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Enhanced Multiqubit Phase Estimation in Noisy Environments by Local Encoding

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The first generation of multiqubit quantum technologies will consist of noisy, intermediate-scale devices for which active error correction remains out of reach. To exploit such devices, it is thus imperative to use passive error protection that meets a careful trade-off between noise protection and resource overhead. Here, we experimentally demonstrate that single-qubit encoding can significantly enhance the robustness of entanglement and coherence of four-qubit graph states against local noise with a preferred direction. In particular, we explicitly show that local encoding provides a significant practical advantage for phase estimation in noisy environments. This demonstrates the efficacy of local unitary encoding under realistic conditions, with potential applications in multiqubit quantum technologies for metrology, multipartite secrecy, and error correction.

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Introduction.—Quantum systems are notoriously fragile due to unavoidable interactions with their environment [1], resulting in decoherence that grows exponentially with system size. This represents a major roadblock for quantum computing [2], quantum communication [3], and quantum metrology [4], rendering noise mitigation [5–7] indispensable. Quantum error correction (QEC) [8–10] schemes in principle achieve full protection against decoherence. However, daunting experimental requirements on the single qubit noise rate and large resource overheads [11] make QEC a long-term vision.

A complementary approach, expected to play a central role in near-term quantum technologies [12], is to relax the fault-tolerance requirement against arbitrary noise, aiming instead at enhanced robustness of quantum systems under experimentally relevant conditions. One of the dominant types of noise is local dephasing along a privileged direction [13–15]. There, simple single-qubit unitary encoding can drastically improve the resilience of quantum resources [16] such as multiqubit Greenberger-Horne-Zeilinger (GHZ) [17] states, where an exponential decay of entanglement [18,19] can be turned into a linear decay [16]. This improvement is crucial for metrology applications such as phase estimation in noisy environments [20,21], whereby the otherwise optimal phase sensitivity of GHZ states [22] becomes asymptotically bounded by a constant [23].

Here, in a state-of-the-art 4-photon experiment at telecom wavelength, we report an in-depth study of the enhanced noise resilience that can be gained from local encoding [16]. Using symmetric informationally complete (SIC) [24] tomographic techniques, we quantify the noise resilience of quantum resources such as coherence and entanglement for all local-unitarily inequivalent classes of 4-qubit graph states [25,26]. Finally, as quantified by the experimental phase variance and the quantum Fisher information [27], we observe that our encoding provides a clear improvement of the 4-qubit GHZ states' usefulness for noisy quantum phase estimation. Notably, the states remain useful even under full dephasing, where the entanglement is always zero. This hints at a key role played by coherence, which is shown instead to be independent from the noise when the encoding is applied.

Multiqubit robustness by local encoding.—Consider an ideal quantum system, subjected to local dephasing noise before being used for an information-processing task such as phase estimation. This occurs, for example, when the system crosses a noisy region before the protocol happens or if its implementation is much faster than the dephasing timescale. We note that the results are the same when the noise acts during the phase estimation task; here however we keep them separate for simplicity. We aim to encode the system before the noise acts in order to increase its resilience against dephasing with as simple an encoding as possible, and then decode the system before it is used. Taking the dephasing to act on the state ρ in the computational basis $\{|0\rangle, |1\rangle\}$, the single-qubit dephasing channel is given by

$$\mathcal{D}(\rho) \doteq \left(1 - \frac{p}{2}\right)\rho + \frac{p}{2}\sigma_z\rho\sigma_z, \tag{1}$$

where $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 0|$ is the Z Pauli matrix, and $0 \leq p \leq 1$ quantifies the noise strength from no noise ($p = 0$) to full dephasing ($p = 1$). Consider now, for instance, an N-qubit GHZ state defined as

$$|\mathrm{GHZ}_N\rangle \doteq \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}). \tag{2}$$

