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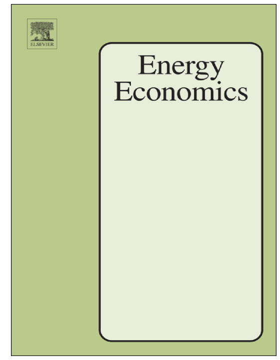
Citation for published version:
Lagomarsino, E 2020, 'Estimating elasticities of substitution with nested CES production functions: Where do we stand?', *Energy Economics*, vol. 88, 104752. https://doi.org/10.1016/j.eneco.2020.104752

Digital Object Identifier (DOI):
10.1016/j.eneco.2020.104752

Published In:
Energy Economics



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PII: S0140-9883(20)30091-8

DOI: https://doi.org/10.1016/j.eneco.2020.104752

Reference: ENEECO 104752

To appear in: *Energy Economics*

Received date: 12 January 2018

Revised date: 31 March 2020

Accepted date: 8 April 2020

Estimating elasticities of substitution with nested CES production functions: Where do we stand?

Elena Lagomarsino*

Department of Economics, Heriot-Watt University, Edinburgh EH144AS, United Kingdom

*Corresponding author. Present address: University of Genoa, Department of Economics, Via Vivaldi 5, 16126 Genova, Italy. E-mail: elena.lagomarsino@unige.it. I am very grateful to Prof. Mark Schaffer, Prof. Karen Turner, Prof. Peter McGregor and Dr. Alessandro Spiganti for the valuable comments.

Keywords: KLEM production functions; substitution elasticities; energy and capital substitution estimation; nested CES

Declarations of interest: none

Abstract

Prompt by an increasing interest by climate change modelers, a rich literature estimating elasticities of substitution in a nested CES production framework has recently developed. This article reviews such literature. We critically describe the nesting structures, estimation approaches, data sources and aggregation, econometric techniques, and types of substitution elasticities used by previous literature, to offer a comprehensive description of the various options available to a researcher attempting this estimation. We also provide suggestions for potential improvements to the estimation process. In particular, we warn researchers to use CES production functions with caution in empirical applications given their restrictive build-in assumptions.

1 Introduction

The first studies on the substitutability between production inputs date back to the 1930s when Hicks (1932) and Robinson (1933) formalized two independent concepts of elasticity of substitution between capital and labour. Energy was recognised as a key input in production only in the 1970s, after the outbreak of the oil crisis. Indeed, the price of oil quadrupled in response to the embargo in 1973, and this prioritized the analysis on a level at which energy could be substituted with other factor inputs. Of particular interest was the relationship between capital and energy: if the two inputs were complements, an increase in energy prices would have led to a downturn in capital formation and, hence, to a slowdown in economic growth; to the contrary, a rapid formation of capital would have balanced the limited use of energy resources and helped to avoid a recession. Today, understanding the relationship between capital and energy is once again highly relevant: on the one hand, the scarcity of non-renewable resources may lead to a sharp increase in their relative price; on the other, the decision on how much to invest in energy-saving technology is driven by the level at which one can substitute away from energy, and this is of utmost interest for climate policies aiming at mitigating greenhouse gas emissions. Starting with Hudson and Jorgenson (1974) and Berndt and Wood (1975), a vast body of applied econometric studies spanning more than four decades has thus attempted to provide empirical estimates of the level of substitution between factor inputs, including energy. This literature has been reviewed by Apostolakis (1990), Thompson (1988), and Koetse et al. (2008), who, however, focused on only empirical econometric papers based on Translog functions, with the clear purpose of comparing results based on the same theoretical foundation. Nevertheless, it has been papers using nested Constant Elasticity of Substitution (henceforth, CES) production technologies that, in recent decades, have exerted the most effort in the estimation of substitution elasticities. In this paper, we review this more recent literature.

Indeed, a rich strand of literature has recently developed to provide estimates based on a CES framework, mostly motivated by the desire to inform computable general equilibrium (henceforth, CGE) models. These models are widely used by academic institutions, governments, and international organizations to simulate and analyse the economy-wide effects of different shocks and policies. However, CGE modellers could no longer rely on parameters borrowed from existing empirical papers, since these were incompatible from the point of view of both the functional form and the set of data employed. In fact, while the older applied econometric papers relied on flexible functional forms for their generality (and, in particular, on the Translog function), the CGE literature uses mostly CES functions for their global validity and increased tractability.[1] Moreover, the older literature has generally estimated production functions at only the broadest levels of disaggregation; on the contrary, industrial disaggregation plays a crucial role in CGE modelling. Results from CGE models applied to climate and environmental analysis have been demonstrated to be highly contingent on the level of substitutability of energy with other factor inputs, particularly capital (see e.g. Jacoby et al., 2006, on climate change mitigation policies and Saunders, 2000, and Turner, 2009, on economy-wide rebound effects). For all these reasons, the estimation of consistent, precise, and reliable substitution elasticities has become of particular interest to CGE researchers.

The papers reviewed are summarised in Table 1. We start this review with a brief introduction to the nested CES functional form in Section 2. Section 3 introduces different types of elasticity and their economic interpretations. Reflecting the different columns of Table 1, the rest of this review is structured around five main aspects that are critical to the estimation of substitution elasticities using CES functions; to each of these aspects, we dedicate a different section. In particular, Section 4 summarises the possible estimation approaches, Section 5 focuses on the choice of a nesting structure, Section 6 describes possible data aggregation and sources, and Section 7 analyses the available econometric techniques. For each of these aspects, we review the literature in light of the choices made so far, the latest advances, and the remaining challenges; we also explain the potential limitations to the use of CES functions in empirical applications, making occasional reference to the older applied econometric literature. We provide conclusions and some promising avenues for future research in Section 8.

Table 1: Studies surveyed and their characteristics (Source: own elaboration.)

Author	Approach	Techn. Change	Nesting	Country Year	Data
Prywes (1986)	Indirect	HN	(((K;E);L;)M)	US 1971-76	20 M.Ind.
Chang (1994)	Indirect	HN	(((K;E);L);M)	Taiwan 1956-71	M.S.
Kemfert	Direct	HN	((K;E);L)	W. Germany	Country 7

[1] Translog function have seldom been used in CGE models (for some exceptions, see Hertel and Mount, 1985, Despotakis and Fisherm 1988, Li and Rose, 1995).

