f = 0 \qquad \text{in } G_T \times \Omega,$$
$$\partial_t u_f(0) = u^1_{f0} \qquad \text{in } G \times \Omega, \tag{14}$$

$$\Pi_\tau \partial_t u_f(t, x, \mathcal{T}_{\widetilde{x}} \omega) = \Pi_\tau \partial_t u_e(t, x) \quad \text{for } (t,x) \in G_T \text{ and } \widetilde{x} \in \Gamma(\omega), \quad \mathcal{P} - a.s. \text{ in } \Omega,$$

and

$$P_e^1(t,x,\omega) = \sum_{k=1}^{3} \partial_{x_k} p_e(t,x) W_p^k(\omega) + \partial_t u_e^k(t,x) W_u^k(\omega) + Q_f(\omega, \partial_t u_f), \tag{15}$$

for all $\varphi \in L^2(G_T; H^1(\Omega))^3$, *with* $\operatorname{div}_\omega \varphi = 0$ *in* $G_T \times \Omega$, *and* $\Pi_\tau \varphi(t,x,\mathcal{T}_{\widetilde{x}}\omega) = 0$ *for* $(t,x) \in G_T$, $\widetilde{x} \in \Gamma(\omega)$ *and* $\mathcal{P}$*-a.s. in* Ω.

Here $\mathbf{e}_\omega(\psi) = \big(1/2(\partial_\omega^j \psi_l + \partial_\omega^l \psi_j)\big)_{j,l=1,2,3}$ *denotes a symmetric gradient for* $\psi \in H^1(\Omega)^3$, $\vartheta_e = \int_\Omega \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega)$, *and* $\operatorname{div}_\omega \psi = \partial_\omega^1 \psi_1 + \partial_\omega^2 \psi_2 + \partial_\omega^3 \psi_3$.

Remark. Notice that the equations for correctors (9) and (11), as well as problem (14) for $\partial_t u_f$ are formulated in the weak form as integral identities. This is due to the fact that the equations are define on $\Omega_e \subset \Omega$ and $\Omega_f \subset \Omega$, respectively, and have strong formulation only for $\mathcal{P}$-a.a. realizations $\omega \in \Omega$.

The homogenized coefficients in reaction-diffusion-convection equations that will be obtained by deriving macroscopic equations for microscopic problem (2) and (4), are defined as

$$\begin{aligned} D_{b,\mathrm{eff}}^{ij}(b_{e,3}) &= \int_\Omega \Big[D_b^{ij}(b_{e,3}) + (D_b(b_{e,3})\, w_b^j)_i \Big] \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega), \\ D_{\mathrm{eff}}^{ij}(b_{e,3}) &= \int_\Omega \big[D^{ij}(b_{e,3},\omega) + \big(D(b_{e,3},\omega)\, w^j\big)_i \big] \,\mathrm{d}\mathcal{P}(\omega), \end{aligned} \tag{16}$$

where $D(b_{e,3},\omega) = D_e(b_{e,3})\chi_{\Omega_e}(\omega) + D_f \chi_{\Omega_f}(\omega)$ for $\omega \in \Omega$, with $w_b^j \in L^2_{\mathrm{pot}}(\Omega)$ and $w^j \in L^2_{\mathrm{pot},\Gamma}(\Omega)$ are solutions of the cell problems

$$\int_\Omega D_b(b_{e,3})(w_b^j + e_j)\,\zeta\, \chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) = 0 \quad \text{for all } \zeta \in L^2_{\mathrm{pot}}(\Omega), \tag{17}$$

and

$$\int_\Omega D(\omega, b_{e,3})(w^j + e_j)\,\eta\, \mathrm{d}\mathcal{P}(\omega) = 0 \quad \text{for all } \eta \in L^2_{\mathrm{pot},\Gamma}(\Omega). \tag{18}$$

The effective velocity is defined by

$$u_{\mathrm{eff}}(t,x) = \int_\Omega \big(\mathcal{G}(\partial_t u_f) - D_f Z(t,x,\omega)\big)\, \chi_{\Omega_f}\, \mathrm{d}\mathcal{P}(\omega),$$

where $Z \in L^\infty(G_T; L^2_{\mathrm{pot}}(\Omega))$ satisfies

$$\int_\Omega (D_f Z - \mathcal{G}(\partial_t u_f))\,\zeta\, \chi_{\Omega_f}\, \mathrm{d}\mathcal{P}(\omega) = 0 \quad \text{for all } \zeta \in L^2_{\mathrm{pot}}(\Omega), \text{ for a.a. } (t,x) \in G_T. \tag{19}$$

Theorem 3.2. *A sequence of solutions of microscopic problem (2) and (4) converges to a solution $b_e, c \in L^2(0,T;H^1(\Omega))$, with $\partial_t b_e, \partial_t c \in L^2(0,T;(H^1(\Omega))')$, of the macroscopic equations*

$$
\begin{aligned}
&\vartheta_e \partial_t b_e - \operatorname{div}(D_{b,\text{eff}}(b_{e,3})\nabla b_e) = \int_\Omega g_b(c, b_e, \mathbb{U}(b_e,\omega)\mathbf{e}(u_e))\chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) + \int_\Omega R(b_e)\mathrm{d}\boldsymbol{\mu}(\omega) && \text{in } G_T,\\
&\partial_t c - \operatorname{div}(D_{\text{eff}}(b_{e,3})\nabla c - u_{\text{eff}} c) = \vartheta_f g_f(c) + \int_\Omega g_e(c, b_e, \mathbb{U}(b_e,\omega)\mathbf{e}(u_e))\chi_{\Omega_e} \mathrm{d}\mathcal{P}(\omega) && \text{in } G_T,\\
&D_{b,\text{eff}}(b_{e,3})\nabla b_e \cdot n = F_b(b_e) && \text{on } (\partial G)_T,\\
&(D_{\text{eff}}(b_{e,3})\nabla c - u_{\text{eff}} c)\cdot n = F_c(c) && \text{on } (\partial G)_T,\\
&b_e(0,x) = b_{e0}(x), \quad c(0,x) = c_0(x) && \text{in } G,
\end{aligned}
\tag{20}
$$

where $\vartheta_j = \int_\Omega \chi_{\Omega_j}(\omega)\,\mathrm{d}\mathcal{P}(\omega)$, for $j = e, f$, and

$$
\mathbb{U}(b_e,\omega) = \{\mathbb{U}_{klij}(b_e,\omega)\}_{k,l,i,j=1}^3 = \left\{ b_{kl}^{ij} + W_{e,\text{sym},kl}^{ij} \right\}_{k,l,i,j=1}^3,
$$

with W_e^{ij} being solutions of cell problems (9) and $\mathbf{b}_{kl} = (b_{kl}^{ij})_{i,j=1}^3$, where $\boldsymbol{b}_{kl} = e_k \otimes e_l$.

Here $\boldsymbol{\mu}$ is the Palm measure of the random measure μ_ω of surfaces $\Gamma(\omega)$, see e.g. [23] for the definition of Palm measure.

4. *A priori* estimates

Considering assumptions on G_j^ε, with $j = e,f$, in the same way as in the periodic case [57], for $\mathcal{P}$-a.a. realizations $\omega \in \Omega$, we obtain the existence, uniqueness and *a priori* estimates, uniform in ε, for solutions of microscopic problem (2)–(4).

Lemma 4.1. *Under assumption 2 there exists a unique weak solution of microscopic problem (2)–(4).*

Proof. Sketch. For each realization ω the proof of the existence and uniqueness results follows the same steps as the proof of theorem 7 in [57]. □

Lemma 4.2. *Under assumptions 2 solutions of microscopic problem (2)–(4) satisfy* a priori *estimates for elastic displacement u_e^ε, pressure p_e^ε, and fluid flow velocity $\partial_t u_f^\varepsilon$*

$$
\begin{aligned}
&\|u_e^\varepsilon\|_{L^\infty(0,T;H^1(G_e^\varepsilon))} + \|\partial_t u_e^\varepsilon\|_{L^2(0,T;H^1(G_e^\varepsilon))} + \|\partial_t^2 u_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)} \leqslant C,\\
&\|p_e^\varepsilon\|_{L^\infty(0,T;H^1(G_e^\varepsilon))} + \|\partial_t p_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)} \leqslant C,\\
&\|\partial_t u_f^\varepsilon\|_{L^\infty(0,T;L^2(G_f^\varepsilon))} + \|\partial_t^2 u_f^\varepsilon\|_{L^2(G_{f,T}^\varepsilon)} + \varepsilon\|\nabla\partial_t u_f^\varepsilon\|_{L^2(G_{f,T}^\varepsilon)} + \|p_f^\varepsilon\|_{L^2(G_{f,T}^\varepsilon)} \leqslant C,
\end{aligned}
\tag{21}
$$

and for the concentration of calcium c_e^ε and c_f^ε and densities of pectins and calcium–pectin cross-links b_e^ε we obtain

$$
\begin{aligned}
&\|b_e^\varepsilon\|_{L^2(0,T;H^1(G_e^\varepsilon))} + \|b_e^\varepsilon\|_{L^\infty(0,T;L^\infty(G_e^\varepsilon))} + \varepsilon^{1/2}\|b_e^\varepsilon\|_{L^2(\Gamma_T^\varepsilon)} \leqslant C,\\
&\|c_j^\varepsilon\|_{L^2(0,T;H^1(G_j^\varepsilon))} + \|c_j^\varepsilon\|_{L^\infty(0,T;L^4(G_j^\varepsilon))} \leqslant C, \qquad j = e, f,
\end{aligned}
\tag{22}
$$

and

$$\|\theta_h b_e^\varepsilon - b_e^\varepsilon\|_{L^2((0,\tilde{T})\times G_e^\varepsilon)} + \|\theta_h c_j^\varepsilon - c_j^\varepsilon\|_{L^2((0,\tilde{T})\times G_j^\varepsilon)} \leqslant Ch^{1/4}, \qquad j = e, f, \tag{23}$$

for $\tilde{T} \in (0, T-h]$ *and for* $\mathcal{P}$*-a.a.* $\omega \in \Omega$*, where the constant* C *is independent of* ε *and* $\theta_h v(t,x) = v(t+h,x)$ *for* $(t,x) \in (0, T-h] \times G_j^\varepsilon$*, with* $j = e,f$.

Proof. For $\mathcal{P}$-a.a. realizations $\omega \in \Omega$ the proof of the *a priori* estimates follows the same lines as in [57, lemma 6]. □

We shall denote $c^\varepsilon(t,x) = c_e^\varepsilon(t,x)\,\chi_{G_e^\varepsilon} + c_f^\varepsilon(t,x)\chi_{G_f^\varepsilon}$.

Using the assumptions on the random microscopic structure of G_e^ε and G_f^ε we obtain the following extension results for functions defined on G_e^ε and on a subdomain $\widetilde{G}_{ef}^\varepsilon \subset G$, which will be specified below.

Lemma 4.3.

(*a*) *There exist extensions* $\overline{b}_e^\varepsilon$ *and* $\overline{c}_e^\varepsilon$ *of* b_e^ε *and* c_e^ε*, respectively, from* $L^2(0,T;H^1(G_e^\varepsilon))$ *to* $L^2(0,T;H^1(G))$ *such that*

$$\|\overline{b}_e^\varepsilon\|_{L^2(G_T)} \leqslant C\|b_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)}, \quad \|\nabla\overline{b}_e^\varepsilon\|_{L^2(G_T)} \leqslant C\|\nabla b_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)}, \tag{24}$$

$$\|\overline{c}_e^\varepsilon\|_{L^2(G_T)} \leqslant C\|c_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)}, \quad \|\nabla\overline{c}_e^\varepsilon\|_{L^2(G_T)} \leqslant C\|\nabla c_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon)}. \tag{25}$$

(*b*) *There exists an extension* $\overline{c}^\varepsilon$ *of* c^ε *from* $L^2(0,T;H^1(\widetilde{G}_{ef}^\varepsilon))$ *to* $L^2(0,T;H^1(G))$ *such that*

$$\begin{aligned} &\|\overline{c}^\varepsilon\|_{L^2(G_T)} \leqslant C\left(\|c_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon\cap\widetilde{G}_{ef,T}^\varepsilon)} + \|c_f^\varepsilon\|_{L^2(G_{f,T}^\varepsilon\cap\widetilde{G}_{ef,T}^\varepsilon)}\right), \\ &\|\nabla\overline{c}^\varepsilon\|_{L^2(G_T)} \leqslant C\left(\|\nabla c_e^\varepsilon\|_{L^2(G_{e,T}^\varepsilon\cap\widetilde{G}_{ef,T}^\varepsilon)} + \|\nabla c_f^\varepsilon\|_{L^2(G_{f,T}^\varepsilon\cap\widetilde{G}_{ef,T}^\varepsilon)}\right). \end{aligned} \tag{26}$$

Here $\widetilde{G}_{ef}^\varepsilon = G\backslash\widetilde{G}^\varepsilon$*, with* $\widetilde{G}^\varepsilon = \widetilde{\Gamma}_{\varepsilon\sigma}^\varepsilon(\omega)\cap\widetilde{G}_e^\varepsilon$*, where* $\widetilde{\Gamma}_{\varepsilon\sigma}^\varepsilon(\omega)$ *is a* $\varepsilon\sigma$*-neighbourhood of* $\widetilde{\Gamma}^\varepsilon$ *for* $\mathcal{P}$*-a.a. realizations* $\omega \in \Omega$ *and* $0 < \sigma < d_{\dim}/4$*, with* $d_{\min}$ *being the minimal distance between connected components of* $G_f(\omega)$*.*

Proof. The uniform boundedness of the diameter of cell walls and cell interiors, independent on realizations $\omega \in \Omega$, implies the existence of the corresponding extension operators, see [3]. □

Extensions for u_e^ε and p_e^ε are defined in the similar way as for b_e^ε.

