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Citation for published version:

Digital Object Identifier (DOI):
10.1016/j.coastaleng.2020.103737

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
Coastal Engineering

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Aquaculture farms as nature-based coastal protection: Random wave attenuation by suspended and submerged canopies

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Abstract

As the frequency and intensity of storms increase, a growing need exists for resilient shore protection techniques that have both environmental and economic benefits. In addition to producing seafood, aquaculture farms may also provide coastal protection benefits either alone or with other nature-based structures. In this paper, a generalized three-layer frequency dependent theoretical model is derived for random wave attenuation due to presence of biomass within the water column. The biomass can be characterized as submerged, emerged, suspended and floating canopies that can consist of natural aquatic vegetation with potential aquaculture systems of kelp or mussels. The present analytical solutions can reduce to the solutions by Mendez & Losada (2004), Chen & Zhao (2012) and Jacobsen et al. (2019) for submerged rigid aquatic vegetation. The present theoretical model incorporates the motion of these canopies using a cantilever-beam model for slender components and a buoy-on-rope model for elements with concentrated mass and buoyancy. Analytical results are compared with existing laboratory and field datasets for submerged and suspended canopies.

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The theoretical model was then used (in a case study at a field site in Northeastern US) to investigate the capacity of suspended mussel farms with submerged aquatic vegetation (SAV) to dissipate wave energy during a recent storm event. Compared to a dense SAV meadow in shallower water, the suspended aquaculture farms more effectively attenuate random waves with a smaller peak period and the higher frequency components of wave spectrum. The performance of suspended aquaculture farms is less affected by water level changes due to tides, surge and sea level rise, while the wave attenuation performance of SAV decreases with increasing water level due to decreased wave motion near the sea bed. Incorporating suspended aquaculture farms offshore significantly enhance the coastal protection effectiveness of SAV-based living shorelines and extend the wave attenuation capacity over a wider wave period and water level range. The combination of suspended aquaculture farms and traditional living shorelines provides a more effective nature-based coastal defense strategy than the traditional living shorelines alone.

Keywords: Wave attenuation, Random waves, Suspended canopy, Vegetation, Aquaculture farm, Natural coastal defense

1. Introduction

Approximately 40% of the world’s population lives within 100 kilometers of the coast (MEA 2005; Ferrario et al., 2014), and 71% of the coastal population lives within 50 kilometers of an estuary (UNEP 2006). While coastal communities benefit from proximity to seascapes, they are more vulnerable to natural coastal hazards and extreme events from the sea. For example, from 1900 to 2017, 197 hurricanes with 206 landfalls in the USA caused about 2 trillion USD damage (normalized to 2018 value by considering the effects of inflation, wealth, and population), or annually about 17 billion USD (Weinkle et al. 2018). Due to climate change, more frequent and severe storms and rising sea level are likely to occur (Izaguirre et al. 2011; Tebaldi et al. 2012; Ondiviela et al. 2014).

To mitigate storm damage, hard structures such as seawalls, breakwaters,
and bulkheads have been used as coastal defenses. These structures, however, may aggravate land subsidence due to soil drainage, inhibit natural accumulation of sediments by tides and waves, adversely impact water quality, and cause coastal habitat loss (Syvitski et al., 2009; Currin et al., 2010; Pace, 2011; Temmerman et al., 2013; Sutton-Grier et al., 2015). Additionally, these conventional hard engineering defenses are also seriously challenged due to their continual and costly maintenance, as well as their reconstruction and reinforcement to keep up with increasing flood risk are becoming unsustainable (Temmerman et al., 2013). Natural and nature-based infrastructure may be a viable alternative to hardened shoreline protection system with added economic and ecological benefits and ability to adapt to sea level rise and climate change (Borsje et al., 2011; Gedan et al., 2011; Temmerman et al., 2013).

As an example of nature-based infrastructure, living shorelines including a variety of wetland plants, aquatic vegetation, kelp beds and oyster reefs have become a complement to hardened shoreline stabilization. Unlike many hardened coastal protection techniques, living shorelines can mitigate storm damage and erosion while enhancing productive habitat, improving water quality, producing food and adapting to rising sea level (Currin et al., 2010; Scyphers et al., 2011; Davis et al., 2015; Bilkovic et al., 2016; Gittman et al., 2016; Saleh & Weinstein, 2016; Vuik et al., 2016; Moosavi, 2017; Leonardi et al., 2018; Möller, 2019). The protection of coastal ecosystems by wave attenuation is more effective in areas with relatively small tidal ranges (Bouma et al., 2014). Living shorelines at exposed, high-energy sites require structure such as breakwater or sill offshore to damp incident wave energy to sustain health growth of the living organisms (McGehee, 2016).

Aquaculture systems may also act as nature-based infrastructure to attenuate wave energy and produce food at the same time. For example, Plew et al. (2005) observed that a 650 m × 2450 m mussel farm reduced wave energy by approximately 5%, 10%, and 17% at wave frequencies of 0.1, 0.2, and 0.25 Hz, respectively at low sea state. It was found that densely grown kelp may have advantageous wave attenuation characteristics (Mork, 1996). For instance, Mork
[1996] observed a 70% to 85% wave energy reduction across a 258 m long kelp bed (dominated by *Laminaria hyperborea*) with the highest wave attenuation observed during low tide. Unlike the natural kelp beds rooted at the seabed, cultivated kelp is suspended near the surface from a longline ([Peteiro & Freire, 2013] [Peteiro et al., 2016] [Walls et al., 2017] [Campbell et al., 2019] [Grebe et al., 2019] [Zhu et al., 2019]), as shown on Fig. 1. Near surface cultivated kelp may damp more wave energy than bottom-rooted kelp since the wave motion decreases towards the bottom. Kelp can also absorb carbon to mitigate climate change impacts and reduce nutrients to improve water quality, therefore, increase the growth rate of marine species ([Duarte et al., 2017] [Campbell et al., 2019]. Other environmental benefits of kelp and seaweed farming include recycling inorganic nutrients and preventing eutrophication conditions ([Yang et al., 2015] [Stévant et al., 2017] [Xiao et al., 2017] [Campbell et al., 2019]).

Both mussels and kelp are often farmed near the surface on a horizontal type mooring system (Fig. 1). The wave attenuation characteristics of these aquaculture farms can be modeled in a similar way as natural, bottom-rooted submerged and emergent canopies such as kelp forests, seagrasses and salt marshes. In this

![Figure 1: Canopy classification (from left to right): suspended aquaculture farms, floating wetlands, submerged plants, and emergent plants (figure credit: Yu-Ying Chen).](image-url)
study, the aquaculture structures are treated as suspended canopies according to the classification shown on Fig. 1. The classification is based on the vertical position in the water column and the “plant” height relative to the water depth (e.g., Plew, 2011; Huai et al., 2012; Chen et al., 2016; Zhu & Zou, 2017). The horizontal mussel and kelp farms shown on Fig. 1 are placed at an elevation with optimum light, temperature and nutrient conditions within the water column to achieve maximum growth. Fig. 1 also shows a row of nature-based floating wetlands and natural submerged and emergent plants.

Extensive studies have been dedicated to better understanding and predicting wave attenuation by submerged and emergent vegetation as reviewed later in section 2.1. To model the wave attenuation by suspended canopies, Plew et al. (2005) developed a two-layer analytical solution for a floating longline mussel farm based on energy conservation equation with linear wave theory (Dalrymple et al., 1984). They represented random wave conditions using root-mean square wave height and peak wave period. Zhu & Zou (2017) extended the two-layer solution by Kobayashi et al. (1993) for submerged vegetation to a generalized three-layer theoretical solution for suspended and submerged vegetation. Zhu & Zou (2017) found that the wave attenuation by a submerged canopy decreases while the wave attenuation by a floating canopy increases with increasing wave frequency. The wave attenuation by a suspended canopy first increases and then decreases with increasing wave frequency. Combining an OpenFOAM (Higuera et al., 2013) hydrodynamics model with an immersed element vegetation model, Chen & Zou (2019) observed a strong jet formed at the top of a submerged flexible canopy in the opposite direction as the wave. Using a SWASH (Simulating WAves till SHore, Zijlema et al., 2011) model, Chen et al. (2019) investigated the wave-driven circulation cell induced by suspended canopies and found that the vertical position of the canopy also has significant effects on the wave-driven current in the canopy. Recently, SWASH was improved by Suzuki et al. (2019) to consider the drag of horizontal vegetation stems, vegetation canopy porosity and vegetation inertia, which can influence the wave dissipation. The effects of vegetation porosity on wave dissipation is of importance for dense vegetation. As
time domain numerical models, OpenFOAM and SWASH are able to simulate waves with arbitrary frequency shape. However, the existing analytical models developed for suspended canopies in single characteristic frequency waves still need to be extended to frequency dependent models for random waves, which are a better representation of field conditions.

The objective of this study is to develop a generalized three-layer frequency dependent theoretical model for random wave attenuation by submerged and suspended canopies. The analytical wave attenuation model is coupled with cantilever-beam and buoy-on-rope vegetation models to consider the motion of canopies with different type components. The coupled flow and vegetation model is validated with laboratory experimental datasets for submerged canopies (Jacobsen et al., 2019) and laboratory and field datasets for suspended canopies (Seymour & Hanes, 1979). The validated coupled model is then applied in the field near Saco, Maine in the Northeastern USA to investigate the potential of a mussel farm to damp storm during the January 2015 North American blizzard. The effectiveness of using a suspended aquaculture farm alone and in combination with submerged aquatic vegetation (SAV) close to shore for wave attenuation is also investigated.