The entanglement of GHZ states affected by independent and identical single-qubit dephasing of strength p is known to decay exponentially with N. More precisely, for the dephased GHZ state $\rho_N(p) \doteq \mathcal{D}^{\otimes N}(|\mathrm{GHZ}_N\rangle\langle\mathrm{GHZ}_N|)$ it holds that $E(\rho_N(p)) \leq (1-p)^N E(\rho_N(0))$ for any convex entanglement quantifier E [18,19]. However, this scaling can be drastically improved [16] by encoding the state using local Hadamard gates H, defined by $H|0\rangle \doteq |+\rangle$ and $H|1\rangle \doteq |-\rangle$ with $|\pm\rangle \doteq (1/\sqrt{2})(|0\rangle \pm |1\rangle)$. The resulting encoded GHZ state

$$|\mathrm{GHZ}_N^{\mathrm{enc}}\rangle \doteq H^{\otimes N}|\mathrm{GHZ}_N\rangle = (1/\sqrt{2})(|+\rangle^{\otimes N} + |-\rangle^{\otimes N}) \tag{3}$$

has the same entanglement properties, yet its entanglement decay rate becomes independent of N and linear in p. Formally, the dephased encoded state $\rho_N^{\mathrm{enc}}(p) \doteq \mathcal{D}^{\otimes N}(|\mathrm{GHZ}_N^{\mathrm{enc}}\rangle\langle\mathrm{GHZ}_N^{\mathrm{enc}}|)$ satisfies the bound $E(\rho_N^{\mathrm{enc}}(p)) \geq E(\rho_2(p))$ for all N and thus possesses at least as much resilience as the two-qubit state $|\mathrm{GHZ}_2\rangle$. A similar enhancement can be extended for arbitrary graph states [26], which play a crucial role in measurement-based quantum computing and quantum error correction, see Supplemental Material (SM) [28] for details.

Experimental setup.—We now test these passive error protection techniques in a state-of-the-art photonic platform, Fig. 1. In general, it is worth noting that the method could be applied to any other experimental architecture commonly dealing with dephasing noise, from spin qubits to trapped ions, without any increase in experimental complexity. Qubits are encoded in the horizontal $|h\rangle = |0\rangle$ and vertical $|v\rangle = |1\rangle$ polarization states of single photons. These are generated at 1550 nm via collinear type-II spontaneous parametric down-conversion in a 22 mm long periodically poled KTP (PPKTP) crystal, pumped with a 1.6 ps pulsed laser at 775 nm [29]. After spectral filtering with a bandwidth of 3 nm, the source generates ~3075 pairs/mW/s with a symmetric heralding efficiency of ~55%. Embedding the crystal within a Sagnac interferometer [30] enables the generation of high-quality entangled states of the form

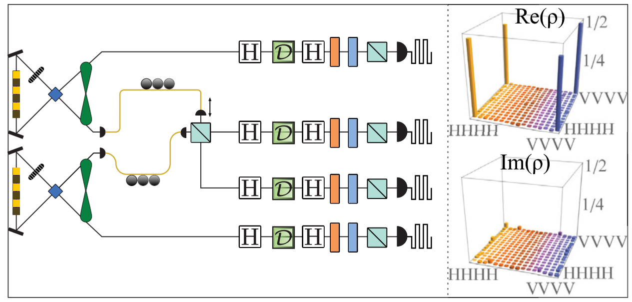

FIG. 1. Experimental setup. Preparation of 4-qubit GHZ states and encoding stage. Pairs of photons at 1550 nm are generated via spontaneous parametric down-conversion in periodically poled KTP (PPKTP) crystals. The GHZ state is obtained from interfering photons from two entangled pairs on a polarizing beam splitter (PBS). The state is then locally encoded, dephased, decoded, and measured using a combination of quarter-wave plate (QWP), half-wave plate (HWP), PBS, and superconducting nanowire single photon detectors (SNSPDs) with fourfold coincidence detection. On the right, the real and imaginary part of the experimental density matrices (without dephasing) are shown.

