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An Achievability Bound of Energy per Bit for Stabilized Massive Random Access Gaussian Channel

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***Abstract*—Developing communication standards highlight a scenario called massive Machine-Type Communication within the framework of the Internet of Things. Under this scenario, a large number of devices with autonomous power supplies will operate in a random multiple access mode. It is crucial to keep the system stable, while consuming minimal energy. We describe a model of a system with a Gaussian Random Access Channel and consider an algorithm allowing message retransmissions. We provide a stability condition for the system and introduce an optimization problem to determine an achievability bound for energy per bit.**

***Index Terms*—massive random access, GMAC, energy per bit, stability, achievability bound, mMTC.**

I. Introduction

Massive Machine-Type Communications (mMTC) is one of the main scenarios of the currently deployed 5G communication technology which is also expected to prove relevant in further standards (including 6G), wtih potentially even larger number of devices and network load ([1], [2]). The main requirements for this scenario are: a large number of devices, network stability, low power consumption, reliable message delivery. The stability issue is caused by the need to reliably deliver data from a large number of devices ([3]), and it should be achieved while keeping the energy consumption of devices as low as possible. The mMTC scenario is, for instance, attributed to the Internet of Things systems. These systems have a large number of transmitting devices (for example, sensors) that operate from an autonomous power source ([4], [5]). In such systems devices need to operate on battery power, underlining the importance of both stability and energy costs. In mMTC scenarios, multiple random access methods are used to access a common channel resource of a large number of devices. An overview of studies containing research in this area is given in [6] and [7].

For a long time, multiple access information theory and random multiple access theory (collision resolution) have been developed independently [8], [9]. This has been noted in [9] but remained the case for a long time after. To the best of our knowledge, the first attempts to join the approaches and to consider energy consumption have been made in [10] and [11]. The authors considered the scenario with a large number of users in a network, with some of them transmitting, and a transmission to the base station needs to be successful with a certain given probability. The number of transmitting devices is assumed to be constant and known. The authors consider several algorithms for multiple access, including ALOHA, and derive achievability bounds of the energy per bit to noise power spectral density ratio $\frac{E_b}{N_0}$. The achievability bound does not mean a specific solution, but only the fact that such a solution exists. The definition of the term of achievability is discussed in detail in [12]. The models of [10] [11] do not allow for retransmissions of messages that were not delivered successfully. This, and the presence of losses in their model, makes stability guaranteed.

We consider a system with dynamic exogenous arrivals of users, so the number of users is random and unbounded. We also allow for an unbounded, random, number of retransmissions of messages, until they are successfully received. Our model thus combines those considered in [10] and [13]. We first establish a condition guaranteeing stability. Using this condition, we then present a method for calculating the achievability bound by the energy per bit costs for a finite code length and stable operation of the system under conditions of a given spectral efficiency. The energy per bit analysis takes into account the effect of retransmissions on energy costs.

The remainder of this work is organized as follows. Section II presents a model of a system with a Massive Random Access Gaussian Channel and introduces spectral efficiency. In Section III, we consider the condition for the stability of the system, determine the achievability bound taking into account average number of transmissions per user and the optimization problem. In addition, in Section IV we give several numerical examples using solutions to the optimization problem. Finally, we draw the main conclusions in section V.

II. System model

We consider a system of random multiple access with acknowledgements using an algorithm type ALOHA. Before formalizing the optimization problem under consideration, we introduce the system of assumptions.

Assumption 1: Each message contains k bits of information. Based on the k bits using the modulation and coding scheme (MCS) A, a signal containing n samples is generated, and in what follows we will call this signal a packet.

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Assumption 2: Users and the base station have synchronization, both by samples and by slots. Time slots are numbered and slot t corresponds to time interval [t, t + 1) (we shall say slot t instead of slot number t). We also assume that it takes exactly one slot to send a single packet, i.e. n samples.