(1998)				1960-93 1970-88	M.Ind.
van der Werf (2008)	Indirect	FS	((K;L);E)	12 countries 19781996	Country 7 M.Ind.
Okagawa and Ban (2008)	Indirect	Null	((K;L);E);M)	14 countries 1995-2004	19 Ind.
Turner et al. (2012)Indirect	FS	((E;L);K)	UK 1970-2005	Country	27 Ind.
Baccianti (2013)	Indirect	FS	-	27 countries 1995-2008	33 Ind.
Su et al. (2012)	Direct	HN	((E;L);K)	China 1953-2006	Country
Shen and Whalley (2013)	Direct	HN	((E;L);K)	China 1979-2006	Country
Koesler and Schymura (2015)	Direct/Linear.	HN	(((K;L);E);M)	40 countries 1995-2006	35 Ind.
Dissou et al. (2015)	Indirect	FS	((E;L);K)	Canada 1962-97	10 M.Ind.
Henningsen et al. (2019)	Direct	HN	-	Germany 1991-2004	Country 8 Ind.
Feng and Zhang (2018)	Direct/Linear.	HN	((K;E);L)	China 2000-15	9 Ind.
Antoszewski (2019)	Indirect	FS	(((K;L);E);M)	26 countries 1995-2009	34 Ind.

Notes: HN stands for Hicks neutral, FS for factor specific; M.Ind. for manufacturing industries, M.S. for manufacturing sector, Ind. for industries.

2 The CES functional form

The CES functional form was originally introduced by the Stanford group around Arrow et al. (1961). While several attempts have been made by production theorists to develop an n-input generalization, only two stand out: Blackorby and Russell's (1989) one-level n-input function and Sato's (1967) nested n-input function. The former is a straightforward extension of the two-input case where n inputs are combined at the same level of production and share the same degree of substitutability. In the latter, n inputs are nested at different levels of production according to a pre-determined structure and eventually combined to form the final output.

In contrast to the one-level CES, which imposes the same elasticity of substitution among all factors, nested CES functions are characterized by a high level of adaptability because they allow for different degrees of substitutability between inputs and unlimited freedom in the composition of the nested structures. Following Sato (1967), let the set of n inputs $\{x\}=\{x_1,\ldots,x_n\}$ be partitioned into S bundles, $\{x^{(1)},x^{(2)},\ldots,x^{(S)}\}$, where $\{x^{(s)}\}=\{x_1^{(s)},\ldots,x_{N_s}^{(s)}\}$. In each bundle, N_s inputs are combined in a CES function to produce an intermediate output X_s. The two-level n-input nested CES is then given by:

$$Q=\gamma\left(\sum_{s=1}^{S}\alpha_s X_s^{-\rho}\right)^{-\frac{\nu}{\rho}} \qquad \text{with} \qquad X_s=\gamma_s\left(\sum_{i\in N_s}\beta_i^{(s)}\left(x_i^{(s)}\right)^{-\rho_s}\right)^{-\frac{1}{\rho_s}}, \tag{1}$$

where Q is output, $\gamma\geq 0$ and $\gamma_s\geq 0$ are efficiency parameters, $\alpha_s>0$ and $\beta_i^{(s)}>0$ are share parameters, with $\sum_s\alpha_s=1$ and $\sum_i\beta_i^{(s)}=1 \quad \forall s$, $\nu>0$ is the scale parameter,[2] and $\rho>-1$ and $\rho_s>-1$ are the substitution parameters.[3] The intra-class elasticity of substitution (i.e. the elasticity of substitution within the sth group) is given by $\sigma_s=1/(1+\rho_s)$ and the inter-class elasticity of substitution (i.e. the elasticity of substitution among input groups) is given by $\sigma=1/(1+\rho)$. Sato's (1967) representation of a nested CES is limited to two levels, however these functions can be organized in more levels by further partitioning at least one of the X_s bundles.

To formally represent a nested CES function, one needs to define a nesting structure for the

[2] The scale parameter represents the degree of homogeneity of the function (in this case, ν th degree). All the empirical literature reviewed in this paper assumed a unitary scale parameter, indicating constant returns to scale. Therefore, hereafter, we consider only production functions with $\nu=1$.

[3] When $\rho=0$, the CES reduces to a Cobb-Douglas form; when $\rho\to\infty$, the CES reduces to a Leontief form; when $\rho\to-1$, the CES becomes a linear production function.

inputs that fixes the order and the level at which these are combined with one another. In the rest of the paper, we use the conventional notation whereby all the inputs that form an intermediate output are aggregated in parenthesis. For instance, in the simplest form of a two-level three-input CES, the feasible nesting structures are represented by $((x_1;x_2);x_3))$, $((x_1;x_3);x_2))$, and $((x_2;x_3);x_1))$. As an example, in the first formulation equation (1) would reduce to:

$$Q = \gamma\left(\alpha\left(\beta(x_1)^{-\rho_1} + (1-\beta)(x_2)^{-\rho_1}\right)^{\frac{\rho}{\rho_1}} + (1-\alpha)(x_3)^{-\rho}\right)^{-\frac{1}{\rho}}, \tag{2}$$

where we have dropped non-essential subscripts and superscripts for ease of reading. We present the different choices made by the reviewed literature regarding nesting structures in Section 5.

Nested CES functions are homogeneous, homothetic, and strongly separable in their inputs: thanks to these characteristics, they are also globally "well-behaved" (i.e. output is monotonically increasing and isoquants are convex). Since this ensures the convergence of their computational models, nested CES functions are by far the most popular functional form among CGE researchers. Nevertheless, as will be further explained in Section 5, the tractability of nested CES functions comes at the detriment of generality; indeed, their maintained assumptions entail that elasticities of substitution are both independent of output and input quantities and constrained by pre-determined rules, which depend on the selected nesting structure.

3 Elasticities of substitution with nested CES

The concept of elasticity of substitution was introduced by Hicks (1932) in relation to a two-input production function. Formally, it is measured as the ratio of the two inputs with respect to the ratio of their marginal products and provides *'a measure of the ease with which the varying factor can be substituted for others'* holding output constant (Hicks, 1932, p.117).

Allen (1934) made two separate generalizations of this concept for a multi-factor production function. The first led to the development of the so-called partial Hicks elasticity of substitution (HES), also known as direct elasticity, which is computed by applying the original elasticity of substitution to each pair of inputs, holding output and the quantities of the other inputs constant. Thus, HES are obtained by computing:

$$\sigma_{ij}^{HES} \equiv \frac{\partial \ln\left(\frac{x_i}{x_j}\right)}{\partial \ln\left(\frac{f_j}{f_i}\right)} = \frac{\partial \ln\left(\frac{x_i}{x_j}\right)}{\partial \ln\left(\frac{P_j}{P_i}\right)}, \tag{3}$$

where f_i and f_j are the partial derivatives of the production function with respect to input i and j respectively (i.e. the marginal products of the inputs) and P_i is the price of input i; when quantities of inputs are optimal, $f_i = P_i$. Because other inputs quantities are not allowed to adjust, it is considered a short-run elasticity.

