Where is the Water? – A Physics Analysis to Identify Possible Processes in Hydraulic Fracturing

ARMA 20–1962

Where is the Water? – A Physics Analysis to Identify Possible Processes in Hydraulic Fracturing

Couples, G.D.
Institute of Geoenergy Engineering, Heriot-Watt University, Edinburgh, Scotland, UK

ABSTRACT: The present operative concept model for hydraulic fracturing (see Miskimins 2019) is shown to be physically impossible. The conceptualisation that underpins current practice is little changed from the time of its introduction by Hubbert and Willis (1957), and imagines a planar fracture opened within the rock mass by the injection of high-pressure fluid into the rock. Current practice typically uses very large volumes of water that exceed the void space created inside the notional fracture. The opening of the fracture causes new strains within the rocks that bound the fracture, with energy budget consequences. The excess water is often implied to go into pre-existing void spaces, also with energy consequences. Both of these explanations involve consequent energy changes that are more than an order of magnitude larger than the input energy amount, so they have to be impossible. Alternate process models, based on activation of natural fracture systems, require less energy, and so are deemed to be the likely reality

1. INTRODUCTION

This paper presents an approach that is based on assessing the budget of extensive (conserved) energy during a hydraulic fracture stimulation operation. The Appendix provides a brief discussion of extensive/intensive energy expressions, and how these relate to familiar state parameters. The rock mass possesses, pre-stimulation, some distribution of extensive energy forms: in the pore fluid, in the rock framework, and in the contained heat. During stimulation, the cooler and pressurised fluid that is pumped into the well affects the energy budget. The pre-treatment energy contained within the rocks, the energy effects in the rock that are associated with the injection of fluids, and any energy that is 'activated' from the prior state (e.g. seismicity, or changes in the potential energy related to changes in elevation), all must balance with the energy supplied by the injection fluids. The conservation of energy provides a means of assessing ideas and inferences about what actually takes place during the treatment.

This paper considers the energy-budget consequences of the process models for hydraulic fracturing that are commonly adopted (Miskimins 2019), and the associated explanations that describe what is imagined to occur in the rock mass affected by the stimulation. The approach in this paper is to first introduce a simple model involving a classical bi-wing fracture, and to examine the extensive energy changes required in the rock mass that arise as a consequence of the opening of the fracture (this relates to the so-called 'stress shadow' effect). The volume of water-based fluid that is injected regularly exceeds the volume inside a typical bi-wing fracture model, so the excess water is often said to enter into the pores of the rock matrix (this relates to the notion of 'bleed off'). That proposed water entry into the matrix also has energy consequences that can be calculated, and the resulting analysis here shows that the 'rock matrix' cannot be the only, or main, place where the excess water is located. The downfall of the simple concept-model, which fails to observe the principle of energy conservation, demands that other solutions be developed. Examples of these alternate ideas, which involve activation of natural fractures in the rock mass, are examined to show how the energy budget can be used to narrow the uncertainty of the final solution.

The choice taken herein is to assess the ideas in a manner that illustrates how the energy forms are quantified, starting with a simple model in which the rock mass is treated as an elastic solid, and the fracture is idealized to a regular geometric shape to allow a reduced-dimension discussion. The next complexity is to consider the energy contained within the fluid phase, if the 'lost' injection fluid is forced into the void spaces within a static rock framework. The next analysis emphasises the changes in energy perspectives arising from relaxing the continuum perspective of the rock mass, which is necessary to allow for arrays of rock-mass discontinuities. Here, in the non-continuum context, the interactions between rock and fluid are briefly considered. The final brief consideration is concerned with thermal effects related to the injection. In each case considered here, the simplifications are deliberate so as to emphasise a single key point; a real analysis would likely move beyond the 1D arguments and elementary formulations.

The paper aims to achieve two objectives. One is to introduce the energy-based analysis method, and how this perspective requires a re-examination of the conceptual models adopted. The second objective is to elucidate the inadequacies of the common ideas applied to hydraulic fracturing, and to point towards a better paradigm.

2. APPLICATION OF THE ENERGY-BUDGET APPROACH

2.1 Elastic Rock, Bi-Wing Fracture

For the purpose of the following analysis, a bi-wing vertical hydraulic fracture is assumed to consist of rectangular panels, where the height and lateral extent of each wing are constant and in a common plane (Fig. 1). The aperture of the fracture varies in a linear fashion from the well to the lateral fracture tip. For simplicity in explaining the approach (fewer parameters to define), the fracture aperture is taken to be constant over its height. For the purposes of illustration, the hydraulic fracture aperture at the wellbore is 12 mm; the height is 41 m; and the lateral half-length is 177 m.

When the fracture is opened by the rapidly-injected fluid, the wall rocks move apart. At the midpoint of each wing, the aperture is 6 mm, so the movement of each wall is 3 mm. The magnitude of that displacement in the x-direction (denoted as u) diminishes away from the fracture plane, and the gradient of the displacement determines the x-component of the strain:

$$\varepsilon_x = \frac{\delta u}{\delta x} \tag{1}$$

We assume that the displacement associated with the fracture-wall movement is dissipated to u=0 within the distance of 25 m (nominally half the fracture-stage distance). The variation in displacement, from $u = 0.003m$ at the fracture wall, to $u = 0.0m$ at the plane of symmetry, is unknown. Some possible displacement functions, not assessed here from any theoretical standpoint, are a linear, a parabolic, and an exponential variation. These possibilities define models of how ε_x varies with position:

$$u(x) = ax;\ \frac{\delta u}{\delta x} = \varepsilon_x = a \tag{2}$$

$$u(x) = bx^2;\ \frac{\delta u}{\delta x} = \varepsilon_x = 2bx \tag{3}$$

$$u(x) = ce^x;\ \frac{\delta u}{\delta x} = \varepsilon_x = ce^x \tag{4}$$

where $a = -0.00012; b = -4.8 \times 10^{-6}; c = -4.166 \times 10^{-14}$ (for the parameters stated).