Lemma 4.4. *For extensions of* b_e^ε*,* c_e^ε*,* u_e^ε*,* p_e^ε *from* $G_{e,T}^\varepsilon$ *to* G_T *and* c^ε *from* $\widetilde{G}_{ef,T}^\varepsilon$ *to* G_T *(denoted again by* b_e^ε*,* c_e^ε*,* u_e^ε*,* p_e^ε*, and* c^ε*) we have the following estimates*

$$\begin{aligned} &\|u_e^\varepsilon\|_{H^1(0,T;H^1(G))} + \|\partial_t^2 u_e^\varepsilon\|_{L^2(G_T)} + \|p_e^\varepsilon\|_{L^\infty(0,T;H^1(G))} + \|\partial_t p_e^\varepsilon\|_{L^2(G_T)} \leqslant C, \\ &\|b_e^\varepsilon\|_{L^2(0,T;H^1(G))} + \|c_e^\varepsilon\|_{L^2(0,T;H^1(G))} + \|c^\varepsilon\|_{L^2(0,T;H^1(G))} \leqslant C, \\ &\|\theta_h b_e^\varepsilon - b_e^\varepsilon\|_{L^2((0,\tilde{T})\times G)} + \|\theta_h c_e^\varepsilon - c_e^\varepsilon\|_{L^2((0,\tilde{T})\times G)} + \|\theta_h c^\varepsilon - c^\varepsilon\|_{L^2((0,\tilde{T})\times G)} \leqslant Ch^{1/4}, \end{aligned} \tag{27}$$

where the constant C is independent of ε. An extension of $\partial_t u_f^\varepsilon$ from $G_{f,T}^\varepsilon$ to G_T, constructed below and denoted again by $\partial_t u_f^\varepsilon$ satisfies the following estimates

$$\|\partial_t u_f^\varepsilon\|_{L^\infty(0,T;L^2(G))} + \|\partial_t^2 u_f^\varepsilon\|_{L^2(G_T)} + \varepsilon\|\nabla\partial_t u_f^\varepsilon\|_{L^2(G_T)} \leqslant C, \tag{28}$$

where the constant C is independent of ε. Also we have that

$$\|\widetilde{p}^\varepsilon\|_{L^2(G_T)} + \|p_f^\varepsilon\|_{L^2((0,T)\times G_f^\varepsilon)} \leqslant C, \quad \text{where } \widetilde{p}^\varepsilon = \begin{cases} p_f^\varepsilon & \text{in } \ (0,T)\times G_f^\varepsilon, \\ p_e^\varepsilon & \text{in } \ (0,T)\times (G\backslash G_f^\varepsilon), \end{cases} \tag{29}$$

and the constant C does not depend on ε.

Proof. The estimates for b_e^ε, c_e^ε, c^ε, u_e^ε, and p_e^ε follow directly from estimates (21)–(23), lemma 4.3, and the linearity of extension considered in lemma 4.3.

Using geometrical assumptions on $G_f(\omega)$, for $\mathcal{P}$-a.a. $\omega\in\Omega$, we can extend $\partial_t u_f^\varepsilon$ from G_f^ε to G in the following way. For each connected component $G_{f,j}(\omega)$ of $G_f(\omega)$, with $j\in\mathbb{N}$, we can consider a σ-neighbourhood $G_{f,j}^\sigma(\omega)$ of $G_{f,j}(\omega)$, where $\sigma = d_{\min}/4$ and $d_{\min}$ is the minimal distance between $G_{f,j}(\omega)$ for $j\in\mathbb{N}$. Then since $\partial_t u_f^\varepsilon \in L^2(0,T;H^1(G_f^\varepsilon))$, i.e. $\partial_t u_f^\varepsilon \in L^2(0,T;H^{1/2}(\Gamma^\varepsilon))$, there exists $\partial_t\widetilde{u}_f^j \in L^2\left(0,T;H^1(G_{f,j}^\sigma(\omega)\backslash G_{f,j}(\omega)\right)$satisfying the problem

$$\begin{aligned} &\mathrm{div}_y\partial_t\widetilde{u}_f^j = 0 && \text{in } G_{f,j}^\sigma(\omega)\backslash G_{f,j}(\omega), \\ &\partial_t\widetilde{u}_f^j = \partial_t u_f^\varepsilon(t,\varepsilon y) && \text{on } \Gamma_j(\omega), \\ &\partial_t\widetilde{u}_f^j = 0 && \text{on } \partial G_{f,j}^\sigma(\omega) \end{aligned} \tag{30}$$

for $\mathcal{P}$-a.a. realizations $\omega\in\Omega$ and $j\in\mathbb{N}$, see e.g. [67, theorem 2.4, lemma 2.4]. Each $\partial_t\widetilde{u}_f^j$ we extend by zero to $G_e(\omega)\backslash G_{f,j}^\sigma(\omega)$. Considering a scaling $y = x/\varepsilon$ in $\partial_t\widetilde{u}_f^j$ and collecting all $\partial_t\widetilde{u}_f^j$ for $j\in\mathbb{N}$ we obtain an extension $\partial_t\overline{u}_f^\varepsilon$ of $\partial_t u_f^\varepsilon$ from G_f^ε to G such that $\partial_t\overline{u}_f^\varepsilon \in L^2(0,T;H^1(G))$ and

$$\begin{aligned} &\mathrm{div}\,\partial_t\overline{u}_f^\varepsilon = 0 \quad \text{in } G, \\ &\|\partial_t u_f^\varepsilon\|_{L^2(G_T)} + \varepsilon\|\nabla\partial_t u_f^\varepsilon\|_{L^2(G_T)} \leqslant C, \end{aligned} \tag{31}$$

where the constant C is independent of ε.

Similar to the periodic case to show the *a priori* estimates for p_f^ε we consider the first and third equations in (3) and use the *a priori* estimates for u_e^ε, p_e^ε, and $\partial_t u_f^\varepsilon$ to obtain

$$\begin{aligned} \langle p_f^\varepsilon, \mathrm{div}\ \phi\rangle_{G_{f,T}^\varepsilon} + \langle p_e^\varepsilon, \mathrm{div}\ \phi\rangle_{G_{e,T}^\varepsilon} &= \langle \varepsilon^2\mu\,\mathbf{e}(\partial_t u_f^\varepsilon), \mathbf{e}(\phi)\rangle_{G_{f,T}^\varepsilon} + \langle \rho_f\partial_t^2 u_f^\varepsilon, \phi\rangle_{G_{f,T}^\varepsilon} \\ &\quad + \langle \rho_e\partial_t^2 u_e^\varepsilon, \phi\rangle_{G_{e,T}^\varepsilon} + \langle \mathbf{E}^\varepsilon(b_{e,3}^\varepsilon)\mathbf{e}(u_e^\varepsilon), \mathbf{e}(\phi)\rangle_{G_{e,T}^\varepsilon} \\ &\quad + \langle p_e^\varepsilon n - F_u, \phi\rangle_{(\partial G)_T} \\ &\leqslant C\|\phi\|_{L^2(0,T;H^1(G))^3}, \end{aligned} \tag{32}$$

with $\phi\in L^2(0,T;H^1(G))^3$. Here we used the extension of p_e^ε from G_e^ε to G, see lemma 4.3, and the trace estimate $\|p_e^\varepsilon\|_{L^2((0,T)\times\partial G)} \leqslant C_1\|p_e^\varepsilon\|_{L^2(0,T;H^1(G))} \leqslant C_2\|p_e^\varepsilon\|_{L^2(0,T;H^1(G_e^\varepsilon))}$.

For any $q \in L^2(G_T)$ there exists $\phi \in L^2(0,T;H^1(G))^3$ satisfying

$$\operatorname{div} \phi = q \quad \text{in} \quad G, \qquad \phi \cdot n = \frac{1}{|\partial G|}\int_G q(\cdot,x)\,\mathrm{d}x \quad \text{on} \quad \partial G$$

and $\|\phi\|_{L^2(0,T;H^1(G))^3} \leqslant C\|q\|_{L^2(G_T)}$. Thus using (29), the definition of the L^2-norm, and the *a priori* estimates for p_e^ε we obtain

$$\|\widetilde{p}^\varepsilon\|_{L^2(G_T)} \leqslant C \quad \text{and} \quad \|p_f^\varepsilon\|_{L^2((0,T)\times G_f^\varepsilon)} \leqslant C,$$

where the constant C is independent of ε. □

5. Convergence results

From *a priori* estimates derived in lemma 4.4 we obtain corresponding strong and stochastic two-scale convergences for a subsequence of solutions of microscopic problem (2)–(4). First we recall the definition of the stochastic two-scale convergence introduced in [74].

Definition 5.1. Let G be a domain in $\mathbb{R}^3$, $\mathcal{T}_x$ be an ergodic dynamical system, and $\widetilde{\omega}$ be a 'typical realization'. Then, we say that a sequence $\{v^\varepsilon\} \subset L^2(0,T;L^2(G))$ converges stochastically two-scale to $v \in L^2(G_T;L^2(\Omega,d\mathcal{P}))$ if

$$\lim_{\varepsilon\to 0}\sup\int_0^T\int_G |v^\varepsilon(t,x)|^2\,\mathrm{d}x\;\mathrm{d}t < \infty \tag{33}$$

and

$$\begin{aligned}&\lim_{\varepsilon\to 0}\int_0^T\int_G v^\varepsilon(t,x)\varphi(t,x)\psi(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})\;\mathrm{d}x\mathrm{d}t \\ &\quad= \int_0^T\int_G\int_\Omega v(t,x,\omega)\varphi(t,x)\psi(\omega)\,\mathrm{d}\mathcal{P}(\omega)\,\mathrm{d}x\mathrm{d}t\end{aligned} \tag{34}$$

for all $\varphi \in C_0^\infty([0,T)\times G)$ and $\psi \in L^2(\Omega)$.

As a 'typical realization' we denote such realization $\omega \in \Omega$ that Birkhoff's theorem is satisfied for $\mathcal{T}_x\omega$, i.e.

$$\lim_{\ell\to\infty}\frac{1}{\ell^3|A|}\int_{\ell A} g(\mathcal{T}_x\omega)\;\mathrm{d}x = \int_\Omega g(\omega)\,\mathrm{d}\mathcal{P}(\omega)$$

$\mathcal{P}$-a.s. for all bounded Borel sets A, $|A| > 0$, and all $g(\omega) \in C(\Omega)$. Let us note that realizations are typical $\mathcal{P}$-a.s., see e.g. [74].

Using compactness properties of stochastic two-scale convergence, see [74], we obtain the following result.

Lemma 5.2. *There exist functions* $u_e \in H^1(0,T;H^1(G)) \cap H^2(0,T;L^2(G))$, $p_e \in L^2(0,T;H^1(G)) \cap H^1(0,T;L^2(G))$, $U_e^1, \partial_t U_e^1 \in L^2(G_T;L^2_{\text{pot}}(\Omega))^3$, $P_e^1 \in L^2(G_T;L^2_{\text{pot}}(\Omega))$, *and*

$\partial_t u_f\, \chi_{\Omega_f}$, $\nabla_\omega \partial_t u_f\, \chi_{\Omega_f}$, $\partial_t^2 u_f\, \chi_{\Omega_f}$, $p_f\, \chi_{\Omega_f} \in L^2(G_T \times \Omega)$, *such that, up to a subsequence,*

$$\begin{aligned} &u_e^\varepsilon \to u_e && \textit{strongly in } H^1(0,T;L^2(G)),\\ &p_e^\varepsilon \to p_e && \textit{strongly in } L^2((0,T)\times G),\\ &\partial_t^2 u_e^\varepsilon \rightharpoonup \partial_t^2 u_e, \quad \partial_t p_e^\varepsilon \rightharpoonup \partial_t p_e && \textit{stochastically two}-\textit{scale},\\ &\nabla u_e^\varepsilon \rightharpoonup \nabla u_e + U_e^1 && \textit{stochastically two}-\textit{scale},\\ &\nabla p_e^\varepsilon \rightharpoonup \nabla p_e + P_e^1 && \textit{stochastically two}-\textit{scale}, \end{aligned} \tag{35}$$

and for fluid velocity and pressure we have

$$\begin{aligned} &\chi_{G_f^\varepsilon} \partial_t u_f^\varepsilon \rightharpoonup \chi_{\Omega_f} \partial_t u_f && \textit{stochastically two}-\textit{scale},\\ &\varepsilon \chi_{G_f^\varepsilon} \nabla \partial_t u_f^\varepsilon \rightharpoonup \chi_{\Omega_f} \nabla_\omega \partial_t u_f && \textit{stochastically two}-\textit{scale},\\ &\chi_{G_f^\varepsilon}\, p_f^\varepsilon \rightharpoonup \chi_{\Omega_f}\, p_f && \textit{stochastically two}-\textit{scale}. \end{aligned} \tag{36}$$

Proof. The estimates (27), the compactness of the embedding of $H^1(0,T;L^2(G)) \cap L^2(0,T;H^1(G))$ in $L^2(G_T)$, and the compactness theorem for stochastic two-scale convergence, see e.g. [74], yield the convergence results in (35).

For the extension of u_f^ε from G_f^ε to G we have the stochastic two-scale convergence of $\partial_t u_f^\varepsilon \rightharpoonup \partial_t u_f$ and $\varepsilon \nabla \partial_t u_f^\varepsilon \rightharpoonup \nabla_\omega \partial_t u_f$, with $\partial_t u_f, \nabla_\omega \partial_t u_f \in L^2(G_T \times \Omega)$, respectively. Additionally we have that $U_e^1\, \chi_{\Omega_e}$, $P_e^1\, \chi_{\Omega_e}$, $\partial_t u_f\, \chi_{\Omega_f}$, and $\nabla_\omega \partial_t u_f\, \chi_{\Omega_f}$ do not depend on the extension of u_e^ε, p_e^ε from G_e^ε to G and of $\partial_t u_f^\varepsilon$ from G_f^ε to G. The estimate and definition of $\widetilde{p}^\varepsilon$ in (29) and (32) ensure the stochastic two-scale convergence of $\chi_{G_f^\varepsilon} p_f^\varepsilon$. □

In the following lemma, we shall use the same notation for b_e^ε, c_e^ε and their extensions from G_e^ε to G, whereas the extension for c^ε from $\widetilde{G}_{ef}^\varepsilon$ to G will be denoted by $\overline{c}^\varepsilon$.