The second generalization, which was re-investigated by Allen (1938) and extended to the n-factor case by Uzawa (1962), came to be known as partial Allen elasticity of substitution (AES). This is given by:[4]

$$\sigma_{ij}^{AES} \equiv \frac{\sum_{k=1}^{n} P_k x_k}{P_j x_j} \frac{\delta x_i / x_i}{\delta P_j / P_j}, \tag{4}$$

where output and all input prices except P_j are held constant. Following Sato (1967), equation (4) can be read as follows: the percentage increase in the quantity of input i due to an increase in P_j is given by σ_{ij}^{AES} multiplied by its percentage expenditure share. AES is symmetric around zero: a negative AES indicates input complementarity whereas a positive one indicates input substitutability.

Berndt and Wood (1979), Prywes (1986), and Koetse (2006) provide an insight into the difference in interpretation between HES and AES based on the concepts of engineering and economic elasticity of substitution. When the price of one input, e.g. P_j, increases, two effects may take place: first, a substitution effect such that the quantity of that input x_j is substituted for another input's quantity x_i; second, an income effect according to which real income declines and thus the quantity demanded of each input, including x_i, is reduced *ceteris paribus*. While engineering elasticities take into account only substitution effects, economic elasticities also include income effects: as the income effect is negative, economic elasticities are always larger

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[4] AES can also be formulated thus: $\sigma_{ij}^{AES} = \sum_{k=1}^{n} f_k x_k (x_i x_j)^{-1} | D_{ij} | / | D |$ where $| D |$ is the bordered Hessian determinant of the production function and $| D_{ij} |$ represents the cofactor of the ik th term in the Hessian matrix.

than the engineering ones. The primary difference between HES and AES is that the former is an engineering elasticity whereas the latter is an economic elasticity.[5]

The relationship between HES and AES and the intra- and inter-class elasticities of substitution in a nested CES (σ_s and σ, respectively) was explained in Sato's (1967) seminal paper and is summarized in Table 2. While inter- and intra-class elasticities of substitution are constant, partial elasticities may be varying with inputs. Indeed, HES are constant and equal to σ_s if inputs belong to the same subset and AES are constant and equal to σ if inputs belong to two different subsets; however, the remaining partial elasticities are not constant and depend on the expenditure shares of the factor inputs in the subset (θ^s, as defined by Sato, 1967, p. 203). On the one hand, this information should warn CGE researchers against using estimates obtained from production functions that do not match the one in their model;[6] on the other, one can use these relationships to compare results based on alternative functional forms.

Table 2: HES and AES partial elasticities in nested CES. (Source: own elaboration.)

	HES	AES
$\sigma_{ij}\quad i,j \in N_s, i \neq j$	σ_s	$\sigma + (\sigma_s - \sigma)/\theta^s$
$\sigma_{ij}\quad i \in N_s, j \in N_r$	Harmonic mean of σ_s, σ_r, and σ	σ

Notes: the harmonic mean is calculated using the expenditure shares of the factor inputs, θ^s, as

[5] Another type of elasticity frequently used in empirical papers is the Morishima elasticity of substitution (MES) (Morishima, 1967, Blackorby and Russell, 1989), which informs on the percentage change in two inputs ratio given a percentage change in the price of one of the two inputs. It is an asymmetric engineering elasticity and can be written in terms of AES as $\sigma_{ij}^{MES} = f_j x_j / f_i x_i \left(\sigma_{ij}^{AES} - \sigma_{jj}^{AES}\right);$ factors that are AES substitutes are MES substitutes, factors that are AES complements might become MES substitutes. Despite the growing attention, MES is hardly of interest for CGE research because it cannot be included as a constant parameter (indeed, both the intra- and inter-class MES vary with input and output levels). Frondel (2011) argues that HES, AES, and MES are seldom relevant for practical purposes: the simpler cross-price elasticity (an economic elasticity) is much more intuitive and more suited to economic interpretation (see also Frondel and Schmidt, 2002).

[6] Particular care should be taken when using estimates obtained with Translog functions. Indeed, from a review of the main empirical efforts based on Translog cost functions, Frondel and Schmidt (2002) maintain that these studies are limited in their ability to measure factor substitution relationships, as the sign of the estimated cross-price elasticities are strongly dependent on the magnitudes of the input cost shares.

weights.

Most papers reviewed provided estimates for only the constant intra- and inter-class elasticities, σ_s and σ; only Prywes (1986) and Chang (1994) also provided AES estimates for inputs belonging to the same subset, whereas no paper provides HES estimates for inputs belonging to different subsets. For comparison, Table 3 reports the estimated industrial energy-capital intra-class elasticities of substitution found by the surveyed studies, considering a $((K;E);L)$ nesting structure. Although values vary among the papers considered, we observe that elasticities are often lower than one.Prywes (1986) obtained negative elasticities, a theoretical impossibility, but emphasized that most of them were not significantly different from zero. Only van der Werf (2008) and Okagawa and Ban (2008) tested the hypotheses of Cobb-Douglas and Leontief production function and found some evidence in favour of them.

Table 3: Estimated σ_s between energy and capital. (Source: own elaboration.)

Sectors	Prywes	Kemfert	van der Werf	O&B	Dissou et al.	Baccianti
Food	-0.24	0.85	0.96^C	0.39	0.66	0.57
Textiles	0.45		1.04^C	0.17^L	0.46	0.63
Paper and printing	-0.15	0.33	0.89^C	0.37^L	0.47	0.55
Chemicals	-0.39	0.93		0.04^L	0.43	0.50
Non metallic min.	-0.13	0.04	0.95^C	0.35	0.47	0.58
Primary metal	0.00	0.34	0.89^C	0.29^L	0.37	0.37
Machinery	0.31			0.12^L	0.44	0.48
Electrical eq.	0.15			0.25^L		0.61
Transportation eq.	0.45	0.61	1.01^C	0.09^L	0.44	0.63
Construction			0.10	0.99^L		0.98

Notes: O&B stands for Okagawa and Ban (2008). The superscripts C and L indicate when an econometric test for Cobb-Douglas or Leontief production function was performed, respectively. In the reviewed papers, the name of industries could vary or be further disaggregated. The industry “Food” for Prywes (1986) and van der Werf (2008) is “Food and Tobacco” ; the industry “Textile” for Prywes (1986) is an average of the values in “Textile mill products” and “Appearel and other textile products" ; the industry "Primary metal" for Kemfert (1998) and van der Werf (2008) is "Iron and steel" ; the industry "Transportation eq." for Kemfert (1998) is "Vehicles".