2. Theory

2.1. Background on analytical wave attenuation models

Theoretical models have been developed to study the wave attenuation characteristics of submerged and emergent canopies by Dalrymple et al. (1984) and Kobayashi et al. (1993). Both studies represented the canopy as arrays of rigid, homogeneous cylinders subject to monochromatic wave action. Assuming the wave energy loss as the work performed by the drag of vegetation, Dalrymple et al. (1984) obtained the wave decay coefficient by solving the energy conservation equation using linear wave theory. By solving the linearized incompressible Euler equations with assumptions of exponentially decayed wave height along the canopy and linearized drag, Kobayashi et al. (1993) obtained the same wave
decay coefficient as Dalrymple et al. (1984). The wave decay coefficient is an explicit function of hydrodynamic conditions and canopy characteristics including blade length, width, and canopy density (defined as blade number per unit area).

Both these analytical solutions have been widely used for calculating wave attenuation by submerged vegetation. The solution by Dalrymple et al. (1984) was modified by Mendez & Losada (2004) to consider random non-breaking and breaking waves propagating over a mildly sloped vegetation seabed by using the unmodified Raleigh distribution method and assuming a narrow-banded wave spectrum. The modification developed by Mendez & Losada (2004) has been implemented in the SWAN (Simulating WAves Nearshore) model by Suzuki et al. (2012), and the MDO (Mellor-Donelan-Oey) wave model for wind-generated waves and swells in deep and shallow waters by Marsooli et al. (2017). Recently, Losada et al. (2016) extended Mendez & Losada (2004) solution for combined wave and currents. The solution by Mendez & Losada (2004) was also used by Garzon et al. (2019) to analyze the wave attenuation by Spartina Saltmarshes in the Chesapeake Bay under storm surge conditions. These models based on the Mendez & Losada (2004) approach are limited to ideal narrow-banded waves. If applied to wide-banded waves, the Mendez & Losada (2004) based models would overestimate the dissipation for the wave components with higher frequency than the characteristic peak frequency and underestimate the dissipation for the wave components with lower frequency than the characteristic peak frequency (Jacobsen et al., 2019).

To investigate the spectral distribution of energy dissipation, Chen & Zhao (2012) developed two analytical frequency dependent wave attenuation models for random waves and rigid vegetation by implementing the energy dissipation of random waves in Hasselmann & Collins (1968) and the joint distribution of wave heights and wave periods proposed by Longuet-Higgins (1983). To derive the wave attenuation solution based on the random waves in Hasselmann & Collins (1968), Chen & Zhao (2012) used the root mean square velocity to linearize the drag force following Madsen et al. (1988) such that $|u| \approx \sqrt{2\sigma_u}$, where $u$ is
the horizontal wave velocity and $\sigma_u$ is the standard deviation of $u$. Recently, Jacobsen et al. (2019) obtained a frequency distributed wave dissipation model by linearizing the drag force such that $|u| \approx \sqrt{8/\pi \sigma_u}$ estimated from 270,000 numerical cases under JONSWAP spectrum so the linearization-induced mean error $|u|/\left(\sqrt{8/\pi \sigma_u}\right) - 1$ is less than 0.01%. Borgman (1967) obtained the same result $|u| \approx \sqrt{8/\pi \sigma_u}$ by minimizing the mean square of the difference between the nonlinear drag and linearized drag for $u$ in normal distribution. The Borgman (1967) method for the drag linearization can also be used for other probability distributions of $u$.

Since most vegetation are flexible, the wave-induced motion of vegetation would reduce the relative velocity between wave-induced flow and vegetation and therefore the drag force, yielding less wave attenuation than rigid vegetation (Mullarney & Henderson 2010; van Veelen et al. 2020). To consider the effects of vegetation motion, one common practice is using a reduced bulk drag coefficient (e.g., Paul & Amos 2011; Jadhav et al. 2013; Pinsky et al. 2013; Anderson & Smith 2014; Hu et al. 2014; Zeller et al. 2014; Möller et al. 2014; Losada et al. 2016; Wu et al. 2016; Marsooli et al. 2017; Nowacki et al. 2017; Garzon et al. 2019; van Veelen et al. 2020). The bulk drag coefficient should be dependent on the Cauchy number ($Ca$) incorporating the blade flexural rigidity related to vegetation motion. However, most of the empirical formulae in literature for the bulk drag coefficient of flexible vegetation are expressed as a function of Reynolds number ($Re$) or Keulegan–Carpenter number ($KC$) without incorporating the blade flexural rigidity. Consequently, the empirical formulae of bulk drag coefficient have different expressions for vegetation with different flexural rigidities for the same set of $Re$ and $KC$ numbers. This introduces uncertainty in modelling wave attenuation by flexible vegetation. To apply the original (unreduced) drag coefficient as previous studies and incorporate the effects of blade motion at the same time, Luhar et al. (2017) proposed a reduced, effective blade length instead of a reduced drag coefficient to incorporate the effects of blade motion. The empirical formula for the effective blade length is dependent on blade flexural rigidity, therefore, can be readily applied to vege-
tation of various flexural rigidities. The formula for the effective blade length was recently modified by considering the effects of rigid sheath of seagrass (Lei & Nepf, 2019b) and applied to combined waves and currents conditions (Lei & Nepf, 2019a). These empirical approaches do not need to resolve blade motion and therefore improve the computational efficiency by reducing the iterative computation for coupling wave and vegetation motion. These approaches, however, require numerous datasets to derive the formulae for bulk drag coefficient and effective blade length. If the blade motion is directly resolved by the model, then the original unreduced drag coefficient and blade length can be used directly without modification. Therefore, the number of experiments and model runs to calibrate the bulk drag coefficient and the uncertainty associate with the bulk drag coefficient are reduced.

To resolve the blade motion, Asano et al. (1992) simplified the blade motion as an oscillator with one degree of freedom by assuming blade deflection is linearly distributed along the length and also averaging deflection along the length. This method was then extended to consider irregular waves, wave reflection, and evanescent modes by Méndez et al. (1999) for submerged vegetation. To analyze the depth dependence of the blade deflection as well as its effects on wave dissipation, Mullarney & Henderson (2010) modeled the blade as a continuous beam with Euler-Bernoulli techniques, where the governing equation for the blade motion is simplified as a balance between the flexural rigidity-induced restoring force and the drag force, assuming the inertia force and buoyancy are negligible. Recently, Henderson (2019) extended this model by including buoyancy but still neglected the inertia force, therefore the model is valid only for blades with small cross sectional area. In addition, the mass of vegetation influences the natural frequency of the vegetation and further impacts the blade motion as well as the resonant conditions. To fully consider the gravity, buoyancy, structural damping, bending stiffness, virtual buoyancy, friction, drag and inertia forces, numerical models are often used to simulate the blade dynamics (e.g., Zeller et al., 2014; Zhu & Chen, 2015; Luhar & Nepf, 2016; Leclercq & de Langre, 2018; Zhu et al., 2018; Chen & Zou, 2019; Zhu et al.)
Recently, Zhu et al. (2020) used a cable model to capture the asymmetric “whip-like” blade motion and proposed mechanisms for the asymmetric blade motion in symmetric waves.

To derive a generalized wave attenuation model for suspended and submerged canopies, the water column is divided into 3 layers with model set-up in section 2.2. The effects of canopy motion are incorporated by resolving the motion of individual canopy component using a cantilever-beam model or a buoy-on-rope model based on the type of the canopy component in section 2.3. These two structural dynamics models consider inertia force and are therefore applicable for large diameter structure such as mussel droppers. The frequency dependent theoretical wave attenuation model incorporating canopy motion is developed in section 2.4.

2.2. Model set-up

The mathematical approach is based on the three-layer model set-up shown on Fig. 2. As shown on Fig. 2, the horizontal coordinate, $x$, is positive in the direction of wave propagation (assumed to be perpendicular to the coast), with $x = 0$ at the leading edge of the canopy. The horizontal length of the canopy is defined as $L_v$ such that $x = L_v$ at the end of the canopy. The vertical coordinate, $z$, is positive upward with $z = 0$ at the still water level (SWL).

The water column is divided into three layers with Layer 1 above the canopy, Layer 2 within the canopy, and Layer 3 below the canopy. The initial static thicknesses for each layer are denoted by $d_1$, $d_2$, $d_3$, respectively. The thickness of Layer 2 ($d_2$) also named the canopy height, is defined as the average submerged length of the canopy components. The water depth from the SWL is defined as $h = d_1 + d_2 + d_3$, where the seafloor is located at $z = -h$ and assumed to be horizontal. This generalized three-layer model can be used to analyze the wave attenuation characteristics of the following four types of canopy configurations: (1) submerged ($d_1 \neq 0$ and $d_3 = 0$), (2) emergent ($d_1 = 0$ and $d_3 = 0$), (3) suspended in the water column ($d_1 \neq 0$ and $d_3 \neq 0$), and (4) floating on the surface ($d_1 = 0$ and $d_3 \neq 0$).
Figure 2: Definition sketch of variables and coordinate system for the three-layer theoretical model of waves propagating over a canopy. The coordinate system \((x, z)\) with the origin at the leading edge of the canopy \((x = 0)\) and the still water level \((\text{SWL}, z = 0)\), where \(x\) is positive in the wave propagation direction from left to right and \(z\) is positive upward. The water column is divided into three layers by the canopy. The thicknesses of layer 1, 2, and 3 are denoted as \(d_1\), \(d_2\), and \(d_3\), respectively. The canopy length is \(L_c\). The water depth from the SWL is defined as \(h = d_1 + d_2 + d_3\).