Lemma 5.3. *There exist functions* $b_e, c \in L^2(0,T;H^1(G))$, $b_e \in L^\infty(0,T;L^\infty(G))$, $c \in L^\infty(0,T;L^4(G))$, *and correctors* $B_e^1 \in L^2(G_T;L^2_{\text{pot}}(\Omega))$ *and* $C^1 \in L^2(G_T;L^2_{\text{pot},\Gamma}(\Omega))$, *such that, up to a subsequence,*

$$\begin{aligned} &b_e^\varepsilon \to b_e, \quad c^\varepsilon \to c && \textit{strongly in } L^2(G_T),\\ &\nabla b_e^\varepsilon \rightharpoonup \nabla b_e + B_e^1 && \textit{stochastically two}-\textit{scale},\\ &\nabla c^\varepsilon \rightharpoonup \nabla c + C^1 && \textit{stochastically two}-\textit{scale}, \quad \textit{as } \varepsilon \to 0. \end{aligned} \tag{37}$$

Proof. The estimates in (27), together with compactness results for stochastic two-scale convergence, see [74], ensure that for every 'typical' realization $\widetilde{\omega} \in \Omega$ there exist $b_e, c_e, c \in L^2(0,T;H^1(G))$ and B_e^1, C_e^1, $\overline{C}^1 \in L^2(G_T;L^2_{\text{pot}}(\Omega))$, such that $\nabla b_e^\varepsilon \rightharpoonup \nabla b_e + B_e^1$, $\nabla c_e^\varepsilon \rightharpoonup \nabla c_e + C_e^1$, and $\nabla \overline{c}^\varepsilon \rightharpoonup \nabla c + \overline{C}^1$ stochastically two-scale. Estimates (22) and (27) and the compactness of the embedding of $H^1(G)$ in $L^2(G)$, together with the Kolmogorov compactness theorem, see e.g. [16, 53], yield the strong convergence $b_e^\varepsilon \to b_e$, $c_e^\varepsilon \to c_e$ and $\overline{c}^\varepsilon \to c$ in $L^2(G_T)$ for $\mathcal{P}$-a.a. realizations $\widetilde{\omega} \in \Omega$. Since $G_{e,T}^\varepsilon \cap \widetilde{G}_{ef,T}^\varepsilon \neq \emptyset$, $c_e^\varepsilon(t,x) = \overline{c}^\varepsilon(t,x)$ for a.a. $(t,x) \in G_{e,T}^\varepsilon \cap \widetilde{G}_{ef,T}^\varepsilon$, and c_e and c are independent of $\omega \in \Omega$, we obtain that $c_e(t,x) = c(t,x)$ for a.a. $(t,x) \in G_T$ and $\mathcal{P}$-a.s in Ω.

From the estimates for $c^\varepsilon = c_e^\varepsilon \chi_{G_e^\varepsilon} + c_f^\varepsilon \chi_{G_f^\varepsilon}$ in (22) we obtain that there exists $C^1 \in L^2(G_T;L_{\text{pot},\Gamma}(\Omega))$ such that $\nabla c^\varepsilon \rightharpoonup \nabla c + C^1$ stochastically two-scale. □

6. Derivation of macroscopic equations for flow velocity and elastic deformations.

To show the convergence of boundary terms we shall prove the relation between convergence with respect to $\mathcal{P}$ in G and Palm measure $\boldsymbol{\mu}$ on the oscillating surfaces Γ^ε.

Definition 6.1. [23] The Palm measure of the random stationary measure μ_ω is the measure $\boldsymbol{\mu}$ on $(\Omega, \mathcal{F})$ defined as

$$\boldsymbol{\mu}(F) = \int_\Omega \int_{\mathbb{R}^3} \chi_{[0,1)^3}(x) \chi_F(\mathcal{T}_x \omega)\, \mathrm{d}\mu_\omega(x)\, \mathrm{d}\mathcal{P}(\omega) \quad \text{ for } F \in \mathcal{F}.$$

Lemma 6.2. *For $u \in H^1(\Omega, \mathcal{P})$ we have that $u \in L^2(\Omega, \boldsymbol{\mu})$, where $\boldsymbol{\mu}$ is the Palm measure of the random stationary measure μ_ω of surfaces $\Gamma(\omega)$ for realizations $\omega \in \Omega$, and the embedding is continuous.*

Proof. Consider $u \in H^1(\Omega, \mathcal{P})$ and a random stationary measure μ_ω given by $\mathrm{d}\mu_\omega(x) = \mathbf{1}_{\Gamma(\omega)} \mathrm{d}\sigma(x)$, where $\mathrm{d}\sigma(x)$ is the standard surface measure. By $\boldsymbol{\mu}$ we denote the Palm measure of the random stationary measure μ_ω. Let $\mathcal{Q}_\rho$ be the ball in $\mathbb{R}^3$ of radius ρ centered at the origin. Since $u \in H^1(\Omega, \mathcal{P})$, then a.s. $u(\mathcal{T}_x \omega) \in H^1_{\mathrm{loc}}(\mathbb{R}^3)$. Under our assumptions by the trace theorem there exist $\delta > 0$ and $C > 0$ such that

$$\int_{\Gamma(\omega)\cap \mathcal{Q}_\rho} |u(\mathcal{T}_x\omega)|^2 \mathrm{d}\sigma(x) \leqslant C \int_{\mathcal{Q}_{\rho+\delta}} |u(\mathcal{T}_x\omega)|^2 \mathrm{d}x + C \int_{\mathcal{Q}_{\rho+\delta}} |\nabla u(\mathcal{T}_x\omega)|^2 \mathrm{d}x \tag{38}$$

$\mathcal{P}$-a.s. in Ω. We divide the left- and the right-hand sides of this relation by ρ^3 and pass to the limit, as $\rho \to \infty$. By the Birkhoff theorem we obtain

$$\int_\Omega |u(\omega)|^2\, \mathrm{d}\boldsymbol{\mu} \leqslant C \left[\int_\Omega |u(\omega)|^2\, \mathrm{d}\mathcal{P} + \int_\Omega |\nabla_\omega u(\omega)|^2\, \mathrm{d}\mathcal{P} \right].$$

This yields the desired statement. □

Proof of Theorem 3.1. To derive macroscopic equations for the system of poro-elastic and Stokes equations, first we consider as test functions in (5) the following functions

- $\phi(t,x) = \varepsilon \phi_1(t,x) \phi_2(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})$, $\phi_1 \in C^1_0(G_T)$, $\phi_2 \in C^1_\mathcal{T}(\Omega)^3$
- $\psi(t,x) = \varepsilon \psi_1(t,x) \psi_2(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})$, $\psi_1 \in C^1_0(G_T)$, $\psi_2 \in C^1_\mathcal{T}(\Omega)$, $\eta_1 \in C^1_0(G_T)$
- $\eta(t,x) = \varepsilon \eta_1(t,x) \eta_2(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})$, $\eta_2 \in C^1_\mathcal{T}(\Omega)^3$, and $\phi_1(t,x) \Pi_\tau \phi_2(\mathcal{T}_{\widetilde{x}}\widetilde{\omega}) = \eta_1(t,x) \Pi_\tau \eta_2(\mathcal{T}_{\widetilde{x}}\widetilde{\omega})$

  for $(t,x) \in G_T$, $\widetilde{x} \in \Gamma(\widetilde{\omega})$, and $\mathcal{P}$-a.a. realizations $\widetilde{\omega} \in \Omega$. To apply stochastic two-scale convergence of u^ε_e, p^ε_e, and $\partial_t u^\varepsilon_f$, we rewrite the boundary integrals over Γ^ε in the weak formulation (5) as volume integrals

$$\begin{aligned}
&\langle \rho_e \partial_t^2 u^\varepsilon_e, \phi \chi_{G^\varepsilon_e} \rangle_{G_T} + \langle \mathbf{E}^\varepsilon(b^\varepsilon_e) \mathbf{e}(u^\varepsilon_e), \mathbf{e}(\phi) \chi_{G^\varepsilon_e} \rangle_{G_T} + \langle \nabla p^\varepsilon_e, \phi\, \chi_{G^\varepsilon_e} \rangle_{G_T} + \langle \rho_p \partial_t p^\varepsilon_e, \psi\, \chi_{G^\varepsilon_e} \rangle_{G_T} \\
&\quad + \langle K^\varepsilon_p \nabla p^\varepsilon_e - \partial_t u^\varepsilon_e, \nabla \psi \chi_{G^\varepsilon_e} \rangle_{G_T} - \langle \partial_t u^\varepsilon_f, \nabla \psi \chi_{G^\varepsilon_f} \rangle_{G_T} + \langle \nabla p^\varepsilon_e, \eta\, \chi_{G^\varepsilon_f} \rangle_{G_T} + \langle p^\varepsilon_e, \operatorname{div}\, \eta\, \chi_{G^\varepsilon_f} \rangle_{G_T} \\
&\quad + \langle \rho_f \partial_t^2 u^\varepsilon_f, \eta\, \chi_{G^\varepsilon_f} \rangle_{G_T} + \mu\, \varepsilon^2 \langle \mathbf{e}(\partial_t u^\varepsilon_f), \mathbf{e}(\eta)\, \chi_{G^\varepsilon_f} \rangle_{G_T} - \langle p^\varepsilon_f, \operatorname{div}\, \eta\, \chi_{G^\varepsilon_f} \rangle_{G_T} \\
&\quad = \langle F_u, \phi \rangle_{(\partial G)_T} + \langle F_p, \psi \rangle_{(\partial G)_T}.
\end{aligned} \tag{39}$$

Here we have used the relation $\operatorname{div} \partial_t u^\varepsilon_f = 0$ in $G^\varepsilon_{f,T}$ and the fact that $\chi_{G^\varepsilon_j}(x,\omega) = \chi_{\Omega_j}(\mathcal{T}_{x/\varepsilon}\omega)$ $\mathcal{P}$-a.s. in Ω, where $j = e, f$. Using the convergence results in lemma 5.2 and passing to the limit $\varepsilon \to 0$ we obtain

$$\langle \widetilde{\mathbf{E}}(\omega, b_{e,3})(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}), \phi_1 \mathbf{e}_\omega(\phi_2)\, \chi_{\Omega_e}\rangle_{G_T\times\Omega} + \langle \widetilde{K}_p(\omega)(\nabla p_e + P^1_e) - \partial_t u_e, \psi_1 \nabla_\omega \psi_2\, \chi_{\Omega_e}\rangle_{G_T\times\Omega}$$
$$- \langle \partial_t u_f, \psi_1 \nabla_\omega \psi_2 \chi_{\Omega_f}\rangle_{G_T\times\Omega} + \langle p_e, \eta_1 \mathrm{div}_\omega \eta_2\, \chi_{\Omega_f}\rangle_{G_T\times\Omega} - \langle p_f, \eta_1 \mathrm{div}_\omega \eta_2\, \chi_{\Omega_f}\rangle_{G_T\times\Omega} = 0. \tag{40}$$

Letting first $\psi_1 \equiv 0$ and $\eta_1 \equiv 0$ and then $\phi_1 \equiv 0$ and $\eta_1 \equiv 0$ we obtain the equations for the correctors U^1_e and P^1_e, i.e.

$$\left\langle \widetilde{\mathbf{E}}(\omega, b_{e,3})(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}})\chi_{\Omega_e}, \phi_1\, \mathbf{e}_\omega(\phi_2)\right\rangle_{G_T\times\Omega} = 0, \tag{41}$$

and

$$\left\langle \left(\widetilde{K}_p(\omega)(\nabla p_e + P^1_e) - \partial_t u_e\right)\chi_{\Omega_e} - \partial_t u_f\, \chi_{\Omega_f}, \psi_1\, \nabla_\omega \psi_2\right\rangle_{G_T\times\Omega} = 0. \tag{42}$$

From (40) considering $\phi_1 \equiv 0$ and $\psi_1 \equiv 0$ also yields

$$p_f\, \chi_{\Omega_f} = p_e\, \chi_{\Omega_f} \quad \text{in} \quad G_T \times \Omega.$$

Next, choosing in (5) test functions of the form $(\phi(t,x), \psi(t,x), \eta(t,x,x/\varepsilon))$, where

- $\phi \in C^\infty(\overline{G}_T)^3$ and $\psi \in C^\infty(\overline{G}_T)$,
- $\eta(t,x,x/\varepsilon) = \eta_1(t,x)\,\eta_2(\mathcal{T}_{x/\varepsilon}\omega)$, where $\eta_1 \in C^\infty(\overline{G}_T)$, $\eta_2 \in C^1_{\mathcal{T}}(\Omega)^3$, with $\mathrm{div}_\omega \eta_2 = 0$ for $\mathcal{P}$-a.a. $\omega \in \Omega$, and $\eta_1(t,x)\,\Pi_\tau \eta_2(\mathcal{T}_{\widetilde{x}}\omega) = \Pi_\tau \phi(t,x)$ for $(t,x) \in G_T$, $\widetilde{x} \in \Gamma(\omega)$, and $\mathcal{P}$-a.s. in Ω,

we obtain

$$\begin{aligned}
&\langle \rho_e \partial^2_t u^\varepsilon_e, \phi\, \chi_{G^\varepsilon_e}\rangle_{G_T} + \langle \mathbf{E}^\varepsilon(b^\varepsilon_e)\mathbf{e}(u^\varepsilon_e), \mathbf{e}(\phi)\, \chi_{G^\varepsilon_e}\rangle_{G_T} + \langle \nabla p^\varepsilon_e, \phi\, \chi_{G^\varepsilon_e}\rangle_{G_T}\\
&\quad + \langle \rho_p \partial_t p^\varepsilon_e, \psi\, \chi_{G^\varepsilon_e}\rangle_{G_T} + \langle K^\varepsilon_p \nabla p^\varepsilon_e - \partial_t u^\varepsilon_e, \nabla\psi\, \chi_{G^\varepsilon_e}\rangle_{G_T} - \langle \partial_t u^\varepsilon_f, \nabla\psi\, \chi_{G^\varepsilon_f}\rangle_{G_T}\\
&\quad + \langle \nabla p^\varepsilon_e, \eta_1\, \eta_2\rangle_{G_T} - \langle \nabla p^\varepsilon_e, \eta_1\, \eta_2\, \chi_{G^\varepsilon_e}\rangle_{G_T} + \langle p^\varepsilon_e, \mathrm{div}_x \eta_1\, \eta_2\, \chi_{G^\varepsilon_f}\rangle_{G_T}\\
&\quad + \langle \rho_f \partial^2_t u^\varepsilon_f, \eta_1\, \eta_2\, \chi_{G^\varepsilon_f}\rangle_{G_T} + \mu\, \varepsilon^2 \langle \mathbf{e}(\partial_t u^\varepsilon_f), [\mathbf{e}(\eta_1)\eta_2 + \varepsilon^{-1}\eta_1 \mathbf{e}_\omega(\eta_2)]\, \chi_{G^\varepsilon_f}\rangle_{G_T}\\
&\quad - \langle p^\varepsilon_f, \mathrm{div}_x \eta_1\, \eta_2\, \chi_{G^\varepsilon_f}\rangle_{G_T} = \langle F_u, \phi\rangle_{(\partial G)_T} + \langle F_p, \psi\rangle_{(\partial G)_T}.
\end{aligned} \tag{43}$$

