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***Abstract*—In the context of next-generation radio interferometers, we are facing a big challenge of how to economically process data. The classical averaging method to reduce the data is not optimal and can even produce false negative errors in some cases, such as fast radio transients (FRT) imaging, where temporal resolution is required. We propose a robust dimensionality reduction method for FRT imaging in the framework of compressed sensing-based imaging algorithms. For each time slice of FRT imaging, our dimensionality reduction defines a linear embedding operator to reduce the space spanned by the left singular vectors of the measurement operator, which can be considered as a fast approximation of singular value decomposition (SVD). The preliminary results on simulated FRT showcase that the proposed dimensionality reduction can simultaneously reduce the data significantly and recover FRT correctly, while the averaging technique causes the FRT dilution problem.**

Fast radio transients (FRT) detection and imaging are active fields of research in radio astronomy and have seen the development of detection pipeline in recent years. FRT imaging, unlike steady sources imaging, suffer from the imaging rate of the instrument.

In the context of next-generation radio interferometers such as Square Kilometre Array (SKA), tremendous data will be produced, which addresses a computational challenge for imaging. Modern imaging methods based on optimization have showcase the good image reconstruction quality and this philosophy has been extended in FRT imaging ([2]). We present herein a dimensionality reduction method to tackle big data challenge, which can be incorporated with FRT imaging methods as an upstream data pre-processing module.

The FRT imaging problem can be summarized as $\mathbf{y}_t = \mathbf{\Phi}_t\mathbf{x}_t + \mathbf{n}_t$, where $\mathbf{y}_t \in \mathbb{C}^M$ is a vector of continuous visibilities of M measurements at time frame t, corrupted by additive i.i.d. Gaussian noise $n \in \mathbb{C}^M$ and $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$ is the measurement operator to measure the sky $\mathbf{x}_t \in \mathbb{R}^N$ ($M \gg N$) at time frame t. According to the study of dimensionality reduction in [1], the reduced imaging problem with an embedding linear reduction operator $\mathbf{R}$ can be expressed as $\mathbf{R}\mathbf{y} = \mathbf{R}\mathbf{\Phi}_t\mathbf{x} + \mathbf{R}\mathbf{n}$. An ideal reduction operator should preserve the noise statistics while reducing the data. By extending the linear reduction operator proposed by Kartik *et al.* [1] to our FRT imaging case, we can incorporate the FRT sparse imaging method ([2]) with the dimensionality reduction as a pre-processing step.

As the measurement operator is time varying in FRT imaging, we only reduce the spatial dimensionality for each time frame. Therefore, at time t, the optimal dimensionality reduction $\mathbf{R}_t$ is a projection to the left singular vectors $\mathbf{U}_t$ of the measurement operator $\mathbf{\Phi}_t$ by selecting largest singular values according to a reduction or threshold level. Thus, $\mathbf{R}_t = \mathbf{U}_{t}{}^{\dagger}_{th} = \mathbf{\Sigma}_{t}{}^{-1}_{th}\mathbf{V}_{t}{}^{\dagger}_{th}\mathbf{\Phi}_{t}{}^{\dagger}_{th}$, so the data size is reduced to $M_{th} = N_{th} < N$. However, due to the fact that SVD is very computational demanding, a fast approximation of SVD is therefore leveraged such that $\mathbf{V}_t{}^{\dagger} \approx \mathbf{F}$ and $\mathbf{\Sigma}_t{}^2 \approx \mathrm{Diag}(\mathbf{F}\mathbf{\Phi}_t^{\dagger}\mathbf{\Phi}_t\mathbf{F}^{\dagger})$ owing to the fact that the matrix $\mathbf{F}\mathbf{\Phi}_t^{\dagger}\mathbf{\Phi}_t\mathbf{F}^{\dagger}$ is diagonal dominant. Therefore, the final reduction operator at time frame t is given by $\mathbf{R}_t = \mathbf{\Sigma}_t{}^{-1}\mathbf{S}_{th}\mathbf{F}\mathbf{\Phi}_t{}^{\dagger}$, where the selection matrix $\mathbf{S}_{th}$ selects singular values larger than th. By performing such dimensionality reduction for each time frame, we obtain a reduced data set so that we can run the convex optimization-based algorithm for image reconstruction.

In the numerical experiments, we compare the results by using our dimensionality reduction with classical averaging (on time) and all data points. For all of these three cases, we use the same convex-based optimization method for image reconstruction. Our sky model cube is of size $32 \times 32 \times 256$ with 256 time frames each of them represents 45 seconds. We create 5 FRT whose "lifespan" are between 1.5min (2 snapshots) and 4.5min (6 snapshots) and they randomly appear during the observation (top-left of fig. 1). The realistic uv-coverage is simulated via MeqTree tool by using SKA 197 antennas configuration. The additive noise is directly injected on the visibilities with SNR=-30dB (SNR $= 20\log_{10}(||\mathbf{y}_0||_2/||\mathbf{n}||_2)$). To fairly compare the results, the total data size is the same for our reduction method and averaging. By a reduction factor of 4, we can observe that our reduction method (bottom-left of fig. 1) does not degrade reconstruction quality, achieving almost the same result of all data (top-right of fig. 1). However, the averaging (bottom-right of fig. 1) showcases the problem of dilution where the FRT can not be detected at all.

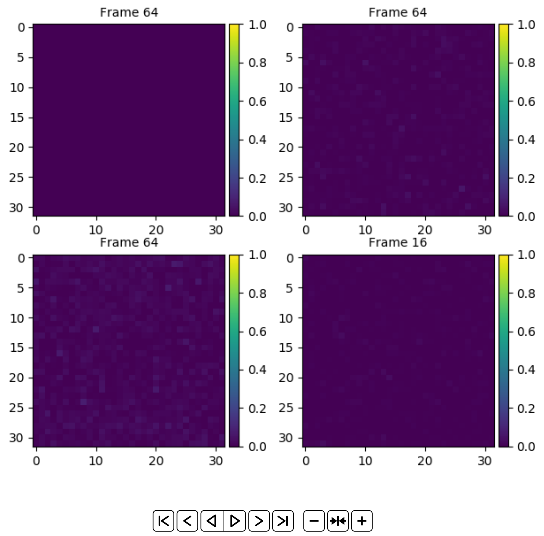

Fig. 1. Numerical experiments on simulated FRT (transients are present between frame 64 and 128). Left-to-right then top-to-bottom: sky model, reconstruction with all data, reconstruction with 25% data by using proposed reduction method and reconstruction with 25% data by using averaging.

References

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