Combining Code-Domain and Power-Domain NOMA for Supporting Higher Number of Users

Combining Code-domain and Power-domain NOMA for Supporting Higher Number of Users

Naveen Mysore Balasubramanya, Ankit Gupta and Mathini Sellathurai
Heriot-Watt University, Edinburgh, U.K.

***Abstract*—Non-orthogonal multiple access (NOMA) is one of the key technologies being evaluated for the fifth generation (5G) wireless communications. In this paper, a novel NOMA mechanism combining both the code-domain and power-domain techniques is proposed to potentially support a higher number of users. In particular, considering the code-domain NOMA method of sparse code multiple access (SCMA) as the baseline, new low data rate (LDR) users are added on top of it using power-domain NOMA. The optimization problem involving resource and power allocation in such a system is solved such that the overall achievable sum-rate is maximized. Simulation results indicate that the proposed mechanism not only supports higher number of users, but also demonstrates a higher achievable sum-rate than the original SCMA-based system.**

***Index Terms*—NOMA, SCMA, resource allocation, integer programming, successive convex approximation**

I. Introduction

The fifth generation (5G) wireless communications are evaluating advanced technologies for achieving enhanced mobile broadband (eMBB), massive machine type communications (mMTC) and ultra-reliable low latency communications (URLLC). One of the key technologies which has interested all the communities of 5G is the non-orthogonal multiple access (NOMA) [1], [2]. NOMA operates on the principle of sharing time-frequency resources between users by separating them in another domain. Generally, this domain of separation falls into two regimes - power-based and code-based, leading to power-domain NOMA and code-domain NOMA mechanisms, respectively. The long term evolution (LTE) standard uses a variant of power-domain NOMA method called the multi-user superposition transmission (MUST) [1], [2] and for 5G both the code-domain and the power-domain NOMA methods are being evaluated.

A prominent code-domain NOMA technique being considered for 5G is the sparse code multiple access (SCMA) [3], where a finite number of users are allowed to interfere with each other. However, each user is assigned a unique codebook, which is complex-valued, multidimensional and sparse in nature. In SCMA encoding, these codebooks are used to spread the modulated symbols from the users over the allocated resources, and in decoding they assist in simultaneous detection of multiple users. The set of codebooks in the SCMA system is represented using a binary pattern matrix called the factor graph matrix. This nomenclature arises from the fact that the SCMA decoding utilizes the message passing algorithm (MPA) for multi-user detection, which in-turn employs a factor graph based approach. The two key elements in the factor graph matrix are the number of users interfering per resource (denoted by d_f) and the number of resources allocated per user (denoted by d_v). The complexity of the MPA increases exponentially with the number of interfering users. Hence d_f is chosen to be a small value [3], [4]. The ratio of the number of users to the number of resources ($(L/K) \times 100\%$) is called the loading factor of an SCMA system. A common example for SCMA consists of 4 resources being shared by 6 users with $d_v = 2$ and $d_f = 3$ (please see Appendix), corresponding to an loading factor of 150% [3].

SCMA can be applied to both grant-free and grant-based scenarios. In grant-free scenarios, each SCMA user randomly picks a codebook pattern for transmission. The performance analysis includes the study of collision detection and collision avoidance methods required when two or more SCMA users pick the same codebook pattern [5], [6]. In grant-based scenarios, each SCMA user is allocated a unique codebook and the performance is analyzed based on how to determine the unique mapping between the SCMA users and codebooks, such that the overall achievable rate in the system is maximized. But this problem of SCMA resource allocation is exponentially complex to be solved optimally, since n codebook patterns can be assigned to exactly n SCMA users in $n!$ ways. In literature, solutions to this problem are proposed through opportunistic mechanisms [7] and game-theoretic methods [8]. In order to increase the number of users supported by a grant-based SCMA system, one has to increase the number of codebook patterns. However, adding new codebook patterns increases the decoding complexity and reduces the reliability of the system [9]. Therefore, supporting higher number of users also requires an increase in the number of available resources to maintain feasible decoding complexity and reliability.