Letting $\varepsilon \to 0$ and using the stochastic two-scale and strong convergences of u^ε_e and p^ε_e, the strong convergence of b^ε_e, and the stochastic two-scale convergence of $\partial_t u^\varepsilon_f$ we obtain

$$\begin{aligned}
&\langle \rho_e \partial^2_t u_e, \phi\, \chi_{\Omega_e}\rangle_{G_T,\Omega} + \langle \widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right), \mathbf{e}(\phi)\, \chi_{\Omega_e}\rangle_{G_T,\Omega} + \langle \nabla p_e + P^1_e, \phi\, \chi_{\Omega_e}\rangle_{G_T,\Omega}\\
&\quad + \langle \rho_p \partial_t p_e, \psi\, \chi_{\Omega_e}\rangle_{G_T,\Omega} + \langle \widetilde{K}_p(\omega)(\nabla p_e + P^1_e) - \partial_t u_e, \nabla\psi\, \chi_{\Omega_e}\rangle_{G_T,\Omega} - \langle \partial_t u_f, \nabla\psi\, \chi_{\Omega_f}\rangle_{G_T,\Omega}\\
&\quad + \langle \nabla p_e, \eta_1\eta_2\, \chi_{\Omega_f}\rangle_{G_T,\Omega} + \langle P^1_e, \eta_1\eta_2\rangle_{G_T\times\Omega} - \langle P^1_e, \eta_1\eta_2\, \chi_{\Omega_e}\rangle_{G_T,\Omega}\\
&\quad + \langle \rho_f \partial^2_t u_f, \eta_1\eta_2\, \chi_{\Omega_f}\rangle_{G_T,\Omega} + \mu\langle \mathbf{e}_\omega(\partial_t u_f), \eta_1 \mathbf{e}_\omega(\eta_2)\, \chi_{\Omega_f}\rangle_{G_T,\Omega} = \langle F_u, \phi\rangle_{(\partial G)_T} + \langle F_p, \psi\rangle_{(\partial G)_T}.
\end{aligned} \tag{44}$$

Here we used the fact that $\chi_{\Omega_f}\, p_f = \chi_{\Omega_f}\, p_e$ in $G_T \times \Omega$. Since $P^1_e \in L^2(G_T; L^2_{\mathrm{pot}}(\Omega))$ and $\eta_1 \in C(\overline{G}_T)$, $\eta_2 \in L^2_{\mathrm{sol}}(\Omega)$ we obtain that

$$\langle P^1_e, \eta_1\, \eta_2\rangle_{G_T,\Omega} = 0.$$

The stochastic two-scale convergence of $\partial_t u_f^\varepsilon$ and the fact that $\partial_t u_f^\varepsilon$ is divergence-free in G_T (we identify here $\partial_t u_f^\varepsilon$ with its extension constructed in lemma 4.4) imply

$$
\begin{aligned}
0 = \lim_{\varepsilon\to 0}\langle \operatorname{div}\, \partial_t u_f^\varepsilon, \varepsilon\eta(t,x,x/\varepsilon)\rangle_{G_T} &= -\lim_{\varepsilon\to 0}\langle \partial_t u_f^\varepsilon, \varepsilon\nabla_x\eta + \nabla_\omega \eta\rangle_{G_T} \\
&= -\langle \partial_t u_f, \nabla_\omega\eta\rangle_{G_T\times\Omega} = \langle \operatorname{div}_\omega \partial_t u_f, \eta\,\rangle_{G_T\times\Omega}.
\end{aligned}
$$

Thus $\operatorname{div}_\omega \partial_t u_f = 0$ a.e. in G_T and $\mathcal{P}$-a.s. in Ω.

Choosing $\phi \equiv 0$ and $\psi \equiv 0$, and taking $\eta = \eta_1\eta_2$, where $\eta_1 \in C^1_0(G_T)$ and $\eta_2 \in C^1_{\mathcal{T}}(\Omega)^3$, with $\operatorname{div}_\omega \eta_2 = 0$ and $\Pi_\tau \eta_2(\mathcal{T}_x\omega) = 0$ on $\Gamma(\omega)$ $\mathcal{P}$-a.s. in Ω, we conclude that $\partial_t u_f$ is a solution to problem (14). Taking $\eta = \eta_1\eta_2$, with $\eta_2 = $ const and $\eta_1 \in C^1_0(0,T; C^1(\overline{G}))^3$ as a test function in (14) yields

$$
\langle P_e^1, \eta_1 \chi_{\Omega_e}\rangle_{G_T,\Omega} = \langle \rho_f \partial_t^2 u_f + \nabla p_e, \eta_1\chi_{\Omega_f}\rangle_{G_T,\Omega}. \tag{45}
$$

Next we have to determine the boundary conditions for tangential components of $\partial_t u_f$ on $\Gamma(\omega)$ for $\mathcal{P}$-a.a. $\omega \in \Omega$. From *a priori* estimates for $\partial_t u_e^\varepsilon$ and $\partial_t u_f^\varepsilon$ we have that

$$
\begin{aligned}
\varepsilon \|\partial_t u_e^\varepsilon\|^2_{L^2(\Gamma_T^\varepsilon)} &\leqslant C_1\left(\|\partial_t u_e^\varepsilon\|^2_{L^2(G^\varepsilon_{e,T})} + \varepsilon^2\|\nabla \partial_t u_e^\varepsilon\|^2_{L^2(G^\varepsilon_{e,T})}\right) \leqslant C_2,\\
\varepsilon \|\partial_t u_f^\varepsilon\|^2_{L^2(\Gamma_T^\varepsilon)} &\leqslant C_3\left(\|\partial_t u_f^\varepsilon\|^2_{L^2(G^\varepsilon_{f,T})} + \varepsilon^2\|\nabla \partial_t u_f^\varepsilon\|^2_{L^2(G^\varepsilon_{f,T})}\right) \leqslant C_4,
\end{aligned}
$$

where the constants C_j, with $j = 1, 2, 3, 4$, are independent of ε. Thus using lemmas 6.2 and 8.1 and the fact that $\partial_t u_f \in L^2(G_T; H^1(\Omega))$ and $\partial_t u_e \in L^2(0,T; H^1(G))$ we obtain

$$
\begin{aligned}
\int_{G_T}\int_\Omega \Pi_\tau \partial_t u_f(t,x,\omega)\psi_1(t,x)\psi_2(\omega)\, \mathrm{d}\boldsymbol{\mu}\, \mathrm{d}x\mathrm{d}t &= \lim_{\varepsilon\to 0}\varepsilon\int_{\Gamma_T^\varepsilon} \Pi_\tau \partial_t u_f^\varepsilon(t,x)\psi_1(t,x)\psi_2(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})\, \mathrm{d}\sigma^\varepsilon\, \mathrm{d}t\\
= \lim_{\varepsilon\to 0}\varepsilon\int_{\Gamma_T^\varepsilon} \Pi_\tau \partial_t u_e^\varepsilon(t,x)\psi_1(t,x)\psi_2(\mathcal{T}_{x/\varepsilon}\widetilde{\omega})\, d\sigma^\varepsilon\, \mathrm{d}t &= \int_{G_T}\int_\Omega \Pi_\tau \partial_t u_e(t,x)\psi_1(t,x)\psi_2(\omega)\, d\boldsymbol{\mu}\, \mathrm{d}x\mathrm{d}t
\end{aligned}
$$

for $\psi_1 \in C^1_0(G_T)$, $\psi_2 \in C^1(\Omega)$ and typical realizations $\widetilde{\omega} \in \Omega$. Thus for each typical realization $\widetilde{\omega} \in \Omega$ we have

$$
\Pi_\tau \partial_t u_f = \Pi_\tau \partial_t u_e \quad \text{on } G_T \times \Gamma(\widetilde{\omega}).
$$

Considering first $\phi \in C_0^\infty(G_T)^3$, $\psi \in C_0^\infty(G_T)$, and then $\phi \in C^\infty(\overline{G}_T)^3$, $\psi \in C^\infty(\overline{G}_T)$, and using equality (45) together with

$$
U_e^1 = \sum_{k,l=1}^3 \mathbf{e}(u_e(t,x))_{kl} W_e^{kl}(t,x,\omega), \tag{46}
$$

where W_e^{kl} are solutions of the first equations in (9), yield the macroscopic equations for u_e:

$$
\begin{aligned}
\vartheta_e\rho_e\partial_t^2 u_e - \operatorname{div}\left(\mathbf{E}^{\hom}(b_{e,3})\mathbf{e}(u_e)\right) + \nabla p_e + \int_\Omega \rho_f \partial_t^2 u_f\, \chi_{\Omega_f} \mathrm{d}\mathcal{P}(\omega) &= 0 \qquad \text{in } G_T,\\
\mathbf{E}^{\hom}(b_{e,3})\mathbf{e}(u_e)\, n &= F_u \qquad \text{on } (\partial G)_T,
\end{aligned} \tag{47}
$$

where $\mathbf{E}^{\hom}$ is defined by (8), as well as the equation

$$\vartheta_e \rho_p \partial_t p_e - \operatorname{div}\left(\int_\Omega \left[\left(\widetilde{K}_p(\omega)(\nabla p_e + P_e^1) - \partial_t u_e\right)\chi_{\Omega_e} - \partial_t u_f\, \chi_{\Omega_f}\right] \mathrm{d}\mathcal{P}(\omega)\right) = 0 \qquad \text{in } G_T,$$
$$\left(\int_\Omega \left[\left(\widetilde{K}_p(\omega)(\nabla p_e + P_e^1) - \partial_t u_e\right)\chi_{\Omega_e} - \partial_t u_f\, \chi_{\Omega_f}\right] \mathrm{d}\mathcal{P}(\omega)\right)\cdot n = F_p \qquad \text{on } (\partial G)_T, \tag{48}$$

together with problem (42) for P_e^1. The structure of the problem (42) suggests that P_e^1 should be of the form

$$P_e^1(t,x,\omega) = \sum_{k=1}^3 \frac{\partial p_e}{\partial x_k}(t,x)\, W_p^k(\omega) + \sum_{k=1}^3 \partial_t u_e^k(t,x)\, W_u^k(\omega) + Q_f(\omega, \partial_t u_f), \tag{49}$$

where W_p^k and W_u^k are solutions of cell problems (9), and Q_f is a solution of problem (11). Substituting the right-hand side of (49) for P_e^1 in (48) we obtain the macroscopic equations for p_e in (12), where K_p^{hom} and K_u are defined in (8). □

7. Strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)$, ∇p_e^ε, and $\partial_t u_f^\varepsilon$.

Due to the presence of nonlinear functions depending on $\mathbf{e}(u_e^\varepsilon)$ and $\partial_t u_f^\varepsilon$ in equations for b_e^ε, c_e^ε, and c_f^ε, in order to derive the macroscopic equations for b_e and c we have to show that $\mathbf{e}(u_e^\varepsilon)$ and $\partial_t u_f^\varepsilon$ converge stochastically two-scale strongly.

Lemma 7.1. *For a subsequences of $\{u_e^\varepsilon\}$, $\{p_e^\varepsilon\}$ and $\{\partial_t u_f^\varepsilon\}$ as in lemma 5.2 (denoted again by $\{u_e^\varepsilon\}$, $\{p_e^\varepsilon\}$, and $\{\partial_t u_f^\varepsilon\}$) we have*

$$\begin{aligned}
&\chi_{G_e^\varepsilon}\mathbf{e}(u_e^\varepsilon) \to \chi_{\Omega_e}\,(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}) && \textit{strongly stochastic two} - \textit{scale},\\
&\chi_{G_e^\varepsilon}\nabla p_e^\varepsilon \to \chi_{\Omega_e}\,(\nabla p_e + P_e^1) && \textit{strongly stochastic two} - \textit{scale},\\
&\chi_{G_f^\varepsilon}\partial_t u_f^\varepsilon \to \chi_{\Omega_f}\partial_t u_f && \textit{strongly stochastic two} - \textit{scale}.
\end{aligned} \tag{50}$$

Proof. Similar to the periodic case [57], to show the strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)$, p_e^ε, and $\partial_t u_f^\varepsilon$ we prove the convergence of the energy related to the equations for u_e^ε, p_e^ε, and $\partial_t u_f^\varepsilon$. We consider a monotone decreasing function $\varrho : \mathbb{R}_+ \to \mathbb{R}_+$, e.g. $\varrho(t) = \mathrm{e}^{-\gamma t}$ for $t \in \mathbb{R}_+$, and define the energy functional for the microscopic problem (3) and (4) as

$$\begin{aligned}
\mathcal{E}^\varepsilon(u_e^\varepsilon, p_e^\varepsilon, \partial_t u_f^\varepsilon) = {}& \frac{1}{2}\rho_e \|\partial_t u_e^\varepsilon(s)\varrho(s)\|^2_{L^2(G_e^\varepsilon)} - \rho_e \langle \varrho'(\cdot)\varrho(\cdot)\, \partial_t u_e^\varepsilon, \partial_t u_e^\varepsilon\rangle_{G^\varepsilon_{e,s}}\\
&+ \frac{1}{2}\langle \mathbf{E}^\varepsilon(b^\varepsilon_{e,3})\mathbf{e}(u_e^\varepsilon)(s), \mathbf{e}(u_e^\varepsilon)(s)\varrho^2(s)\rangle_{G_e^\varepsilon}\\
&- \frac{1}{2}\left\langle \left(2\varrho'(\cdot)\varrho(\cdot)\mathbf{E}^\varepsilon(b^\varepsilon_{e,3}) + \varrho^2 \partial_t \mathbf{E}^\varepsilon(b^\varepsilon_{e,3})\right)\mathbf{e}(u_e^\varepsilon), \mathbf{e}(u_e^\varepsilon)\right\rangle_{G^\varepsilon_{e,s}}\\
&+ \frac{1}{2}\rho_p \|p_e^\varepsilon(s)\varrho(s)\|^2_{L^2(G_e^\varepsilon)} - \rho_p\langle \varrho'(\cdot)\varrho(\cdot), |p_e^\varepsilon|^2\rangle_{G^\varepsilon_{e,s}} + \langle K_p^\varepsilon \nabla p_e^\varepsilon \varrho(\cdot), \nabla p_e^\varepsilon \varrho(\cdot)\rangle_{G^\varepsilon_{e,s}}\\
&+ \frac{1}{2}\rho_f \|\partial_t u_f^\varepsilon(s)\varrho(s)\|^2_{L^2(G_f^\varepsilon)} - \rho_f\langle \varrho'(\cdot)\varrho(\cdot)\, \partial_t u_f^\varepsilon, \partial_t u_f^\varepsilon\rangle_{G^\varepsilon_{f,s}}\\
&+ \mu\|\varepsilon\varrho(\cdot)\mathbf{e}(\partial_t u_f^\varepsilon)\|^2_{L^2(G^\varepsilon_{f,s})}
\end{aligned} \tag{51}$$

for $s \in (0, T)$ and $\mathcal{P}$-a.a. $\omega \in \Omega$. Considering $\partial_t u_e^\varepsilon\, \varrho^2$, $p_e^\varepsilon\, \varrho^2$, and $\partial_t u_f^\varepsilon\, \varrho^2$ as test functions in (5) yields the equality

$$\begin{aligned}
\mathcal{E}^\varepsilon(u_e^\varepsilon, p_e^\varepsilon, \partial_t u_f^\varepsilon) &= \frac{1}{2}\rho_e \|\partial_t u_e^\varepsilon(0)\|^2_{L^2(G_e^\varepsilon)} + \frac{1}{2}\langle \mathbf{E}^\varepsilon(b_{e,3}^\varepsilon)\mathbf{e}(u_e^\varepsilon)(0), \mathbf{e}(u_e^\varepsilon)(0)\rangle_{G_e^\varepsilon} \\
&\quad + \frac{1}{2}\rho_f \|\partial_t u_f^\varepsilon(0)\|^2_{L^2(G_f^\varepsilon)} \\
&\quad + \frac{1}{2}\rho_p \|p_e^\varepsilon(0)\|^2_{L^2(G_e^\varepsilon)} + \langle F_u, \partial_t u_e^\varepsilon\, \varrho^2\rangle_{(\partial G)_T} + \langle F_p, p_e^\varepsilon\, \varrho^2\rangle_{(\partial G)_T}.
\end{aligned} \tag{52}$$