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|h\rangle|v\rangle - |v\rangle|h\rangle), \tag{4}$$

with typical fidelities $F(\rho_e, \rho_t) = (\mathrm{Tr}[\sqrt{\sqrt{\rho_t}\rho_e\sqrt{\rho_t}}])^2 = 99.62^{+0.01}_{-0.04}\%$ where ρ_e and ρ_t are the experimental and target state, respectively. The measured purity is $P = 99.34^{+0.01}_{-0.09}\%$ and entanglement as measured by the concurrence [31] is $\mathcal{C} = 99.38^{+0.02}_{-0.10}\%$. The photons are detected using superconducting nanowire single-photon detectors (SNSPDs) with an efficiency of ~80% and processed using a time-tagging module with a resolution of 156 ps. Using two such photon-pair sources in the setup of Fig. 1, we can prepare the 4-qubit GHZ state $|\mathrm{GHZ}_4\rangle$ of Eq. (2) by subjecting one photon of each entangled pair to nonclassical interference on a polarizing beam splitter (PBS), which transmits horizontal and reflects vertically polarized photons. This implements a so-called type-I fusion gate [32] for which we achieved a visibility of $91.80^{+1.73}_{-1.73}\%$, translating into a purity of $P = 87.09^{+1.15}_{-2.18}\%$ and fidelity of $F = 92.53^{+0.63}_{-1.23}\%$ for the 4-qubit GHZ state. The states are generated at a measured rate of 47.6 Hz using 60 mW pump power.

Single qubit dephasing of Eq. (1) is experimentally implemented in a controllable manner by applying the identity channel for a time $1 - p/2$ and the σ_z channel for a time $p/2$. For simplicity, these operations together with encoding and decoding, are applied as appropriate rotations of the measurement frame. The density matrices of the experimentally generated states are then reconstructed using maximum-likelihood quantum state tomography. The tomography is performed using the set of SIC measurements [24], which reduces the number of measurements compared to the standard Pauli basis by a factor $(2/3)^N$, leading to improved precision at equal acquisition time.

Experimental noise protection.—In this section we experimentally investigate the effect of the encoding proposed on two paradigmatic quantum resources: quantum entanglement and coherence. The former is quantified by the negativity [33], the latter instead using the recently developed resource theory of multilevel coherence [34,35], see SM [28] for details. The results for both these figures of merit are shown in Fig. 2. The top panel shows that the negativity of the encoded $|\mathrm{GHZ}_4^{\mathrm{enc}}\rangle$ states is significantly more resilient against dephasing than the nonencoded states. Moreover, the inset in Fig. 2 shows how the enhancement for a fixed amount of dephasing becomes more significant as the number of qubits increases, instead of the exponential decay observed for the nonencoded states [16]. As shown in the SM [28], such noise resilience is notably achieved by increasing the amount of entanglement with the environment. In fact, the encoded states experience a higher loss of purity than the nonencoded ones.