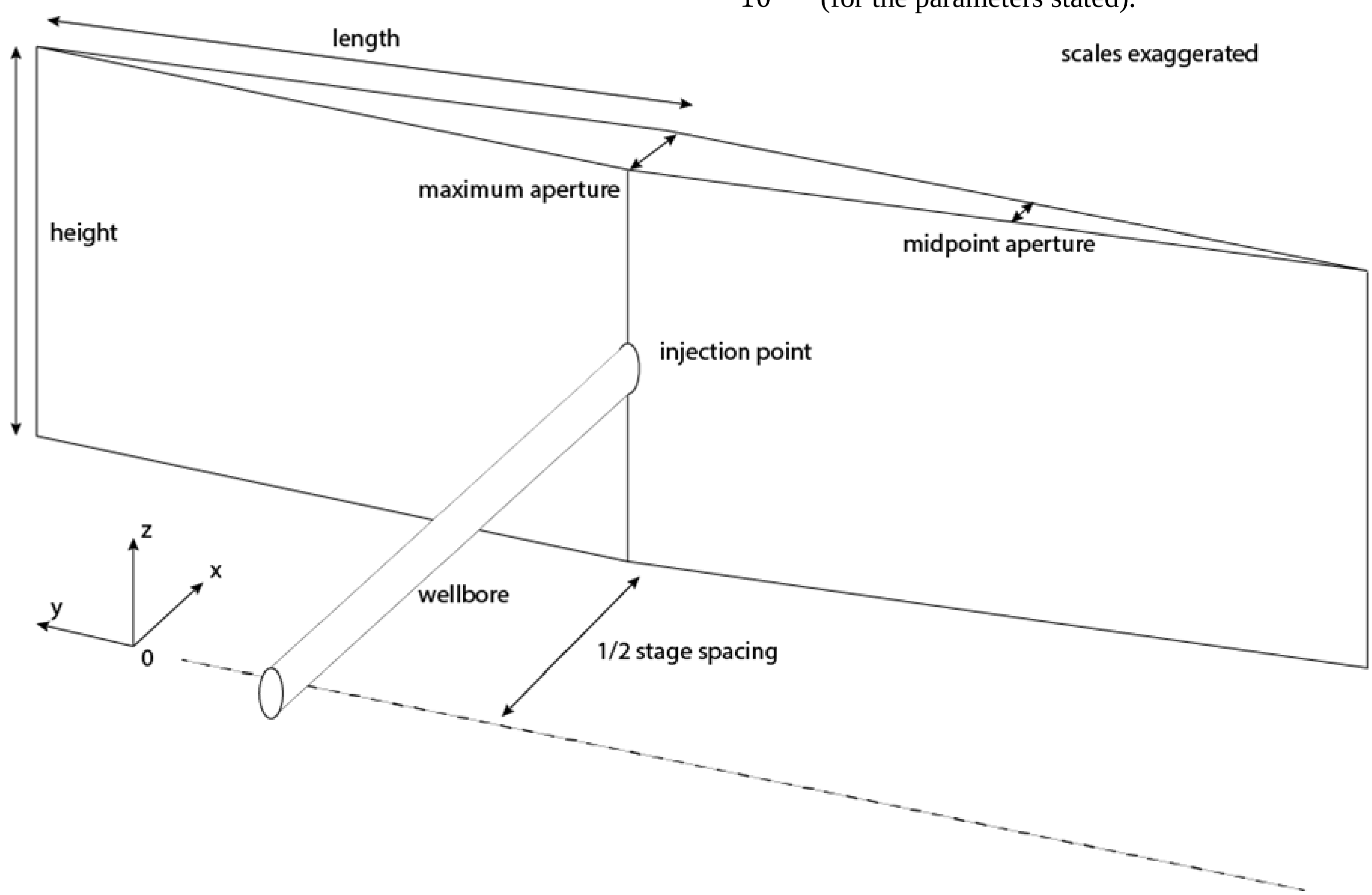

Fig. 1. Sketch of simplified model geometry.

Using Eq. (A9), and assigning the rock stiffness to be E=11.3GPa, the x-direction component of intensive elastic strain energy varies with the x-coordinate (normal to the fracture), depending upon the distribution model assumed (Fig. 2A). If we consider a horizontal body of rock, of square section $1m^2$, and extending from the fracture plane to the symmetry plane, its volume is $25m^3$ (ignoring that the length of that rock column is only 24.997m when the aperture is 3mm). Taking the intensive x-direction strain energy at the centre of each 1 m cube of rock, the extensive energy within the whole column is the sum of the extensive energies in each of those blocks. The extensive x-direction elastic strain energy associated with the three displacement functions varies (Fig. 2B), with the constant-strain case giving the lowest amount (50.85 kJ), followed by the parabolic (67.8 kJ) and exponential (541 kJ) cases. Since the expression of intensive energy involves the square of the strain term, strain concentrations, if not highly localised to small volumes so that the extensive energy amount is minimized, are not favourable. Because Nature is expected to accomplish a required deformation with the least energy cost, the linear distribution of displacement is a suitable model, and that will be used in the following analysis.

Now, we wish to derive an estimate of the total elastic strain energy in the affected rock volume (still only considering the x-direction strains, and simplifying to assume that the strains are strictly limited to the rock volume that lies ±25m to both sides of the fracture plane). We use the strain and energy values from the midpoint location of the fracture, since a single value is useful to simplify the calculations, which are used to make the point: that the energy consequences of forming the hydraulic fracture have to be included in the budget. The extensive strain energy of the rock column located at the point of maximum aperture is four times the midpoint energy (due to the square of the strain term), so using the midpoint value as a full-fracture average for total area of the bi-wing fracture is a safe simplification. Likewise, since the strain energies related to the y- and z-direction constraints are not included here, the key point is not adversely affected by the simplification.