4 Estimation approaches

Three approaches have been employed by the literature to estimate nested CES production functions: the direct approach based on its non-linear estimation, the indirect approach based on a cost minimization (or, equivalently, a profit maximization) problem, and the approximation approach based on its Kmenta's (1967) linearisation.

The direct approach consists in using a non-linear least squares estimation based on ad-hoc non-linear optimization algorithms. Some of these algorithms were made available for empirical applications by Henningsen and Henningsen (2011); nevertheless, researchers often encountered problems in finding adequate starting values for the non-linear procedure and in making it reach numerical convergence, particularly with complex nested CES structures (Henningsen and Henningsen, 2012). Even though the direct approach has been used several times by the CGE literature to estimate substitution elasticities, Henningsen et al. (2019) gave a warning about the reliability of some of the published results. Indeed, in an attempt to replicate and update Kemfert's (1998) findings using both her original estimation approach and more up-to-date techniques, Henningsen et al. (2019) could not obtain statistically accurate estimates of the production function parameters. Therefore, they advised against the use of the direct estimation methods in empirical applications if long time-series and high independent variation of the input quantities are not available.

The indirect approach, which has been the most popular so far (see Table 1), relies on the assumptions of profit maximizing (cost minimizing) behaviour and price exogeneity, and involves the collection of data on both quantities and prices of inputs and output. Prywes (1986), Chang (1994), and Okagawa and Ban (2008) employed it to estimate a three-level four-input nested CES of the form $(((x_1;x_2);x_3);x_4))$, which we use as an example to illustrate this approach.[7] This consists of solving a cost minimizing problem at each of the three levels - $(x_1;x_2)$, $((x_1;x_2);x_3)$,

[7] Van der Werf (2008) further elaborated the indirect approach, deriving conditional factor demands and adding some algebraic steps to identify biased technical progress coefficients. A detailed description is provided in Section 4.1.

and $(((x_1;x_2);x_3);x_4))$ - starting with the inner one. In the example considered, the inner level cost minimization problem is given by

$$\min_{x_1,x_2\geq 0} \quad P_1x_1 + P_2x_2 \tag{5a}$$

$$\text{subject to:} \qquad X_{12} = \left(\beta_{12}\left(x_1\right)^{-\rho_{12}} + (1-\beta_{12})\left(x_2\right)^{-\rho_{12}}\right)^{-\frac{1}{\rho_{12}}}, \tag{5b}$$

where X_{12} is the intermediate output obtained by combining x_1 and x_2, β_{12} is the inner share, and ρ_{12} is the substitution parameter. The inner elasticity is given by $\sigma_{12} = 1/(1+\rho_{12})$. From the first order conditions and using a log-transformation, the following equation is derived:

$$\ln\left(\frac{x_1}{x_2}\right) = \sigma_{12}\ln\left(\frac{\beta_{12}}{(1-\beta_{12})}\right) + \sigma_{12}\ln\left(\frac{P_2}{P_1}\right). \tag{6}$$

Equation (6), with the addition of a disturbance term, is then estimated.

This procedure is repeated for the two upper levels of production, using the intermediate output obtained from the estimated coefficients of the previous level as one of the inputs, and the Lagrangian multiplier as its price (since prices are assumed exogenous). For instance, the Lagrangian multiplier of X_{12} is given by:

$$P_{12} = \left(\beta_{12}^{\sigma_{12}} P_1^{1-\sigma_{12}} + (1-\beta_{12})^{\sigma_{12}} P_2^{1-\sigma_{12}}\right)^{1/(1-\sigma_{12})}. \tag{7}$$

The elasticities of substitution are represented by the estimated coefficients attached to the logarithm of the ratio between prices, and the share parameters can be derived from the constant term.

Finally, the linearisation method was rarely chosen by the literature surveyed because of the associated approximation bias (Thursby and Lovell, 1978). Koesler and Schymura (2015) compared estimates obtained from the Kmenta's (1967) estimation and the non-linear approach and concluded that the former performs less well in terms of goodness of fit statistics, particularly when the elasticity of substitution is far from the point around which the approximation is made. Feng and Zhang (2018) also estimated a linearised nested CES and found the resulting coefficients to be inconsistent with economic theory.

4.1 Technical change

Technical change can be seen as a shift upward of the production function due to an increase of factor effectiveness; in CGE models, the shift can be assumed exogenously or modelled. Among the CGE models, climate change research has focused particularly on investigating the welfare effects of endogenous technical change, given alternative climate policy scenarios. For this purpose, and since the level at which inputs can be substituted for one another influences investments in new technologies (van der Werf, 2008), it is important that empirical researchers provide elasticities of substitution estimated alongside technical change parameters.

As shown in the third column of Table 1, technical change has usually been incorporated in the production function through an exogenous Hicks-neutral term, often represented by a monotonic and non-decreasing function of time. To provide an example of this formulation, the outer nest of the CES function in (1) can be modified as follows:

$$Q = e^{\lambda t}\gamma\left(\sum_{s=1}^{S}\alpha_s X_s^{-\rho}\right)^{-\frac{1}{\rho}}, \tag{8}$$

where λ denotes growth in technical progress and t represents a time trend. Two valuable insights are offered by the literature that chose this representation of technical change. First, Kemfert (1998) and Koesler and Schymura (2015) found intersectoral differences in the estimates of the λ parameter, indicating that an aggregated coefficient would not be able to account for the high variation of industrial data and structure effects. Secondly, Kemfert (1998) found that technical change rates vary significantly with CES nesting structures.

Conversely, few authors considered in this review included input-specific technical change parameters in their production functions: Table 4 shows their estimated value of the technical change parameters. Even though this representation of technical change is more realistic, particularly as it allows for the acknowledgment of technological progress, it has not often been used because of the identification problem connected with the estimation of the associated CES. One method to overcome this issue was proposed by van der Werf (2008), who modified a three-input nested CES to include biased technical change as follows:

$$Q = \left(\alpha\left(\beta(A_{x_1}x_1)^{-\rho_1} + (1-\beta)(A_{x_2}x_2)^{-\rho_1}\right)^{\frac{\rho}{\rho_1}} + (1-\alpha)(A_{x_3}x_3)^{-\rho}\right)^{-\frac{1}{\rho}}, \tag{9}$$

where $A_{x_i} = \exp\{\gamma_{x_i}t\}$ for $i = \{1,2,3\}$ are the factor-specific technical change parameters. The cost function associated with the production function can be obtained from the first order conditions; Shephard's lemma is then applied to obtain conditional factor demands. Rearranging and using a log-transformation, van der Werf (2008) obtained the following factor share equations:

$$\ln\left(\frac{x_1}{X_{12}}\right) = \sigma_{12}\ln\beta + (\sigma_{12}-1)\ln A_{x_1} + \sigma_{12}\ln\left(\frac{P_{12}}{P_{x_1}}\right) \tag{10a}$$

$$\ln\left(\frac{x_2}{X_{12}}\right) = \sigma_{12}\ln(1-\beta) + (\sigma_{12}-1)\ln A_{x_2} + \sigma_{12}\ln\left(\frac{P_{12}}{P_{x_2}}\right) \tag{10b}$$

$$\ln\left(\frac{x_3}{Q}\right) = \sigma\ln(1-\alpha) + (\sigma-1)\ln A_{x_3} + \sigma\ln\left(\frac{P_Q}{P_{x_3}}\right) \tag{10c}$$

$$\ln\left(\frac{X_{12}}{Q}\right) = \sigma\ln\alpha + \sigma\ln\left(\frac{P_Q}{P_{12}}\right). \tag{10d}$$