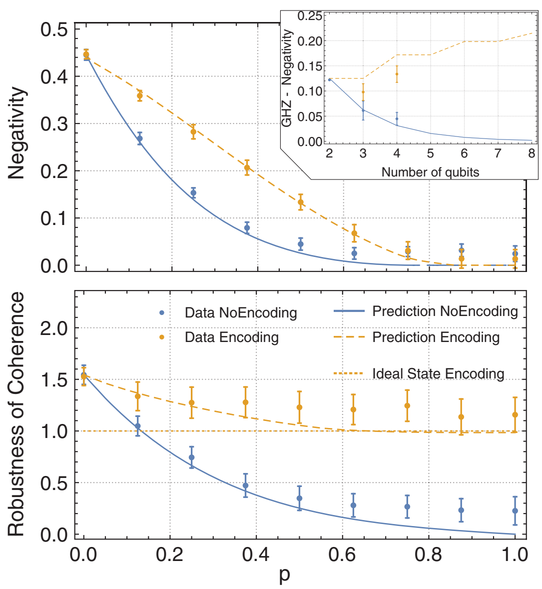

FIG. 2. Resilience enhancement of negativity in the partition (1|234) and the robustness of coherence R_{C_1}. Shown is the negativity (top) and coherence (bottom) of the GHZ with (dashed-orange) and without (solid-blue) encoding. The solid (dashed) lines depict the theoretical predictions with the experimental nonencoded (encoded) input state. In the top-right inset the trend of the GHZ negativity is shown in terms of the number of qubits and at fixed noise $p = 0.5$. With the encoding proposed, the entanglement is best protected when the number of qubits increases. For the coherence only, the theory prediction starting with an ideal input encoded state is shown with a dotted-orange curve. Note that experimental imperfections tend to lead to additional coherence terms compared to the ideal GHZ state. The robustness of coherence reflects this as higher initial values of coherence and nonvanishing coherence for all dephasing strengths. Error bars represent 3σ statistical confidence regions obtained from a Monte Carlo routine taking into account the Poissonian counting statistics.

The bottom panel of Fig. 2 shows how coherence of the encoded states not only is protected from the action of noise entirely unaffected from it. On the other hand, for the nonencoded states, coherence decays exponentially. Intuitively, this may be understood based on the distribution of coherence within the state. Concentrating all coherence on two terms (coherence rank 2), such as the GHZ_4 state, leaves the state vulnerable to dephasing. In contrast, maximally distributing the coherence (coherence rank 2^N), as in the encoded state, achieves increased resilience. In the latter case, the decoding map (a nonfree operation in the resource theory of coherence) can under certain conditions recover a significant amount of coherence, see SM [28] for details. We remark that, multilevel coherence is independent of entanglement measures and can unlock information on the encoding effects otherwise inaccessible. This, as we will see in the next section, provides useful insights on phase estimation in a noisy environment. Finally, noise protection is also achieved for the linear cluster state, see SM [28].

Enhanced phase estimation.—We now exploit our passive error correction for quantum metrology, by performing a 4-qubit phase estimation task [36] in a noisy environment. The goal is to estimate an unknown phase ϕ imparted on a probe state ρ by the unitary $U_\phi = e^{-(i/2)\phi\sigma_z}$ by measuring the evolved state $\rho_\phi \doteq U_\phi^{\otimes N}\rho U_\phi^{\dagger\otimes N}$. It is well known that, just as NOON states, GHZ states can reach Heisenberg scaling and are therefore optimal for phase estimation, if properly measured [4,36]. In the presence of dephasing noise, however, this task becomes much more challenging, and different inequivalent strategies can be devised [37,38]. We now show how our local encoding can significantly enhance the metrology performance of a 4-qubit GHZ state under such conditions. To assess the performance of phase estimation we study the expectation value

$$\mathrm{Tr}[\rho_\phi(|+\rangle\langle+|)^{\otimes 4}]_{\mathrm{GHZ}} = \frac{1}{16}[(p-1)^4\cos(4\phi)+1],$$

for noise of strength p, showing that for maximal dephasing, i.e., $p = 1$, no phase information can be recovered. Conversely, if the encoding of Eq. (3) is used, we find improved performance for all p. Most strikingly, the encoding preserves phase sensitivity even under full dephasing:

$$\mathrm{Tr}[\rho_\phi(|+\rangle\langle+|)^{\otimes 4}]_{\mathrm{GHZ}^{\mathrm{enc}}} = \frac{1}{128}[4\cos(2\phi) + \cos(4\phi) + 11].$$

Although entanglement is recognized as a fundamental resource for phase estimation [39,40], it is remarkable that phase sensitivity is observed even in the full dephasing regime, whereby the entanglement is always zero, even for the encoded states. It follows that the phase sensitivity observed is instead provided by the coherence only, which as we have previously seen is left untouched by the dephasing. Only when both coherence and entanglement are zero (as for the nonencoded states) is the phase sensitivity completely suppressed. This suggests, at least for this instance, the coherence to be a useful resource whereas the entanglement is not.