Derived from 'core-through' operations (which obtain rock samples from previously-stimulated rock volumes), an idea sometimes heard is that multiple strands of parallel fractures replace the single-fracture ideal model, whose aperture then is assigned to separate single fractures located within a cluster. If we apply a similar analysis as above, and assume that five fractures in a cluster each have 1/5 of the total aperture, but the cluster occurs within 1m (1/25 of the horizontal stage distance used in the calculations above), then we can look at two possibilities. In one, the displacement gradients are larger (Eq (1)) in the thin plates of rock between the clustered fractures, and thus the intensive energy is higher in those plates, but the volume of strained rock is smaller, and the total effect on extensive energy is nil. The second option is that there is no strain in the plates, and the total displacement (sum of 4.5 (four full and half of the central one) individual apertures) is now imposed onto the fully-intact mass of rock that extends across to the plane of symmetry (where we say that displacement becomes zero). That extensive energy calculation follows the method described below, and results in almost the same number as for the single-fracture case. Thus, the idea of fracture clustering does not have a major impact on the key point being made.

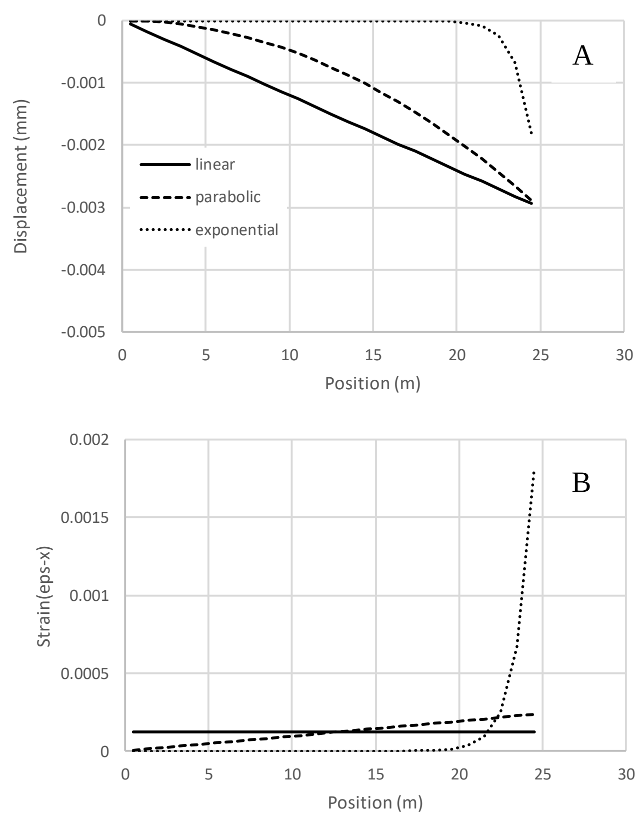

Fig. 2. (A) Rock displacements due to opening of hydraulic fracture. (B) Resulting contractional strains. Cases described in text.

We are able to calculate the energy cost associated with causing strains in the wall-rocks of the fracture as follows. We multiply the total fracture extent (area) by the factor of two to account for the strains occurring in both walls of the fracture. That area, multiplied by the extensive energy in the midpoint rock column, gives a total (minimum) energy cost of 1.48 GJ linked to those strains. Considering only the pressure (29 MPa) and volume (1243 m^3) of the injection fluid, the extensive energy provided by the injection process is only 209 MJ (calculated at the surface, so this number does not account for friction losses in the well). If we wish to taper the top and bottom of the fracture, the amount of strain energy induced would be reduced, but if we consider a fracture shape with significant geometry variation, it would be better to calculate the strain over each unit area, and sum up all of them to derive the total to get a better number. If we feel the need to do that, then we might as well add in the strain energies induced in the fracture-parallel directions also, or even run a full numerical simulation (with displacement boundary conditions!). However, our point here is not to achieve precision, but to derive a more general understanding of the rock-mass energy consequences of the opening of the fracture.

There is a large discrepancy between the amount of strain energy created in the rock and the amount of energy contained in the injection fluid. This means that the fluid injection process does not provide enough energy to push the wall rocks out of the way so that the fracture opens to a finite aperture. The classic conceptual model of hydraulic fracturing fails the test of energy conservation.

2.2 *Pushing Water Into the Rock Matrix*

Another important discrepancy arises if we look at the void volume within the idealised fracture (174 m^3), and compare that against the volume of injected fluid (1243 m^3). The fracture is not able to hold all of the injected water+proppant, and an additional volume of void space is needed beyond the amount attributed to the hydraulic fracture space. We could pose the possibility that the fracture is actually larger than the sizes given, thus 'solving' the volume problem. But, the energy content of the wall rocks scales with the fracture dimensions, so the energy cost of straining the additional volume of rock, while creating a positive fracture aperture over the added fracture area, far exceeds the energy provided by the injected fluid volume and pressure. Even setting aside the energy discrepancy between the injected fluid volume and the volume of the fracture, it is an inescapable deduction that much of the injection fluid goes somewhere other than inside the single fracture that is usually assumed to be created from the injection.

The mis-match between the volume of injected water, and the fracture volume, is not a startling observation, since the recognition of that discrepancy has given rise to an 'explanation' that is commonly offered: the excess water is suggested to enter into the pore system of the rock matrix. Here, we examine that idea from the energy perspective.