Using first differentiation and a few algebraic steps (van der Werf, 2008, p. 2,969), a system of four linear equations, independent from the unobservable X_{12} and P_{12}, is obtained. His estimation results of the system with imposed cross-equation restrictions showed both energy-saving and labour-saving technical change and rejected Hicks-neutral total factor productivity for all the industries and countries considered. Turner et al. (2012) and Dissou et al. (2015) replicated van der Werf's (2008) approach.

Table 4: Technical change coefficients' estimates (TCE)

Authors	TCE	Type	Nested structure
Kermfert (1998)	2.20%	Neutral	((K;E);L)
	0.69%	Neutral	((K;L);E)
	0.64%	Neutral	((E;L);K)
van der Werf (2008)	1.2-2.8%	Energy augmenting	((K;L);E)
	3%	Labour augmenting	((K;L);E)
	-2.4%	Capital augmenting	((K;L);E)
Su et al. (2012)	0.1%-0.9%	Neutral	((K;L);E), ((K;E);L), ((E;L);K)
Feng and Zhang (2018)	5%-7%	Neutral	((K;E);L)

Baccianti (2013), instead, followed the recommendations of the dynamic macroeconomic literature and normalised the CES production function by converting variables in an indexed number form, in order to identify the input-specific technical change parameters. Indeed, Klump et al. (2007), León-Ledesma et al. (2010), and Klump et al. (2012) showed the empirical benefits of estimating normalised two-input CES functions with biased technical change, both in terms of the identification and interpretation of parameters.

More recently, Antoszewski (2019, pp. 280-281) illustrated a new approach to estimate elasticities of substitution with non-neutral technical change. He considered a normalized three-input nested CES function and estimated the first order conditions derived from a profit maximizing problem at each nest, similarly to Prywes (1986). Even though this approach does not allow for the identification of single factor-specific parameters but only their absolute difference, it produces estimates of elasticities corrected for technical change.

Finally, it should be noted that the approaches of both van der Werf (2008) and Antoszewski (2019) are in line with the macroeconomic notion of embodied technical change, whereby technical progress is embodied in the characteristics of new capital plants and equipment and, as such, depends on the rate of investment in capital goods. This concept has been analysed by the empirical literature focusing on energy efficiency (Berndt et al., 1993, Kratena, 2007), but should be further investigated in relation to input elasticities of substitution. For example, it should be evaluated whether energy savings could be realized in the long run by changing the capital stock when the level of energy-capital substitution in the short run is low.

5 Nested structures

When attempting the estimation of a nested CES production function, a researcher is confronted with the selection of a specific nesting structure for the CES, which defines how inputs are combined with each other. For comparison purposes, the fourth column of Table 1 reports the nesting chosen by the papers surveyed in this review. Interestingly, it appears that the inner nests $(K;E)$, $(K;L)$, and $(L;E)$, have been selected in approximately equal proportions, despite different economic interpretations.[8] While surveying the nesting structures adopted in a set of well-known CGE models, Feng and Zhang (2018) noted that $((K;L);E)$ has been the most popular form, $((K;E);L)$ has been chosen in only three models, including the well-known

[8] In particular, whereas the first is consistent with Berndt and Wood's (1975) theory of utilized capital and the second represents value-added, $(L;E)$ lacks an intuitive economic interpretation.

GTAP-E model by Truong et al. (2002), and that $((E;L);K)$ has never been selected.

The selection of a nesting structure represents a crucial decision. From a theoretical standpoint, the choice of the nesting structure implies determined relations between inputs because of the maintained assumption of strong separability. Indeed, nested inputs must share the same degree of substitutability with the input(s) outside the nest: for example, in the case of the three-input two-level nested CES of the form $((x_1;x_2);x_3)$, strong separability constrains x_1 and x_2 to equally substitute for x_3. From an empirical standpoint, Lecca et al. (2011) showed that CGE results are sensitive to the choice of input nesting in the production function and Feng and Zhang (2018) confirmed that CGE simulations using different nesting structures can lead to divergent results.

Despite its importance, the literature has not yet analysed seriously the best way to tackle this choice. Different rationales have been followed: either this choice is made *a priori*, according to some pre-existing belief, or a data-driven approach is used, where statistical tests are employed to assess which functional form represents the best fit for the data. In particular, the early study of Prywes (1986) on US manufacturing industries assumed a three-level CES production function of the form $(((K;E);L);M)$, without presenting any theoretical or empirical evidence in support of this choice. Chang (1994), analysing the Taiwan manufacturing sector, chose the same nesting structure but mentioned the use of the R^2 statistic as a discriminating criterion, without reporting formal results. In principle, this data-driven approach consists of estimating all the feasible nesting structures separately and selecting the one with the highest R^2 (Kemfert, 1998). This approach was used by all subsequent studies using a three-input production function.

Despite its popularity, this selection method was questioned by Baccianti (2013) and Dissou et al. (2015). They noted that the use of an R^2 statistic with an indirect method based on conditional factor demands is not recommended because the final comparison is made between models based on different dependent and explanatory variables. Moreover, Dissou et al. (2015) warned that goodness of fit measures should be interpreted with caution in the context of a system of equations. We underline that the use of this selection method should be discouraged under a direct estimation approach as well. Indeed, it has long been known within the statistical and econometric literature that the R^2 statistic is inadequate for comparing competing non-linear models because it is based on the underlying assumption that the model being fit is linear (Spiess and Neumeyer, 2010).

Furthermore, we want to stress that any selection criteria, such as goodness of fit measures, should be used only when the researcher is convinced that the set of competing models considered includes the true one. With restrictive production functions such as the CES, we cannot exclude the possibility that another functional form could provide a better representation of the true input-output relationship. If that is the case, goodness of fit results would mistakenly lead to the selection of a nesting structure, even when none of the alternatives represents the "best" characterization of the true production function.