At many sites, sea surface profiles are better represented by random waves, which can be formulated as a superposition of monochromatic waves with a set of random phases. Thus, the water elevation can be expressed as

\[
\eta = \sum_{i=1}^{\infty} a_i \cos (k_i x - \omega_i t + \psi_i),
\]

where \(t\) is time, \(a_i\) is the wave amplitude, \(k_i\) is the wave number, \(\omega_i\) is the angular frequency and \(\psi_i\) is the random phase of the \(i\)th monochromatic wave component. As a sum of infinite independent random variables, the water elevation tends toward a normal distribution according to the central limit theorem. Assuming that the random phase is distributed uniformly on \((0, 2\pi)\), the water elevation is normally distributed with a zero mean \(\langle \eta \rangle = 0\), where \(\langle \rangle\) indicates expected value) and a variance of \(\sigma_{\eta}^2 = \langle \eta^2 \rangle = \sum_{i=1}^{\infty} a_i^2/2 = \int_0^\infty S_{\eta\eta}(\omega, x) d\omega\), where \(S_{\eta\eta}(\omega, x)\) is the wave spectrum. For the convenience of expression, the index of summation \(i\) is omitted and \(\omega\) is used to indicate the summation such
that

\[ \eta = \sum_{\omega} a(kx - \omega t + \psi). \]  

(2)

According to linear wave theory [Dean & Dalrymple 1991], the wave number and angular frequency satisfy the dispersion relation, \( \omega^2 = gk \tanh kh \), where \( g \) is the gravitational acceleration. The wave orbital velocity \( (u) \) at a given level \( z \) is then written as

\[ u = \sum_{\omega} a\omega \Gamma \cos(kx - \omega t + \psi), \]  

(3)

where \( \Gamma = \cosh k(h + z) / \sinh kh \) when \( z \leq \eta \) and \( \Gamma = 0 \) when \( z > \eta \).

2.3. Models for the motion of canopy components

The wave-induced motion of a canopy component is simulated by different models depending on the morphology and physical properties of the species. In this paper, we introduce cantilever-beam and buoy-on-rope models. The cantilever-beam model is applicable for slender species such as vegetation blades, kelp blades, and mussel droppers (Fig. 3). The buoy-on-rope model is applicable for species with concentrated mass and buoyancy supported by a tethered stipe whose mass and stiffness can be ignored, e.g., the bull kelp, *Nereocystis luetkeana* (Fig. 3).
2.3.1. Cantilever-beam model

The individual component of the canopies such as seagrass meadow, kelp forest and mussel farms is modeled as a slender cantilever beam (Fig. 3), referred as a blade hereinafter. A typical blade having the averaged geometrical and physical properties of the canopy components is used to represent the canopy components. To simulate the large-amplitude deflection of a flexible blade, Zhu et al. (2020) introduced a cable model that can capture the asymmetric “whip like” motion of a flexible blade (Luhar & Nepf, 2016). To obtain the analytical solution for the horizontal displacement ($\xi$) of the blade, the governing equations in Zhu et al. (2020) are linearized by assuming a small-amplitude motion such that the vertical displacement of the blade is negligible. The horizontal displacement, $\xi(s, t)$ is a function of time $t$ and the distance $s$ along the blade length from the fixed end. The relation between the local coordinate $s$ and the global coordinate $z$ is given by

$$z = \begin{cases} -d_1 - d_2 + s, & \text{blade fixed at the bottom end}, \\ -d_1 - s, & \text{blade fixed at the tip end}. \end{cases}$$ (4)

Neglecting tension and buoyancy, the linearized governing equation is given by

$$\rho_v A_c \dddot{\xi} + EI \dddot{\xi} = \rho_w A_c \ddot{u} + \frac{1}{2} C_d \rho_w b |u - \ddot{\xi}| (u - \ddot{\xi}) + C_m \rho_w A_c (\dot{u} - \dot{\xi}),$$ (5)

where the dot (\(\dot{\cdot}\)) indicates derivative with respect to $t$, the prime (\(\prime\)) indicates derivative with respect to $s$, $\rho_v$ is the water density, $\rho_w$ is the blade mass density, $b$ is the projected blade width, $A_c$ is the blade cross sectional area, $E$ is the Young’s modulus of the blade, $I$ is second moment of the blade cross sectional area, $C_d$ is the drag coefficient and $C_m$ is the added mass coefficient. The terms on the right-hand side of (5) are virtual buoyancy, drag and added mass force per unit length modified from the Morison formula (Morison et al., 1950). To obtain an analytical solution to (5), the nonlinear drag $1/2 C_d \rho_w b |u - \ddot{\xi}| (u - \ddot{\xi})$ is linearized as $c(u - \ddot{\xi})$, where the linearization coefficient ($c$) is calculated using the Borgman (1967) method. Substituting (3) into (5) yields

$$m \dddot{\xi} + c \dddot{\xi} + EI \dddot{\xi} = \sum_\omega a_\omega \Gamma [c \cos(kx - \omega t + \psi) + \omega m_1 \sin(kx - \omega t + \psi)],$$ (6)
where \( m = (\rho + C_m \rho_o) A_c \) and \( m_l = (1 + C_m) \rho_u A_c \). The boundary conditions for a cantilever beam are given by \( \xi(0, t) = 0, \xi'(0, t) = 0, \xi''(l, t) = 0 \) and \( \xi'''(l, t) = 0 \). Using a normal mode approach (Rao, 2007), the solution for the blade displacement is obtained in Appendix A as

\[
\xi = \sum_\omega a_\Gamma [\gamma_s \sin(kx - \omega t + \psi) + \gamma_c \cos(kx - \omega t + \psi)],
\]

where \( \gamma_s \) and \( \gamma_c \) are the transfer functions given by

\[
\gamma_s = \frac{\omega}{\Gamma} \sum_{n=1}^{\infty} \phi_n \frac{\omega I_n (\lambda_n^2 - \omega^2) - D_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2}
\]

and

\[
\gamma_c = \frac{\omega}{\Gamma} \sum_{n=1}^{\infty} \phi_n \frac{D_n (\lambda_n^2 - \omega^2) + \omega I_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2},
\]

where \( \phi_n = (\cos \mu_n l + \cosh \mu_n l) (\sin \mu_n s - \sinh \mu_n s) + (\sin \mu_n l + \sinh \mu_n l) (\cosh \mu_n s - \cos \mu_n s) \) is the \( n \)th normal mode of the cantilever beam with \( \mu_n \) being the \( n \)th solution of \( 1 + \cos \mu l \cosh \mu l = 0, \lambda_n = \mu_n^2 \sqrt{\int_0^l E I \phi_n^2 ds / \int_0^l m \phi_n^2 ds} \) is the \( n \)th natural frequency of the blade, \( 2\zeta_n \lambda_n = \int_0^l \phi_n^2 ds / \int_0^l m \phi_n^2 ds, D_n = \int_0^l c_\phi \phi_n ds / \int_0^l m \phi_n^2 ds \) and \( I_n = \int_0^l m l \Gamma \phi_n ds / \int_0^l m \phi_n^2 ds \). Since \( \Gamma \) is expressed in terms of \( z \) and \( \phi_n \) is expressed in terms of \( s \), the relation between \( s \) and \( z \) in (4) is required to calculate the integral \( \int_0^l \Gamma \phi_n ds \).

The relative velocity of flow to blade \( u_r = u - \xi \) is given by

\[
u_r = \sum_\omega a_\Gamma [(1 + \gamma_s) \cos(kx - \omega t + \psi) + \gamma_c \sin(kx - \omega t + \psi)].
\]

According to the central limit theorem, the relative velocity also asymptotically approaches a normal distribution with zero mean (\( \langle u_r \rangle = 0 \)) and the variance

\[
\sigma_{u_r}^2 = \langle u_r^2 \rangle = \int_0^\infty \omega^2 \Gamma^2 \left[ (1 + \gamma_s)^2 + \gamma_c^2 \right] S_{\eta\eta}(\omega, x) d\omega.
\]

Hence, the probability density function of \( u_r \) is given by

\[
p(u_r) = \frac{1}{\sigma_{u_r} \sqrt{2\pi}} e^{-\frac{u_r^2}{2\sigma_{u_r}^2}}.
\]

Using the Borgman (1967) method, the linearization coefficient \( c \) is obtained by minimizing the mean square difference between the nonlinear and linearized
drag so that \( \partial \int_{-\infty}^{\infty} (1/2C_d \rho_w b |u_r| u_r - cu_r)^2 p (u_r) du_r / \partial c = 0, \) yielding

\[
c = \frac{1}{2} C_d \rho_w b \int_{-\infty}^{\infty} |u_r| u_r^2 p (u_r) du_r = \frac{1}{2} C_d \rho_w b \sqrt{\frac{8}{\pi} \sigma_{u_r}}. \tag{13}
\]

The linearization coefficient can be obtained iteratively through the following procedure. Starting from a static blade, an initial \( c \) is calculated from equation (13) with (11) by assuming \( \gamma_s = 0 \) and \( \gamma_c = 0 \). Once the blade displacement is obtained, \( c \) can be recalculated from (13) and (11) with (8) and (9). Using the new value of \( c \), the blade displacement can be updated. The procedure is repeated until a convergent solution is achieved.