The pressure of water is an indicator of its state of volumetric strain (see Appendix). To calculate the resultant pressure, in the case of adding the 'lost' injection water into already-water-filled rock matrix pores, we need to determine the un-strained volume (at zero pressure) of the pre-stimulation pore water. Then, we add the excess injected-water volume, and compress the combined volume into the pore space. Note: for this analysis, we assume (for simplicity) that the pore volume is fixed, with no alteration of the porosity due to pressure changes or other mechanical loads. We assume that the appropriate gross rock volume is given as the total fracture area times the inter-stage distance ($41m \times 348m \times 25m \times 2 = 725700m^3$). Multiplying by the porosity (0.08, or 8%) gives the pore volume $43542m^2$. If this water volume is de-compressed to surface conditions (Eq. A3), it increases to $44047m^2$. Now, we add the 'excess' of injection water $1243m^3 - 176m^3 = 1067m^3$ (using the difference between the injected volume and the surface-conditions water volume needed to fill the fracture space). This combined volume $45114m^2$ is then compressed to fit within the matrix pore space, resulting in a calculated pore pressure of 87 MPa.

The calculated matrix pore pressure far exceeds the pressure of the injection process, so the conclusion is clear: the excess injection volume is not 'lost' into the matrix pore spaces. Although we could calculate how much of the injection-water volume could be forced into the matrix until the pore pressure was equal to the injection pressure, that process would take time in order to reach the eventual equilibrium quantities calculated. Given that the point of hydraulic fracturing is to create flow pathways, due to poor-quality reservoir rock, that time-to-equilibrium would be much longer than the short-duration injection period. The idea, that excess injection water is somehow accounted within the matrix pore space, is implausible.

If the pore-filling fluid of the matrix is not water, but instead is oil or gas, then the analysis is less straightforward, due to the higher compressibility of these fluids. The resulting equations are not linear (not examined here), but they lead to a similar conclusion: the already-in-place fluids exist in a state of elevated intensive energy, and increasing their energy due to injection is limited by the extensive energy amount of the injection fluid. We note that many reservoirs that are candidates for hydraulic fracturing do not have significant free phases of hydrocarbons, with much of the oil or gas that is trapped being present in solution, or within organic matter whose pores would be unavailable to any invading water anyway. The key point here is that excess water cannot be easily pushed into the matrix pores.

Spontaneous imbibition of the water is another idea that warrants examination. This mechanism could potentially operate in cases with water-wet pores that are filled (to residual water saturation) with a non-wetting fluid phase. Imbibition, in such cases, will be limited by the energy cost of compressing the already-pressurised non-wetting phase (similar to the point made above). At the lower end of the water saturation, the rate of influx is likely to be very slow due to low intrinsic permeability and to the low relative permeability to water at these end-point saturations. The flux rate of water is unlikely to be sufficient to solve the short-time volume deficit underlying this discussion. A full analysis of this hypothesised process is an interesting exercise of multiphase transient flow, but one that is beyond the scope of this paper.

2.3 *Considering Fluid-Rock Mechanical Interactions*

The assessments examined above, using simplified physics principles, rule out the options – of larger bi-wing fractures, or pushing fluid into the rock matrix – as solutions to the 'volume problem' apparent in hydraulic fracturing. Those options have energy costs that far exceed the energy content of the injection process. In order to consider or formulate suggested answers that may well involve contemplation of more-complex process interactions, it is helpful to ensure that the conceptual basis of any idea is well-grounded in physics. Here, we note an additional and little-considered factor that is related to the interaction between 'solid' geomechanics and pore fluids.

Micro-mechanics models of geomaterials, in which the rock framework and pore space are explicitly represented (as has become standard in the subject of 'digital-rock physics'; see Andrä et al 2013; Bultreys et al 2016), allow the derivation of coupled behaviours where the framework components and the fluid-filled pores are considered as discrete features (Ahmed et al 2019). In addition to their role in developing fundamental understanding, such models provide a means of deriving bulk or up-scaled physical responses that can be transformed into a continuum-equivalent set of material laws (Couples 2019). Here, we consider only reversible process interactions, for which the term 'elastic' seems appropriate, and thus the fluid-rock interactions are commonly labelled as poro-elastic.

Prior theoretical perspectives on poro-elasticity were derived via an enriched-continuum approach in which the material laws were based on enhancements of classic continuum laws, with added parameters argued to account for interactions between solid and fluid (such as the theory of Biot 1941). The micro-mechanics approach shows that the traditional continuum-based perspective of poro-elastic behaviour is ill founded, being argued from assertions that do not hold, as well as errors in assembling the equations of energy balance. In the micro-mechanics framework, there are substantial dependencies on the material texture as well as the bulk boundary conditions (Couples 2019). The 'Law of Effective Stress', as commonly expressed with a parameter α that modifies the fluid pressure term, obscures the fact that the stress state is strongly affected by changes in fluid pressure (in a typical confined situation).

Here, we examine the extensive energy budget, using a micro-mechanics perspective, of fluid-solid mechanical interactions. For a simple illustration, we examine the changes in the extensive energy contents of a macro-scale porous rock that is constrained so as to permit only uni-axial strains (here, in the z-direction), while the fluid pressure is increased $\Delta P > 0$ by the introduction of fluid mass (the derivation of the relevant equations for this this case is detailed in Couples 2019):

$$\overline{\Delta\varepsilon_y} = \overline{\Delta\varepsilon_x} \equiv 0;\ \Delta\sigma_z = 0;\ \Delta P \neq 0 \tag{5}$$

$$\begin{aligned}\Delta\varepsilon_y^{rod} &= \tfrac{1}{E}\left[\Delta\sigma_y - v(\Delta\sigma_x + \Delta\sigma_z)\right]\\ &= \tfrac{1}{E}\left[\Delta\sigma_y - v(\Delta\mathrm{P} + \Delta\mathrm{P})\right]\\ &= \tfrac{1}{E}\left(\Delta\sigma_y - 2v\Delta P\right)\end{aligned} \tag{6}$$