Three recent papers provide alternative empirical approaches for selecting nesting structures. Zha and Zhou (2014) proposed a two-step method: first, a Translog function and the relative elasticities of substitution are estimated; second, a nested CES framework is built where the inputs in the inner nest are those that exhibit, on average, the largest Translog elasticity. Conversely, Dissou et al. (2015) based their approach on the cost minimization cross-equation restrictions characterizing the CES indirect estimation method based on a system of first order conditions. They proposed choosing the nesting structure for which the estimated coefficients meet the cross-equation restrictions.[9] Finally, Feng and Zhang (2018) expanded Chang's (1994) approach and defined three selection criteria for studies based on direct non-linear estimation methods: goodness of fit, goodness of convergence, and compliance to economic meaning. The first is assessed by means of the R^2 statistic, the second is measured as the distance between the estimated elasticities and the initial point assumed for the estimation, and the third is a check on the compliance of the estimated CES parameters to their meaningful theoretical ranges.

Although these latest approaches represent interesting attempts at improving the nesting selection process, not only do they lack theoretical support but also have limitations in applicability. Three shortcomings of the existing methods seem especially relevant. First, whereas this issue has been approached in only a three-input scenario, the choice becomes increasingly demanding as the number of inputs increases: indeed, the number of feasible combinations grows exponentially, as exemplified in Table 5. Second, the non-nested structure, $(x_1; x_2; x_3)$, has been disregarded in most studies: only van der Werf (2008) and Dissou et al. (2015) formally tested for the equivalence of intra- and inter-class elasticities, and Baccianti (2013) was the only paper

[9] van der Werf (2008) also performed this test and rejected at least one restriction for most of the industries/countries considered.

estimating the one-level CES alongside the nested ones. Interestingly, his findings showed that the non-nested CES was not rejected in 5 out of 7 industries and 8 out of 12 countries, supporting the importance of including this specification alongside the other feasible options. Third, conditional on the availability of sectoral data, nesting structures should be tested at a disaggregated level because the best nesting specification could differ across industries. Although assuming a single structure certainly simplifies CGE modelling, it might not provide a realistic representation of the production side of the economy, and it does not allow for the investigation of the different impacts that, for instance, climate policies can have on the various sectors.

Table 5: Three- and four-input feasible nesting structures. (Source: own elaboration.)

Three-input	Four-input			
$(x_1;x_2;x_3)$	$(x_1;x_2;x_3;x_4)$	$((x_1;x_2;x_3);x_4)$	$(((x_1;x_2);x_3);x_4)$	$(((x_1;x_2);x_4);x_3)$
$((x_1;x_2);x_3)$	$((x_1;x_2);(x_3;x_4))$	$((x_1;x_2;x_4);x_3)$	$(((x_1;x_3);x_2);x_4)$	$(((x_1;x_3);x_4);x_2)$
$((x_1;x_3);x_2)$	$((x_1;x_3);(x_2;x_4))$	$((x_1;x_3;x_4);x_2)$	$(((x_1;x_4);x_3);x_2)$	$(((x_1;x_4);x_2);x_3)$
$((x_2;x_3);x_1)$	$((x_1;x_4);(x_2;x_3))$	$((x_2;x_3;x_4);x_1)$	$(((x_2;x_3);x_1);x_4)$	$(((x_2;x_3);x_4);x_1)$
			$(((x_3;x_4);x_1);x_2)$	$(((x_3;x_4);x_2);x_1)$

We believe that a new potential selection approach could be based on a test on the maintained separability hypotheses on which the nested CES is founded. The test would be carried out exploiting the Translog functional form, which represents a second-order linear approximation of a CES that does not share the same built-in properties. A researcher could estimate the Translog function and test whether the estimated coefficients satisfy the conditions required for strong separability and homogeneity.

As an example, we show the formulation of the test hypothesis for the three-input case.[10] Consider the following Translog, characterized by inputs symmetry and Hicks neutrality:

$$\ln Q = \ln a_0 + \sum_{i=1}^{3} a_i \ln x_i + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} \ln x_i \ln x_j . \quad (11)$$

[10] However, we refer the interested reader to Berndt and Christensen (1973) and Hoff (2014) for a more detailed presentation of the separability restrictions required. Following the separability definition proposed by Berndt and Christensen (1973), this approach can be extended to the four-input case.

Linear homogeneity in inputs, which corresponds to the assumption of constant returns to scale, is satisfied when:[11]

$$\sum_{i=1}^{3} a_i = 1 \tag{12a}$$

$$\sum_{j=1}^{3} a_{ij} = 0 \quad \forall i \in \{1,\ldots,n\}. \tag{12b}$$

Inputs x_i and x_j are strongly separable from a given input x_k, if the following set of constraints are jointly satisfied (Berndt and Christensen, 1973):[12]

$$a_i a_{jk} - a_j a_{ik} = 0 \tag{13a}$$

$$a_{jj} a_{kk} - a_{jk}^2 = 0. \tag{13b}$$

A Wald test on constraints (12) and (13) informs on whether the Translog is approximating a CES production function with a $((x_i; x_j); x_k)$ structure and constant returns to scale. The test should be repeated for all the feasible nested structures. The following three outcomes are possible: i) failure to reject for only one nested structure, ii) failure to reject for more than one nested structure, and iii) rejection of the hypotheses for all the nested structures tested. In the first case, data support a precise nesting structure; in the second case, the researcher can decide among alternative production specifications; in the third case, results seem to indicate that a nested CES might not be the appropriate functional form to describe production technology given the data under analysis.[13]

5.1 A note on CES built-in assumptions

Even though the approach described for the selection of the appropriate nesting structure has not

[11] If the researcher wants to introduce non-constant returns, then the right hand side of equation (12) can take on any positive value. However, it should be noted that the scale parameter ν of the CES should then be imposed equal to it.

[12] It should be noted that Danny et al. (1978) criticized Berndt and Christensen's (1973) separability definition for being too restrictive. According to their own definition of approximate separability, a Translog needs to satisfy only constraint (13a) to approximate a CES with a $((x_i; x_j); x_k)$ nested structure (as in equation (2)).

[13] Nevertheless, rejection could also follow from the approximation error implicit in the use of the Translog functional form. This error would increasingly bias the results of the test as one moves away from the approximation point.

yet been empirically employed, a similar one was used by the applied econometric literature cited above to test data for constant returns to scale (i.e. linear homogeneity) and separability. In particular, weak and strong separability were tested for two purposes: to check whether a value-added aggregate could be separated from other inputs or to verify whether a further disaggregation of the energy input was justified.