Due to assumptions on $\widetilde{\mathbf{E}}$ and $\partial_t \widetilde{\mathbf{E}}$ there exists such $\gamma > 0$ that

$$\Big(2\gamma \widetilde{\mathbf{E}}_1(\omega, \mathcal{K}(\eta)) - \partial_t \widetilde{\mathbf{E}}_1(\omega, \mathcal{K}(\eta))\Big) A \cdot A \geqslant 0 \quad \text{for all symmetric matrices } A \text{ and } \eta \in \mathbb{R},$$
$$\text{and } \mathcal{P} - \text{a.a. } \omega \in \Omega.$$

The weak stochastic two-scale convergence of $(\mathbf{E}^\varepsilon(b_{e,3}^\varepsilon))^{1/2}\mathbf{e}(u_e^\varepsilon)$, $(2\gamma \mathbf{E}^\varepsilon(b_{e,3}^\varepsilon) - \partial_t \mathbf{E}^\varepsilon\ (b_{e,3}^\varepsilon))^{1/2}\mathbf{e}(u_e^\varepsilon)$, and $(K_p^\varepsilon)^{1/2}\nabla p_e^\varepsilon$, as $\varepsilon \to 0$, and the lower-semicontinuity of the norm ensure

$$\begin{aligned}
&\frac{\rho_e}{2}\|\partial_t u_e(s)\varrho(s)\, \chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} + \gamma\rho_e \|\partial_t u_e\, \varrho \chi_{\Omega_e}\|^2_{L^2(G_s\times\Omega)} \\
&\quad + \frac{1}{2}\langle \widetilde{\mathbf{E}}(\omega, b_{e,3})\varrho^2(s) \left(\mathbf{e}(u_e(s)) + U^1_{e,\mathrm{sym}}(s)\right)\ \chi_{\Omega_e}, \mathbf{e}(u_e(s)) + U^1_{e,\mathrm{sym}}(s)\rangle_{G,\Omega} \\
&\quad + \frac{1}{2}\langle \varrho^2 \left(2\gamma\, \widetilde{\mathbf{E}}(\omega, b_{e,3}) - \partial_t \widetilde{\mathbf{E}}(\omega, b_{e,3})\right) (\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}})\chi_{\Omega_e}, \mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\rangle_{G_s,\Omega} \\
&\quad + \frac{\rho_p}{2}\|p_e(s)\varrho(s)\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} + \gamma\rho_p\|p_e \varrho\chi_{\Omega_e}\|^2_{L^2(G_s\times\Omega)} \\
&\quad + \langle \varrho^2 \widetilde{K}_p(\omega)(\nabla p_e + P_e^1)\chi_{\Omega_e}, \nabla p_e + P_e^1\rangle_{G_s,\Omega} \\
&\quad + \frac{\rho_f}{2}\|\partial_t u_f(s)\varrho(s)\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} + \gamma\rho_f\|\partial_t u_f\ \varrho\chi_{\Omega_f}\|^2_{L^2(G_s\times\Omega)} \\
&\quad + \mu\|\mathbf{e}_\omega(\partial_t u_f)\ \varrho\, \chi_{\Omega_f}\|^2_{L^2(G_s\times\Omega)} \\
&\quad \leqslant \liminf_{\varepsilon\to 0} \mathcal{E}^\varepsilon(u_e^\varepsilon, p_e^\varepsilon, \partial_t u_f^\varepsilon) \leqslant \limsup_{\varepsilon\to 0} \mathcal{E}^\varepsilon(u_e^\varepsilon, p_e^\varepsilon, \partial_t u_f^\varepsilon) = \frac{\rho_e}{2}\|\partial_t u_e(0)\, \chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} \\
&\quad + \frac{1}{2}\Big\langle \widetilde{\mathbf{E}}(\omega, b_{e,3}) \left(\mathbf{e}(u_e)(0) + U^0_{e,\mathrm{sym}}\right)\ \chi_{\Omega_e}, \mathbf{e}(u_e)(0) + U^0_{e,\mathrm{sym}}\Big\rangle_{G,\Omega} + \frac{\rho_p}{2}\|p_e(0)\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} \\
&\quad + \frac{\rho_f}{2}\|\partial_t u_f(0)\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} + \langle F_u, \partial_t u_e\, \varrho^2\rangle_{(\partial G)_s,\Omega} + \langle F_p, p_e\, \varrho^2\rangle_{(\partial G)_s,\Omega}.
\end{aligned} \tag{53}$$

Here we also used the strong convergence of b_e^ε and the stochastic two-scale convergence of ∇p_e^ε, $\mathbf{e}(u_e^\varepsilon)$, $\partial_t u_f^\varepsilon$, and $\varepsilon\mathbf{e}(\partial_t u_f^\varepsilon)$. Considering the limit equations for u_e, U_e^1, p_e, P_e^1, and $\partial_t u_f$ and taking $(\partial_t u_e \varrho^2, p_e \varrho^2, \partial_t u_f \varrho^2)$ as a test function yield

$$
\begin{aligned}
&\frac{\rho_e}{2}\|\partial_t u_e(s)\varrho(s)\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} - \frac{\rho_e}{2}\|\partial_t u_e(0)\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} + \gamma\rho_e\|\partial_t u_e\,\varrho\,\chi_{\Omega_e}\|^2_{L^2(G_s\times\Omega)}\\
&\quad + \langle\widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right), \mathbf{e}(\partial_t u_e)\,\varrho^2\chi_{\Omega_e}\rangle_{G_s,\Omega} + \langle\nabla p_e + P^1_e, \partial_t u_e\chi_{\Omega_e}\rangle_{G_s,\Omega}\\
&\quad + \frac{\rho_p}{2}\|p_e(s)\varrho(s)\,\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} - \frac{\rho_p}{2}\|p_e(0)\,\chi_{\Omega_e}\|^2_{L^2(G\times\Omega)} + \gamma\rho_p\|p_e\,\varrho\,\chi_{\Omega_e}\|^2_{L^2(G_s\times\Omega)}\\
&\quad + \left\langle\left[\widetilde{K}_p(\omega)(\nabla p_e + P^1_e) - \partial_t u_e\right]\chi_{\Omega_e} - \partial_t u_f\,\chi_{\Omega_f}, \nabla p_e\,\varrho^2\right\rangle_{G_s,\Omega}\\
&\quad + \frac{\rho_f}{2}\|\partial_t u_f(s)\,\varrho(s)\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} - \frac{\rho_f}{2}\|\partial_t u_f(0)\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} + \gamma\rho_f\|\partial_t u_f\,\varrho\,\chi_{\Omega_f}\|^2_{L^2(G_s\times\Omega)}\\
&\quad + \mu\langle\mathbf{e}_\omega(\partial_t u_f), \mathbf{e}_\omega(\partial_t u_f)\varrho^2\chi_{\Omega_f}\rangle_{G_s,\Omega} + \langle\nabla p_e, \partial_t u_f\,\varrho^2\,\chi_{\Omega_f}\rangle_{G_s,\Omega} - \langle P^1_e\chi_{\Omega_e}, \partial_t u_f\varrho^2\rangle_{G_s,\Omega}\\
&\quad = \langle F_u, \partial_t u_e\,\varrho^2\rangle_{(\partial G)_s} + \langle F_p, p_e\,\varrho^2\rangle_{(\partial G)_s} \qquad (54)
\end{aligned}
$$

for $s \in (0, T)$. Taking P^1_e as a test function in the equation for P^1_e yields

$$
\langle P^1_e, \partial_t u_f\,\varrho^2\,\chi_{\Omega_f}\rangle_{G_s,\Omega} = \langle\widetilde{K}_p(\omega)(\nabla p_e + P^1_e) - \partial_t u_e, P^1_e\varrho^2\,\chi_{\Omega_e}\rangle_{G_s,\Omega}. \qquad (55)
$$

Since $P^1_e \in L^2(G_T; L^2_{\mathrm{pot}}(\Omega))$ and $\partial_t u_f \in L^2(G_T; L^2_{\mathrm{sol}}(\Omega))$ we obtain

$$
\langle P^1_e, \partial_t u_f\,\varrho^2\rangle_{G_s,\Omega} = 0 \quad \text{and} \quad \langle P^1_e, \partial_t u_f\,\varrho^2\,\chi_{\Omega_e}\rangle_{G_s,\Omega} = -\langle P^1_e, \partial_t u_f\,\varrho^2\,\chi_{\Omega_f}\rangle_{G_s,\Omega}.
$$

Considering equation (41) for the corrector U^1_e and taking $\partial_t U^1_e\,\varrho^2$ as a test function imply

$$
\begin{aligned}
&\langle\widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right), \mathbf{e}(\partial_t u_e)\,\varrho^2\,\chi_{\Omega_e}\rangle_{G_s,\Omega}\\
&\quad = \langle\widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right), (\mathbf{e}(\partial_t u_e) + \partial_t U^1_{e,\mathrm{sym}})\,\varrho^2\,\chi_{\Omega_e}\rangle_{G_s,\Omega}\\
&\quad = \frac{1}{2}\left\langle\widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e(s)) + U^1_{e,\mathrm{sym}}(s)\right)\varrho^2(s)\,\chi_{\Omega_e}, \mathbf{e}(u_e(s)) + U^1_{e,\mathrm{sym}}(s)\right\rangle_{G,\Omega}\\
&\quad - \frac{1}{2}\left\langle\widetilde{\mathbf{E}}(\omega, b_{e,3})\left(\mathbf{e}(u_e(0)) + U^1_{e,\mathrm{sym}}\right)\chi_{\Omega_e}, \mathbf{e}(u_e(0)) + U^1_{e,\mathrm{sym}}\right\rangle_{G,\Omega}\\
&\quad + \frac{1}{2}\left\langle\left(2\gamma\widetilde{\mathbf{E}}(\omega, b_{e,3}) - \partial_t\widetilde{\mathbf{E}}(\omega, b_{e,3})\right)\varrho^2\left(\mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right)\chi_{\Omega_e}, \mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}\right\rangle_{G_s,\Omega}.
\end{aligned}
\qquad (56)
$$

Thus we obtain that

$$
\mathcal{E}(u_e, p_e, \partial_t u_f) \leqslant \liminf_{\varepsilon\to 0}\mathcal{E}^\varepsilon(u^\varepsilon_e, p^\varepsilon_e, \partial_t u^\varepsilon_f) \leqslant \limsup_{\varepsilon\to 0}\mathcal{E}^\varepsilon(u^\varepsilon_e, p^\varepsilon_e, \partial_t u^\varepsilon_f) = \mathcal{E}(u_e, p_e, \partial_t u_f),
$$

and, hence the strong stochastic two-scale convergence stated in lemma. □

8. Derivation of macroscopic equations for b_e and c.

Using strong stochastic two-scale convergence of $\mathbf{e}(u^\varepsilon_e)$ and $\partial_t u^\varepsilon_f$ we derive macroscopic equations for concentrations of pectins b_e and calcium c. First we shall prove convergence of sequences defined on the boundaries of the random microstructure Γ^ε.