Assumption 3: The system has one base station and multiple users. In our model, users and messages are equivalent. Assume that at the beginning of a time slot t, ξ_t new packets (or, equivalently, new users) arrive in the system. We assume that $\{\xi_t\}_{t=1,2,...}$ is a sequence of independent random variables, each having a Poisson distribution with a parameter Λ.

Assumption 4: Consider a time-discrete communication channel with Additive white Gaussian noise (Gaussian Multiple Access Channel, GMAC), defined as $Y = \sum_{i=1}^{K_t} X_i + Z$, where Y is the channel output, X_i is the i-user signal, K_t is the number of users transmitting to the channel in slot t (which is a random variable unlike work [10]), Z is additive white Gaussian noise, i.e. $Z \sim \mathcal{N}(0,1)$. The signals transmitted by the users contain n samples, and the average energy constraint of each signal X_i is nP.

Assumption 5: By the beginning of slot t, all users and base station know how many users have messages ready for transmission - let us denote this number by M_t [1] Each user transmits its message with probability $p = \frac{G}{M_t}$, independently of everything else, where $0 < G \leq 1$ is the algorithm parameter. If exactly one packet is sent in a slot, then it is successfully received with the probability P_d and leaves the system. Otherwise it stays in the system. If several users send packets in a given time slot, all packets will not be delivered successfully and will stay in the system.

Assumption 6: The probability of successful transmission of a packet P_d is defined as $P_d = 1 - P_e(A,n,k,P)$, where $P_e(A,n,k,P)$ is the probability of decoding error due to AWGN for given k, A, n *and* P (see assumption 1 and 4).

Remark 1: Since the variance of noise in the model under consideration is equal to 1, the SNR in times is equal to $\left(P\ [in\ times] = \frac{P\ [watt]}{1\ [watt]}\right)$. In the remainder, in order to keep notation consistent with the papers of Polyanskiy, by P we will mean SNR in times.

The model introduced is characterized by the following set of parameters: $\Lambda\ \left[\frac{message}{slot}\right]$, $k\ [bit]$, $n\ [samples]$, $P\ [in\ times]$. The system is also characterized by a method of generating signals from n samples at k bits, which is determined by the MSC A. In order to compare the results obtained in this paper with the results presented in [10] and [11], we introduce

$$\rho \triangleq \frac{\Lambda k}{n}. \tag{1}$$

This value characterizes the average number of bits transmitted per sample. In what follows we shall refer to ρ as system spectral efficiency.

[1]The assumption is not practical in many scenarios. In practice the number of users is not known and needs to estimated by the base station and then sent to new users; users can then observe the channel to update their estimates, see, e.g. [14]. It is known that estimating the number of users does not change stability conditions, see, e.g. [15]. We therefore make this assumption in order to facilitate our theoretical analysis of stability.

If the transmission of the message is repeated on average S times, then the average energy spent on sending it, in view of assumption 4, is defined as nPS. Then the average spent energy-per-bit can be calculated as (see also [10], [11])

$$\frac{E_b}{N_0} \triangleq \frac{nPS}{2k}. \tag{2}$$

III. System analysis

A. Stability analysis

Before formulating the stability problem, we introduce the following definition.

Definition 1: Following [16], we define the maximum throughput of a random multiple access system with retransmissions as the maximum throughput at which the average delay in the system is finite: $\Lambda_{Thr} = \sup\{\Lambda : D < \infty\}$, where D is the average delay.

Let us also introduce, in line with (1), the maximum spectral efficiency as:

$$\rho_{Thr} \triangleq \frac{\Lambda_{Thr} k}{n} \tag{3}$$

It is assumed that the number of information bits k to be transmitted and the required maximum spectral efficiency ρ_{Thr} for which the system must be stable are specified, and one needs to determine the (average) minimum value of $\frac{E_b}{N_0}$ and the number of samples n at which the system is stable. In order to solve this problem,we first specify the stability condition.