2.3.2. Buoy-on-rope model

The bull kelp (\textit{Nereocystis luetkeana}) is used as an example to describe the buoy-on-rope model (Denny et al., 1997), which is also used for other species such as \textit{Macrocystis pyrifera} (Utter & Denny, 1996). The pneumatocyst (the ball-shape “float” structure) of \textit{Nereocystis luetkeana} is modeled as a buoy and the stipe is modeled as a rope (Fig. 3). Therefore, the canopy component is modeled as a buoy attached to seabed by a thin, straight, non-buoyant rope. The inertia, drag and buoyancy act at the buoy center, \( z_c = -d_1 - d_2/2 \), where the canopy height \( d_2 \) is the diameter of the buoy. The horizontal displacement of the buoy and the fluid velocity at the buoy center is used to calculate the forces. The governing equation for buoy-on-rope model is given by

\[
\rho_v V \dddot{\xi} + \frac{(\rho_w - \rho_v) V}{R} \ddot{\xi} = \rho_w V \dot{u} (z_c) + \frac{1}{2} C_d \rho_w A_p \left| u (z_c) - \dot{\xi} \right| \left[ u (z_c) - \dot{\xi} \right] + C_m \rho_w V \left[ \dot{u} (z_c) - \dot{\xi} \right] + C_m \rho_w V \left[ \dot{u} (z_c) - \dot{\xi} \right], \tag{14}
\]

where \( R \) is the length of the tethered rope, \( V \) is the volume of the buoy with projected area of \( A_p \). Similarly, the nonlinear drag force \( 1/2C_d \rho_w A_p |u (z_c) - \dot{\xi}| \left| u (z_c) - \dot{\xi} \right| \) is linearized as \( C \left[ u (z_c) - \dot{\xi} \right] \), where \( C \) is obtained using the Borgman (1967) method. Substituting (3) into (14) yields

\[
M \dddot{\xi} + C \ddot{\xi} + K \xi = \sum_\omega a_\omega \Gamma (z_c) [C \cos(kx - \omega t + \psi) + \omega M f \sin(kx - \omega t + \psi)], \tag{15}
\]
where \( M = (\rho_v + C_m \rho_w) V, \) \( K = (\rho_w - \rho_v) V g / R, \) and \( M_I = (1 + C_m) \rho_w V. \) The solution for (15) is

\[
\xi = \sum \omega a \Gamma [\gamma_s \sin(kx - \omega t + \psi) + \gamma_c \cos(kx - \omega t + \psi)],
\]

where \( \gamma_s \) and \( \gamma_c \) are the transfer functions given by

\[
\gamma_s = \frac{\Gamma (z_c) \omega M_I (\lambda^2 - \omega^2) - C(2\zeta \lambda \omega)}{(\lambda^2 - \omega^2)^2 + (2\zeta \lambda \omega)^2},
\]

and

\[
\gamma_c = \frac{\Gamma (z_c) \omega C (\lambda^2 - \omega^2) + \omega M_I (2\zeta \lambda \omega)}{(\lambda^2 - \omega^2)^2 + (2\zeta \lambda \omega)^2},
\]

where \( \lambda = \sqrt{K/M} \) and \( 2\zeta \lambda = C/M. \) Similarly, the relative velocity \( u_r = u - \xi \) asymptotically approaches a normal distribution with zero mean and the variance \( \sigma_{u_r}^2 \) in a similar expression as (11) except for the transfer functions \( \gamma_s \) and \( \gamma_c, \) which are calculated using (17) and (18). Thus, the linearization coefficient \( (C) \) is given by

\[
C = \frac{1}{2} C_d \rho_w A_p \sqrt{\frac{8}{\pi} \sigma_{u_r}},
\]

which is obtained iteratively using the same procedure for the cantilever-beam model.

2.4. Solutions for random wave attenuation

Following Dalrymple et al. [1984], Kobayashi et al. [1993], and Mendez \& Losada [2004], the wave attenuation is assumed to come from the work of the canopy-induced drag force. The inertia force has a negligible contribution to wave attenuation since the mathematical expectation of the work due to the inertia force is zero because the relative acceleration and the relative velocity are out of phase in linear waves. The vertical frictional force is assumed negligible when compared with the horizontal drag force. The wave reflection from the canopy is also assumed negligible since the wave reflection has limited contributions to the wave attenuation for both submerged vegetation (Mendez \& Losada [2004]) and suspended canopies (Seymour \& Hanes [1979]). Some wave
energy is converted into the kinematic and potential energy of the canopy at the beginning. However, once the canopy motion becomes steady, the energy needed to maintain the steady motion can be assumed negligible because the structural damping of the canopy components is negligible (Asano et al., 1992; Méndez et al., 1999). Lacking data, the velocity reduction in the canopy (Lowe, 2005), the sheltering effects (Raupach & Thom, 1981; Abdelrahman, 2007; Etminan et al., 2019), and the porosity effects (Mei et al., 2011; Nepf, 2011; Liu et al., 2015; Arnaud et al., 2017; Suzuki et al., 2019) are not considered. Using the linearized drag force, the energy conservation equation can be written as

\[
\frac{\partial}{\partial x} \int_0^\infty \rho_w g S_{\eta\eta}(\omega, x) c_g d\omega = - \int_{-d_1}^{-d_1} \left\{ N \frac{1}{2} C_d \rho_w b \sqrt{\frac{8}{\pi}} \sigma_u u_r^2 \right\} dz, \tag{20}
\]

where \( c_g = (\omega/k)(1+2kh/\sinh 2kh)/2 \) is the group velocity and \( N \) is the number of canopy components per unit horizontal area (also referred to as the canopy density). Substituting (11) into (20) yields the transmitted wave spectrum at distance \( x \) in relation to the incident wave spectrum at \( x = 0 \),

\[
S_{\eta\eta}(\omega, x) = S_{\eta\eta}(\omega, 0) e^{-2\beta(\omega)x}, \tag{21}
\]

where the frequency dependent decay coefficient \( \beta(\omega) \) is given by

\[
\beta(\omega) = \frac{2\sqrt{2} N k^2 \sinh^2 kh}{\sqrt{\pi} \omega (2kh + \sinh 2kh)} \int_{-d_1}^{-d_1} C_d \sigma_u \Gamma^2 \left[ (1 + \gamma_x)^2 + \gamma_c^2 \right] dz. \tag{22}
\]

The transfer functions \( \gamma_x \) and \( \gamma_c \) are selected based on the structural dynamics model used for the canopy motion. To evaluate the effect of the canopies on wave attenuation, the wave spectral dissipation ratio (SDR) and wave energy dissipation ratio (EDR) are used and defined as

\[
SDR = 1 - \frac{S_{\eta\eta}(\omega, L_v)}{S_{\eta\eta}(\omega, 0)}, \tag{23}
\]

and

\[
EDR = 1 - \frac{\int_0^\infty S_{\eta\eta}(\omega, L_v) d\omega}{\int_0^\infty S_{\eta\eta}(\omega, 0) d\omega}, \tag{24}
\]

respectively.
3. Model-Data comparison

3.1. Submerged canopy

The model results were first compared with the laboratory experiments by Jacobsen et al. (2019) for a submerged canopy consisting of artificial vegetation. The wave conditions were based on a single peaked JONSWAP spectrum with a peak enhancement factor $\gamma = 3.3$ and peak wave period $T_p = 1.15$ s. The incident significant wave height at the leading edge of the canopy was $H_{s0} = 3.7$ cm. The water depth was $h = 0.685$ m.

The artificial vegetation was made of 4 mm-wide polypropylene blades with $\rho_v \approx 920$ kg/m$^3$ and $E \approx 0.3$ GPa (Ghisalberti & Nepf, 2002). Four blades were taped to a 6 mm-diameter PVC dowel and 60 mm above the bed. The canopy was 7.5 m long with a density of 566 dowels/m$^2$ therefore 2264 blades/m$^2$. The blade length was 20, 40, and 60 cm such that $d_2/h = \{0.38, 0.67, 0.96\}$. The blade thickness was 0.12, 0.2, 0.5 and 1.0 mm for the 20 cm-long blade and 0.5 mm for the other blades. More details of the experiments can be found in Jacobsen et al. (2019).

Based on the datasets for rigid flat plates in oscillatory flows (Keulegan & Carpenter, 1958; Sarpkaya & O’Keefe, 1996) with $1.7 \leq KC \leq 118.2$, Luhar & Nepf (2016) derived the drag coefficient and added mass coefficient,

$$C_d = \max(10KC^{-1/3}, 1.95)$$

and

$$C_m = \min(C_{m1}, C_{m2}),$$

where $C_{m1} = \begin{cases} 
1 + 0.35KC^{2/3}, & KC < 20, \\
1 + 0.15KC^{2/3}, & KC \geq 20 
\end{cases}$ and $C_{m2} = 1 + (KC - 18)^2/49$ as described in Luhar (2012). Equations (25) and (26) are robust in calculating the hydrodynamic forces acting on flexible blades in regular waves (Zhu et al., 2020). To apply (25) and (26) to random waves, the $KC$ number is calculated using the significant relative velocity (2$\sigma_u$) as $KC = 2\sigma_u T_p/b$.