Lemma 8.1. *Consider the random measure μ_ω denoting the surface measure of $\Gamma(\omega)$ and define $d\mu^\varepsilon_\omega(x) = \varepsilon^3\, d\mu_\omega(x/\varepsilon)$.*

(a) *If $\|b^\varepsilon\|_{L^p(G^\varepsilon_{e,T})} + \|\nabla b^\varepsilon\|_{L^p(G^\varepsilon_{e,T})} \leqslant C$ and $b^\varepsilon \to b$ stochastic two-scale, $b \in L^p(0,T; W^{1,p}(G))$, with $p \in (1,\infty)$, then for any $\phi \in C^\infty(0,T; C_0^\infty(\mathbb{R}^3))$ and any $\psi \in C(\Omega)$ we have*

$$\lim_{\varepsilon\to 0}\int_{G_T} b^\varepsilon(t,x)\,\phi(t,x)\psi(\mathcal{T}_{x/\varepsilon}\omega)\mathrm{d}\mu^\varepsilon_\omega(x)\,\mathrm{d}t = \int_{G_T}\int_\Omega b(t,x)\phi(t,x)\psi(\omega)\mathrm{d}\boldsymbol{\mu}(\omega)\ \mathrm{d}x\mathrm{d}t \tag{57}$$

and

$$\int_{G_T}\int_\Omega |b|^p d\boldsymbol{\mu}(\omega)\,\mathrm{d}x\mathrm{d}t \leqslant C\int_{G_T}\int_\Omega |b|^p \mathrm{d}\mathcal{P}\,\mathrm{d}x\mathrm{d}t. \tag{58}$$

(b) *If $\|b^\varepsilon\|_{L^p(G^\varepsilon_{e,T})} + \varepsilon\|\nabla b^\varepsilon\|_{L^p(G^\varepsilon_{e,T})} \leqslant C$ and $b^\varepsilon \to b$ stochastic two-scale, $b \in L^p(G_T; W^{1,p}(\Omega,\mathrm{d}\mathcal{P}))$, with $p \in (1,\infty)$, then convergence (57) holds, and*

$$\int_{G_T}\int_\Omega |b|^p d\boldsymbol{\mu}(\omega)\,\mathrm{d}x\mathrm{d}t \leqslant C. \tag{59}$$

Proof. For $\mathcal{P}$-a.a. realizations $\omega \in \Omega$, using the assumptions on the geometry of G^ε_e and the trace inequality in each $G^\sigma_{e,j} = G^\sigma_{f,j}(\omega)\backslash G_{f,j}(\omega)$, see proof of lemma 4.4 for the definition of $G^\sigma_{f,j}(\omega)$, applying the scaling x/ε and summing up over j we obtain

$$\int_{G_T} |b^\varepsilon|^p \mathrm{d}\mu^\varepsilon_\omega(x)\,\mathrm{d}t = \varepsilon\int_{\Gamma^\varepsilon_{e,T}} |b^\varepsilon|^p \mathrm{d}\sigma^\varepsilon\,\mathrm{d}t \leqslant C_1\int_{G^\varepsilon_{e,T}} |b^\varepsilon|^p\,\mathrm{d}x\mathrm{d}t + C_2\varepsilon^p\int_{G^\varepsilon_{e,T}} |\nabla b^\varepsilon|^p\,\mathrm{d}x\mathrm{d}t \leqslant C. \tag{60}$$

Moreover, in the case (a) the limit function b does not depend on ω, its trace on $\Gamma^\varepsilon_{e,T}$ is well defined, and

$$\varepsilon^p\int_{G^\varepsilon_{e,T}} |\nabla b^\varepsilon - \nabla b|^p\,\mathrm{d}x\mathrm{d}t \xrightarrow[\varepsilon\to 0]{} 0. \tag{61}$$

Choosing $\phi(x,t) = \phi_1(t)\phi_2(x)$ we conclude that $\hat{b}^\varepsilon(x) = \int_0^T b^\varepsilon(x,t)\phi_1(t)\,\mathrm{d}t$ converges in $L^p(G)$ strongly to $\hat{b}(x) = \int_0^T b(x,t)\phi_1(t)\,\mathrm{d}t$, and

$$\int_G |\hat{b}^\varepsilon - \hat{b}|^p \mathrm{d}\mu^\varepsilon_\omega(x) \leqslant C_3\int_{G^\varepsilon_e} |\hat{b}^\varepsilon - \hat{b}|^p\,\mathrm{d}x + C_4\varepsilon^p\int_{G^\varepsilon_e} |\nabla\hat{b}^\varepsilon - \nabla\hat{b}|^p\,\mathrm{d}x \xrightarrow[\varepsilon\to 0]{} 0.$$

This yields (57).

In the case (b), for $b \in L^p(G_T; W^{1,p}(\Omega,\mathrm{d}\mathcal{P}))$ using the same arguments as in the proof of lemma 6.2 one can show that $b \in L^p(G_T; L^p(\Omega,\boldsymbol{\mu}))$. This yields (59).

To justify (57) we regularize measures μ_ω as follows. Let $k = k(x)$ be a non-negative symmetric $C_0^\infty(\mathbb{R}^d)$ function such that $\int_{\mathbb{R}^d} k(x)\,\mathrm{d}x = 1$, where here $d = 3$. We set

$$d\mu_{\omega,\delta}(x) = \rho^\delta(\mathcal{T}_x\omega)\mathrm{d}x \quad \text{with} \quad \rho^\delta(\omega) = \delta^{-d}\int_{\mathbb{R}^d} k\left(\frac{y}{\delta}\right)\mathrm{d}\mu_\omega(y).$$

It is easy to check that a.s. for any test functions $\phi \in C^\infty(0,T;C_0^\infty(\mathbb{R}^d))$ and $\psi \in C(\Omega)$ we have

$$\lim_{\delta\to 0}\lim_{\varepsilon\to 0}\sup\left|\int_{G_T} b^\varepsilon(t,x)\,\phi(t,x)\psi(\mathcal{T}_{x/\varepsilon}\omega)d\mu^\varepsilon_\omega(x)\,\mathrm{d}t - \int_{G_T} b^\varepsilon(t,x)\,\phi(t,x)\psi(\mathcal{T}_{x/\varepsilon}\omega)d\mu^\varepsilon_{\omega,\delta}(x)\,\mathrm{d}t\right| = 0.$$

The Palm measure of $d\mu_{\omega,\delta}(x)$ is $d\boldsymbol{\mu}_\delta(\omega) = \rho^\delta(\omega)d\mathcal{P}$. Since for each $\delta > 0$ the measure $\boldsymbol{\mu}_\delta$ is absolutely continuous with respect to $d\mathcal{P}$ and the density ρ^δ is bounded, the two-scale limit of b^ε with respect to $d\mu^\varepsilon_{\omega,\delta}$ is b that is

$$\lim_{\varepsilon\to 0}\int_{G_T} b^\varepsilon(t,x)\,\phi(t,x)\psi(\mathcal{T}_{x/\varepsilon}\omega)d\mu^\varepsilon_{\omega,\delta}(x)\,\mathrm{d}t = \int_{G_T}\int_\Omega b(t,x,\omega)\phi(t,x)\psi(\omega)d\boldsymbol{\mu}_\delta(\omega)dxdt.$$

By the trace theorem a.s.

$$\lim_{\varepsilon\to 0}\sup\|b^\varepsilon\|_{L^p(G_T,d\mu^\varepsilon_\omega)} \leqslant C.$$

Therefore, for a subsequence b^ε stochastically two-scale converge in $L^p(G_T, d\mu^\varepsilon_\omega)$ to some function $B \in L^p(G_T; L^p(\Omega, d\boldsymbol{\mu}))$. As was proved in [74], the measures $d\boldsymbol{\mu}_\delta$ converge weakly to the measure $d\boldsymbol{\mu}$. Using one more time the same arguments as in the proof of lemma 6.2 we obtain

$$\lim_{\delta\to 0}\int_{G_T}\int_\Omega b(x,t)\phi(t,x)\psi(\omega)\mathrm{d}\boldsymbol{\mu}_\delta(\omega)\ \mathrm{d}x\mathrm{d}t = \int_{G_T}\int_\Omega b(x,t)\phi(t,x)\psi(\omega)\mathrm{d}\boldsymbol{\mu}(\omega)\ \mathrm{d}x\mathrm{d}t.$$

Passing to the limit $\delta \to 0$ and combining the above relations, we conclude that

$$\int_{G_T}\int_\Omega b(x,t)\phi(t,x)\psi(\omega)\mathrm{d}\boldsymbol{\mu}(\omega)\ \mathrm{d}x\mathrm{d}t = \int_{G_T}\int_\Omega B(x,t)\phi(t,x)\psi(\omega)\mathrm{d}\boldsymbol{\mu}(\omega)\ \mathrm{d}x\mathrm{d}t.$$

In view of arbitrariness of ϕ and ψ this implies that $B = b$ in $L^p(G_T; L^p(\Omega, d\boldsymbol{\mu}))$. □

Using the convergence on the oscillating boundary Γ^ε proved in lemma 8.1 we can now derive macroscopic equations for b_e and c.

Proof of Theorem 3.2. We can rewrite the microscopic equation for b_e^ε as

$$\begin{aligned}&-\langle b_e^\varepsilon\,\chi_{G_e^\varepsilon}, \partial_t\varphi_1\rangle_{G_T} + \langle D_b^\varepsilon(b_{e,3}^\varepsilon)\nabla b_e^\varepsilon, \nabla\varphi_1\,\chi_{G_e^\varepsilon}\rangle_{G_T} = \langle b_{e0}\,\chi_{G_e^\varepsilon}, \varphi_1(0)\rangle_G \\ &+ \langle g_b(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)), \varphi_1\,\chi_{G_e^\varepsilon}\rangle_{G_T} + \varepsilon\langle R(b_e^\varepsilon), \varphi_1\rangle_{\Gamma_T^\varepsilon} + \langle F_b(b_e^\varepsilon), \varphi_1\rangle_{(\partial G)_T}\end{aligned} \tag{62}$$

for $\varphi_1 = \phi_1(t,x) + \varepsilon\phi_2(t,x)\phi_3(\mathcal{T}_{x/\varepsilon}\omega)$, where $\phi_1 \in C^\infty(\overline{G}_T)$, with $\phi_1(T,x) = 0$ for $x \in \overline{G}$, $\phi_2 \in C_0^\infty(G_T)$, and $\phi_3 \in C^1_{\mathcal{T}}(\Omega)$.

From the *a priori* estimates for b^ε and assumptions on R we have that

$$\varepsilon\int_0^T\int_{\Gamma^\varepsilon}|R(b_e^\varepsilon)|^2\mathrm{d}\sigma^\varepsilon\,\mathrm{d}t \leqslant C,$$

where the constant C is independent of ε. Thus considering the stochastic two-scale convergence we obtain that there exists $\widetilde{R} \in L^2(G_T \times \Omega,\ \mathrm{d}t \times\ \mathrm{d}x \times d\boldsymbol{\mu}(\omega))$

$$\lim_{\varepsilon\to 0}\varepsilon\int_0^T\int_{\Gamma^\varepsilon}R(b_e^\varepsilon)\,\varphi_1\,\mathrm{d}\sigma^\varepsilon\,\mathrm{d}t = \lim_{\varepsilon\to 0}\int_0^T\int_G R(b_e^\varepsilon)\,\varphi_1\,\mathrm{d}\mu^\varepsilon_\omega(x)\,\mathrm{d}t = \int_0^T\int_G\int_\Omega \widetilde{R}\varphi_1\,\mathrm{d}\boldsymbol{\mu}(\omega)\,\mathrm{d}x\mathrm{d}t,$$

where μ_ω is the random measure of $\Gamma(\omega)$. Using the assumptions on the geometry and on the function R together with the strong convergence of b_e^ε in $L^2(G_T)$ we have that

$$\varepsilon\int_0^T\int_{\Gamma^\varepsilon}|R(b_e^\varepsilon)-R(b_e)|^2\mathrm{d}\sigma^\varepsilon\,\mathrm{d}t\leqslant C\int_{G^\varepsilon_{e,T}}\left[|b_e^\varepsilon-b_e|^2+\varepsilon^2|\nabla(b_e^\varepsilon-b_e)|^2\right]\,\mathrm{d}x\mathrm{d}t\to 0\text{ as }\varepsilon\to 0.$$

Then using the strong convergence of b_e, the continuity of R and the convergence result in lemma 8.1 we obtain that $\widetilde{R}=R(b_e)$ $\mathcal{P}$-a.s. in $G_T\times\Omega$.

Taking the stochastic two-scale limit and using the strong convergence of b_e^ε and c_e^ε and the strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)$, shown in lemma 7.1, we obtain

$$\begin{aligned}&-\langle\vartheta_e b_e,\partial_t\phi_1\rangle_{G_T}+\langle D_b(b_{e,3})(\nabla b_e+B_e^1)\chi_{\Omega_e},\nabla\phi_1+\phi_2\nabla_\omega\phi_3\rangle_{G_T,\Omega}=\langle\vartheta_e b_{e0},\phi_1(0)\rangle_G\\&\quad+\langle g_b(c,b_e,\mathbf{e}(u_e)+U^1_{e,\mathrm{sym}})\chi_{\Omega_e},\phi_1\rangle_{G_T,\Omega}+\int_0^T\int_G\int_\Omega R(b_e)\,\phi_1\mathrm{d}\boldsymbol{\mu}(\omega)\,\mathrm{d}x\mathrm{d}t+\langle F_b(b_e),\phi_1\rangle_{(\partial G)_T}.\end{aligned}\tag{63}$$

To show the convergence of $g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon))$ we considered an approximation of $U^1_{e,\mathrm{sym}}\in L^2(G_T\times\Omega)$ by $U_\delta\in C(G_T;C_\mathcal{T}(\Omega))$, such that $U_\delta\to U^1_{e,\mathrm{sym}}$ in $L^2(G_T\times\Omega)$ as $\delta\to 0$. For $\mathcal{P}$-a.a. $\omega\in\Omega$ we define $U_\delta^\varepsilon(t,x)=U_\delta(t,x,\mathcal{T}_{x/\varepsilon}\omega)$. Using the strong stochastic two-scale convergence of U_δ^ε to U_δ we obtain

$$\lim_{\delta\to 0}\lim_{\varepsilon\to 0}\|U_\delta^\varepsilon\|_{L^2(G_T)}=\lim_{\delta\to 0}\|U_\delta\|_{L^2(G_T\times\Omega)}=\|U^1_{e,\mathrm{sym}}\|_{L^2(G_T\times\Omega)}.\tag{64}$$

Then for $\phi_2\in C_0([0,T)\times G)$ and $\phi_3\in C_\mathcal{T}(\Omega)$ we can write

$$\begin{aligned}&\langle g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon)),\phi_2(t,x)\phi_3(\mathcal{T}_{x/\varepsilon}\omega)\,\chi_{G_e^\varepsilon}\rangle_{G_T}\\&=\langle g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon))-g_b(c,b_e,\mathbf{e}(u_e)+U_\delta^\varepsilon),\phi_2\phi_3(\mathcal{T}_{x/\varepsilon}\omega)\,\chi_{G_e^\varepsilon}\rangle_{G_T}\\&\quad+\langle g_b(c,b_e,\mathbf{e}(u_e)+U_\delta^\varepsilon),\phi_2\phi_3(\mathcal{T}_{x/\varepsilon}\omega)\,\chi_{G_e^\varepsilon}\rangle_{G_T}.\end{aligned}$$

