Modulation instability of discrete angular momentum in fiber rings

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Modulation Instability of Discrete Angular Momentum in Fiber Rings

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Abstract: We present an analysis of modulation instability in a ring of coupled optical fibers. Plane waves are shown to be unstable to perturbations carrying discrete angular momenta, both for normal and anomalous group velocity dispersion.

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Modulation instability (MI) of plane waves in anomalously-dispersive fibers with a self-focusing Kerr nonlinearity is one of the most well-known phenomena in nonlinear optics [1]. It has been observed in higher dimensional systems, in particular as an azimuthal instability of optical vortices in continuous self-focusing Kerr media [2]. Circular arrays of coupled optical fibres have been shown to support supermodes carrying angular momentum as discrete optical vortices [3]. The nonlinear optical properties of fiber rings have been examined previously, particularly in the context of $\mathscr{PT}$-symmetry breaking [4] and the stability of modes in fibers with only a few cores [5]. Here we show that plane wave supermodes of fiber arrays as shown in figure 1 are unstable in the presence of perturbations and that the gain spectra of these perturbations depends on their angular momenta.

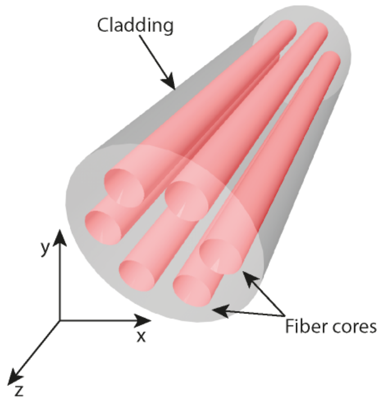

Fig. 1. Six core fibre ring. Light couples between cores through the overlap of their evanescent fields.

We model light propagation in the N-core fiber array with N coupled (1+1) dimensional nonlinear Schrödinger equations; the equation for the field E_n in the n^{th} fiber is

$$i\partial_z E_n = \frac{\beta_2}{2}\partial_t{}^2 E_n - \gamma |E_n|^2 E_n - \Delta\left(E_{n+1} + E_{n-1}\right) \tag{1}$$

Following the standard MI derivation in Agrawal [1], we consider a plane-wave solution of (1)

$$\overline{E}_n = \sqrt{P_0}\exp\left(i2\pi mn/N + ik_0 z\right) \tag{2}$$

as a pump, perturbed by a pair of weak excitations coupled through the nonlinearity

$$a_n(z,t) = a_S \exp\left(i\left(Kz + 2\pi ln/N - \Omega t\right)\right) + a_I \exp\left(-i\left(Kz + 2\pi(l-2m)n/N - \Omega t\right)\right) \tag{3}$$

These perturbations will grow exponentially if their wavenumber has an imagninary component, which requires the perturbation's detuning Ω from the pump to lie between two critical frequencies, Ω_{C1} and Ω_{C2} . If this holds, then the perturbations grow exponentially in power at a rate

$$G(\Omega,\Delta) = \mathrm{Re}\left(\sqrt{4\gamma^2 P_0{}^2 - \left(\beta_2\Omega^2 + 2\gamma P_0 + 2\Delta_{l,m} + 2\Delta_{2m-l,m}\right)^2}\right) \tag{4}$$

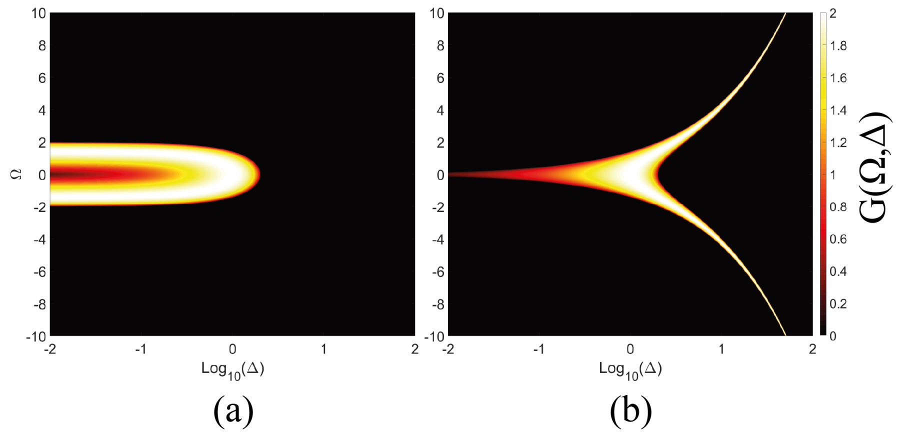

Fig. 2. Gain spectra for perturbations carrying angular momentum $|l| = 1$ as a function of the detuning Ω and the fiber coupling strength Δ. **(a)** Anomalous GVD $\beta_2 = -1$; **(b)** Normal GVD $\beta_2 = 1$