In this paper, we introduce a novel *NOMA mechanism that retains the number of resources in the uplink grant-based SCMA system and increases the number of supported users by adding one new low data rate (LDR) user per resource using power-domain NOMA*. A common example for such LDR users are Internet-of-things (IoT) users in 5G mMTC requiring delay tolerant services with reduced data rates. In the proposed mechanism, the SCMA signals are decoded considering the LDR signals as interference and the LDR user signals are decoded after cancelling the SCMA user signals. We perform uplink resource and power allocation in order to maximize the overall achievable rate. The main contributions of this paper are summarized below.

- We formulate the optimization problem for the resource and power allocation for the proposed NOMA mechanism and solve the problem by splitting it into independent sub-problems. Particularly, we split the problem into four parts - i) LDR power allocation, ii) LDR resource allocation, iii) SCMA resource allocation and iv) SCMA power allocation.
- We show that the LDR users can be assigned the maximum possible transmit power when there is adequate interference in the original SCMA system. For the LDR user resource allocation, we propose a binary-integer linear programming based solution to find the optimal allocation.
- Since finding an optimal allocation for the SCMA user resource allocation is exponentially complex, we use the heuristic algorithm proposed in our previous work [10]. For the SCMA power allocation problem, we propose a solution using the successive convex approximation technique.

II. System Model and Problem Formulation

Following the paradigms and the nomenclature of LTE systems, we use one physical resource block (PRB) pair as the minimum unit of resource allocation. On the time-frequency frame grid, a PRB pair occupies 12 subcarriers $\times$ 1 subframe [11]. Since the subcarrier spacing in LTE is 15 kHz and the duration of one subframe is 1 ms, a PRB pair denotes a region occupying a bandwidth of 180 kHz over an interval of 1 ms [11]. We refer to a PRB pair as the "resource" throughout this paper. The system model corresponds to the uplink scenario, where K resources have to be shared by L SCMA users and K LDR users. Thus, the number of resources remains the same (K), but number of users supported by our proposed mechanism increases to $(K + L)$.

Let $\mathcal{K} = \{1, \dots, K\}$ denote the set of available resources and $\mathcal{L} = \{1, \dots, L\}$ denote the set of SCMA users. The allocation of the k^{th} resource to the l^{th} SCMA user is indicated by the binary variable Λ_{lk}. Similarly, $\mathcal{D} = \{1, \dots, K\}$ represents the set of LDR users and their resource allocation is denoted by Ω_{dk}, where $d \in \{1, \dots K\}$. Also, we assume that the channels of the LDR users are inferior to those of the SCMA users. The base station first detects and decodes the SCMA user signals considering the LDR transmission as interference, subtracts these components from the received signal and then proceeds for LDR user detection and decoding. This assumption is in line with the power-domain NOMA model, wherein successive interference cancellation (SIC) techniques are used to detect the signals from different users [1]. Therefore, the proposed mechanism can be regarded as a combination of code-domain and power-domain NOMA techniques.

The instantaneous signal-to-interference-plus-noise ratio (SINR) of the SCMA and the LDR users are calculated as

$$\Upsilon_{lk}^{S} = \frac{\Lambda_{lk} P_{lk}^{S} \left|h_{lk}\right|^2}{\sum\limits_{j=1, j\neq l}^{L} \Lambda_{jk} P_{jk}^{S} \left|h_{jk}\right|^2 + \sum\limits_{d=1}^{K} \Omega_{dk} P_{dk}^{D} \left|g_{dk}\right|^2 + \sigma^2}, \quad (1)$$

$$\Upsilon_{dk}^{D} = \frac{\Omega_{dk} P_{dk}^{D} \left|g_{dk}\right|^2}{\sigma^2}, \quad (2)$$

where σ^2 is the noise variance, h_{lk} and g_{dk} are the channel coefficients of the l^{th} SCMA user and the d^{th} LDR user on the k^{th} resource, respectively. The corresponding transmission power values are indicated by P_{lk}^{S} and P_{dk}^{D}, respectively.