Assumptions on g_b, together with (64), the strong convergence of U_δ to $U^1_{e,\mathrm{sym}}$, and the strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)$, imply

$$\lim_{\delta\to 0}\lim_{\varepsilon\to 0}\langle g_b(c,b_e,\mathbf{e}(u_e)+U_\delta^\varepsilon),\phi_2\,\phi_3\,\chi_{G_e^\varepsilon}\rangle_{G_T}=\langle g_b(c,b_e,\mathbf{e}(u_e)+U^1_{e,\mathrm{sym}}),\phi_2\phi_3\,\chi_{\Omega_e}\rangle_{G_T,\Omega}$$

and

$$\begin{aligned}&|\langle g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon))-g_b(c,b_e,\mathbf{e}(u_e)+U_\delta^\varepsilon),\phi_2\phi_3\chi_{G_e^\varepsilon}\rangle_{G_T}|\\&\quad\leqslant C\left[\|c_e^\varepsilon-c\|_{L^2(G_T)}+\|b_e^\varepsilon-b_e\|_{L^2(G_T)}+\|\mathbf{e}(u_e^\varepsilon)-(\mathbf{e}(u_e)+U_\delta^\varepsilon)\|_{L^2(G^\varepsilon_{e,T})}\right]\\&\quad\leqslant C(\varepsilon)+C\left[\|\mathbf{e}(u_e^\varepsilon)\,\chi_{G_e^\varepsilon}\|^2_{L^2(G_T)}+\|(\mathbf{e}(u_e)+U_\delta^\varepsilon)\chi_{G_e^\varepsilon}\|^2_{L^2(G_T)}-2\left\langle\mathbf{e}(u_e^\varepsilon)\,\chi_{G_e^\varepsilon},\mathbf{e}(u_e)+U_\delta^\varepsilon\right\rangle_{G_T}\right]^{1/2}\to 0\end{aligned}$$

as $\varepsilon\to 0$ and $\delta\to 0$. Assumptions on g and *a priori* estimates for b_e^ε, c_e^ε, and $\mathbf{e}(u_e^\varepsilon)$ ensure that

$$\|g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon))\|_{L^2(G^\varepsilon_{e,T})}\leqslant C,$$

where the constant C is independent of ε. Thus

$$g_b(c_e^\varepsilon,b_e^\varepsilon,\mathbf{e}(u_e^\varepsilon))\rightharpoonup g_b(c_e,b_e,\mathbf{e}(u_e)+U^1_{e,\mathrm{sym}})\quad\text{stochastically two}-\text{scale}.$$

Considering $\phi_1 = 0$ and using the linearity of the resulted equation we obtain

$$B_e^1(t,x,\omega) = \sum_{j=1}^{3} \partial_{x_j} b_e(t,x) w_b^j(\omega)$$

and the unit cell problem (17) for w_b^j. Choosing $\phi_2 = 0$ yields macroscopic equations for b_e.

Taking $\varphi_2(t,x) = \psi_1(t,x) + \varepsilon\psi_2(t,x)\psi_3(\mathcal{T}_{x/\varepsilon}\omega)$ with $\psi_1 \in C^\infty(\overline{G}_T)$, $\psi_1(T,x) = 0$ for $x \in \overline{G}$, $\psi_2 \in C_0^\infty(G_T)$, and $\psi_3 \in C^1_{\mathcal{T},\Gamma}(\Omega)$ as a test function in (7) we obtain

$$\begin{aligned}
&- \langle c_e^\varepsilon \chi_{G_e^\varepsilon}, \partial_t \varphi_2 \rangle_{G_T} + \langle D_e(b_{e,3}^\varepsilon) \nabla c_e^\varepsilon, \nabla\varphi_2\, \chi_{G_e^\varepsilon} \rangle_{G_T} - \langle g_c(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)), \varphi_2 \chi_{G_e^\varepsilon} \rangle_{G_T} \\
&- \langle c_f^\varepsilon \chi_{G_f^\varepsilon}, \partial_t \varphi_2 \rangle_{G_T} + \langle D_f \nabla c_f^\varepsilon, \nabla\varphi_2 \chi_{G_f^\varepsilon} \rangle_{G_T} - \langle \mathcal{G}(\partial_t u_f^\varepsilon) c_f^\varepsilon, \nabla\varphi_2 \chi_{G_f^\varepsilon} \rangle_{G_T} \\
&- \langle g_f(c_f^\varepsilon), \varphi_2 \chi_{G_f^\varepsilon} \rangle_{G_T} = \langle c_{e0}^\varepsilon\, \chi_{G_e^\varepsilon}, \varphi_2(0) \rangle_G + \langle c_{f0}^\varepsilon\, \chi_{G_f^\varepsilon}, \varphi_2(0) \rangle_G + \langle F_c(c_e^\varepsilon), \varphi_2 \rangle_{(\partial G)_T}.
\end{aligned}$$

In the same way as for g_b, using the strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)\chi_{G_e^\varepsilon}$ and $\partial_t u_f^\varepsilon\, \chi_{G_f^\varepsilon}$, the strong convergence of b_e^ε and c^ε, and assumptions on g_e and $\mathcal{G}$, we obtain

$$\begin{aligned}
\chi_{G_e^\varepsilon}\, g_e(c_e^\varepsilon, b_e^\varepsilon, \mathbf{e}(u_e^\varepsilon)) &\rightharpoonup \chi_{\Omega_e}\, g_e(c_e, b_e, \mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}) \qquad && \textit{stochastically two} - \textit{scale}, \\
\chi_{G_f^\varepsilon}\, \mathcal{G}(\partial_t u_f^\varepsilon) &\rightharpoonup \chi_{\Omega_f}\, \mathcal{G}(\partial_t u_f) && \textit{stochastically two} - \textit{scale}.
\end{aligned}$$

Thus applying the stochastic two-scale and the strong convergences of b_e^ε, and c^ε together with strong stochastic two-scale convergence of $\mathbf{e}(u_e^\varepsilon)\, \chi_{G_e^\varepsilon}$ and $\partial_t u_f^\varepsilon\, \chi_{G_f^\varepsilon}$, yields

$$\begin{aligned}
&- \langle c, \partial_t \psi_1 \rangle_{G_T,\Omega} + \langle D(b_{e,3})(\nabla c + C^1), \nabla\psi_1 + \psi_2 \nabla_\omega \psi_3 \rangle_{G_T,\Omega} - \Big\langle \mathcal{G}(\partial_t u_f) c\, \chi_{\Omega_f}, \nabla\psi_1 \\
&+ \psi_2 \nabla_\omega \psi_3 \rangle_{G_T,\Omega} = \langle g_f(c)\, \chi_{\Omega_f}, \psi_1 \rangle_{G_T,\Omega} + \langle g_c(c, b_e, \mathbf{e}(u_e) + U^1_{e,\mathrm{sym}}) \chi_{\Omega_e}, \psi_1 \rangle_{G_T,\Omega} \\
&+ \langle c_0, \psi_1(0) \rangle_{G,\Omega} + \langle F_c(c), \psi_1 \rangle_{(\partial G)_T}.
\end{aligned} \tag{65}$$

Considering $\psi_1 = 0$ yields

$$\langle D(b_{e,3})(\nabla c + C^1) - \mathcal{G}(\partial_t u_f)\, c\, \chi_{\Omega_f}, \psi_2 \nabla_\omega \psi_3 \rangle_{G_T \times \Omega} = 0.$$

From here we obtain that

$$C^1(t,x,\omega) = \sum_{j=1}^{3} \partial_{x_j} c(t,x) w^j(\omega) + c(t,x) Z(t,x,\omega)\, \chi_{\Omega_f}, \tag{66}$$

where $w^j \in L^2_{\mathrm{pot},\Gamma}(\Omega)$, with $j = 1,2,3$, and $Z \in L^\infty(G_T; L^2_{\mathrm{pot}}(\Omega))$ are solutions of the cell problems (18) and (19). Then considering $\psi_2 = 0$ and first $\psi_1 \in C_0^1(G_T)$ and then $\psi_1 \in C_0^1(0,T; C^1(\overline{G}))$, and using the expression (46) for the corrector U_e^1 we obtain the macroscopic equation and the boundary conditions for c in (20). The equations for b_e and c and the fact that $b_e, c \in L^2(0,T; H^1(G))$ imply that $\partial_t b_e, \partial_t c \in L^2(0,T; (H^1(G))')$. Thus $b_e, c \in C([0,T]; L^2(G))$ and using equations (63) and (65) we obtain that b_e and c satisfy initial conditions. □

9. Well-posedness of the macroscopic problem

In the same way as in the case of periodic microstructure [57], using fixed point iteration we show existence of an unique solution of the limit problem.

Lemma 9.1. *There exists a unique weak solution of the limit problem* (12)–(14), (20).

Proof. First we show estimates for two iterations $(u_e^{j-1}, \partial_t p_e^{j-1}, \partial_t u_f^{j-1})$, (b_e^{j-1}, c^{j-1}) and $(u_e^j, \partial_t p_e^j, \partial_t u_f^j)$, (b_e^j, c^j) for limit problem (12)–(14), (20).

We begin with the equations for fluid flow velocity $\partial_t u_f$ and for elastic displacement u_e. Taking $\partial_t\widetilde{u}_f - \partial_t\widetilde{u}_e$ as a test function in the equation for the difference $\partial_t\widetilde{u}_f^j$ and $\partial_t\widetilde{u}_e$ as a test function in the equations for the difference $\widetilde{u}_e^j$ we obtain

$$\begin{aligned}
&\rho_e\|\partial_t\widetilde{u}_e^j(s)\|^2_{L^2(G)} + \left\langle \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1})\mathbf{e}(\widetilde{u}_e^j(s)), \mathbf{e}(\widetilde{u}_e^j(s))\right\rangle_G - \left\langle \partial_t\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1})\mathbf{e}(\widetilde{u}_e^j), \mathbf{e}(\widetilde{u}_e^j)\right\rangle_{G_s}\\
&\quad + 2\left\langle (\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1}))\, \mathbf{e}(u_e^j(s)), \mathbf{e}(\widetilde{u}_e^j(s))\right\rangle_G\\
&\quad - 2\left\langle \partial_t(\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1}))\, \mathbf{e}(u_e^j) + (\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1}))\,\partial_t\mathbf{e}(u_e^j), \mathbf{e}(\widetilde{u}_e^j)\right\rangle_{G_s}\\
&\quad + \rho_f\|\partial_t\widetilde{u}_f^j(s)\,\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} + 2\mu\|\mathbf{e}_\omega(\partial_t\widetilde{u}_f^j)\,\chi_{\Omega_f}\|^2_{L^2(G_s\times\Omega)} + 2\langle \nabla\widetilde{p}_e^j, \partial_t\widetilde{u}_e^j\chi_{\Omega_e} + \partial_t\widetilde{u}_f^j\,\chi_{\Omega_f}\rangle_{G_s,\Omega}\\
&\quad = 2\langle \widetilde{P}_e^{1,j}, \partial_t\widetilde{u}_f^j\,\chi_{\Omega_e} - \partial_t\widetilde{u}_e^j\,\chi_{\Omega_e}\rangle_{G_s,\Omega} + \rho_f\|\partial_t\widetilde{u}_f^j(0)\,\chi_{\Omega_f}\|^2_{L^2(G\times\Omega)} + \rho_e\|\partial_t\widetilde{u}_e^j(0)\|^2_{L^2(G)}\\
&\qquad + \left\langle \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1})\,\mathbf{e}(\widetilde{u}_e^j(0)), \mathbf{e}(\widetilde{u}_e^j(0))\right\rangle_G + 2\left\langle (\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1}))\,\mathbf{e}(u_e^j(0)), \mathbf{e}(\widetilde{u}_e^j(0))\right\rangle_G,
\end{aligned} \tag{67}$$

where $\widetilde{u}_e^j = u_e^j - u_e^{j-1}$, $\widetilde{p}_e^j = p_e^j - p_e^{j-1}$, $\widetilde{u}_f^j = u_f^j - u_f^{j-1}$, and $\widetilde{P}_e^{1,j} = P_e^{1,j} - P_e^{1,j-1}$. The equation (12) for p_e^j and p_e^{j-1} yields

$$\begin{aligned}
\rho_p\|\widetilde{p}_e^j(s)\|^2_{L^2(G)} + 2\langle K_p^{\mathrm{hom}}\nabla\widetilde{p}_e^j, \nabla\widetilde{p}_e^j\rangle_{G_s} &= 2\langle K_u\,\partial_t\widetilde{u}_e^j + Q(\partial_t u_f^j) - Q(\partial_t u_f^{j-1}), \nabla\widetilde{p}_e^j\rangle_{G_s}\\
&\quad + \rho_p\|\widetilde{p}_e^j(0)\|^2_{L^2(G)}.
\end{aligned} \tag{68}$$

Due to the assumptions in A1 on $\mathbf{E}$, the definition of the macroscopic elasticity tensor $\mathbf{E}^{\mathrm{hom}}$ and the properties of a solution W_e^{kl}, with $k, l = 1, 2, 3$, of the corresponding cell problems in (9), we have

$$\|\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1})\|_{L^\infty(G_s)} + \|\partial_t(\mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j}) - \mathbf{E}^{\mathrm{hom}}(b_{e,3}^{j-1}))\|_{L^\infty(G_s)} \leqslant C\|\widetilde{b}_e^j\|_{L^\infty(0,s;L^\infty(G))}$$

for $s \in (0, T]$, where $\widetilde{b}_e^j = b_e^j - b_e^{j-1}$. The expression (15) for $P_e^{1,j}$ and $P_e^{1,j-1}$ and the estimates for the H^1-norm of the solutions of the cell problems (9) and (11) yield

$$\|\widetilde{P}_e^{1,j}\|_{L^2(G_s\times\Omega)} \leqslant C\left(\|\nabla\widetilde{p}_e^j\|_{L^2(G_s)} + \|\partial_t\widetilde{u}_e^j\|_{L^2(G_s)} + \|\partial_t\widetilde{u}_f^j\,\chi_{\Omega_f}\|_{L^2(G_s\times\Omega)}\right).$$

Adding (67) and (68), and applying the Hölder and Gronwall inequalities yield

$$\|\partial_t \widetilde{u}_e^j\|_{L^\infty(0,s;L^2(G))} + \|\mathbf{e}(\widetilde{u}_e^j)\|_{L^\infty(0,s;L^2(G))} + \|\widetilde{p}_e^j\|_{L^\infty(0,s;L^2(G))} + \|\nabla \widetilde{p}_e^j\|_{L^2(G_s)}$$
$$+ \|\partial_t \widetilde{u}_f^j\, \chi_{\Omega_f}\|_{L^\infty(0,s;L^2(G\times\Omega))} + \|\mathbf{e}_\omega(\partial_t \widetilde{u}_f^j)\, \chi_{\Omega_f}\|_{L^2(G_s\times\Omega)} \leqslant C\|\widetilde{b}_e^j\|_{L^\infty(0,s;L^\infty(G))} \tag{69}$$

for all $s \in (0, T]$ and the constant C does not depend on s and solutions of the macroscopic problem.