Experimentally, we applied a phase shift $\phi \in [0, \pi]$ to each qubit, by rotating the measurement frame accordingly, and reconstructed the expectation values $\mathrm{Tr}[\rho_\phi(|+\rangle\langle+|)^{\otimes 4}]$ as a function of ϕ for a range of p, see Fig. 3(a). The results clearly show a steeper slope of $\mathrm{Tr}[\rho_\phi(|+\rangle\langle+|)^{\otimes 4}]$ for the encoded state when compared to the nonencoded state for all nonzero values of ϕ. This directly translates into a more sensitive phase estimator in the encoded case. Moreover, we emphasize that the encoded fringes preserve at least half the visibility of the $p = 0$ case, even for $p = 1$ where instead, without our encoding, the nonencoded fringes flatten to a constant value. In other words, whereby phase estimation would be normally impossible, our encoding makes it feasible again.

This qualitative behavior is turned into a quantitative result by measuring the experimental variance of the estimated phase at the point where the fringes are the steepest for different values of p, according to

$$\mathrm{Var}[\phi] = \frac{\mathrm{Var}[\epsilon]}{|\frac{d}{d\phi}\epsilon|^2}, \tag{5}$$

where ϵ is the measured average value of our estimator $\epsilon \equiv \langle +_1 +_2 +_3 +_4 \rangle$. The results are shown in Fig. 3(b).

More in general however, the primary figure of merit in quantum metrology is the so-called *quantum Fisher information* (QFI) [27]. In the noiseless case, the statistical deviation $\delta\phi$ in the estimation of ϕ, is bounded as $\delta\phi \geq 1/(\sqrt{\nu \mathcal{F}(\rho_\phi)})$ [41], where ν is the number of repetition runs in the estimation and $\mathcal{F}(\rho_\phi)$ is the QFI, measuring the maximum amount of information about ϕ that can be extracted from ρ_ϕ. For separable states $\mathcal{F}(\rho_\phi) \leq N$, which means that the QFI is bounded by the shot-noise limit (SNL), while for GHZ states the QFI attains the optimal value $\mathcal{F}_{\max} = N^2$, known as the Heisenberg limit. On the other hand, the QFI of a locally dephased GHZ state