The vegetation blade is modeled as a cantilever beam so that the wave attenuation model incorporating the cantilever beam model is used to calculate
the wave decay coefficient, $\beta$. The model results using frequency dependent $C_d$ and $C_m$ in (25) and (26) as well as constant $C_d = 1.95$ and $C_m = 1$ are compared with the datasets of Jacobsen et al. (2019) on Fig. 4. It is noted that

![Graphs showing comparisons of calculated frequency ($f$) dependent wave decay coefficient ($\beta$) by the present model and the data (black dotted lines) from Jacobsen et al. (2019). The model results using frequency dependent and constant drag coefficient ($C_d$) and added mass coefficient ($C_m$) are denoted by red solid and blue dashed lines, respectively. The submerged canopies with blade lengths ($l$) of 20, 40 and 60 cm and thicknesses ($d$) of 0.12, 0.20, 0.5 and 1.0 mm are subjected to random waves of JONSWAP spectrum with peak enhancement factor $\gamma = 3.3$, peak wave period $T_p = 1.15$ s (vertical dashed black line) and incident significant wave height of 3.7 cm at a water depth $h = 0.685$ m with normalized blade length ($l/h$) of 0.38 (a-d), 0.67 (e) and 0.96 (f). The canopy density is 566 shoots/m$^2$ (2264 blades/m$^2$).]
$C_d = 1.95$ is the minimum drag coefficient for (25).

The model results are in a good agreement with the data with the root-mean-square-error (RMSE) of about 0.002 for the 20 cm-long blades ($l/h = 0.38$) with thickness $d \leq 0.5$ mm (Fig. 4a-c). For the thickest blades with $d = 1$ mm, the decay coefficient is slightly overestimated for the lower frequency wave components ($f < 0.8$ Hz) resulting in a larger RMSE of 0.0032 (Fig. 4d). One possible reason is that the drag coefficient calculated using equation (25) might be overestimated for the thicker blades whose thickness-width ratio has reached 0.25 and much larger than the thickness-width ratio ($< 0.1$) of the experimental plates for the formula (25). The thicker blades are expected to have a smaller drag coefficient due to increased Reynolds number. Thus, the model results can be improved by using a smaller drag coefficient. For instance, the RMSE for the 1 mm-thick blades is reduced to 0.0016 by using $C_d = 1.95$ (Fig. 4d).

For the longer blades that are nearly emergent ($l/h \geq 0.67$), the model results calculated with frequency dependent hydrodynamic coefficients underestimate the observation with RMSE=0.0046 and 0.0073 for $l = 40$ cm ($l/h = 0.67$) and 60 cm ($l/h = 0.96$), respectively, as shown on Fig. 4e and f) possibly due to the simplification of the cantilever beam model. Neglecting the large deflection-induced geometrical non-linearity, net buoyancy, and the net buoyancy-induced tension would underestimate the restoring capacity of the blades. Thus, the simplified model may overestimate the blade motion resulting in a smaller wave attenuation. This underestimation of wave attenuation is more obvious for longer blades because the effects of the large deflection-induced geometrical non-linearity, the net buoyancy and the net buoyancy-induced tension are more significant for longer blades. Compared to the shorter blade ($l = 20$ cm), the longer blade ($l \geq 40$ cm) is more flexible, therefore, the blade motion follows the flow more closely so that the relative velocity between the longer blade and flow is smaller, resulting in a larger $C_d$. Therefore, using a smaller $C_d = 1.95$ enhances the underestimation as indicated by RMSE=0.0055 and 0.0177 for the $l = 40$ cm ($l/h = 0.67$) and 60 cm ($l/h = 0.96$), respectively on Fig. 4e and f). A more precise formula for the hydrodynamic coefficients in random
waves is desired for the nearly emergent canopies with \(l/h \geq 0.67\). However, the hydrodynamic coefficients in (25) and (26) as well as the constant hydrodynamic coefficients work well for the submerged vegetation \((l/h \leq 0.38)\) with a small RMSE of about 0.002.

3.2. Suspended canopy

The model results were also compared with the laboratory and field experiments by Seymour & Hanes (1979) for a suspended canopy consisting of spherical buoys. The field experiments for a suspended canopy consisting of arrays of tethered sphere buoys (Fig. 5) were conducted in San Diego Bay, California, USA. The half-scale model tests for the field experiments were conducted in the 40-m long Wind Wave Channel at the Hydraulics Laboratory of Scripps Institution of Oceanography (Seymour & Hanes 1979). The properties of the canopies in the laboratory and field experiments are shown in Table 1.

For the laboratory experiments, the incident significant wave height was 0.069 – 0.176 m and the peak frequency was 0.19 – 0.883 Hz. For the field experiments, two storms were observed on Jan 22, 1976 and Feb 9, 1976. The measured significant wave height was 0.17 – 0.44 m. The drag coefficient and
Table 1: Properties for the suspended canopies consisting of sphere components in the laboratory and field experiments (Seymour & Hanes [1979]).

<table>
<thead>
<tr>
<th></th>
<th>In the lab (half scale)</th>
<th>In the field (full scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere mass density [kg/m$^3$]</td>
<td>40</td>
<td>85</td>
</tr>
<tr>
<td>Sphere diameter [cm]</td>
<td>15.8</td>
<td>29.2</td>
</tr>
<tr>
<td>Depth of sphere center [cm]</td>
<td>7.36 – 15.24</td>
<td>21.9</td>
</tr>
<tr>
<td>Effective tether length [cm]</td>
<td>83.8</td>
<td>168.0</td>
</tr>
<tr>
<td>Sphere spacing (along canopy length) [cm]</td>
<td>31.6</td>
<td>58.41</td>
</tr>
<tr>
<td>Sphere spacing (along canopy width) [cm]</td>
<td>31.6</td>
<td>58.41</td>
</tr>
<tr>
<td>Canopy width (perpendicular to wave direction) [m]</td>
<td>2.39</td>
<td>46</td>
</tr>
<tr>
<td>Canopy length (along wave direction) [m]</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>Water depth [m]</td>
<td>1.78</td>
<td>8</td>
</tr>
</tbody>
</table>

The added mass coefficient for the tethered spheres are assumed as $C_d = 0.5$ and $C_m = 0.5$, respectively.

The calculated transmitted wave spectrum and spectral dissipation ratio ($SDR$) are shown on Fig. [4]. The calculated transmitted wave spectrum follows the shape of the incident wave spectrum. The $SDR$ for the suspended canopy first increases and then decreases with increasing wave spectrum as expected.

The comparison between the calculated and measured energy dissipation ratio ($EDR$) is shown on Fig. [7]. Good agreement (RMSE=0.073) between model and data indicates that the present generalized analytical solutions are also applicable to suspended aquaculture farms with simple structures in other forms, such as cylinders, as long as the appropriate hydrodynamic coefficients are available.

4. Case study at the field site

The present frequency dependent theoretical model is now applied to analyze the wave attenuation capacity of suspended aquaculture farms at a field site and compared with that of submerged aquatic vegetation, as well as a combination of these two nature-based shore protection schemes.
Figure 6: Comparisons between calculated and measured transmitted wave spectrum \( S_{\eta_\eta} \) as well as spectral dissipation ratio \( SDR \) versus wave frequency \( f \) for suspended canopies with spheres in (a) laboratory and (b) field experiments by Seymour & Hanes (1979). The incident significant wave height is \( H_{\phi_0} \) and the peak period is \( T_p \).

Figure 7: Comparisons between the calculated and measured wave energy dissipation ratio \( EDR \) for laboratory (blue +) and field experiments (red \( \times \)) by Seymour & Hanes (1979).
The study site (42°28′2″N, 70°21′2″W) is located at Saco Bay, Maine, USA as shown on Fig. 8 with a water depth of about 10.6 m. The January 2015 North American blizzard was a powerful and destructive extratropical storm that swept across the Saco Bay and along the coast of the Northeastern United States in January of 2015. To assess coastal flood risk and sea level rise effects during this storm event, Xie et al. (2019) constructed an integrated atmosphere-ocean-coast and overtopping-drainage modeling framework based on the coupled tide, surge and wave model, SWAN+ADCIRC. The wave spectrum and water level conditions output from the SWAN+ADCIRC model (Xie et al., 2019) are used to drive the present theoretical model. The canopies are oriented to be parallel to the dominant wave direction so that the present 1-D solutions can be applied.

4.1. Properties for the mussel farm and submerged aquatic vegetation

The mussel farm is simplified as arrays of cylinders which represent the mussel droppers. The cylinders have similar mechanical and hydrodynamic performances to the actual droppers. In this study, the geometric and physical properties of the cylinders are based on the measurements of the live droppers at the University of New Hampshire nearshore multi-trophic aquaculture site in the Gulf of Maine, USA (Knysh et al., 2020). The measured dropper diameter
was 0.13 m, mass per unit length was 7.53 kg/m, and flexural rigidity was 0.79
N/m² on July 3 (Knysh et al., 2020), which was about 160 days (years are not
considered) after the January 2015 North American blizzard arrived at the study
site. The geometrical properties of the droppers in January were estimated by
assuming a growth rate of 7.5% per 40 days (Lauzon-Guay et al., 2006; Gagnon
& Bergeron, 2017), which indicates that the diameter of the mussel dropper in
January was about 75% of that in July. Therefore, the cylinder diameter was
taken as \( b = 0.10 \) m, the mass per unit length was assumed as \( \rho_v A_c = 4.46 \) kg/m,
and the flexural rigidity was assumed as \( EI = 0.28 \) N/m². The cylinders are
assumed to be submerged half meter below the water surface so that \( d_1 = 0.5 \)
m. The length of the mussel dropper is assumed as \( l = 8 \) m following Plew
et al. (2005) and Stevens et al. (2007) for a similar water depth. A sparse
configuration with 0.06 droppers/m² (Plew et al., 2005; Gagnon & Bergeron,
2017) and a dense configuration with 0.125 droppers/m² (e.g., mussel droppers
are 0.5 m apart and the longline interval is about 16 m) are compared. Following
Plew et al. (2009), Dewhurst (2016) and Knysh et al. (2020), the drag coefficient
and added mass coefficient are assumed as \( C_d = 1.3 \) and \( C_m = 1 \), respectively,
which are also comparable to the values in Raman-Nair & Colbourne (2003),
Raman-Nair et al. (2008), Stevens et al. (2008), Plew et al. (2009), Gagnon &
Bergeron (2017) and Landmann et al. (2019).