$$\Delta\varepsilon_y^{junction} = \tfrac{1}{E}\left[\Delta\sigma_y - v(\Delta\sigma_x + \Delta\sigma_z)\right]$$

$$= \tfrac{1}{E}\left[\Delta\sigma_y - v\left(\Delta\sigma_y + 0\right)\right] = \tfrac{1}{E}\left[\Delta\sigma_y(1-v)\right] \tag{7}$$

$$0 = t\left(\Delta\sigma_y - 2v\Delta P\right) + (1-t)\Delta\sigma_y(1-v) \tag{8}$$

$$\text{therefore } \Delta\sigma_y = \Delta\sigma_x = \frac{2tv\Delta P}{1-v+vt} \tag{9}$$

$$\begin{aligned}\Delta\varepsilon_z^{junction} &= \tfrac{1}{E}\left[0 - v\left(\frac{2tv\Delta P}{1-v+vt} + \frac{2tv\Delta P}{1-v+vt}\right)\right]\\ &= -\frac{4tv^2\Delta P}{E(1-v+vt)}\end{aligned} \tag{10}$$

$$\Delta\varepsilon_y^{rod} = \tfrac{1}{E}\left[0 - v(\Delta\mathrm{P} + \Delta\mathrm{P})\right] = -\frac{2v\Delta P}{E} \tag{11}$$

$$\overline{\Delta\varepsilon_z} = -\frac{2tv\Delta P}{E} - \frac{4tv^2\Delta P}{E(1-v+vt)} = -\frac{2tv(1+v+vt)\Delta P}{E(1-v+vt)} \tag{12}$$

Note that these expressions relate to the micro-mechanics analysis, which is formulated to address changes (hence the Delta symbols that modify state parameters). The overbar signifies a bulk parameter that can be associated with the macro-scale. The material properties (Young's modulus $E = 7.4GPa$ and Poisson ratio $v = 0.26$) are those of the micro-components of the rock framework. The parameter $t = 0.06$ relates the geometry of the lattice configuration of this simplified analytical model, and results in a porosity of just over 1%.

The arbitrary increase in pressure ΔP causes increases in the (principal) stress components aligned with the directions of strain constraints (here, x- and y-directions). These increased stress components occur in both the connecting elements ('rods' in the terminology used in Couples 2019) and in the places where rods meet, called 'junctions'. Using the expressions described in the Appendix, the intensive strain energies in each of these components can be determined using the elastic parameters of the micro-scale framework components. In a prismatic model (dimensions: X= 150 mm, Y= 200 mm, Z= 300 mm), an increase in pressure $\Delta P = 22MPa$ results in a total extensive energy change of 103.5 J, while only ~9 J of that increase is within the pore water (the exact energy change in the water depends upon the preceding pressure state).

Relative to the analysis in Section 2.2, for fluid ingress into the rock matrix, any volume of excess injection water that enters the matrix would 'cost' much more – in energy terms – than was calculated by the simplified assessment of net change of pressure in the pore fluid. Since that analysis already revealed that there would be an upper limit on the volume that could enter the matrix, due to 'running out' of driving energy, the inclusion of the extra energy costs, related to stress changes resulting from fluid-rock interactions, further discounts that long-standing idea about injection-water invasion into the rock mass.

2.4 New Ideas Are Needed

The discrepancy between injected water volume, and the volume of the classic hydraulic fracture that is usually assumed, or even fracture clusters, demands a different physical explanation than those examined above. The injection process has to create new void volumes during the time of stimulation, and this new void volume must continue to exist into at least a short time thereafter (until well production allows some of the reservoir fluids to leave, and perhaps longer). It is not reasonable to suppose that the solution to the space conundrum lies in the over- or under-burden, since these regions equally already have pressurised pore fluids in the rock matrix. Thus, the same analysis (as above) applies there, with the same conclusion. Instead, the process to create new void volume must operate within the reservoir unit itself.

An obvious candidate process to be examined is the creation of more fractures. This idea usually involves the assumption of mode I fractures, with the individual discontinuities having characteristics much like the presumed main hydraulic fracture. If such a fracture array were to be created, with a distribution across the rock mass, the same energy effects considered above would occur. Namely, the fracture apertures thus created would cause elastic strains in the adjacent rock, and thus energy changes within that rock-mass volume. One would also need to ask how the fluid would be able to enter into these fractures, since they would (might) not be directly connected to the wellbore. The notion, that such an array of fractures could answer the need for new and immediate volume, is unlikely to survive a full energy audit – which is not pursued here.

A different type of candidate process, but one still dependent on discontinuities, concerns the re-activation of pre-existing discontinuities, including (possibly) slip along frictional bedding contacts. Systems with intersecting discontinuities respond to far-field displacement loading by the shifting of individual blocks, wedge-opening of discontinuities, and substantial relaxation (unloading) of intact blocks (Fig. 3). The presence of the discontinuities allows the system to achieve bulk strains with lowered levels of internal stresses, which is equivalent to treating that mass as having a much-lowered stiffness, and thus the contained extensive energy is significantly reduced in comparison with an equivalent continuum. Thus, the energy cost of deforming that mass is less than with a non-fractured rock body. These systems lead to (in simulations) the activation of the discontinuities, resulting in openings with connected pathways to enable fluids to access them. A further effect from discontinuities is relevant: frictional bedding planes in a deforming rock mass provide additional process opportunities that allow the deforming rock to achieve bulk strains at low energy costs (Couples and Lewis 1998).

