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Research Article

Bright Soliton Solution of (1+1)-Dimensional Quantum System with Power-Law Dependent Nonlinearity

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We study the nonlinear dynamics of (1+1)-dimensional quantum system in power-law dependent media based on the nonlinear Schrödinger equation (NLSE) incorporating power-law dependent nonlinearity, linear attenuation, self-steepening terms, and third-order dispersion term. The analytical bright soliton solution of this NLSE is derived via the F-expansion method. The key feature of the bright soliton solution is pictorially demonstrated, which together with typical analytical formulation of the soliton solution shows the applicability of our theoretical treatment.

1. Introduction

In the fascinating study of physical reality, nonlinear phenomena have attracted significant attention. Besides experimental observation of nonlinear behaviors, for the theoretical investigation of nonlinearity, the nonlinear Schrödinger equation (NLSE) has proved to be a reliable model under certain experimental settings because it is closely related to many nonlinear problems and people have conducted extensive research on finding solutions [1–15] of particular nonlinear features. For a practical theoretical treatment, the one-dimensional scenario of the NLSE model is typical not only because of the integrability of the system under many parametric settings, but also because that actual experimental observation identifies stable soliton behavior under such a scenario. Therefore, the (1+1)-dimensional case of the NLSE is the focus of many theoretical works, including this study.

For the analytical study of various categories of the (1+1)-dimensional NLSE, many approaches have been proposed, such as the Bäcklund transformation method [16], homotopy analysis method [17], variational iteration method, and Exp-function method [2]. For a typical analytical solution investigation of the NLSE in this study, we utilize the F-expansion method, which features the Jacobian elliptic function expansion to solve NLSE for typical traveling wave solutions. It is a very effective method to solve partial differential equations such as NLSE. Basically, it is implemented by expressing the solutions of the equations in the form of power expansion of Jacobian function [18, 19]. Moreover, for the nonlinear study of the NLSE, the soliton-type solution is usually the ultimate goal of the analytical solution search. This usually comes from the desirable features of soliton. Because it is wave solution of a nonlinear wave equation, it can propagate for a long distance without deformation, and its amplitude, shape, and speed remain unchanged when it meets other similar solitary waves. In this paper, we study the soliton behavior of the NLSE by modeling the nonlinear dynamics in power-law dependent media such as an optical fiber system, where the group velocity dispersion term and nonlinear term are present and adjustable. The linear attenuation term, self-steepening term, and third-order dispersion term also exist [20]. We identify that, under an appropriate experimental setting, the bright soliton solutions can be derived via the F-expansion method, and the typical bright soliton feature is pictorially demonstrated.

This paper is organized as follows. The next section gives the NLSE model formulation of this study, Section 3 describes the procedural detail of the F-expansion method, Section 4 gives the concrete calculation steps of solving the NLSE for its solition solution, and the last section gives the conclusive remarks.

2. Model and Method

2.1. NLSE Model in Power-Law Dependent Media. For studying the soliton dynamics in a (1+1)-dimensional power-law dependent media [2, 10, 15, 20], the NLSE in dimensionless format is

$$\begin{aligned} &i\psi_t + a(t)\psi_{xx} + b(t)|\psi|^{2m}\psi \\ &\quad = i\alpha(t)]\psi + i\lambda(t)\left(|\psi|^{2m}\psi\right)_x - i\gamma(t)\psi_{xxx} \end{aligned} \tag{1}$$

The independent variables in ψ include the space x and time t coordinates, $i\psi_t$ describes the evolution of wave function, $a(t)\psi_{xx}$ is the term of group velocity dispersion, $b(t)|\psi|^{2m}\psi$ is the nonlinear term, $a(t)$ and $b(t)$ are the real valued functions of time t, $\alpha(t)$ is the time-dependent coefficient of linear attenuation term, $\lambda(t)$ is the time-dependent coefficient of self-steepening term, and $\gamma(t)$ is the time-dependent coefficient of the third-order dispersion term. Here, we consider the case where the coefficient parametric functions of various terms in (1) do not vary with time within a certain period but can be freely adjusted according to the experimental setting.