The objective of the resource and power allocation is to maximize the achievable sum-rate of the system given by

$$\mathcal{R}\big(\mathbf{\Lambda}, \mathbf{P}^S, \mathbf{\Omega}, \mathbf{P}^D\big) = \mathcal{R}^S\big(\mathbf{\Lambda}, \mathbf{P}^S, \mathbf{\Omega}, \mathbf{P}^D\big) + \mathcal{R}^D\big(\mathbf{\Omega}, \mathbf{P}^D\big), \quad (3)$$

where

$$\mathcal{R}^S\big(\mathbf{\Lambda}, \mathbf{P}^S, \mathbf{\Omega}, \mathbf{P}^D\big) = \sum_{l=1}^{L}\sum_{k=1}^{K} \log_2\left(1 + \Upsilon_{lk}^{S}\right), \quad (4)$$

$$\mathcal{R}^D\big(\mathbf{\Omega}, \mathbf{P}^D\big) = \sum_{d=1}^{D}\sum_{k=1}^{K} \log_2\left(1 + \Upsilon_{dk}^{D}\right). \quad (5)$$

The primal optimization problem can be formulated as

$$\begin{aligned}
(\textbf{OP Main}) \quad & \max_{\mathbf{\Lambda}, \mathbf{P}^S, \mathbf{\Omega}, \mathbf{P}^D} \quad \mathcal{R}\big(\mathbf{\Lambda}, \mathbf{P}^S, \mathbf{\Omega}, \mathbf{P}^D\big) \\
& \text{s.t} \quad (C.1) \quad \bar{F}_1(\mathbf{\Lambda}) = 1, \quad \forall l, k; \\
& \quad (C.2) \quad \sum_{k=1}^{K} P_{lk}^{S} \le P_{\max}^{S}, \quad \forall l; \\
& \quad (C.3) \quad P_{lk}^{S} \ge P_{\min}^{S}, \quad \forall l, k; \\
& \quad (C.4) \quad \sum_{k=1}^{K} \Omega_{dk} = 1, \quad \forall d; \\
& \quad (C.5) \quad \sum_{d=1}^{K} \Omega_{dk} = 1, \quad \forall k; \\
& \quad (C.6) \quad \sum_{k=1}^{K} P_{dk}^{D} \le P_{\max}^{D}, \quad \forall d; \\
& \quad (C.7) \quad P_{dk}^{D} \ge P_{\min}^{D}, \quad \forall d, k,
\end{aligned} \quad (6)$$

where $(C.1)$ to $(C.3)$ represent the SCMA user constraints and $(C.4)$ to $(C.7)$ represent the LDR user constraints. The function $\bar{F}_1(\mathbf{\Lambda})$ returns a "1" only if the resource allocation constraints of the SCMA users are satisfied. For example, using the matrix $\mathbf{F}$ in the [3], a pattern under consideration is valid if its row-wise logical AND with each of the allocated patterns results in a single "1". The LDR resource allocation is such that each user gets exactly one exclusive resource, which is represented by constraints $(C.4)$ and $(C.5)$. The maximum and the minimum transmission power values of the SCMA users are indicated by $P_{\max}^{S}$ and $P_{\min}^{S}$, respectively. The corresponding parameters for LDR users are denoted by $P_{\max}^{D}$ and $P_{\min}^{D}$, respectively.

We approach the task of maximizing the achievable sum-rate by starting with the optimal resource allocation and power allocation for LDR users, followed by the corresponding allocations for SCMA users. This is because the LDR users are detected after subtracting the SCMA user signals and maximization of their achievable sum-rate solely depends on their own resource and power allocation metrics, i.e., $\mathcal{R}^D$ only depends on $\mathbf{\Omega}$ and $\mathbf{P}^D$.