The method employed was to test restrictions, such as equations (12) and (13), on the estimated coefficients. For what concerns constant returns, Iqbal (1986), Khiabani and Hasani (2010), and Haller and Hyland (2014) found that they were not statistically supported. With regards to separability and the first purpose, Medina and Vega-Cervera's (2001) results indicated that a value-added aggregate could be separated from other inputs, and Griffin and Gregory (1976) obtained the same results for both their US and European models. To the contrary, Roy et al. (2006) found no evidence in favour of value-added separability; Berndt and Wood (1975) and Chung (1987), looking at the same dataset and using two different approaches, failed to reject the separability conditions for only the $((K;E);(L;M))$ case. With regards to separability and the second purpose, this was investigated by Hazilla and Kopp (1986), Moghimzadeh and Kymn (1986), Garofalo and Malhotra (1988), and Hisnanick and Kyer (1995); only Hisnanick and Kyer (1995) did not reject the relative restrictions.

These findings highlight that nested CES built-in assumptions are often not met in empirical applications. Therefore, we suggest that results obtained from the estimation of production functions based on a CES functional form should be taken with caution because they may lead to unrealistic conclusions if the maintained assumptions on which they are based are not supported by the data.

6 Data

6.1 Aggregation bias

Applied econometric studies on substitution elasticities have generally estimated aggregate production functions for the manufacturing sector of the country they were analysing.[14] Thus, they assumed that all industries within this sector shared the same elasticities of substitution. In the

[14] With few exceptions, i.e. Field and Grebenstein (1980), Dargay (1983), Hazilla and Kopp (1986), and Iqbal (1986).

context of CGE modelling, industrial disaggregation plays a crucial role because it provides a near approximation to the real economy, allowing for the evaluation of the effect of demand or supply shocks on the different production sectors. Accordingly, all the papers surveyed in this review, apart from Chang (1994), broke apart the manufacturing sector into several industries, finding evidence of high intersectoral variation in the values of the estimated elasticities of substitution.

Nonetheless, industries are, in turn, aggregates of sub-industries: for instance, the industry "non-metallic minerals" comprises several sub-industries such as cement, glass, and stone. Alexeeva-Talebi et al. (2012) emphasized that, because of aggregation, important details on the structural characteristics (e.g. energy intensity) of the sub-industries can be neglected and that this leads to biased CGE results. Afterwards, Oberfield and Raval (2014) demonstrated how, from a macroeconomic standpoint, aggregated elasticities of substitution can be seen as the sum of a "micro" elasticity and an elasticity of demand, where the former is a measure of the substitution between inputs within sub-industries, and the latter captures reallocation between more or less factor-intensive sub-industries. They conclude that results based on "macro" industries should be interpreted with caution and that further research should endeavour to disentangle the effects of these two components of aggregated elasticities.

A convincing solution to aggregation biases is the one proposed by Prywes (1986) who, in a study on the US, estimated the production function of twenty, two-digit SIC industries based on the four-digit SIC breakdown of each of them, gathering data on 450 sub-industries. However, this approach is of course subject to data availability.

6.2 Input aggregation

The research on inputs substitutability in the last forty years has focused mainly on aggregate capital and energy, with the aim of shedding light on the nature of their relationship, given the conflicting evidence found in the earliest applied works. But capital and energy, such as labour and material inputs, are themselves aggregates, in the sense that all of them are made of different components. For instance, the energy aggregate can comprise natural gas, electricity, and oil, among others, whereas the capital input can be broken apart into machines and structures. The disaggregation of the energy input can be particularly relevant for CGE models attempting to analyse the impact of policies connected with the use of different sources of energy (e.g. coal, oil, and natural gas) in production (see e.g. Burniaux et al., 1992).

As far as we know, there have been no attempts at the estimation of the elasticity between energy sub-inputs using nested CES functions. However, in the empirical econometric literature, Fuss (1977), Prindyck (1979), Turnovsky et al. (182), and Iqbal (1986) were motivated by the idea of revealing the importance of disaggregating the energy input. They estimated Translog sub-models where they broke apart the energy input into four or six components with the aim of estimating interfuels' substitutability and found evidence of heterogeneity in the relation between the different components of energy and capital. Subsequently, this point was further explored using inference tests on separability restrictions to support the hypothesis of the energy input disaggregation. Moghimzadeh and Kymn (1986) and Hisnanick and Kyer (1995) performed separability tests on a five-input Translog cost function (considering capital, labour, electric energy, non-electric energy, and material) for the US. Both showed that the electric and non-electric partition is statistically justified, but they reached discordant conclusions on the nature of the relationships between inputs.

6.3 Building a dataset

The estimation of nested CES production functions depends on the availability of output and input quantity time-series, as well as the relative prices time-series (if the indirect approach is employed). Before 2000, gathering data for even a single country, but more than one industry, was demanding: authors had to deal with multiple national sources and accounts and this increased the probability of measurement errors; it was often the case that authors had to build their own measures by, for instance, employing Divisia quantity and price indexes.[15] In 2000, the European Commission and the OECD introduced the EU-KLEM and WIOD databases, making international and European time-series on production output, inputs, and prices for several industrial sectors available. In this section, we provide an overview of which data, for each input, have been employed by the existing literature to describe the production technology.

Starting with output, most authors have measured it using time-series on gross output. Exceptions are represented by van der Werf (2008), who built a measure based on the sum of value-added and the value of energy, and Su et al. (2012) and Shen and Whalley (2013), who used

[15] Divisia index are continuous index number series constructed from aggregates variations that can have different measurement units.

real GDP time-series. Henningsen et al. (2019) casted doubt on Kemfert's (1998) decision (later imitated by Feng and Zhang, 2018) of measuring output using only gross value-added, underlying how this approach leads to unreliable substitution estimates.

With regard to labour quantity, this has generally been expressed in the number of employees or man-hours; for its price, average employee compensation or hourly wage were usually employed. Both the EU-KLEM and WIOD databases provide a quality-adjusted measure of labour quantity (termed labour services), which takes into account different levels of productivity of the worker (i.e. high, medium, and low skilled).

The quantity of energy has commonly been measured through an index of gross energy consumption and reported in several different measurement units. The relative price was either an average of the total expenditure or a price Divisia index.

Since the measurement of quantity of materials is challenging, most studies considered specifications with only labour, capital, and energy (see column 4 of Table 1): among the papers surveyed in this review, half are based on the four-input production function (thus including materials), but none provided clear information on materials data-gathering. The EU-KLEM provides an estimate of quantity based on national supply and use tables.