2.2. The F-Expansion Method. The F-expansion method is used to solve general partial differential equations of the following form:

$$G(u, u_t, u_x, u_{xx}, \ldots) = 0 \tag{2}$$

where $u(x,t)$ is the unknown function to be solved and G is the polynomial of $u(x,t)$ and its partial derivatives of various orders. The F-expansion method is implemented through the use of the polynomial of base function $F(\xi)$ to express $u(x,t)$, where $F(\xi)$ is defined as

$$\begin{aligned} \left(\frac{dF(\xi)}{d\xi}\right)^2 &= F(\xi)^4 + b_3F(\xi)^3 + b_2F(\xi)^2 + b_1F(\xi) \\ &\quad + b_0 \end{aligned} \tag{3}$$

where

$$\xi = px + qt \tag{4}$$

Here p, q, b_0, b_1, b_2, b_3 are parametric constants to be determined in the problem-solving steps. $u(x,t)$ is expressed through $F(\xi)$ and it takes the following form:

$$u(x,t) = \sum_{i=0}^{m} h_i(t)F^i(\xi) \tag{5}$$

where the highest order number m is determined by balancing the highest order of nonlinearity and highest order of derivatives. By inserting (5) into (2) and using the definition of $F(\xi)$, we obtain a polynomial of $F(\xi)$. After setting the coefficients of various terms of the polynomial, we obtain a set of ordinary differential equations (ODEs) of time t. Solving the ODEs consistently, we obtain the analytical expressions of the parametric constants in (3) and $h_i(t)$ in (5), and (2) is solved accordingly.

3. Procedural Details and Results for Reaching Solition Solution of NLSE in Power-Law Dependent Media

To solve (1), we can assume that $\psi(x,t)$ takes the following traveling wave format:

$$\psi(x,t) = \varphi(\xi)e^{i(Ax+Bt)} \tag{6}$$

Substituting (6) into (1), we get

$$i\psi_t = \left(iq\varphi' - A\varphi\right)e^{i(At+Bx)} \tag{7a}$$

$$\psi_x = \left(\varphi' p + iB\varphi\right)e^{i(At+Bx)} \tag{7b}$$

$$\psi_{xx} = \left(\varphi'' p^2 + 2iB\varphi' p - B^2\varphi\right)e^{i(At+Bx)} \tag{7c}$$

$$\psi_{xxx} = \left(\varphi''' p^3 + 3iB\varphi'' p^2 - 3B^2\varphi' p - iB^3\varphi\right)e^{i(At+Bx)} \tag{7d}$$

Plugging (7a), (7b), (7c), and (7d) into (1), we get

$$\begin{aligned} &\left(iq\varphi' - A\varphi\right) + a(t)\left(\varphi'' p^2 + 2iB\varphi' p - B^2\varphi\right) \\ &\quad + b(t)\varphi^{2m+1} \\ &\quad = i\alpha(t)\varphi + i\lambda(t)(2m+1)\varphi^{2m}\varphi' p \\ &\quad - \lambda(t)(2m+1)B\varphi^{2m+1} + i\gamma(t)\varphi''' p^3 \\ &\quad - 3B\gamma(t)\varphi'' p^2 - 3iB^2\gamma(t)\varphi' p + B^3\gamma(t)\varphi \end{aligned} \tag{8}$$

The real part of (8) is

$$\begin{aligned} &[b(t) + \lambda(t)(2m+1)B]\varphi^{2m+1} \\ &\quad - \left[A + a(t)B^2 + \gamma(t)B^3\right]\varphi \\ &\quad + \left[a(t)p^2 + 3\gamma(t)Bp^2\right]\varphi'' = 0 \end{aligned} \tag{9}$$

The imaginary part of (8) is

$$\begin{aligned} &\lambda(t)(2m+1)p\varphi^{2m}\varphi' + \alpha(t)\varphi \\ &\quad - \left[q + 2a(t)Bp + 3\gamma(t)B^2p\right]\varphi' + \gamma(t)p^3\varphi''' \\ &\quad = 0 \end{aligned} \tag{10}$$

We define

$$T(\varphi) = \left[\frac{d\varphi(\xi)}{d\xi}\right]^2 = \left(\varphi'\right)^2 \tag{11}$$

So

$$\frac{dT}{d\xi} = \frac{dT}{d\varphi}\frac{d\varphi}{d\xi} = \varphi'\frac{dT}{d\varphi} \tag{12}$$

and

$$\varphi'' = \frac{d\varphi'}{d\xi} = \varphi' \frac{d\varphi'}{d\varphi} = \frac{1}{2}\frac{d\varphi'^2}{d\varphi} = \frac{1}{2}\frac{dT}{d\varphi} = \frac{1}{2}T' \quad (13)$$