The SAV is modeled as a rectangular plate based on the properties of Zostera
marina, which is a common SAV in the Gulf of Maine, USA (Mattila et al., 1999;
Gaeckle & Short, 2002; Beal et al., 2004; Neckles et al., 2005; Beem & Short,
2009; Newell et al., 2010). The length of Zostera marina ranges from 10 to 150
cm and the shoot density is about 50 – 1100 shoots/m² with 3 – 7 blades per
shoot (Abdelrhman, 2007; Beem & Short, 2009; Boström & Bonsdorff, 1997;
Gaeckle & Short, 2002; Mattila et al., 1999; Ondiviela et al., 2014). In January,
however, the averaged blade length of Zostera marina is about 16 cm (Gaeckle
& Short, 2002; Ondiviela et al., 2014). Thus, the SAV blade length is assumed as
16 cm. The corresponding blade width and thickness as well as the sheath length
and width are estimated using the empirical formula provided by Abdelrhman,
[2007], which yields a blade width of 3.7 mm, blade thickness of 0.11 mm, sheath length of 8 cm and sheath width of 3.4 mm. Following [Abdelrahman, 2007], the mass density is assumed $\rho_v = 700 \text{ kg/m}^3$. The Young’s modulus is assumed $E = 0.26 \text{ GPa}$ for the blades based on the measurements by [Fonseca et al., 2007]. The sheath is considered rigid following [Lei & Nepf, 2019b]. A sparse SAV meadow with 200 shoots/m$^2$ and a dense meadow with 400 shoots/m$^2$ are used in the study to investigate the variation of wave attenuation with vegetation density. The number of blades per shoot is assumed to be 5 so that there are 1000 blades/m$^2$ for the sparse configuration and 2000 blades/m$^2$ for the dense configuration. Due to small blade width and large significant wave height and peak period in a storm event, the calculated $KC$ number is greater than 135, yielding a constant drag coefficient of 1.95 based on equation (25). The data comparison in section 3.1 has shown that $C_d = 1.95$ and $C_m = 1$ work well for submerged vegetation with $l/h < 0.38$ (Fig. 4). Therefore, the drag coefficient and added mass coefficient are assumed $C_d = 1.95$ and $C_m = 1$, respectively, for this case study. The properties of the mussel farm and SAV meadow are summarized in Table 2. In this study, both mussel droppers and SAV are modeled as cantilever beams.

<table>
<thead>
<tr>
<th>Mussel farm</th>
<th>SAV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canopy component</td>
<td>Mussel dropper</td>
</tr>
<tr>
<td>Component properties</td>
<td>Length: 8 m</td>
</tr>
<tr>
<td></td>
<td>Diameter: 0.10 m</td>
</tr>
<tr>
<td></td>
<td>$EI = 0.28 \text{ N/m}^2$</td>
</tr>
<tr>
<td></td>
<td>$\rho_v A_c = 4.46 \text{ kg/m}$</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Canopy density</td>
<td>Sparse: 0.060 droppers/m$^2$</td>
</tr>
<tr>
<td></td>
<td>Dense: 0.125 droppers/m$^2$</td>
</tr>
<tr>
<td>Drag coefficient ($C_d$)</td>
<td>1.3</td>
</tr>
<tr>
<td>Added mass coefficient ($C_m$)</td>
<td>1</td>
</tr>
</tbody>
</table>
4.2. Mussel farm and SAV at the same water depth

The time evolution of tide, storm tide, storm surge at the study site during the January 2015 North American blizzard is given by the SWAN+ADCIRC model [Xie et al., 2019] and shown on Fig. 9(a). The tidal range is around 3.3 m and the largest storm surge is about 0.8 m at the study site. The incident significant wave height ($H_{s0}$) and corresponding peak wave period ($T_p$) for every

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Figure 9: Time evolution of (a) tide, storm tide and storm surge during the January 2015 North American blizzard, (b) significant wave height ($H_{s0}$) and the corresponding peak wave period ($T_p$), (c) and (d) calculated wave energy dissipation ratio ($EDR$) by the suspended mussel farm (blue lines) and submerged aquatic vegetation (SAV, red lines) using the wave spectrum data. The canopy lengths are $L_v = 100$ m in (c) and $L_v = 200$ m in (d). The canopy densities are shown in the legend.
30 minutes are shown on Fig. [9]b). At the study site during the storm, the significant wave height reached 3.6 m with peak wave periods ranging from 5.2 s to 13.5 s.

The wave attenuation by the mussel farm and SAV at the same still water depth of 10.6 m during the storm is calculated with the wave spectral data from the SWAN+ADCIRC model. The calculated wave energy dissipation ratio (EDR) is shown on Fig. [9]c and d). The EDR of both SAV and mussels increases with incident significant wave height. However, the EDR decreases with water level resulting in an oscillating wave attenuation with the same period of the tidal cycle. This periodic behavior is more obvious for SAV because the mussels are less influenced by the tidal change since the mussels can move up and down with the buoys. The largest wave attenuation value occurs at the highest wave height during low tide. The larger (Lv = 200 m) and denser (0.125 droppers/m²) mussel farm provides a more pronounced wave attenuation with EDR up to 0.32 (Fig. [9]), which is a bit more than that of the same size (Lv = 200 m) but sparse (200 shoots/m²) SAV with EDR up to 0.26. However, for the denser (400 shoots/m²) SAV with the same size (Lv = 200 m), the EDR can reach to 0.45. For the shorter period waves with Tp < 9 s as shown on Fig. [9]c and d), the mussel farm can damp more wave energy than the same size SAV since SAV at the ocean bottom has little effect on wave attenuation for short period waves whose energy is concentrated near the ocean surface.

The comparisons for the selected wave spectrum as well as the associated spectral dissipation ratio (SDR) at 10:00 UTC (high tide with Hs0 = 2.9 m and Tp = 9.2 s), 16:00 UTC (low tide with Hs0 = 3.5 m and Tp = 13.5 s), and 22:00 UTC (high tide with Hs0 = 3.5 m and Tp = 13.5 s) on Jan 27 are shown on Fig. [10] The SDR of the suspended mussel farm increases with wave frequency until reaching the maximum value, while the SDR of SAV decreases with wave frequency. As a result, the suspended mussel farm shows the advantage of reducing higher frequency (shorter period) wave components over SAV. For example on Fig. [10]a2) with smaller Hs0 and Tp, the SDR of the dense suspended mussel farm (0.125 droppers/m²) is larger than that of dense
Figure 10: Comparisons of wave spectrum ($S_{\eta\eta}$) and wave spectral dissipation ratio ($SDR$) versus wave frequency ($f$) between the suspended mussel farm and submerged aquatic vegetation (SAV) with different canopy densities (shown in legend) at 10:00 UTC (a), 16:00 UTC (b), and 22:00 UTC (c) on Jan 27. The canopy length is 200 m for both canopies. The incident significant wave height and peak period are denoted by $H_{s0}$, and $T_p$, respectively.

SAV (400 shoots/m$^2$) for $f > 0.12$ Hz (wave period $T < 8.3$ s) and sparse SAV (200 shoots/m$^2$) for wave frequency $f > 0.055$ Hz ($T < 18$ s). The $SDR$ of the sparse suspended mussel farm (0.06 droppers/m$^2$) is larger than that of the dense SAV for wave frequency $f > 0.16$ Hz ($T < 6.25$ s) and sparse SAV for $f > 0.12$ Hz ($T < 8.3$s). As $H_{s0}$ and $T_p$ increases, the threshold value of the wave frequency where the $SDR$ of suspended mussel farm is larger than that
of SAV increases to $f > 0.167$ Hz ($T < 6$ s) for the dense mussel farm and the dense SAV, $f > 0.1$ Hz ($T < 10$ s) for the dense mussel farm and the sparse SAV, $f > 0.21$ Hz ($T < 4.8$ s) for the sparse mussel farm and the dense SAV, and $f > 0.17$ Hz ($T < 5.9$ s) for the sparse mussel farm and the sparse SAV as shown on Fig. 10(b2). For the same $H_{s0}$ and $T_p$ at high tide, the SDR of both the mussel farm and SAV decreases due to the increase of water level. The threshold value of the wave frequency where the SDR of suspended mussel farm is larger than that of SAV decreases to $f > 0.147$ Hz ($T < 6.8$ s) for the dense mussel farm and the dense SAV, $f > 0.095$ Hz ($T < 10.5$ s) for the dense mussel farm and the sparse SAV, $f > 0.18$ Hz ($T < 5.5$ s) for the sparse mussel farm and the dense SAV, and $f > 0.15$ Hz ($T < 6.7$ s) for the sparse mussel farm and the sparse SAV as shown on Fig. 10(c2).