The importance of fractured-rock behaviour, in relation to the water-volume issues in hydraulic fracturing, is linked to the characteristic way that broken masses are able to experience strain and become (more) dilated. In terms of the hydraulic fracturing energy budget, the behaviour of fractured rocks provides a potential explanation of the manner by which new void space can be created at low energy cost. This avenue cannot be explored fully at present due to the lack of suitable simulation methods: no simulation tool includes the type of poro-elastic interaction responses that have been identified from the micro-mechanics analysis. The simulations of fractured-rock systems mentioned above are approximately correct for cases with constant pore pressure, but they cannot be simply extended to the case with varying fluid pressures in the intact blocks. Thus, the ideas presented are plausible, but they need to be rigorously assessed via a physics-based simulation analysis.

2.5 Short-Term Thermal Effects

A final consideration concerns the possible role of thermal effects during the stimulation treatment. The cooler injection water can absorb heat from the *in situ* rocks and formation waters, if there is contact, and time to exchange heat. During the stimulation, we have shown that the injection water does not significantly enter the rock matrix, so heat is not taken from the rock framework to warm cooler, introduced pore fluids. If the cooler water is present within fractures, heat can transfer from the rock face to warm those fluids. If there is such an exchange of heat, the rock and pore fluid both relax slightly. As we note above, this does not mean an actual change in shape, but a reduction of stress that is not directly linked to a physical strain (displacement gradient).

If we seek a rough estimate of the maximum scope of the thermal effects, we can assume that the entirety of the injected and cooler water is instantaneously introduced to fracture faces. To illustrate the computation, assume that all fracture apertures are 1.0 mm, thus the total fracture area (in addition to the 'main' fracture) is $\frac{1076m^3}{0.001m} = 1.07 \times 10^6 m^2$. Assume that the reservoir temperature is 80C, and the injected-water temperature is 20C (ΔT=60C), and use the pure-water heat capacity of 4.177 $kJkg^{-1}C^{-1}$, and a water density of 1013 kg m^{-3}. Each square metre of fracture surface needs $2 \times 0.001m^3m^{-2} \times$

$1013kgm^{-3} \times 4.177Jkg^{-1}C^{-1} \times 60C = 507Jm^{-2}$. To heat all of the newly-injected water in the 'fractured rock mass' requires 543MJ of heat. Compared against the ~220GJ of energy in the form of the heat contained within the inter-stage rock volume, the net cooling effect is quite small. The local and short-term reduction of elastic mechanical energy at the walls of fractures cannot be directly linked with aperture changes, but – in a future simulation method – those local and temporary reductions of total energy could provide a means by which the nearby rock mass might experience further inter-block movements, and thus alterations of the fracture-related flow system. The development of suitable simulation tools, which account for the micro-mechanics-based upscaling of material laws, will be important for enabling full-process investigations of such interactions.

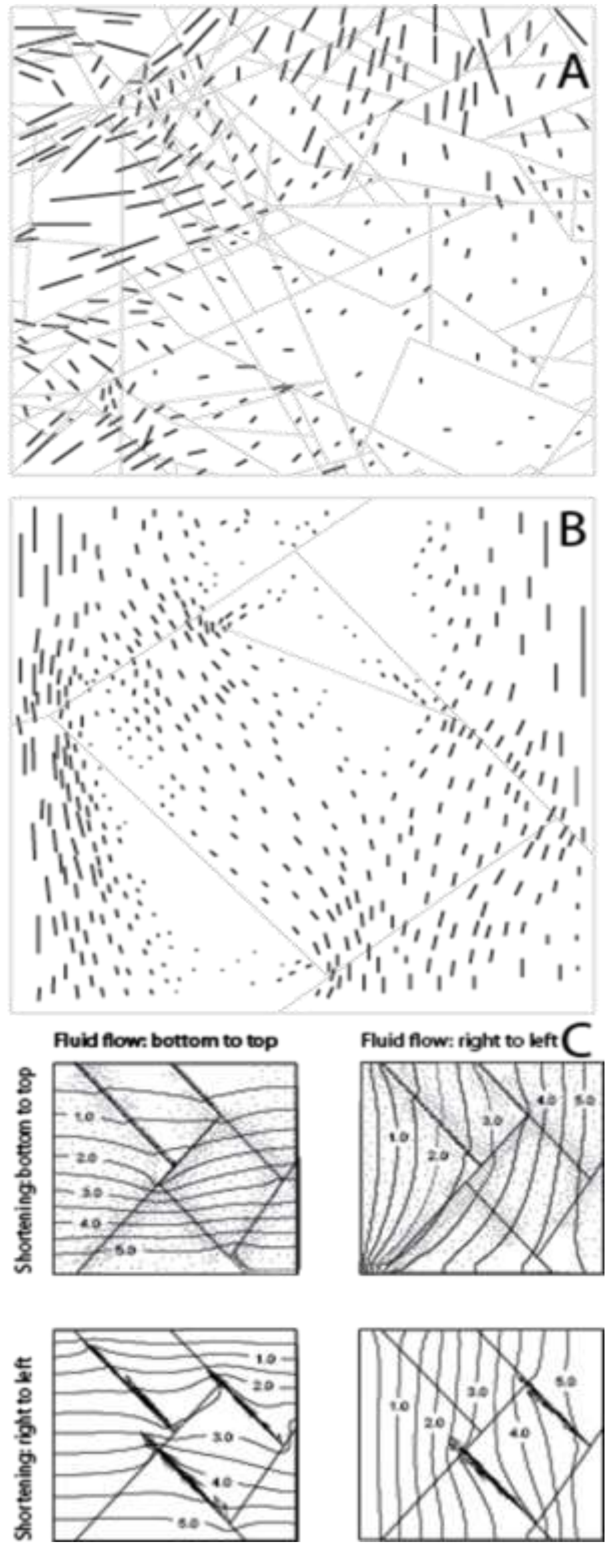

Fig. 3. A,B) Maximum-principal stress trajectories in simulations of model systems with functional discontinuities. (Image A redrawn from Baghbanan and Jing 2008; image B redrawn from Hall et al 2007). (C) Fluid flow simulation through a discontinuity-based model loaded in multiple ways, with contrasting directions of shortening and fluid potential. Note concentrations of flow due to wedge-opening of fractures in some loading cases. (Redrawn from Rouainia et al 2006.)