III. LDR User Resource and Power Allocation

The sub-problem related to the resource and power allocation of LDR users can be derived from **(OP Main)** as follows

$$\begin{aligned}
(\mathbf{OP1})\ \max_{\mathbf{\Omega},\mathbf{P}^D}\quad & \mathcal{R}^D(\mathbf{\Omega},\mathbf{P}^D) \\
(C.4) \quad & \sum_{k=1}^{K} \Omega_{dk} = 1, \qquad \forall d; \\
(C.5) \quad & \sum_{d=1}^{K} \Omega_{dk} = 1, \qquad \forall k; \\
(C.6) \quad & \sum_{k=1}^{K} P_{dk}^D \le P_{\max}^D, \quad \forall d; \\
(C.7) \quad & P_{dk}^D \ge P_{\min}^D, \qquad \forall d,k,
\end{aligned} \tag{7}$$

It is evident that the instantaneous SINR of the LDR users and hence the achievable rate increases with increasing transmission power. This means that the transmission power of the LDR users should be simply set to the highest possible value. However, a high transmission power leads to an increased interference for the SCMA users. This begs the question as how high the transmission power of the LDR users should be so that the overall achievable sum-rate of the SCMA plus LDR system is maximized.

Lemma 1. *The LDR user should be allocated the maximum power to maximize the overall achievable sum-rate since the underlying SCMA model is interference-limited.*

Proof. Each LDR user is assigned one exclusive resource. Therefore, on a given resource, there are d_f SCMA users and one LDR user. Without loss of generality, let us assume user 1, user 2 and user 3 are the three SCMA users. Note that they interfere with each other and also experience interference from the LDR user. The achievable rate on the resource is given by

$$R_1 = \sum_{l=1}^{3} \log\left(1 + \frac{P_l^S |h_l|^2}{\sum\limits_{j=1, j\neq l} P_j^S |h_j|^2 + P_1^D |g|^2 + \sigma^2}\right) + \log\left(1 + \frac{P_1^D |g|^2}{\sigma^2}\right). \tag{8}$$

Setting $a = \frac{P_1^S|h_1|^2}{\sigma^2}$, $b = \frac{P_2^S|h_2|^2}{\sigma^2}$, $c = \frac{P_3^S|h_3|^2}{\sigma^2}$ and $d = \frac{P_1^D|g|^2}{\sigma^2}$,

$$R_1 = \log\left(1 + \frac{a}{b+c+d+1}\right) + \log\left(1 + \frac{b}{a+c+d+1}\right) + \log\left(1 + \frac{c}{a+b+d+1}\right) + \log(1+d). \tag{9}$$

Note that $a, b, c, d \geq 0$. Without LDR users, the achievable rate would be

$$R_2 = \log\left(1 + \frac{a}{b+c+1}\right) + \log\left(1 + \frac{b}{a+c+1}\right) + \log\left(1 + \frac{c}{a+b+1}\right). \tag{10}$$

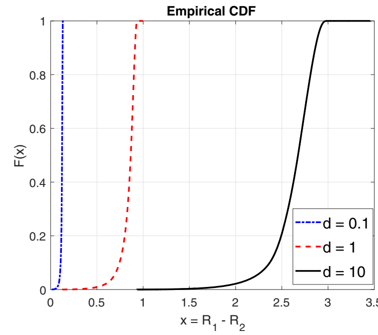

Fig. 1. Dependency of sum-rate on the transmission power of the LDR user.