Capital deserves a special discussion because its measurement has always been troublesome, owing to the complexity of reconciling the theory with the empirical data. According to production theory, the quantity of capital input is represented by the flow of services provided by capital goods. However, there are no readily available measures of the flow of capital services. This, in fact, remains a highly abstract concept: it includes all the explicit and implicit transactions connected with capital goods in each production period. Hence, if the firm owns a particular capital asset such as a machine, the rental price or user cost for this asset in each period is implicit and does not appear in the accountancy books. Moreover, a machine is usually deployed for more than one period, but the explicit transaction cost appears only in the accountancy year in which it has been purchased. Neoclassical theory has linked the quantity of capital services to the quantity of capital goods (i.e. the stock of capital) by defining the quantity of services as a measure of the contribution of the capital stock to the production process in a given year (Jorgenson and Griliches, 1967, Hall and Jorgenson, 1967, Hulten, 1990);[16] the capital stock, in turn, is an aggregate that can

[16] Jorgenson and Griliches (1967) proposed the idea of capacity utilization but it has been demonstrated that this entailed ulterior measurement problems.

include several types of goods such as equipment and structures, intangibles (e.g. software), land, financial assets, and human capital. However, National Accounts traditionally exclude the last two.

The existing literature based on CES production functions has derived the capital stock value using the perpetual inventory method (see Meinen et al., 1998, for an illustration). However, recent papers based on the OECD and EU-KLEM international databases exploited the provided measures of gross capital stock. In particular, the latest approach used by the OECD to estimate capital services is the following: first, calculate the net stock series from investment series using a perpetual inventory model that accounts for age-efficiency profile and depreciation patterns; second, estimate the rental price of each asset (that is, the cost of the asset for one period) to obtain the price of the capital services; finally, use these two steps to generate weights for each input component and combine them.

The price of capital can be estimated using a number of different approaches. In the papers considered in this review, it has been typically derived as forgone interest plus depreciation minus capital gain. However, Hazilla and Kopp (1986) reviewed 34 alternative definitions of capital service price and demonstrated that different capital service price specifications lead to statistically different elasticity estimations.

7 Econometric techniques

In this section, we focus on the econometric techniques employed by those authors that opted for an indirect estimation approach.[17] Apart from Prywes (1986) and Turner et al. (2012), who used cross-sectional, time-series disaggregated data on sub-industries, and Dissou et al. (2015), who limited their analysis to Canada, the remaining authors exploited inter-country variability by building country panel datasets. Below, we illustrate briefly the methods used.[18]

van der Werf (2008) exploited the OECD international sectoral database, covering 12 countries and 7 industries for 19 years. As mentioned in Section 4, his econometric approach

[17] Indeed, a discussion on the various algorithms that can be used with non-linear procedures would be too technical and out of the scope of this review.

[18] We skip Prwes (1986), Chang (1994), and Okagawa and Ban (2008) since they did not explicitly discuss their approach.

consisted of estimating a system of equations separately for country and for industry, including country-industry fixed effects. He then tested for the validity of the fixed effects' restrictions and concluded that a pooled regression was the most efficient for his dataset. The same approach was followed by Turner et al. (2012) for 27 UK sectors over the years 1970-2005.

Baccianti (2013) built a panel dataset on 27 countries for the period 1995-2004 using the WIOD database. He estimated a system of equations for each sector, allowing the technological change parameters to be heterogeneous across countries. He used a generalised methods of moments estimator robust to heteroskedasticity and autocorrelation. Testing random effects against fixed effects, he found results in support of the latter. Even though Antoszewski (2019) used a very similar panel dataset, based on WIOD data, the econometric estimation followed a very different strategy. Instead of a system of equations, the author estimated separately single log-linearised first order conditions equations for each sector using ordinary least squares.[19]

Dissou et al. (2015) proposed a different approach: they estimated the same system but imposed cross-equation restrictions with a Seemingly Unrelated Equation estimator. Indeed, their dataset included only time-series data on 10 manufacturing industries spanning from 1962 to 1997. It should be noted that their work is the first where Augmented Dickey-Fuller unit-root tests were performed to investigate the stationarity of the input and output series. The analysis of time-series stationarity and cointegration should, in fact, be carefully considered in this type of study because results could otherwise be biased and inconsistent.

Two further aspects have not yet been tackled by the literature but are worth considering. First, serial correlation and simultaneous correlation of the error term which, given the typical dimension of the dataset (i.e. a number of sectors higher than the number of years), should be appropriately tested. Second, the possible presence of structural breaks. Column 5 of Table 1 shows that the time span considered in the analysis is generally long and often includes oil crisis periods and, more recently, the financial crisis. In such cases, together with plotting the time-series, structural break tests should be performed because they enable the detection of significant changes in data that could lead to the unreliability of the estimated model.

[19] The author verified the presence of country fixed effects with a likelihood ratio test and rejected it for most of the sectors and first order condition equations.

8 Conclusions

The applied literature analysing the substitution elasticities between production inputs, including energy, is vast and spans more than four decades. Results are highly discordant, particularly for what concerns the nature of the relationship between energy and capital, and the reviews by Apostolakis (1990), Thompson (1988), and Koetse et al. (2008) have attempted to identify the reasons. These surveys, however, focused on only empirical econometric papers based on Translog functions, with the clear purpose of comparing results based on the same theoretical model. Nevertheless, it has been papers using nested CES production technologies that, in recent decades, have exerted the most efforts in the estimation of substitution elasticities, motivated by the desire to inform CGE models. In this paper, we have reviewed this literature.

In particular, we structured our review around five main aspects that are critical to the estimation of nested CES: the interpretation of substitution elasticities, the estimation approach, the choice of a nesting structure, data sources and aggregation, and the econometric technique. For each of these aspects, we have described the choices made so far, with the intention of providing guidelines for future research, underlining matters that have not been properly investigated yet, and encouraging further discussions.

From an empirical standpoint, a step that has been so far overlooked is the running of diagnostic and formal tests on the data. This is crucial, given the new available databases. In particular, stationarity of the time-series of prices and quantities should be checked, as well as co-integration. Furthermore, with panels of industries followed over several years, serial and simultaneous correlations of the error term should be tested, as these datasets are generally characterized by a number of sectors that is bigger than the number of yearly observations available.

From a theoretical standpoint, we stress the need for new selection methods for discriminating between nesting structures, because those proposed so far have either limited applicability or some theoretical flaws. Furthermore, we warn researchers to be careful in employing nested CES functional forms: although their properties are particularly desirable in a CGE framework, their built-in assumptions might not be empirically supported by the dataset. This issue is likely to have important consequences on the reliability of the estimation results.

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Credit author statement

Elena Lagomarsino: Conceptualization, Methodology, Writing- Original draft preparation, Visualization, Writing- Reviewing and Editing

Highlights:

- Nested CES production function should be used with caution in empirical analysis
- The selection of an appropriate nesting structure for nested CES production function should be data-driven
- Econometric techniques for the estimation of nested CES production functions should be ameliorated