3. DISCUSSION AND SUMMARY

The 'elastic continuum' analysis (Section 2.1) did not consider variations in strains associated with the tips of the fracture, nor the impacts of additional complexity in wall-rock strains that would be associated with a non-planar fracture, or for fracture-shape irregularities that would lead to strong gradients in apertures and wall-rock strains. The calculations also ignored the energy changes linked to stress increases in the components parallel to the fracture that would arise from Poisson effects. In terms of making the point, that the hydraulic fracture leads to wall-rock strains and thus energy increases, there was no need to include the whole set of these complexities, since, ultimately, the continuum model has to be discarded.

A comparable selectivity was employed for choosing the rock volume used to calculate the resulting pressure, in the case that the excess fluid is assumed to be forced into the matrix pores. The rigid-frame model used in those calculations is incorrect, since it does not account for the energy costs that are induced by fluid pressure changes (Section 2.3). The arbitrary assignment of the extent of the fluid invasion does not matter relative to the point being made. For both of these simplifications, the energy costs were already neglecting important factors, so there was no value to be gained by making an attempt to define and argue the smaller details – whose inclusion would make the energy discrepancies worse than they already are. These simple cases make that point sufficiently forcefully that ignoring some factors is warranted, allowing the general method to be illustrated.

The description of fluid-rock interactions, with bi-directional effects arising from either pressure changes or bulk strains, is presented in a 1.5D fashion. We accounted for effects arising because of constraints around the prism of rock that we examined, but we did not attempt to perform the analysis in a true 3D strain field. It was not necessary to consider a realistic 3D problem here, since the primary purpose for describing the bi-directional interactions was to highlight the failure of typical approaches, which do not account for the energy cost in the rock framework due to pressure changes.

A comprehensive analysis of fractured-rock mechanics is not possible in this paper. The examples mentioned serve to highlight the general principle that rock masses, which are composed of fracture-bounded blocks, respond to their loading by developing load-carrying arrangements that are somewhat like the force chains that are now understood to dominate the loading responses that develop within un-cemented soils. Simulations of such systems, operated without the type of fluid interactions described herein, possess a much-reduced content of elastic energy, with qualitative and quantitative numbers indicative of energy amounts of ~60% of what would be apparent from the bulk motions and loads while allowing large openings to develop between blocks. The subject of discontinuum geomechanics may well hold the key to developing process models of hydraulic fracturing that operate with a balanced energy budget.

The brief consideration of the thermal effects associated with hydraulic fracturing indicates that this consideration appears to have only secondary importance. But, the noted difference (in ground uplift) between thermally-stimulated reservoirs, and those hydraulically-stimulated, is interesting. Both stimulations induce the rock mass to try to expand. The thermal case can result in ground movement, while the hydraulic case does not seem to do this much, if at all. Two aspects may be significant in explaining the difference. The first one is that in the hydraulic case, the provoking factor (fluid energy) is transferrable to some distance, and may be involved in making pathways to enable its own transfer. That contrasts with heat, which needs conduction to achieve large transfers. The other factor is that the majority of extensive energy in a typical rock (90+%) is linked to its thermal state. In a laterally-equilibrated reservoir, thermal energy is already the dominant form everywhere. When a thermal stimulation occurs, this involves a major addition of energy, much larger than the energy of a hydraulic stimulation, so there is 'spare' energy that is able to lift the entire overburden to produce the ground motion.

Adopting an energy-focused approach to stimulation forces a careful analysis of any proposed process, along with a need to express that process in energy terms. The usual analytical expressions adopted for fracture propagation purposes are not developed from a full-system energy perspective, and the stress changes that are calculated by these methods, in the wall rocks, are not explicitly included within the formulation of the energy budget for the analytical model system. The micro-mechanics perspective forcefully demonstrates the bi-directional mechanical interactions between pore fluids and the rock framework, and provides the basis for understanding the partitioning of energy flux into or out of a fluid-filled porous rock. Many common expressions in rock mechanics allow for an arbitrary change in one factor (stress or pressure) without accounting for the change in other factors, and thus the energy budget is unbalanced when using those expressions.

As shown herein, a physics-based analysis reveals serious or fundamental flaws in the concept models that are in common use to explain hydraulic fracturing. Developing their replacements should prove interesting over the coming years.

4. APPENDIX

Pressurised water has potential energy that can be transformed into work, if the water pressure acts against a reactive object. The pressure is directly associated with the volumetric strain of the water:

$$P = \int K(P) d\varepsilon_{vol} \qquad \text{(A1)}$$

Where K is the bulk modulus, or the inverse of the compressibility.

$$C = {}^{1}/_{K}. \qquad \text{(A2)}$$

The bulk modulus of water is almost constant over the range of pressures of interest here, so the expression can be simplified by treating that modulus as a constant

$$P = K\varepsilon_{vol}. \qquad \text{(A3)}$$

The specific, or intensive, energy of water is given as:

$$U_{water} = \frac{1}{2} P\varepsilon_{vol} = \frac{1}{2}\frac{P^2}{K}, \qquad \text{(A4)}$$

Both $P\ and\ K$ have the unit of $Pa = {}^{N}/_{m^2}$. The specific energy has the units of

$$U_{water} = {}^{Nm}/_{m^3} = {}^{J}/_{m^3} = Pa \qquad \text{(A5)}$$

Caution is warranted, to avoid equating intensive energy and pressure/traction, even though they have the same units. Intensive energy is related to the relevant pressure/traction value via the strain, and thus through the relevant material modulus.