Setting $T = a+b+c+1$ and simplifying we get

$$R_1 - R_2 = \log\left(\frac{(T+d)^3(1+d)}{(T+d-a)(T+d-b)(T+d-c)}\right) - \log\left(\frac{T^3}{(T-a)(T-b)(T-c)}\right). \tag{11}$$

We have to prove that $R_1 - R_2 > 0$ for any $d > 0$. This can be easily verified for the special case when $b = c = 0$. But the general case is unfortunately not tractable for an analytical proof. Therefore, we use numerical simulations to prove that $R_1 - R_2 > 0$ when $d > 0$. We conducted simulations for different possible values of the four variables (a, b, c and d) and obtained the empirical cumulative distribution (CDF) of $R_1 - R_2$ as shown in Fig. 1. It is evident from the x-axis of this CDF that $R_1 - R_2 > 0$. Also, the difference increases with increasing d, suggesting that d has to be chosen to the maximum possible value. Recall that $d = \frac{P_1^D|g|^2}{\sigma^2}$. Since we do not have control over the channel gain (g) and noise power (σ^2), the only way to maximize d is to set the power P_1^D to the maximum possible value, thereby proving the lemma. □

A. LDR Resource Allocation Using Integer Programming

Given that the transmission power of the LDR user on its allocated resource will be set to $P_{\max}^D$, the uplink resource allocation problem for the LDR users is now a transformed version of **(OP1)** with only $(C.4)$ and $(C.5)$ as the contraints. This can be formulated as a binary-integer set-partitioning problem [12] given by

$$\begin{aligned}
(\mathbf{OP1a}) \qquad \max_{\mathbf{y}} \quad & \mathbf{c}^T\mathbf{y} \\
\text{s.t.} \quad & \mathbf{A}\mathbf{y} = \mathbf{1}, \\
& y_i \in \{0,1\},
\end{aligned} \tag{12}$$

where $\mathbf{c}$ is the cost vector, $\mathbf{A}$ is the binary constraint matrix (computed as shown in Eqn. (13)) and $\mathbf{y}$ represents the resource allocation vector.

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_K^1 & \mathbf{I}_K^2 & . & . & . & \mathbf{I}_K^K \\ & & & & & \\ \mathbf{1}_K^T & \mathbf{0}_K^T & . & . & . & \mathbf{0}_K^T \\ \mathbf{0}_K^T & \mathbf{1}_K^T & . & & & \mathbf{0}_K^T \\ . & & & . & & . \\ \mathbf{0}_K^T & . & . & . & . & \mathbf{1}_K^T \end{bmatrix}, \tag{13}$$

where $\mathbf{1}_K$ denotes a column vector of K ones.

The cost vector is obtained by evaluating the throughput of each user on each resource. In this case, there are K users occupying K different resources, leading to a cost vector of length K^2. The constraint matrix, $\mathbf{A}$, is formed by taking into account the possible allocation patterns for each user. Since each user can occupy any of the K resources, the possible allocation patterns for d^{th} user is represented by a $K \times K$ identity matrix, denoted as $\mathbf{I}_K^d$. However, the resource allocation has to ensure that each pattern (column of $\mathbf{I}_K$) is allocated to a different user (denoted by the constraint $\mathbf{Ay} = \mathbf{1}$). This exclusivity constraint can be included by appending K more rows to the binary constraint matrix $\mathbf{A}$ as shown in Eqn. (13).

The solution for this binary-integer linear programming problem can be obtained using standard semi-definite programming techniques [13] or in software through `intlinprog` in MATLAB. The optimal LDR resource allocation $\mathbf{\Omega}^*$ is obtained by choosing the first K rows of $\mathbf{A}$ with columns corresponding to the indices of 1's in $\mathbf{y}$, mathematically represented as

$$\mathbf{\Omega}^* = \mathbf{A}(1:K, \text{index}(\mathbf{y}=1)). \tag{14}$$

IV. SCMA User Resource and Power Allocation

With the LDR user resource and power allocation completed, the sub-problem of SCMA resource and power allocation can be written as

$$\begin{aligned} (\mathbf{OP2}) \quad & \max_{\mathbf{\Lambda},\mathbf{P}^S} \quad \mathcal{R}^S(\mathbf{\Lambda},\mathbf{P}^S) \\ \text{s.t} \quad & (C.1) \quad \bar{F}_1(\mathbf{\Lambda}) = 1 \quad \forall l,k; \\ & (C.2) \quad \sum_{k=1}^{K} P_{lk}^S \le P_{\max}^S, \ \forall l; \\ & (C.3) \quad P_{lk}^S \ge P_{\min}^S, \quad \forall l,k; \end{aligned} \tag{15}$$