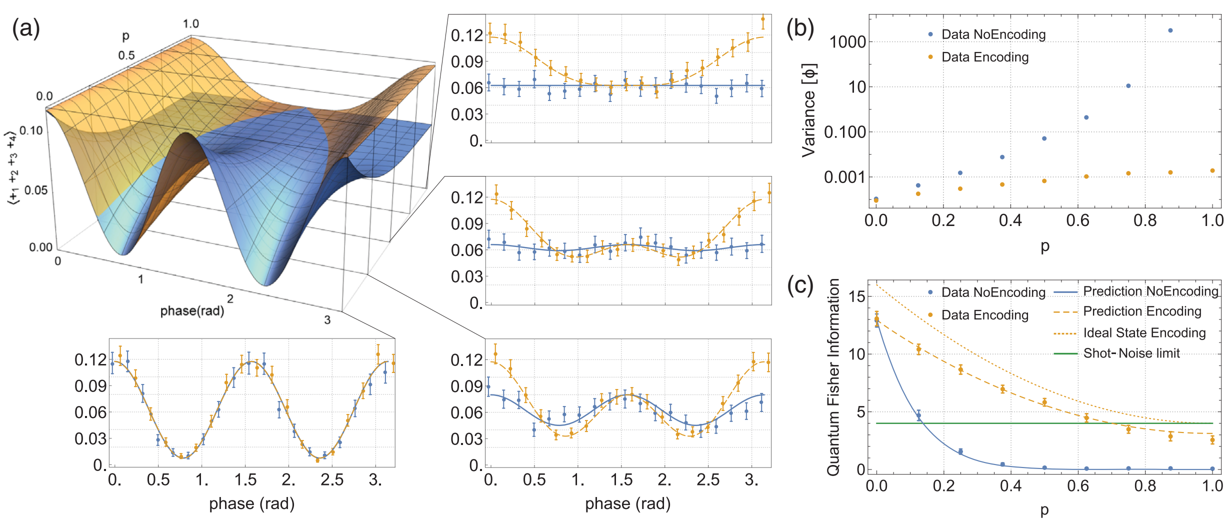

FIG. 3. Phase estimation with and without encoding. (a) Expectation value $\langle +_1 +_2 +_3 +_4 \rangle$ as a function of phase and amount of noise p, for a locally encoded (orange) and a nonencoded (blue) 4-qubit GHZ state. In particular, for values of $p = 0$, 0.25, 0.5, 1 the experimentally measured expectation values as a function of the phase are shown. The theoretical predictions are shown as blue-solid (no encoding) and orange-dashed (encoding) curves, and error bars indicate 3σ statistical uncertainty regions obtained from a Monte Carlo resampling of our Poisson counting statistics. In the absence of noise ($p = 0$) there is no difference between encoded and nonencoded states. With increasing dephasing, however, the advantage of the encoding becomes clear in that the expectation values for nonencoded states decay to zero, but remain nonzero for all p if the local encoding is used. (b) Robustness enhancement of the quantum Fisher information. QFI of the encoded (dashed-orange) and nonencoded (solid-blue) states, compared with the shot-noise limit (solid green). Without encoding, the GHZ state loses its advantage already in the low-noise regime. In contrast, as shown in the figure, the encoding preserves the QFI for all values of dephasing, ideally (dotted-orange), and up to $p = 0.6$ experimentally. (c) Comparison of the phase variance without (blue) and with (orange) encoding, for different noise strengths. Notably, with encoding, the variance observed is up to 2 orders of magnitude smaller than in the case without, where the error on the inferred phase diverges with increasing noise.

$\mathcal{F}(\rho_N(p)) = N^2(1-p)^{2N}$, indicates that for fixed noise strength p the precision of the estimate of ϕ decreases exponentially with N. This drastic decay is turned into a quadratic one for the encoded GHZ state, $\mathcal{F}(\rho_N^T(p)) = N^2(1-p)^2 + 4N[1-(p/2)](p/2)$. The quantum Fisher information is measured experimentally for both the encoded and nonencoded density matrices [42], see Fig. 3(c). In the absence of noise we experimentally observed a value close to the Heisenberg limit $N^2 = 16$ exponentially turned to 0 when the encoding is not applied. On the other hand, encoded GHZ states preserve their quantum advantage for significantly high noise strengths. Here, we have considered that the noise acts before the phase is imprinted on the system. However, the same robust behavior would also be observed if the noise and the phase evolution happen simultaneously [21], or if when the noise happens after the phase is imprinted, thus showing the wide applicability of our approach (see SM [28]). We further note that the protocol works not only for identical and independent dephasing, but also for any uncorrelated noise with a preferred direction (not necessarily the same for all qubits). Extending the protocol to correlated forms of noise is an interesting direction for future research. Finally we note, that in the process of revising our Letter, we became aware of a related work [43] where the authors investigate coherence "freezing" of GHZ states under bit-flip noise in a phase and frequency estimation protocol.

Conclusions.—We have shown that, given knowledge of the dominant noise sources in the experiment, protection of quantum resources is feasible in practice. This is achieved without complex encoding and additional overhead in the number of physical qubits, hence enabling significant improvements. We revealed different behaviors of graph states for all the quantum figure of merits under study, highlighting the importance of a deep understanding of state dynamics in the multiqubit scenario. In particular, we exploited our method in one of the most common quantum information tasks, i.e., phase estimation. Adding dephasing noise, we simulated a realistic implementation of a protocol where without any action no quantum advantage would be observed. With our encoding, we instead observe phase sensitivity up to the full-noise point where only coherence and not entanglement is the enabling resource for phase estimation. We envision our method to be particularly relevant for protecting multiqubit graph states from noise during distribution over quantum networks, before being used in measurement-based quantum computing, or when stored in quantum memories. In conclusion, we successfully demonstrated an alternative route for noise protection which might be further exploited as long as active multiqubit quantum error correction remains out of reach for near-term technology.

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