Substituting (13) into (9), we obtain

$$\lambda_1 \varphi^{2m+1} - \lambda_2 \varphi + \frac{1}{2}\lambda_3 T' = 0 \quad (14)$$

where

$$\lambda_1 = b(t) + \lambda(t)(2m+1)B \quad (15a)$$

$$\lambda_2 = A + a(t)B^2 + \gamma(t)B^3 \quad (15b)$$

$$\lambda_3 = a(t)p^2 + 3\gamma(t)Bp^2 \quad (15c)$$

For the implementation of the F-expansion method, we set $\varphi(\xi) = F(\xi)$, and the F base function is chosen as follows:

$$T = a_m \varphi^{2m+2} + a_2 \varphi^2 \quad (16)$$

Plugging (16) into (14), and performing integration, we get the coefficients as follows:

$$\varphi^{2m+1}: \lambda_1 + (m+1)\lambda_3 a_m = 0 \quad (17a)$$

$$\varphi: \ -\lambda_2 + \lambda_3 a_2 = 0 \quad (17b)$$

From (17a) and (17b), we get

$$a_2 = \frac{\lambda_2}{\lambda_3} \quad (18a)$$

$$a_m = -\frac{\lambda_1}{(m+1)\lambda_3} \quad (18b)$$

Plugging (15a), (15b), and (15c) into (18a) and (18b), we get

$$a_2 = \frac{A + a(t)B^2 + \gamma(t)B^3}{a(t)p^2 + 3\gamma(t)Bp^2} \quad (19a)$$

$$a_m = -\frac{b(t) + \lambda(t)(2m+1)B}{(m+1)\left[a(t)p^2 + 3\gamma(t)Bp^2\right]} \quad (19b)$$

We get φ'' from (9):

$$\varphi'' = -\frac{\lambda_1}{\lambda_3}\varphi^{2m+1} + \frac{\lambda_2}{\lambda_3}\varphi \quad (20)$$

We differentiate both sides of (20):

$$\varphi''' = \frac{d\varphi''}{d\xi} = -\frac{\lambda_1}{\lambda_3}(2m+1)\varphi^{2m}\varphi' + \frac{\lambda_2}{\lambda_3}\varphi' \quad (21)$$

Equation (8) then takes the following form:

$$\lambda_4 \varphi^{2m}\varphi' + \alpha(t)\varphi - \lambda_5 \varphi' + \lambda_6 \varphi''' = 0 \quad (22)$$

Here

$$\lambda_4 = \lambda(t)(2m+1)p \quad (23a)$$

$$\lambda_5 = q + 2a(t)Bp + 3\gamma(t)B^2 p \quad (23b)$$

$$\lambda_6 = \gamma(t)p^3 \quad (23c)$$

Plugging (21) into (22), we get

$$\left[\lambda_4 - \lambda_6 \frac{\lambda_1}{\lambda_3}(2m+1)\right]\varphi^{2m}\varphi' + \alpha(t)\varphi + \left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right]\varphi' = 0 \quad (24)$$

Dividing both sides of (24) by φ, and then taking both derivatives with respect to ξ, we get

$$\left[\lambda_4 - \lambda_6 \frac{\lambda_1}{\lambda_3}(2m+1)\right](2m-1)\varphi^{2m-2}\varphi'^2 + \left[\lambda_4 - \lambda_6 \frac{\lambda_1}{\lambda_3}(2m+1)\right]\varphi^{2m-1}\varphi'' + \left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right]\frac{\varphi'' - \varphi'^2}{\varphi^2} = 0 \quad (25)$$

Substituting (11) and (13) into (25), we get

$$\left[\lambda_4 - \lambda_6 \frac{\lambda_1}{\lambda_3}(2m+1)\right](2m-1)\varphi^{2m-2}T + \left[\lambda_4 - \lambda_6 \frac{\lambda_1}{\lambda_3}(2m+1)\right]\varphi^{2m-1}\frac{1}{2}T' + \left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right]\frac{(1/2)T' - T}{\varphi^2} = 0 \quad (26)$$