Determining the optimal solution to this joint resource and power allocation problem is highly complex. This is because it is exponentially complex to find the optimal solution for SCMA resource allocation part. Moreover, the expression for the SINR of SCMA users has the interference components in the denominator (please see Eqn. (1)), making it non-convex and rendering it difficult to be solved using conventional optimization methods. Therefore, we separate the SCMA resource and power allocation parts. We propose a heuristic algorithm for SCMA resource allocation and solve the power allocation using successive convex approximation.

A. SCMA Resource Allocation Using a Heuristic Method

The heuristic algorithm for SCMA resource allocation was first introduced in our previous work [10]. It was shown to be more than 90% close to the optimal solution obtained through an exhaustive search. We describe the algorithm here for the sake of completion.

The heuristic algorithm operates on one resource at a time. Since a resource can be shared by d_f users in SCMA, we allocate the resource under consideration to the best set of d_f users maximizing the achievable sum-rate. Particularly, let $\mathcal{P}$ denote the set of all possible combinations of choosing d_f out of L SCMA users. Then, for each resource, the heuristic algorithm computes the SINR for all the entries of $\mathcal{P}$ and arranges them in descending order. The resource is assigned to a specific combination which not only maximizes the SINR, but also satisfies the pattern validity (i.e, the function $\bar{F}_1$ should return a "1"). The process is repeated until all the resources are assigned. Algorithm 1 describes the heuristic algorithm. Its complexity is $\mathcal{O}(\tilde{L}K)$, where $\tilde{L} = \binom{L}{d_f}$.

Algorithm 1 SCMA Resource Allocation Heuristic

```
Initialize Λ to 0
Compute P
for k = 1 to K do
   for p = 1 to (L choose d_f) do
      Compute the SINR(k, p) using Eqn. (1)
   end for
   index = Indices of SINR(k, p) sorted in descending order
   for p = 1 to (L choose d_f) do
      Set Λ(k, P(index(p))) = 1
      Check if the pattern is valid: checkFlag = F̄_1(Λ).
      if checkFlag == 1 then
         BREAK
      else
         Set Λ(k, P(index(p))) = 0
      end if
   end for
end for
```

B. SCMA Power Allocation

After LDR user resource allocation ($\mathbf{\Omega}^*$), LDR user power allocation (set to $P_{\max}^D$) and SCMA user resource allocation ($\mathbf{\Lambda}^*$) using the heuristic algorithm, the SCMA power allocation problem can be formulated as

$$\begin{aligned} (\mathbf{OP2a}) \quad & \max_{\mathbf{P}^S} \quad \mathcal{R}^S\left(\mathbf{P}^S\right) \\ \text{s.t} \quad & (C.2) \quad \sum_{k=1}^{K} P_{lk}^S \le P_{\max}^S, \quad \forall l; \\ & (C.3) \quad P_{lk}^S \ge P_{\min}^S, \quad \forall l,k; \end{aligned} \tag{16}$$

Applying change of variables such that $\hat{P}_{lk}^S = \ln(P_{lk}^S)$, the SCMA user SINR is computed as

$$\hat{\Upsilon}_{lk}^S = \frac{\Lambda_{lk} e^{\hat{P}_{lk}^S} \left|h_{lk}\right|^2}{\sum\limits_{j=1, j\neq l}^{L} \Lambda_{jk} e^{\hat{P}_{jk}^S} \left|h_{jk}\right|^2 + \sum\limits_{d=1}^{K} \Omega_{dk} P_{\max}^D \left|g_{dk}\right|^2 + \sigma^2}. \tag{17}$$