Lemma 1: Let the modulation and coding scheme (MCS) A be selected and k, n, G and P be given. Then the maximum throughput can be found as a solution to the equation

$$\rho_{Thr} = \frac{k}{n} G e^{-G} \left(1 - P_e(A,n,k,P)\right) \tag{4}$$

Proof. The sequence M_t (see assumption 5) forms a Markov chain, which is irreducible, aperiodic, and homogeneous. Using the Foster criterion (see, e.g. [17]), it can be shown that this Markov chain is ergodic and therefore $D < \infty$ if

$$\Lambda < G e^{-G} \left(1 - P_e(A,n,k,P)\right). \tag{5}$$

Indeed, assume that (5) holds and fix $\varepsilon > 0$ such that $\Lambda < Ge^{-G}(1 - P_e(A,n,k,P)) - \varepsilon$. If there are M users in the system at the beginning of a time slot, then the number of attempted transmissions in this time slot follows a Binomial distribution with parameters M and G/M. A transmission will then be successful if and only if it is the only one attempted, and it is correctly decoded. Then

$$\begin{aligned}
&\mathbb{E}[M_{t+1} - M_t | M_t = M] \\
&= \Lambda - M\frac{G}{M}\left(1 - \frac{G}{M}\right)^{M-1} \left(1 - P_e(A,n,k,P)\right) \\
&< \Lambda - Ge^{-G}(1 - P_e(A,n,k,P)) + \varepsilon/2 < -\varepsilon/2,
\end{aligned}$$

for large enough values of M. From formula (5) and definition 1, it follows that $\Lambda_{Thr} = Ge^{-G}(1 - P_e(A,n,k,P))$. The result now follows from formula (3).

Corollary 1: Since stability is established for $\rho < \rho_{Thr}$, there exist stationary probabilities for the Markov chain under consideration, i.e.

$$\lim_{t\to\infty} \Pr\{M_t = i\} = \pi_i, \tag{6}$$

where π_i is the stationary probability that i users are in the system.

Since we are considering a system with retransmissions, these transmissions affect the energy consumed by the user on transmitting one bit of information. The following lemma shows how to determine the average number S of transmissions required to successfully transmit one packet.

Lemma 2: In a stable system (i.e. when $\rho < \rho_{Thr}$), the average number of transmissions per user (denoted by S) for a given set of parameters k,n,G can be obtained as:

$$S = \frac{kG(1-\pi_0)}{\rho n}. \tag{7}$$

Remark 2: Before proving the lemma, note that the value of ρ also affects the distribution of the number of active users in the system, that is, π_0.

Proof. We shall utilise rate-balance arguments for stable systems. The average number of transmissions attempted by a user as seen from the departure process is ΛS (there are on average Λ users departing per time slot, due to stability, and each user has on average attempted S transmission). The same average number as seen from the medium at a given time slot is

$$\sum_{i=1}^{\infty} i\frac{G}{i}\pi_i = G(1-\pi_0).$$

The statement follows now from equating the two expressions above and using (1).

Then, in accordance with formula (2) and the obtained average number transmissions S, we can write the following:

$$\frac{E_b}{N_0} = \frac{PG(1-\pi_0)}{2\rho}. \tag{8}$$

Now we introduce the equation describing the stability of the system taking into account retransmissions.

Theorem 1: Let the MCS A be selected and k, n, G and $\frac{E_b}{N_0}$ be given. Then the critical intensity of the input arrival rate can be found as a solution to the equation:

$$\rho_{Thr} = \frac{k}{n}Ge^{-G}\left(1 - P_e\left(A, n, k, 2\frac{E_b}{N_0}\frac{\rho_{Thr}}{G}\right)\right) \tag{9}$$

Proof. Clearly, as $\rho \uparrow \rho_{Thr}$, $\pi_0 \downarrow 0$ and thus, from lemma 2,

$$\lim_{\rho\uparrow\rho_{Thr}} S = \frac{kG}{n\rho_{Thr}}. \tag{10}$$

Using formulas (8) and (10), we can conclude that as

$$\lim_{\rho\uparrow\rho_{Thr}} P = 2\frac{E_b}{N_0}\frac{\rho_{Thr}}{G}. \tag{11}$$

Substituting (11) into (4) we obtain the statement.