The extensive 'elastic' potential energy, expressed in J, is derived by multiplying the intensive energy by the volume over which that state applies $W_{water} = U_{water}Vol$, where the current volume is related to the reference mass by the present density, ρ

$$Vol = {}^{\rho Vol_{ref}}/_{\rho_{ref}}. \qquad \text{(A6)}$$

Intensive energy forms are not conserved, and only relate to the state at some instant. It is incorrect to say that these forms move; rather, it can be said that the distribution of pressure changes. Extensive energy forms are additive, and thus they are conserved in any conversions of energy, as process interactions occur. Extensive energy can be said to move. The choice to express extensive energy with the symbol W is linked to the fact that the potential energy can be transformed into work.

In a linear continuum (as assumed herein for the elements of the rock framework), the (principal) components of the state of stress are functionally linked to the components of elastic strain. Here, the expressions are written with a term to emphasise their use for examining how states change during some process.

$$\Delta\varepsilon_i = \frac{1}{E}[\Delta\sigma_i - v(\Delta\sigma_{ii} + \Delta\sigma_{iii})] \qquad \text{(A7)}$$

$$\Delta\sigma_i = \frac{vE(\Delta\varepsilon_i + \Delta\varepsilon_{ii} + \Delta\epsilon_{iii})}{(1+v)(1-2v)} + \frac{E\Delta\varepsilon_i}{1+v} \qquad \text{(A8)}$$

The specific, or intensive, elastic energy can be expressed in the principal directions, with the total intensive elastic energy being the sum.

$$U_{elas} = \frac{1}{2}(\sigma_i\varepsilon_i + \sigma_{ii}\varepsilon_{ii} + \sigma_{iii}\varepsilon_{iii}) \qquad \text{(A9)}$$

Due to the strict relationship between stress and strain in linear elasticity, the intensive energy can be expressed in terms of either stress or strain only.

$$U_{elas} = \frac{1}{2}\frac{1}{E}(\sigma_i\sigma_i + \sigma_{ii}\sigma_{ii} + \sigma_{iii}\sigma_{iii})$$

$$= \frac{1}{2}\frac{1}{E}\left(\sigma_i^2 + \sigma_{ii}^2 + \sigma_{iii}^2\right) \qquad \text{(A10)}$$

$$U_{elas} = \frac{1}{2}E(\varepsilon_i\varepsilon_i + \varepsilon_{ii}\varepsilon_{ii} + \varepsilon_{iii}\varepsilon_{iii})$$

$$= \frac{1}{2}E\left(\varepsilon_i^2 + \varepsilon_{ii}^2 + \varepsilon_{iii}^2\right) \qquad \text{(A11)}$$

Similarly, as with the intensive energy units for the fluid components, the unit of specific elastic energy has the same units as the Pascal, but it is more appropriate to use the equivalent dimensional statement of ${}^{J}/_{m^3}$. Extensive energy is determined by multiplying the relevant specific energy component by the volume over which that stress component applies.

$$W_{elas} = U_{elas}Vol \qquad \text{(A12)}$$

All of the energy expressions can be written in tensor form so that they can be used in non-principal orientations or in the case of non-isotropic materials.

5. ACKNOWLEDGEMENT

The micro-mechanics investigations that underpin this paper have been progressed with the financial support of a Royal Society Theo Murphy Blue Skies Award. Comments received in relation to oral presentations of parts of this paper have proven helpful in identifying points in need of greater clarification.

6. REFERENCES

1. Ahmed, S, Müller, TM, Madadi, M, and Calo, V. 2019. Drained pore modulus and Biot coefficient from pore-scale digital rock simulations. International Journal of Rock Mechanics and Mining Sciences, 114, 62-70.
2. Andrä, H, Combaret, N, Dvorkin, J, Glatt E, and Han, J *et al.*, 2013. Digital rock physics benchmarks - Part I: Imaging and segmentation. Comput. Geosci., 50: 25-32.
3. Baghbanan, A. and Jing, L. 2008. Stress effects on permeability in a fractured rock mass with correlated fracture length and aperture. *International Journal of Rock Mechanics and Mining Sciences*, 45, 1320-1334.
4. Biot, M.A., 1941. General theory for three-dimensional consolidation. Journal of Applied Physics, 12, 155-164.
5. Bultreys, T, De Boever, W. Cnudde, V. 2016. Imaging and image-based fluid transport modeling at the pore scale in geological materials: A practical

introduction to the current state-of-the-art. Earth Sci Reviews, 155, 93-128.

6. Couples, G.D. 2019. Phenomenological understanding of poro-elasticity via the micro-mechanics of an idealized digital-rock model, Geophysics. 84, doi: 10.1190/geo2018-0577.1
7. Couples, G.D., and Lewis, H., 1998, Lateral variations of strain in experimental forced folds, *Tectonophysics*, **295** 79-91.
8. Hall, S.A., Lewis, H. and Macle, X. 2007. Improved seismic identification of inter-fault damage via a linked geomechanics seismic approach, . In: Lewis, H. and Couples, G.D. (eds) *Relationships Between Damage and Localization*, Geological Society of London, Special Publications, **289**, 187-207.
9. Hubbert, M.K., and Willis, B.D. 1957. Mechanics of hydraulic fracturing. Petroleum Transactions. 210: 153–68.
10. Miskimins, J.L (ed) 2019. Hydraulic Fracturing: Fundamentals and Advancements. Society of Petroleum Engineers, 795pp.
11. Rouainia, M., Lewis, H., Pearce, C., Bicanic, N., Couples, G.D., and Reynolds, M.A. 2006. Hydro-geomechanical modelling of seal behaviour in overpressured basins using discontinuous deformation analysis. *Engineering Geology* **82** 222-233.