Using the successive convex approximation technique, we impose the lower bound as

$$\mathcal{R}\left(\hat{\mathbf{P}}^S\right) \ge \mathcal{R}_{LB}\left(\hat{\mathbf{P}}^S\right) \approx \sum_{l=1}^{L}\sum_{k=1}^{K} \frac{\alpha_{lk}}{\ln(2)} \ln(\hat{\Upsilon}_{lk}^S) + \beta_{lk} \tag{18}$$

where $\alpha_{lk} = \frac{\xi_{lk}^S}{1+\xi_{lk}^S}$ and $\beta_{lk} = \log_2\left(1+\xi_{lk}^S\right) - \alpha_{lk}\log_2\left(\xi_{lk}^S\right)$ for any $\xi_{lk}^S \geq 0$ and the equality is satisfied when

$$\alpha_{lk} = \frac{\hat{\Upsilon}_{lk}^S}{1+\hat{\Upsilon}_{lk}^S}, \tag{19}$$

$$\beta_{lk} = \log_2\left(1+\hat{\Upsilon}_{lk}^S\right) - \alpha_{lk}\log_2\left(\hat{\Upsilon}_{lk}^S\right). \tag{20}$$

Thus the problem $(\mathbf{OP2a})$ transforms into

$$(\mathbf{OP2a'}) \quad \max_{\hat{\mathbf{P}}^S} \quad \mathcal{R}_{LB}\left(\hat{\mathbf{P}}^S\right)$$
$$\text{s.t} \quad (C.2) \quad \sum_{k=1}^{K} e^{\hat{P}_{lk}^S} \leq P_{\max}^S, \qquad \forall l; \tag{21}$$
$$(C.3) \quad e^{\hat{P}_{lk}^S} \geq P_{\min}^S, \qquad \forall l,k;$$

whose Lagrangian can be computed as

$$\mathcal{L}\left(\hat{\mathbf{P}}_S, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\vartheta}\right) = \sum_{l=1}^{L}\sum_{k=1}^{K}\frac{\alpha_{lk}}{\ln(2)}\ln(\hat{\Upsilon}_{lk}^S) + \beta_{lk}$$
$$-\sum_{l=1}^{L}\mu_l\left(\sum_{k=1}^{K}e^{\hat{P}_{lk}^S} - P_{\max}^S\right) - \sum_{l=1}^{L}\sum_{k=1}^{K}\nu_{lk}\left(P_{\min}^S - e^{\hat{P}_{lk}^S}\right) \tag{22}$$

and the dual decomposition problem is solved using

$$\min_{\boldsymbol{\mu},\boldsymbol{\nu}\geq 0} \quad \max_{\hat{\mathbf{P}}^S} \quad \mathcal{L}\left(\hat{\mathbf{P}}_S, \boldsymbol{\mu}, \boldsymbol{\nu}\right) \tag{23}$$

The solution for the SCMA user optimal power allocation is computed in an iterative manner and its updated as

$$P_{lk}^S(m+1) = e^{\hat{P}_{lk}^S}(m+1) = \frac{\Lambda_{lk}\alpha_{lk}}{\ln(2)\left(\mu_l - \nu_{lk}\right)}, \tag{24}$$

and the Lagrangian multipliers are updated as

$$\mu_l(m+1) = \left[\mu_l(m) + \epsilon_\mu\left(\sum_{k=1}^{K}e^{\hat{P}_{lk}^S} - P_{\max}^S\right)\right]^+, \tag{25}$$

$$\nu_{lk}(m+1) = \left[\nu_{lk}(m) + \epsilon_\nu\left(P_{\min}^S - e^{\hat{P}_{lk}^S}\right)\right]^+. \tag{26}$$

where ϵ_μ and ϵ_ν are positive step-sizes and $[x]^+ = \max(0,x)$.

The final algorithm summarizing the entire procedure of resource and power allocation for LDR and SCMA users is presented as Algorithm 2.