B. Achievability bound and optimization problem formulation

To find the achievability bound for the probability of error, we will perform the following steps. From [12] it follows that for a fixed length of n and a fixed error probability ε, there is a code whose cardinality is 2^k. Here k can be calculated as

$$k = nC - \sqrt{nV}Q^{-1}(\varepsilon) + \frac{1}{2}\log_2 n, \tag{12}$$

where $C = \frac{1}{2}\log_2(1+P)$, $V = \frac{P}{2}\frac{P+2}{(P+1)^2}\left(\log_2 e\right)^2$, Q^{-1} is the inverse Q-function. We thus introduce the following.

Definition 2: Assume the number of samples n, the number of bits transmitted by a user k, and a certain value $\frac{E_b}{N_0}$ are specified. Then there exists a MCS for which the following equality holds for the achievability bound of the probability of error:

$$P_e(n,k,P) = Q\left(\frac{n\frac{1}{2}\log_2(1+P) + \frac{1}{2}\log_2 n - k}{\sqrt{n\frac{P}{2}\frac{P+2}{(P+1)^2}}\log_2 e}\right). \tag{13}$$

The following theorem is stated directly from theorem 1 and definition 2.

Theorem 2: Let k, n, G and $\frac{E_b}{N_0}$ be given. Then achievability bound for the critical throughput can be found as a solution to the equation:

$$\rho_{Thr} = \frac{k}{n}Ge^{-G}\left(1 - P_e\left(n, k, 2\frac{E_b}{N_0}\frac{\rho_{Thr}}{G}\right)\right) \tag{14}$$

The proof of Theorem 2 is trivial. In formula (4), we replace the error probability with a fixed MCS A $P_e(A,n,k,P)$ by the achievability bound (13) taking into account formula (11).

In view of theorem 2, we will give a formal statement of the problem to optimize the average $\frac{E_b}{N_0}$:

$$\begin{aligned}
&\text{given}: \ k, \rho_{Thr} \\
&\text{minimaze}: \ \tfrac{E_b}{N_0} \quad \text{in} \quad n, G \\
&\text{subject to}: \\
&\rho_{Thr} = \tfrac{k}{n}Ge^{-G}\left(1 - P_e\left(n, k, 2\tfrac{E_b}{N_0}\tfrac{\rho_{Thr}}{G}\right)\right)
\end{aligned} \tag{15}$$

The solution to the optimization problem (15) is the achievability bound. This problem can be solved using numerical optimization methods.

C. Lower bound

We obtain the following lower bound.

Statement 1: If $k \to \infty$, $n \to \infty$ and $R = \frac{k}{n}$ is fixed, then the value of $\frac{E_b}{N_0}$ for given parameters λ_{Thr} and G satisfies

$$\frac{E_b}{N_0} > f(\rho_{Thr}, G) = \frac{\left(2^{\frac{2\rho_{Thr}}{Ge^{-G}}} - 1\right)G}{2\rho_{Thr}}. \tag{16}$$

For a fixed ρ_{Thr}, let G_L be the value of G minimizing $f(\rho_{Thr}, G)$. From formula (16), it is clear that $\lim_{\rho_{Thr}\to 0} f(\rho_{Thr}, G) = e^G \ln(2)$. This shows that in this case $G_L \to 0$ which is approaching the Shannon limit.

Remark 3: A similar relationship between $\frac{E_b}{N_0}$, G and ρ_{Thr} for the $k \to \infty$ and $n \to \infty$ case was obtained in [18].