Plugging (16) into (26), performing the integration, we get the coefficient of each term as

$$\varphi^{4m}: (3m)a_m\left[\lambda_4 - (2m+1)\lambda_6 \frac{\lambda_1}{\lambda_3}\right] \quad (27a)$$

$$\varphi^{2m}: (2m)a_2\left[\lambda_4 - (2m+1)\lambda_6 \frac{\lambda_1}{\lambda_3}\right] - a_m\left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right] \quad (27b)$$

$$\varphi^{2m-1}: (m+1)a_m\left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right] \quad (27c)$$

$$\varphi^0: -a_2\left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right] \quad (27d)$$

$$\varphi^{-1}: a_2\left[\lambda_6 \frac{\lambda_2}{\lambda_3} - \lambda_5\right] \quad (27e)$$

From Eqs. (27a), (27b), (27c), (27d), and (27e), we get

$$B = \frac{a(t)\lambda(t) - b(t)\gamma(t)}{(2m-2)\lambda(t)\gamma(t)} \quad (28a)$$

$$A = -\left[a(t)B^2 + \gamma(t)B^3\right] \quad (28b)$$

$$\frac{q}{p} = -\left[2a(t)B + 3\gamma(t)B^2\right] \quad (28c)$$

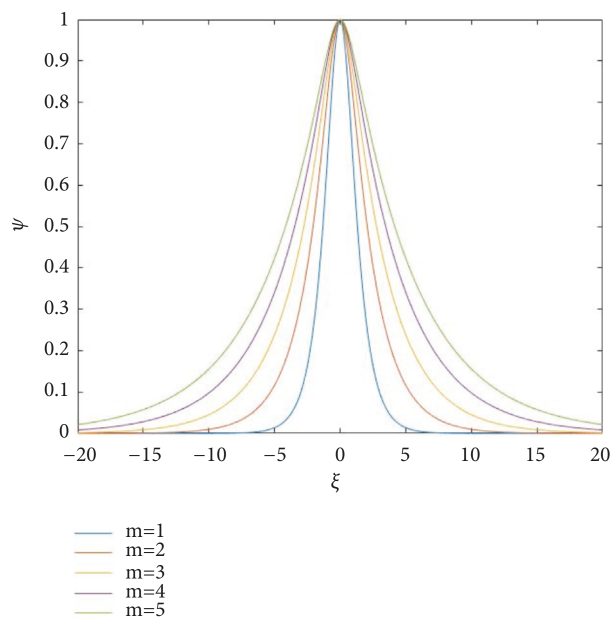

Figure 1: Bright soliton waveform of $|\psi|$ for various power indexes m.

Next we define ϕ as

$$\phi(\xi) = \varphi^{-1} \tag{29}$$

So (16) takes the following form:

$$\frac{d\phi}{\phi^{1-m}\sqrt{a_m + a_2\phi^{2m}}} = -d\xi \tag{30}$$

This equation can be solved with the following analytic solutions:

$$\varphi = \phi^{-1} = \sqrt[m]{\sqrt{\left|\frac{a_2}{a_m}\right|}\operatorname{sech}\left(m\sqrt{a_2}\xi\right)} \tag{31}$$

The Modulus of wave function is

$$\left|\psi(x,t)\right| = \sqrt[m]{\sqrt{\left|\frac{a_2}{a_m}\right|}\operatorname{sech}\left(m\sqrt{a_2}\left(px+qt\right)\right)} \tag{32}$$

Now all of the a_2, a_m, A, B can be expressed by $a(t), b(t), \alpha(t), \lambda(t), \gamma(t)$ (are constants within the timing range under study), p and q, and q/p can be expressed by $a(t), b(t), \alpha(t), \lambda(t), \gamma(t)$.

We can see that the solution (32) obtained by our method is of bright soliton type. The pictorial demonstration of the soliton forms for different power index values m is shown in Figure 1. We see that the NLSE modeling (1+1)-dimensional system in power-law dependent media supports bright soliton solution.

4. Conclusion

This study investigatied the nonlinear dynamics for the (1+1)-dimensional quantum system in the power-law dependent media with the corresponding NLSE model. Using the F-expansion method, we derived typical bright soliton solutions under appropriate parameter settings, with key features pictorially demonstrated. Our theoretical treatment showed that a (1+1)-dimensional quantum system with power-law dependent nonlinearity supports bright soliton behavior. The analytical results obtained can be used to guide relevant experimental observation of the soliton dynamics in the (1+1)-dimensional system with power-law dependent nonlinearity.

Data Availability

Our study is theoretical work consisting of analytical derivation of theoretical model. We do not use previously published data for our work.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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