where $\Delta_{x,m} = \Delta(\cos(2\pi x/N) - \cos(2\pi m/N))$. Example plots of gain spectra in a six-core fibre ring are shown in figure 2, given pump and probe angular momentum $m = 0$ and $|l| = 1$ respectively. In contrast to standard MI, here we see MI can occur for both normal and anomalous GVD. If $\beta_2 < 0$, pertubations with non-zero l see suppressed instability above a threshold coupling strength. With $\beta_2 > 0$, gain initially grows stronger with increasing coupling, until it forks into two sidebands. Higher order angular momenta experience similar instabilities; figure 3 compares the gain sidebands for $|l| = 1,2,3$ **(a)** predicted by eq. (4) in a six-core ring with the spectra in a single core **(b)** obtained by numerically integrating eq. (1). We will also explore how twisting the array can tune the MI gain.

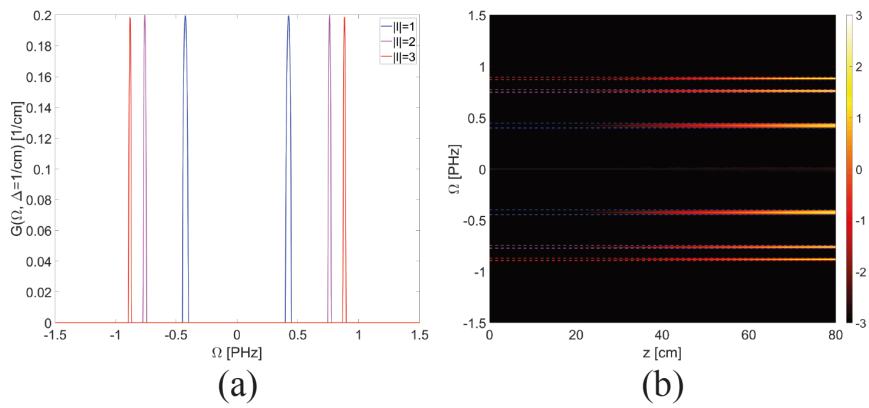

Fig. 3. **(a)** Predicted MI gain for perturbations carrying different angular momenta $|l| = 1,2,3$ in an homogeneously pumped six-core fiber ring with $\beta_2 = 1\text{ps}^2/\text{km}$, $\Delta = 1/\text{cm}$, $\gamma P_0 = 0.1/\text{cm}$ **(b)** Spectrum $|E_1(\Omega,z)|^2$ in a single core of the ring from a numerical simulation of (1). Blue, magenta and red dashed lines indicate the MI critical frequencies for $|l| = 1,2,3$, verifying the accuracy of eq. (4).

References

1. G. P. Agrawal, *Nonlinear Fiber Optics (3rd ed.)* (Academic, 2001).
2. L. T. Vuong *et. al.*, "Collapse of Optical Vortices", *Phys. Rev. Lett.* **96**, 133901 (2006).
3. C. N. Alexeyev, A. V. Volyar and M. A. Yavorsky, "Linear azimuthons in circular fiber arrays and optical angular momentum of discrete optical vortices", *Phys. Rev. A* **80**, 063821 (2009).
4. C. Castro-Castro, Y. Shen, G. Srinivasan, A. B. Aceves and P. G. Kevrekidis, "Light dynamics in nonlinear trimers and twisted multicore fibers", *J. Nonlinear Opt. Phys. Mater.* **25**, 1650042 (2016).
5. J. H. Li, H. Zhou, K. S. Chiang and S. R. Xiao, "Modulation instabilities in equilateral three-core optical fibers", *J. Opt. Soc. Am. B* **33** (11), 2357 (2016).