IV. Numerical results

Figure 1 shows graphs of the dependence of the achievability bound for $\frac{E_b}{N_0}$ on ρ for various values of k and two values of the parameter G. For graphs without markers, the value of G was selected by solving the optimization problem in all parameters (G_{opt}). For graphs with a marker (circle), the value of G when solving the optimization problem was fixed and equal to 1 (optimization was carried out according to all other

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parameters). As can be seen from the figure, optimization with respect to the parameter G makes it possible to better minimize energy costs. The results obtained show that in order to reduce energy costs it is necessary to transmit with parameter G different from unity. It is easy to show that if in (15) we minimize P instead of $\frac{E_b}{N_0}$, then the parameter G equal to unity will be optimal. Moreover, to ensure spectral efficiency $\rho = \frac{e^{-1}}{2}$, the value of G_L (in accordance with assumption 4) is equal to 1 when considering the lower bound. However, the choice of the optimal value of G from the point of view of energy consumption can adversely affect the delay values in the system. One can also see that with an increase in the number of transmitted bits k, the values of the achievability bound approach the lower bound.

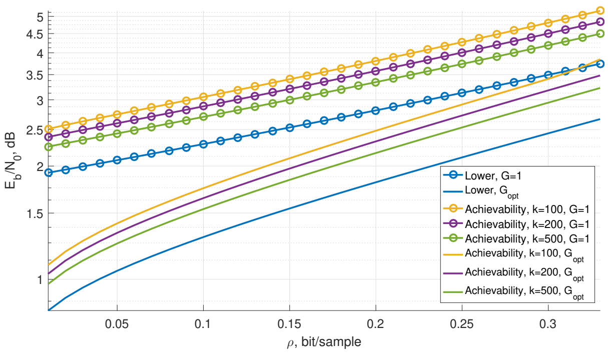

Fig. 1. Dependence of the value of $\frac{E_b}{N_0}$ on ρ for various k and G

Numerical calculations showed that the use of the G_L obtained from (16) in solving the optimization problem (15) worsens the achievability bound for the average $\frac{E_b}{N_0}$. In this case, the loss is negligible, for example, at $k = 100$ it is $\approx 10^{-3}$ and for $k = 500$ it is $\approx 10^{-4}$.

V. Conclusion

We considered a Massive Random Access Gaussian Channel for transmission of messages of size $k \approx 100$ bit, which is typical in the Internet of Things. We introduced a model which combines the model of a multiple random access channel without noise and with a potentially unbounded number of devices ([13]) and the model of a Gaussian noise channel ([10]). We study the question of stability of a system with exogenous arrivals where all packets need to be delivered with probability 1 (as in [13]; note that in [10] packets need to be delivered with a high probability). The combined model of our paper allows to study the effect of energy consumption on stability. It also allowed us to obtain an achievability bound for E_b/N_0 such that the system is stable for a given spectral efficiency ρ. We considered the simple ALOHA algorithm for multiple random access, where each device transmits its message in a time slot with the probability G/M_t, where M_t is the number of devices in the system in the slot, and G is a fixed parameter. We minimise the average energy consumed by the system with respect to G and show that the optimal value of G is decreasing with a decreasing spectral efficiency. Interestingly, it depends only weakly on the size of a message. It is worth noting that similar results may be obtained from [14] and [19] without the use of the number of devices present in the system in a time slot. We believe that the achievability bound obtained here may be improved by the use of random multiple access algorithms with conflict resolutions (see, e.g., [13]) and [16]) which are known to be more efficient than ALOHA. The achievability bound may also be improved with the use of techniques introduced in [11]. We are working on both these exciting approaches.

We assumed an unbounded number of possible retransmissions. In practice this is often not the case. As stability is guaranteed if the number of retransmission is bounded, the interesting question is the interplay between the proportion of lost packets and energy consumption.

References

[1] K. B. Letaief, W. Chen, Y. Shi, J. Zhang, and Y.-J. A. Zhang, “The roadmap to 6g: Ai empowered wireless networks,” *IEEE Communications Magazine*, vol. 57, no. 8, pp. 84–90, 2019.

[2] M. Piran, D. Y. Suh *et al.*, “Learning-driven wireless communications, towards 6g,” *arXiv preprint arXiv:1908.07335*, 2019.

