Accurate Modeling of Coil Inductance for Near Field Wireless Power Transfer

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Abstract—This article presents closed form expressions for the self- and mutual inductances of circular wire wound coils used in near field wireless power transfer systems. The calculation of the radius of the coils, inspired from the Archimedean spiral found in many biological organisms, is used to model the self-inductance of single and multi-layer spiral coils. The value of the mutual inductance is determined by expressing the Taylor expansion of the Neumann’s integral for constant current carrying wires. Formulas for mutual inductance are also derived for misaligned magnetically coupled coils enabling the rapid but accurate calculation of power transfer efficiency in real-life applications. Self- and mutual inductance values are computed using the 3D electromagnetic software package MAXWELL 3D™ and these values demonstrate excellent agreement compared with the proposed models. Wire wound coils of different geometrical configurations have been manufactured to validate experimentally the accuracy of the proposed models. Comparison of analytical and experimental results indicate that the proposed models are capable to accurately predict the self-inductance and mutual coupling rapidly. The proposed modeling paves the way for the time efficient optimization of near field wireless power transfer links.

Index Terms—3D EM solver, multi-layer coil, misalignment, near field wireless power transfer, single-layer coil.

I. INTRODUCTION

Wireless power transfer is utilized today in a wide range of applications, ranging from sophisticated low-power biomedical implants [1]–[4] to high-power electric vehicles [5]–[8]. The successful implementation of a wireless power link depends to some extent on the accurate modeling of the self- and mutual inductances that contribute to the optimization of the coupling coefficient between the primary transmission coil and the secondary receiver coil [9]. Achieving optimum power transfer efficiency and large tolerance to misalignment is today the subject of much research efforts [10]–[15].

Several established methods for the calculation of the self-inductance exist in the literature [10],[16]–[25]. Most of these methods consider the current carrying loop as a set of concentric circles. Consequently, there is always some discrepancy between the analytical expression of the self-inductance and the simulated and experimental results. Many contributions in literature have also been made to calculate the mutual inductance [26]–[29], predominantly in the case of perfectly aligned coils [30]–[32]. Analytical derivation of the mutual inductance for misaligned coils is however rare [24],[33],[34] and only carried out for translational misalignment despite the fact that angular misalignment in some applications can also drastically affect the mutual inductance and thus the power link performance. A complete analysis of the variation of the mutual inductance value that takes into account translational and angular misalignment is therefore needed for the accurate characterization of the performance of the wireless power transfer link.

3D electromagnetic (EM) field solution software is one of the most consistent approaches for computing self- and mutual inductance values [33]. However, the simulation of complex multi-loop and multi-planar wire wound coils (WWC) in a 3D EM solver requires substantial computational time. On the other hand, the previous semi-analytical approaches referenced above require long numerical operations which do not reduce the computational complexity [20],[26],[29],[30],[32]. In the light of these difficulties, this article presents a simple, yet accurate, method to estimate self- and mutual inductances to design and optimize a near field wireless power transfer link. In this study, the self-inductance is calculated using the Archimedean spiral geometry used for the coiling of WWCs. Furthermore, complex geometrical parameters are avoided and computationally efficient models are presented for the calculation of the mutual inductance. The proposed models have been verified by 3D EM simulation and validated experimentally. Various coil models have been designed and simulated in ANSYS MAXWELL 3D™. The software package is a commercial quasi-static solver [35], [36]. The magnetostatic simulation mode has been used to solve the coil models. The self-inductance of the same configurations has been measured using a Fluke PM6306 RCL meter and an auto-balancing bridge technique is adopted to measure the mutual inductance of the physical coils. The experimental
values obtained indicate excellent agreement with the proposed analytical models.