4.3. Mussel farm and SAV at different water depths

The previous section shows the advantages of suspended mussel farms on damping high frequency wave energy over SAV at the same water depth. Usually, SAV colonizes in shallower water as shown on Fig. 1. To compare the performances of the suspended mussel farm and the shallow water SAV meadow, the water depth for SAV is set at 6 m so that maximum $H_{s0}/h = 0.79$ to avoid wave breaking. The water depth for the suspended mussel farm keeps the same at 10.6 m. The wave shoaling is incorporated using shoaling coefficient $K_s(\omega) = \sqrt{c_{gd}(\omega)/c_{gs}(\omega)}$ [Dean & Dalrymple 1991], where $c_{gd}(\omega)$ and $c_{gs}(\omega)$ are the wave group speed at deeper and shallower water depths, respectively. Correspondingly, EDR and SDR are calculated using the shoaled wave energy and wave spectrum. The canopy density is set as 200 shoots/m$^2$ (1000 blades/m$^2$) for SAV meadow and 0.125 droppers/m$^2$ for the mussel farm. The canopy length for SAV meadow is set as $L_v = 100$ m. For the mussel farm, two canopy lengths of 100 m and 200 m are designed for comparison.

The wave attenuations of SAV and mussel farms as well as their combinations are shown on Fig. 11. The wave attenuation by SAV decreases dramatically with increasing water level while the suspended mussel farm is less affected by
Figure 11: (a) Comparisons of wave energy dissipation ratio (EDR) between the suspended mussel farm and submerged aquatic vegetation (SAV). The canopy densities are 0.125 droppers/m$^2$ for the suspended mussel farm and 200 shoots/m$^2$ (1000 blades/m$^2$) for the SAV meadow, respectively. The canopy length is 100 m for the SAV meadow. The canopy length for the mussel farm is shown in the legend. (b1, c1, d1) The incident wave spectrum ($S_{\eta\eta}$) and (b2, c2, d2) Comparisons of wave spectral dissipation ratio (SDR) versus wave frequency ($f$) at 10:00 UTC (A), 16:00 UTC (B) and 22:00 UTC (C) on Jan 27. The incident significant wave height and peak wave period are denoted by $H_{s0}$, and $T_p$, respectively.

The water level change. For example (Fig. 11a), the EDR of SAV decreases by 49% from 0.51 at low tide (Jan 27 16:00 UTC) to 0.26 at high tide (Jan 27 22:00 UTC) with water level increment of 2.5 m while the EDR of the suspended mussel farm decreases by 29%. The combination of the suspended mussel farm
and SAV provides a larger wave attenuation, especially for smaller significant
wave period and low tide (Fig. 11a). For example at 10:00 UTC on Jan 27
with \( T_p = 9.2 \) s and \( H_{s0} = 2.9 \) m, the \( EDR \) of SAV and a large mussel farm
\((L_v = 200 \text{ m})\) is 0.37, which is 1.5 times of the \( EDR = 0.15 \) of SAV. Adding a
small mussel farm \((L_v = 100 \text{ m})\) to SAV can also favorably improve the \( EDR \)
of SAV to 0.27 by 80%. As \( H_{s0} \) and \( T_p \) increases to \( H_{s0} = 3.5 \) m and \( T_p = 13.5 \)
s, the improvements of the wave attenuation of SAV by adding mussel farms are
reduced because the mussel farm is more effective for reducing shorter period
waves. However, the improvements still can reach up to 31% by adding a small
mussel farm and 54% by adding a large mussel farm. The improvements of
combined SAV and mussels hold for wave energy at all frequencies by taking
the advantage of the canopy density of SAV and the vertical position of the
suspended mussel as shown on Fig. 11b2, c2 and d2).

5. Discussion

5.1. Wave attenuation characteristics of suspended aquaculture farms and SAV

Wave attenuation occurs through the drag force which is determined by
the horizontal wave orbital velocity. In shallow water waves, the amplitude of
the horizontal wave orbital velocity is almost uniform with depth. Thus, the
vertical position of the canopy has little effect on attenuating shallow water
waves. Taking the advantages of canopy density, SAV can dissipate more wave
energy than suspended mussel farms for long period waves. However, the wave
attenuation of SAV is influenced by changes of water level. In shallow water, the
wave attenuation of SAV decreases dramatically during high tide, storm surge,
or storm tide (tide plus storm surge), which highlights the weakness of SAV in
protecting coastlines during large storm tide conditions. This implies that severe
erosion by storms may occur during high storm tide levels (in addition, higher
waves may arrive at the shore without breaking during high tide). Therefore,
living shorelines represented by SAV would be less effective during extreme
events.
Suspended aquaculture farms can work as living breakwaters to protect the coast due to their capacity for wave attenuation. The wave attenuation capacity of suspended aquaculture farms is mainly dictated by the canopy density and the size (length) in the wave direction. Unlike SAV, which is limited by water depth due to light and nutrients, the vertical locations of suspended aquaculture farms can be adjusted to optimize their growth. Consequently, there is no depth restriction for suspended farms and they can be quite large, e.g., the suspended mussel aquaculture farm off Gouqi Island in East China Sea has an area of about 8 km$^2$ [Lin et al., 2016]. In theory, the size of suspended aquaculture farms can be designed to achieve optimal wave attenuation. For example, for the incident significant wave height ($H_{s0}$) to be reduced to the transmitted significant wave height ($H_{sT}$) that will allow the living shorelines to thrive and mitigate coastal erosion, the size of the aquaculture farms can be designed as

$$L_v > \frac{1}{\beta(\omega_c)} \ln \frac{H_{s0}}{H_{sT}},$$

where $\omega_c$ is the critical angular frequency such that \( \int_0^\infty S_{\eta\eta}(\omega, 0)e^{-2\beta(\omega)L_v}d\omega = e^{-2\beta(\omega_c)L_v} \int_0^\infty S_{\eta\eta}(\omega, 0)d\omega \). The existence of $\omega_c$ is guaranteed according to the mean value theorem for definite integrals. For narrow-banded waves, $\omega_c$ can be approximated using the peak wave angular frequency, $\omega_c \approx \omega_p = 2\pi/T_p$. The external factors such as the water depth and the vertical position of aquaculture farms should also be considered during the design. For places that are not suitable to establish living shorelines, such as low-nutrient seabeds, the suspended aquaculture farms offer a viable alternative to SAV for nature-based coastal defense.

This work has shown that suspended aquaculture farms can supplement SAV in wave attenuation. Suspended aquaculture farms attenuate shorter peak period waves and high frequency wave components more than SAV. Hence, adding suspended aquaculture farms to SAV-based living shorelines can compensate for the limitations of SAV for attenuating shorter period waves (such as boat wake) and enhance the wave attenuation capacity of SAV-based living shorelines for a wider range of wave frequency. The wave attenuation by SAV decreases during
high tide or storm surge due to the increase in water level. The water level, however, has fewer influences on suspended aquaculture farms since they are located near the surface and move up and down with water level. Therefore, suspended aquaculture farms can enhance the wave attenuation capacity of SAV-based living shorelines during extreme events. The combination of suspended aquaculture farms and traditional living shorelines (such as SAV) is therefore a desirable nature-based coastal defense strategy.

5.2. Simplified analytical solutions

The generalized three-layer frequency dependent theoretical wave attenuation model developed in this paper is applicable to analyze the wave attenuation capacity of submerged, emerged, suspended, and floating canopies for random waves including narrow-banded and wide-banded wave conditions. The present analytical model provided a more precise consideration of the blade motion by incorporating the effects of inertia (neglected in Mullarney & Henderson, 2010; Henderson, 2019) and the mode shape (not considered in Asano et al., 1992; Méndez et al., 1999).

The present model can reduce to previous models for submerged rigid vegetation without motion by setting the transfer functions $\gamma_s$ and $\gamma_c$ as 0. Therefore, the decay coefficient $\beta$ in (22) reduces to the solution for rigid blades and given by

$$
\beta_R(\omega) = \frac{2\sqrt{2}N k^2}{\sqrt{\pi} \omega(2k h + \sinh 2k h)} \int_{-d_1}^{-d_2} C_d b \sigma_u [\cosh k (h + z)]^2 dz,
$$

where $\sigma_u^2 = \int_0^\infty [\omega \cosh k (h + z)/ \sinh k h]^2 S_\eta(\omega, 0) d\omega$. If $d_3 = 0$, the solution in (28) reduces to the solution by Jacobsen et al. (2019) for submerged rigid vegetation. In this model, the nonlinear drag is linearized using the Borgman (1967) method such that $\vert u \vert \approx \sqrt{8/\pi} \sigma_u$. If using the root mean square velocity to linearize the drag force following Madsen et al. (1988) such that $\vert u \vert \approx \sqrt{2} \sigma_u$, the solution reduces to the Hasselmann & Collins (1968) based solution in Chen & Zhao (2012) for submerged rigid vegetation. For idealized narrow-banded waves such that $S_\eta \approx 0$ when $\omega \neq \omega_p$, the damping coefficient in (28) can be...
further simplified as

\[
\beta_{RN} = \frac{1}{12\sqrt{\pi}} C_d b N k_p H_{rms0} \frac{9 \sinh k_p (d_2 + d_3) - 9 \sinh k_p d_3 + \sinh 3k_p (d_2 + d_3) - \sinh 3k_p d_3}{(\sinh 2k_p h + 2kh) \sinh k_p h},
\]

where \( H_{rms0} = \sqrt{\frac{8}{\pi}} \int_0^\infty S_\eta(\omega, 0) d\omega \) is the root mean square incident wave height and \( k_p \) is the peak wave number calculated by solving \( \omega_p^2 = gk_p \tanh k_p h \). For bottom-rooted rigid vegetation such that \( d_3 = 0 \), \( \beta_{RN} \) in (29) reduces to the solution of Mendez & Losada (2004). The relationship between the present and previous models are shown on Fig. 12.