[3] N. H. Mahmood, H. Alves, O. A. López, M. Shehab, D. P. M. Osorio, and M. Latva-aho, “Six key enablers for machine type communication in 6g,” *arXiv preprint arXiv:1903.05406*, 2019.

[4] S. Horsmanheimo, J. Säe, T. Jokela, L. Tuomimäki, E. Nigussie, A. Hjelt, S. Huilla, T. Dönmez, N. Le Bail, and M. Valkama, “Remote monitoring of iot sensors and communication link quality in multisite mmtc testbed,” in *2019 IEEE 30th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*. IEEE, 2019, pp. 1–7.

[5] I. Jovović, I. Forenbacher, and M. Periša, “Massive machine-type communications: An overview and perspectives towards 5g,” in *The 3rd International Virtual Research Conference In Technical Disciplines, RCITD 2015*, vol. 3, 2015.

[6] Y. Wu, X. Gao, S. Zhou, W. Yang, Y. Polyanskiy, and G. Caire, “Massive access for future wireless communication systems,” *arXiv preprint arXiv:1910.12678*, 2019.

[7] M. B. Shahab, R. Abbas, M. Shirvanimoghaddam, and S. J. Johnson, “Grant-free non-orthogonal multiple access for iot: A survey,” *arXiv preprint arXiv:1910.06529*, 2019.

[8] P. Mathys, “A class of codes for a t active users out of n multiple-access communication system,” *IEEE Transactions on Information Theory*, vol. 36, no. 6, pp. 1206–1219, 1990.

[9] R. G. Gallager, “A perspective on multiaccess channels,” *IEEE Transactions on Communications*, vol. 31, pp. 124 – 142, 1985, ISSN: 1557-9654.

[10] Y. Polyanskiy, “A perspective on massive random-access,” in *2017 IEEE International Symposium on Information Theory (ISIT)*. IEEE, 2017, pp. 2523–2527.

[11] O. Ordentlich and Y. Polyanskiy, “Low complexity schemes for the random access gaussian channel,” in *2017 IEEE International Symposium on Information Theory (ISIT)*. IEEE, 2017, pp. 2528–2532.

[12] Y. Polyanskiy, H. V. Poor, and S. Verdú, “Channel coding rate in the finite blocklength regime,” *IEEE Transactions on Information Theory*, vol. 56, no. 5, pp. 2307–2359, 2010.

[13] B. S. Tsybakov and V. A. Mikhailov, “Free synchronous packet access in a broadcast channel with feedback,” *Problemy Peredachi Informatsii*, vol. 14, no. 4, pp. 32–59, 1978.

[14] R. Rivest, “Network control by bayesian broadcast,” *IEEE Transactions on Information Theory*, vol. 33, no. 3, pp. 323–328, 1987.

[15] J. H. Jeon S. W., “Online estimation and adaptation for random access with successive interference cancellation,” in *2020 IEEE International Symposium on Information Theory (ISIT)*. IEEE, 2020, pp. 3049–3054.

[16] B. S. Tsybakov, “Survey of USSR Contributions to Random Multiple-Access Communications,” *IEEE Transactions on Communications*, vol. 31, pp. 143 – 165, 1985, ISSN: 1557-9654.

[17] F. G. Foster, “On the Stochastic Matrices Associated with Certain Queuing Processes,” *The Annals of Mathematical Statistics*, vol. 24, pp. 355 – 360, 1953.

[18] D. Tuninetti and G. Caire, “The throughput of some wireless multiaccess systems,” *IEEE Transactions on Information Theory*, vol. 48, no. 10, pp. 2773–2785, 2002.

[19] O. Galinina, A. Turlikov, S. Andreev, and Y. Koucheryavy, “Stabilizing multi-channel slotted aloha for machine-type communications,” in *2013 IEEE International Symposium on Information Theory*. IEEE, 2013, pp. 2119–2123.