In the same way as in the case of periodic microstructure [57] we obtain the following estimates for $\widetilde{b}_e^j$ and $\widetilde{c}^j$:

$$\|\widetilde{b}_e^j\|_{L^\infty(0,s;L^\infty(G))} + \|\widetilde{c}^j\|_{L^\infty(0,s;L^2(G))} \leqslant C_1 \left[\|\mathbf{e}(\widetilde{u}_e^{j-1})\|_{L^{1+\frac{1}{\sigma}}(0,s;L^2(G))} + \|\partial_t \widetilde{u}_f^{j-1}\, \chi_{\Omega_f}\|_{L^2(G_s\times\Omega)} \right], \tag{70}$$

for $s \in (0, T]$ and any $0 < \sigma < 1/9$, the constant C being independent of s and solutions of the problem, and

$$\|b_e^j\|_{L^\infty(0,T;L^\infty(G))} + \|c^j\|_{L^\infty(0,T;L^\infty(G))} + \|b_e^{j-1}\|_{L^\infty(0,T;L^\infty(G))}$$
$$+ \|c^{j-1}\|_{L^\infty(0,T;L^\infty(G))} \leqslant C.$$

Then combining (69) and (70) and applying a fixed point argument we obtain existence of a uniques solution of the coupled macroscopic problem (12)–(14), (20). □

Acknowledgments

This research was supported by a Northern Research Partnership early career research exchange grant. The research of MP was also supported by the EPSRC First grant EP/K036521/1 “Multiscale modelling and analysis of mechanical properties of plant cells and tissues”.

ORCID iDs

Andrey Piatnitski https://orcid.org/0000-0002-9874-6227
Mariya Ptashnyk https://orcid.org/0000-0003-4091-5080

References

[1] Aarnes J E and Efendiev Y 2008 Mixed multiscale finite element methods for stochastic porous media flows *SIAM J. Sci. Comput.* **30** 2319–39
[2] Asokan B V and Zabaras N 2006 A stochastic variational multiscale method for diffusion in heterogeneous random media *J. Comput. Phys.* **218** 654–76
[3] Acerbi E, Chiadò Piat V, Dal Maso G and Percivale D 1992 An extension theorem from connected sets, and homogenization in general periodic domains *Nonlinear Anal.: Theor. Methods Appl.* **18** 481–96
[4] Armstrong S N and Souganidis P E 2012 Stochastic homogenization of Hamilton-Jacobi and degenerate Bellman equations in unbounded environments *J. Math. Pures Appl.* **97** 460–504
[5] Auriault J-L and Sanchez-Palencia E 1977 Etude du comportement macroscopic dún mileu poreux saturé déformable *J. Mécanique* **16** 575–603

[6] Badia S, Quainib A and Quarteroni A 2009 Coupling Biot and Navier–Stokes equations for modelling fluid-poroelastic media interaction *J. Comput. Phys.* **228** 7986–8014
[7] Band L R, Fozard J A, Godin C, Jensen O E, Pridmore T, Bennett M J and King J R 2012 Multiscale systems analysis of root growth and development: modeling beyond the network and cellular scales *The Plant Cell* **24** 3892–906
[8] Baskin T I and Jensen O E 2013 On the role of stress anisotropy in the growth of stems *J. Exp. Bot.* **64** 4697–707
[9] Bensoussan A and Blankenship G 1988 Controlled diffusions in random medium *Stochastics* **24** 87–120
[10] Biot M A 1972 Theory of finite deformations of porous solids *Indiana Univ. Math.* J **21** 597–620
[11] Biot M A 1962 Generalized theory of acoustic propagation in porous dissipative media *J. Acoust. Soc. Am.* **34** 1256–64
[12] Biot M A 1941 General theory of three-dimensional consolidation *J. Appl. Phys.* **12** 155–64
[13] Bourgeat A, Mikelić A and Piatnitski A 2003 On the double porosity model of a single phase flow in random media *Asymptotic Anal.* **34** 311–32
[14] Bourgeat A and Piatnitski A 2004 Approximations of effective coefficients in stochastic homogenization *Annales de l'Institute Henri Poincare* B **40** 153–65
[15] Bourgeat A, Mikelić A and Wright S 1994 Stochastic two-scale convergence in the mean and applications *J. Reine Angewandte Math.* **456** 19–51
[16] Brezis H 2010 *Functional Analysis, Sobolev Spaces and Partial Differential Equations* (New York: Springer)
[17] Broedersz C P and MacKintosh F C 2014 Modeling semiflexible polymer networks *Rev. Mod. Phys.* **86** 995
[18] Burridge R and Keller J B 1981 Poroelasticity equations derived from microstructure *J. Acoust. Soc. Am.* **70** 114–1146
[19] Bukač M, Yotov I, Zakerzadeh R and Zunino P 2015 Partitioning strategies for the interaction of a fluid with a poroelastic material based on a Nitsches coupling approach *Comput. Methods Appl. Mech. Eng.* **292** 138–70
[20] Caffarelli L A, Souganidis P E and Wang L 2005 Homogenization of fully nonlinear, uniformly elliptic and parabolic partial differential equations in stationary ergodic media *Commun. Pure Appl. Math.* **58** 0319–61
[21] Castell F 2001 Homogenization of random semilinear PDEs *Probab. Theor. Relat. Fields* **121** 492–524
[22] Clopeau T, Ferrin J L, Gilbert R P and Mikelić A 2001 Homogenizing the acoustic properties of the seebed: part II *Math. Comput. Model.* **33** 821–41
[23] Daley D and Vere-Jones D 1988 *An Introduction to the Theory of Point Processes* (Berlin: Springer)
[24] Dal Maso G and Modica L 1986 Nonlinear stochastic homogenization and ergodic theory *J. Reine Angewandte Math.* **368** 28–42
[25] Dal Maso G and Modica L 1986 Nonlinear stochastic homogenization *Ann. Mat. Pura Appl.* **144** 347–89
[26] Davalos-Sotelo R 2005 Determination of elastic properties of clear wood by the homogenization method in two dimensions *Wood Sci. Technol.* **39** 385–417
[27] Dyson R J *et al* 2014 Mechanical modelling quantifies the functional importance of outer tissue layers during root elongation and bending *New Phytol.* **202** 1212–22
[28] Faisal T R, Hristozov N, Rey A D, Western T L and Pasini D 2012 Experimental determination of *Philodendron melinonii* and *Arabidopsis thaliana* tissue microstructure and geometric modeling via finite-edge centroidal Voronoi tessellation *Phys. Rev.* E **86** 031921
[29] Faisal T R, Rey A D and Pasini D 2013 A multiscale mechanical model for plant tissue stiffness *Polymers* **5** 730–50
[30] Fozard J A, Lucas M, King J R and Jensen O E 2013 Vertex-element models for anisotropic growth of elongated plant organs *Front. Plant Sci.* **4** 1–16
[31] Fozard J A, Bennett M J, King J R and Jensen O E 2016 Hybrid vertex-midline modelling of elongated plant organs *Interface Focus* **6** 20160043
[32] Ghysels P, Samaey G, Van Liedekerke P, Tijskens E, Ramon H and Roose D 2010 *Int. J. Multiscale Comput. Eng.* **8** 379–96
[33] Gibson L J and Ashby M F 1999 *Cellular Solids: Structure and Properties* (Cambridge: Cambridge University Press)

[34] Gilbert R P and Mikelić A 2000 Homogenizing the acoustic properties of the seabed: part I *Nonlinear Anal.* **40** 185–212
[35] Gloria A and Otto F 2015 Quantitative estimates on the periodic approximation of the corrector in stochastic homogenization *ESAIM: Proc. Surv.* **48** 80–97
[36] Heida M 2012 Stochastic homogenization of heat transfer in polycrystals with nonlinear contact conductivities *Asymptotic Anal.* **91** 1243–64
[37] Heida M 2011 An extension of the stochastic two-scale convergence method and application *Appl. Anal.* **72** 1–30
[38] Jäger W, Mikelić A and Neuss-Radu M 2011 Homogenization limit of a model system for interaction of flow, chemical reactions, and mechanics in cell tissues *SIAM J. Math. Anal.* **43** 1390–435
[39] Jensen O E and Fozard J A 2015 Multiscale models in the biomechanics of plant growth *Physiology* **30** 159–66
[40] Kosygina E, Rezakhanlou F and Varadhan S R S 2006 Stochastic homogenization of Hamilton–Jacobi–Bellman equations *Commun. Pure Appl. Math.* **59** 1489–521
[41] Kozlov S M 1980 Averaging of random operators *Math. USSR-Sbornik* **37** 167–80
[42] Levy T 1979 Propagation waves in a fluid-saturated porous elastic solid *Int. J. Eng. Sci.* **17** 1005–14
[43] van Liedekerke P, Ghysels P, Tijskens E, Samaey G, Smeedts B, Roose D and Ramon H 2010 A particle-based model to simulate the micromechanics of single-plant parenchyma cells and aggregates *Phys. Biol.* **7** 026006
[44] Lions P-L and Souganidis P E 2005 Homogenization of 'viscous' Hamilton–Jacobi equations in stationary ergodic media *Commun. Part. Differ. Equ.* **30** 335–75
[45] Lions P-L and Souganidis P E 2010 Stochastic homogenization of Hamilton–Jacobi and 'viscous' Hamilton–Jacobi equations with convex nonlinearity-revisited *Commun. Math. Sci.* **8** 627–37
[46] Malek S and Gibson L J 2017 Multi-scale modelling of elastic properties of balsa *Int. J. Solids Struct.* **113-114** 118–31
[47] Mathers W A, Hepworth C, Baillie A L, Sloan J, Jones H, Lundgren M, Fleming A J, Mooney S J and Sturrock C J 2018 Investigating the microstructure of plant leaves in 3D with lab-based x-ray computed tomography *Plant Methods* **14** 99
[48] Mebatsion H K, Verboven P, Ho Q T, Verlinden B E, Mendoza F, Nguyen T A and Nicolaï B M 2006 *Modeling Fruit Microstructure Using an Ellipse Tessellation Algorithm* (Leuven: IUFoST)
[49] Meirmanov A M 2008 Homogenization for a short-time filtration in elastic porous media *Electron. J. Differ. Equ.* **2008** 1–18
[50] Meirmanov A M 2007 Nguetseng's two-scale convergence method for filtration and seismic acoustic problems in elastic porous media *Siberian Math. J.* **48** 519–38
[51] Mikelić A and Wheeler M F 2012 On the interface law between a deformable porous medium containing a viscous fluid and an elastic body *Math. Models Methods Appl. Sci.* **22** 125003
[52] Mimault M, Ptashnyk M, Bassel G W and Dupuy L X 2019 Smoothed particle hydrodynamics for root growth mechanics *Eng. Anal. Bound. Elem.* **105** 20–30
[53] Necas J 1967 *Les méthodes directes en théorie des équations elliptiques* (Prague: Academie)
[54] Nguetseng G 1990 Asymptotic analysis for a stiff variational problem arising in mechanics *SIAM J. Math. Anal.* **21** 1394–414
[55] Nobel P S 1999 *Plant Physiology* (San Diego, CA: Academic)
[56] Papanicolaou G and Varadhan S R S 1979 *Boundary value problem with rapidly oscillating random coefficients* (College Mathematical Society Jámos Bolyai) vol 27 (Hungary: Esztergom) pp 835–73
[57] Piatnitski A and Ptashnyk M 2017 Homogenization of biomechanics models for plant tissues *Multiscale Modelling and Simulations* **15** 339–87
[58] Ptashnyk M and Seguin B 2016 The impact of microfibril orientations on the biomechanics of plant cell walls and tissues *Bull. Math. Biol.* **78** 2135–64
[59] Proseus T E and Boyer J S 2006 Calcium pectate chemistry controls growth rate of *Chara corallina J. Exp. Bot.* **57** 3989–4002
[60] Qing H and Mishnaevsky LJr 2010 3D multiscale micromechanical model of wood: from annual rings to microfibrils *Int. J. Solids Struct.* **47** 1253–67
[61] Sanchez-Palencia E 1980 *Non-Homogeneous Media and Vibration Theory* (Berlin: Springer)
[62] Schopfer P 2006 Biomechanics of plant growth *Am. J. Bot.* **93** 1415–25
[63] Schuster E, Lundin L and Williams M A K 2012 Investigating the relationship between network mechanics and single-chain extension using biomimetic polysaccharide gels *Macromolecules* **45** 4863–9
[64] Showalter R E 2000 Diffusion in poro-elastic media *J. Math. Anal. Appl.* **251** 310–40

[65] Showalter R E 2005 Poroelastic filtration coupled to Stokes flow *Control theory of partial differential equations Lect. Notes Pure Appl. Math.* (vol 242) (Boca Raton, FL: CRC Press) pp 229–41
[66] Silva M J, Hayes W C and Gibson L J 1995 The effects of non-periodic microstructure on the elastic properties of two-dimensional cellular solids *Int. J. Mech. Sci.* **37** 1161–77
[67] Temam R 1977 *Navier-Stokes Equations: Theory and Numerical Analysis* (Amsterdam: North-Holland)
[68] Wijerathne W D C C, Rathnayaka C M, Karunasena H C P, Senadeera W, Sauret E, Turner I W and Gu Y T 2019 A coarse-grained multiscale model to simulate morphological changes of food-plant tissues undergoing drying *Soft Matter* **15** 9001
[69] Wolf S and Greiner S 2012 Growth control by cell wall pectins *Protoplasma* **249** 169–75
[70] Wolf S, Hématy K and Höfte H 2012 Growth control and cell wall signaling in plants *Annu. Rev. Plant Biol.* **63** 381–407
[71] Wright S 2001 On diffusion of a single-phase, slightly compressible fluid through a randomly fissured medium *Math. Methods Appl. Sci.* **24** 805–25
[72] Zhikov V V, Kozlov O A, Oleinik O A and Ngoan K T 1979 Averaging and *G*-convergence of differential operators *Russ. Math. Surv.* **34** 69–147
[73] Zhikov V V, Kozlov O A and Oleinik O A 1994 *Homogenization of Differential Operators and Integral Functionals* (New York: Springer)
[74] Zhikov V V and Pyatnitskii A L 2006 Homogenization of random singular structures and random measures *Izvestiya: Mathematics* **70** 19–67