Figure 12: Relationship between the present solutions (22), (28), (29) and previous solutions by Chen & Zhao (2012), Jacobsen et al. (2019), and Mendez & Losada (2004), where \( \gamma_s \) and \( \gamma_c \) are the transfer function for the motion of canopy component, \( d_3 \) is the thickness of the gap between the canopy and sea bed, \( u \) is the horizontal wave velocity and \( \sigma_u \) is the stand deviation of \( u \).

### 6. Conclusions

A generalized three-layer frequency-dependent theoretical model for the wave attenuation by submerged and suspended canopies subjected to random waves was derived and validated with laboratory and field data. This model incorporates the motion of canopies using a cantilever-beam model for slender canopy
components and a buoy-on-rope model for canopy components with concentrated mass and buoyancy. This frequency-dependent solution was used to demonstrate the shoreline protection capability of suspended mussel farms alone and in combination with submerged aquatic vegetation (SAV) to damp wave energy at a field site in Saco Bay, Maine, USA during a January 2015 Blizzard. The results showed that both suspended mussel farms and SAV have the potential to damp wave energy considerably during storm events. Suspended mussel farms are more effective at damping shorter waves and high frequency wave components of the wave spectrum while dense SAV colonized in shallower water have the advantages of damping longer waves and lower frequency wave components more effectively. However, the wave attenuation of SAV in shallow water decreases dramatically at the peak of storm tide due to increased water level, which decreases the wave motion reaching the ocean bottom. In contrast, suspended aquaculture farms can move up and down with water level change and are less affected by water level change. As a consequence, the combination of suspended aquaculture farms and traditional SAV-based living shorelines provide an optimized nature-based shore protection scheme that can damp more wave energy for a wider wave frequency and water level range.

The research of wave attenuation by suspended and floating canopies is still in its infancy. More laboratory and field experiments data for the hydrodynamic properties of suspended aquaculture farms (e.g., mussels and kelp) as well as wave attenuation are desirable. The present theoretical model assumed the blade motion as a linear vibration with small-amplitude. However, as long as the nonlinear effects of large-amplitude blade motion is negligible, the present theoretical model remains valid. In addition, the bottom friction, bedforms, bottom slope as well as wave-driven currents, wave and current conditions may also be significant for certain types of bottom rooted vegetation (e.g., Jensen et al. 1989, Myrhaug 1995, Zou 2004, Zou & Hay 2003, Smyth & Hay 2002, Maza et al. 2019, Abdolahpour et al. 2017, van Rooijen et al. 2020). Therefore, it is worthwhile to investigate the nonlinear effects of large-amplitude blade motion on the wave damping capacity of suspended canopies as well as the
effects of bottom properties and wave-current conditions in the future work.

Acknowledgments

This work was completed as part of the PhD research of Longhuan Zhu who is supported by National Science Foundation award #1355457 to Maine EP-SCoR at the University of Maine. The authors benefited from discussions with Zhilong Liu and Haifei Chen. The authors wish to thank Stephen Cousins and Shane Moeykens at the University of Maine Advanced Research Computing for the computational support. The authors would like to thank Niels G. Jacobsen for sharing the data used on Fig. 4. The authors would also like to thank the anonymous reviewers for constructive comments that helped to improve this manuscript greatly.

Appendix A. Normal mode solutions for blade displacements in random waves

The governing equation (6) for the blade displacement in random waves is given by

\[ m\ddot{\xi} + c\dot{\xi} + EI\xi^{\prime\prime\prime\prime} = \sum_\omega a_\omega \Gamma \left[ c \cos(kx - \omega t + \psi) + \omega m_I \sin(kx - \omega t + \psi) \right] \quad (A.1) \]

with the boundary conditions \( \xi(0,t) = 0, \xi'(0,t) = 0, \xi''(l,t) = 0, \) and \( \xi'''(l,t) = 0 \) for a cantilever beam. The solution of (A.1) can be written as the linear superposition of components of different frequencies

\[ \xi = \sum_\omega \xi_\omega, \quad (A.2) \]

where \( \xi_\omega \) is the solution of

\[ m\ddot{\xi}_\omega + c\dot{\xi}_\omega + EI\xi''''_\omega = a_\omega \Gamma \left[ c \cos(kx - \omega t + \psi) + \omega m_I \sin(kx - \omega t + \psi) \right]. \quad (A.3) \]

According to normal mode approach [Rao, 2007], the solution of (A.3) can be assumed as a linear superposition of the normal modes of the cantilever beam.
as
\[ \xi_\omega = \sum_n \phi_n(s)q_n(t), \quad (A.4) \]
where \( \phi_n(s) \) is the \( n \)th normal mode and \( q_n \) is the \( n \)th generalized coordinate or modal participation coefficient. The normal modes for a cantilever beam are found from the equation
\[ \phi''' - \mu^4 \phi = 0 \quad (A.5) \]
with boundary conditions \( \phi(0) = 0, \phi'(0) = 0, \phi''(l) = 0, \) and \( \phi'''(l) = 0. \)
Solving (A.5) yields the \( n \)th normal mode,
\[ \phi_n = (\cos \mu_n l + \cosh \mu_n l) (\sin \mu_n s - \sinh \mu_n s) + (\sin \mu_n l + \sinh \mu_n l) (\cosh \mu_n s - \cos \mu_n s), \quad (A.6) \]
where \( \mu_n \) is the \( n \)th solution of
\[ 1 + \cos \mu l \cosh \mu l = 0 \quad (A.7) \]
Using (A.5) associated with the boundary conditions, the normal modes are proved to satisfy the orthogonality conditions,
\[ \int_0^l G(s)\phi_n\phi_m ds = \begin{cases} \int_0^l G(s)\phi_n^2 ds, & n = m, \\ 0, & n \neq m, \end{cases} \quad (A.8) \]
where \( G(s) \) is an arbitrary function. Substituting (A.4) into (A.3) yields
\[ m \sum_n \phi_n\ddot{q}_n + c \sum_n \phi_n\dot{q}_n + EI \sum_n \phi_n'''q_n = a\omega \Gamma \left[c \cos(kx - \omega t + \psi) + \omega m_I \sin(kx - \omega t + \psi)\right]. \quad (A.9) \]
Multiplying (A.9) by \( \phi_m \) and integrating from 0 to \( l \) result in
\[ \sum_n \left( \int_0^l m\phi_n\phi_m ds\ddot{q}_n + \int_0^l c\phi_n\phi_m ds\dot{q}_n + \int_0^l EI\phi_n'''\phi_m dsq_n \right) = a\omega \left[ \int_0^l \Gamma\phi_m ds \cos(kx - \omega t + \psi) + \omega \int_0^l m_I \Gamma\phi_m ds \sin(kx - \omega t + \psi) \right]. \quad (A.10) \]
Substituting (A.5) into (A.10) and using the orthogonality conditions (A.8) yield

\[ \ddot{q}_n + 2\zeta_n \lambda_n q_n + \lambda_n^2 q_n = \alpha \omega [D_n \cos(kx - \omega t + \psi) + \omega I_n \sin(kx - \omega t + \psi)], \]

(A.11)

where \(2\zeta_n \lambda_n = \int_0^l c\phi_n^2 ds / \int_0^l m\phi_n^2 ds\), \(\lambda_n^2 = \mu_n^4 \int_0^l EI\phi_n^2 ds / \int_0^l m\phi_n^2 ds\), \(D_n = \int_0^l c\Gamma \phi_n ds / \int_0^l m\phi_n^2 ds\), and \(I_n = \int_0^l m\Gamma \phi_n ds / \int_0^l m\phi_n^2 ds\). The steady state solution for (A.11) is

\[ q_n = \alpha \omega Q_s \sin(kx - \omega t + \psi) + \alpha \omega Q_c \cos(kx - \omega t + \psi), \]

(A.12)

where

\[ Q_s = \frac{\omega I_n \left( \lambda_n^2 - \omega^2 \right) - D_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2}, \]

(A.13)

and

\[ Q_c = \frac{D_n \left( \lambda_n^2 - \omega^2 \right) + \omega I_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2}. \]

(A.14)

Substituting (A.6) and (A.12) into (A.4) and the result into (A.2) yields the blade displacement,

\[ \xi = \sum_{\omega} a\Gamma [\gamma_s \sin(kx - \omega t + \psi) + \gamma_c \cos(kx - \omega t + \psi)], \]

(A.15)

where the transfer functions \(\gamma_s\) and \(\gamma_c\) are given by

\[ \gamma_s = \frac{\omega}{\Gamma} \sum_{n=1}^{\infty} \phi_n \frac{\omega I_n \left( \lambda_n^2 - \omega^2 \right) - D_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2}, \]

(A.16)

and

\[ \gamma_c = \frac{\omega}{\Gamma} \sum_{n=1}^{\infty} \phi_n \frac{D_n \left( \lambda_n^2 - \omega^2 \right) + \omega I_n 2\zeta_n \lambda_n \omega}{(\lambda_n^2 - \omega^2)^2 + (2\zeta_n \lambda_n \omega)^2}. \]

(A.17)

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