Algorithm 2 Final Algorithm

Set $\mathbf{P}^D = P_{\max}^D$ using **Lemma 1**.
Update LDR resource allocation $\boldsymbol{\Omega}^*$ using Eqn. (14).
Update SCMA resource allocation using **Algorithm 1**.
Initialize $\mathbf{P}^S$ to $P_{\min}^S$
Set the maximum number of iterations $I_{\max}$.
Initialize iteration counter $m = 0$ and $\{\alpha_{lk}, \beta_{lk}\} = (1,0)$.
Initialize the step sizes ϵ_μ and ϵ_ν.
Initialize $\boldsymbol{\mu}(m)$ and $\boldsymbol{\nu}(m)$.
repeat
    Calculate the SINR using Eqn. (17).
    Update $\{\alpha_{lk}, \beta_{lk}\}$ using Eqns. (19) and (20).
    Update $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ using Eqns. (25) and (26).
    Update $\mathbf{P}^S$ using Eqn. (24).
    Set $m \leftarrow m+1$.
until convergence $\mathbf{P}^{S^\star}$ or $m > I_{\max}$.

V. Simulation Results

In this section, we demonstrate the performance analysis of our proposed NOMA mechanism using Monte Carlo simulations. We adopt the basic SCMA configuration with $K = 4$ resources and accommodating $L = 6$ users and add $K = 4$ LDR users on top of the SCMA system using our proposed mechanism. Therefore, the total number of users supported by our system is 10, corresponding to a loading factor of $\left(\frac{K+L}{K}\right) \times 100\% = 250\%$.

The channel gains of the SCMA users ($|h_{lk}|^2$) and those of the LDR users ($|g_{dk}|^2$) are obtained using a typical urban (TU) model. This is one of the standard channel models for performance evaluation in LTE systems [11] and we used the Vienna LTE uplink simulator on MATLAB to obtain these values [14]. Moreover, in line with our system model, we chose the channel gains of LDR users to be worse than those of SCMA users on average, i.e., $\mathbb{E}\left(|g_{lk}|^2\right) < \mathbb{E}\left(|h_{dk}|^2\right)$. The maximum transmit power of the SCMA users ($P_{\max}^S$) and that of the LDR users ($P_{\max}^D$) were set to 23 dBm and 20 dBm, respectively. The minimum transmit levels, ($P_{\min}^S$) and ($P_{\min}^D$), were set to 15 dBm. The experiments were performed for 1000 channel realizations for transmit SNR values between 0 dB and 8 dB. For the SCMA power allocation, $\epsilon_\mu, \epsilon_\nu$ and ϵ_ϑ were set to 0.001, the convergence tolerance value was set to 10^{-5} and $I_{\max}$ was set to 10, while the algorithm converged in 5 iterations on average.

Fig. 2 shows the average achievable sum-rate for different cases. The solid lines indicate the results with optimal power allocation, while the dotted lines correspond to the cases without optimal power allocation. For the case without power allocation, the SCMA user power was equally divided between the allocated resources, i.e., the power on each allocated resource was fixed to $\frac{P_{\max}^S}{d_v}$, which is a common practice in uplink LTE systems [11], [12]. It is clear that the average achievable sum-rate increases when optimal power allocation is performed on this system.

When LDR users have to be included to the SCMA system, we have the option of adding them without or with optimal power allocation for the SCMA users. In both the cases, the average achievable sum-rate of the system is greater than that obtained in the SCMA system with optimal power allocation. This shows that *adding new LDR users is beneficial to the network throughput than merely optimizing the power allocation of the baseline SCMA system*. However, the disadvantage of adding LDR users without optimal power allocation is that the average achievable sum-rate of the SCMA users will be worse than the case without power allocation. It should be noted that the average achievable sum-rate of LDR users is lower than that of the SCMA users, but this is acceptable because a typical LDR user such as an IoT device using 5G mMTC does not demand high data rates.

Fig. 3a and Fig. 3b provide the average achievable sum-rate per SCMA user for transmit SNR = 0 dB and SNR = 8 dB, respectively. The per-user throughput for LDR users is not shown because it was close to the average sum-rate