II. SELF-INDUCTANCE

The self-inductance, $L_{self}$, of a single circular coil of loop radius, $R$, and wire diameter, $w$, can be represented as [10], [20]–[25]

$$L_{self}(R,w) = \mu R \left[ \ln \left( \frac{16R}{w} \right) - 2 \right]$$  \hspace{1cm} (1)$$

where $\mu$ is the permeability of the medium surrounding the coil. For two perfectly aligned coils of radii $R_i$ and $R_j$, with center-to-centre distance, $d_{ij}$, the mutual inductance, $M_{ij}$ can be calculated as [10], [20]–[25]

$$M_{ij} = M(R_i, R_j, d_{ij}) = \frac{2\mu}{\alpha_{ij}} \frac{R_i R_j}{\sqrt{(R_i + R_j)^2 + d_{ij}^2}}$$  \hspace{1cm} (2)$$

with

$$\alpha_{ij} = 2 \sqrt{\frac{R_i R_j}{(R_i + R_j)^2 + d_{ij}^2}}$$  \hspace{1cm} (3)$$

where $K(\alpha_{ij})$ and $E(\alpha_{ij})$ are the complete elliptic integrals of the first and second kind, respectively.

A. Self-inductance of a single layer spiral coil (SLSC)

The continuously varying radius of the loop of a spiral coil needs to be calculated to determine the self-inductance of such a coil. The coil is in fact an Archimedean spiral [37]. As shown in Fig. 1, the calculated radius, $R$, is:

$$R = R_0 + \frac{w + s_l}{2\pi} \theta_R$$  \hspace{1cm} (4)$$

where $R_0$ is the start radius, $\theta_R$ is the angle of revolution (=2$\pi j$ at the $j^{th}$ loop forming thereby the radius $R_j$) and $s_l$ is the space between two consecutive loops.

All the loops of a single-layer coil have a common center so that $d_{ij}=0$. The self-inductance of $N_l$ loops, $L_{self}$, can be calculated from (1) and (2) as

$$L_{self} = \sum_{i=1}^{N_l} L(R_i, w) + \sum_{i=1}^{N_l} \sum_{j=1}^{N_l} M(R_i, R_j, 0) \times (1 - \delta_{ij})$$  \hspace{1cm} (5)$$

where $\delta_{ij}=1$ for $i=j$; $\delta_{ij}=0$, otherwise.

The behaviour of the self-inductance is compared in Fig. 2 for different outer diameters (or number of loops) of a coil, for wire diameter $w=200 \mu\text{m}$, starting radius $R_0=2.19 \text{mm}$ and spacing $s_l=10 \mu\text{m}$, against results obtained with the 3D-EM field solution software package MAXWELL 3D$^\text{TM}$. The case of the ideal coil of same outer diameter but with concentric circles (labelled ideal CC) is also provided [10], [20]. The maximum percentage of variation with respect to the 3D EM simulation is also indicated on the same figure (right vertical axis). For the proposed model, the variation is less than 3% whereas the use of concentric circles produces a variation of around 45%. Therefore, the proposed model offers better accuracy in the approximation of $L_{self}$.

B. Self-inductance of a Multi-Layer Helical Coil (MLHC)

Fig. 3 shows the model of a multi-loop, multi-layer helical coil where $s_l$ represents the spacing between two consecutive layers in the vertical direction. $L_i$ and $T_j$ are the $i^{th}$ loop and $j^{th}$ layer, respectively.

![Fig. 1. Parameters to calculate the varying radius of a 4-loop single layer Archimedean spiral.](image1)

![Fig. 2. $L_{self}$ as a function of the outer diameter of the coil. The proposed new model is benchmarked against the 3D EM modeling software package and the idealized model where all circles are considered concentrical. The % variation with respect to the 3D EM values is also indicated for the ideal and proposed models.](image2)

![Fig. 3. Multi-loop and multi-layer helical coil. The spacing between two loops has been exaggerated to indicate the interconnection between the loops of the MLHC.](image3)
In this model, a loop $L_1$ of radius $R_i$ is formed initially with $T_{ij}$ layers going downwards. At the bottom of the coil, a second loop $L_2$ of radius $R_i$ is formed with $T_{ij}$ layers going upwards. At the top of the coil, the same process continues for loop $L_i (= L_j)$ going downwards, etc. In this case, $\theta_k=2\pi (i-1)$ is used to estimate the radius of the $L_i^{th}$ loop using equation (4). The distance between the reference layer $T_1$ and $T_j$, $d_{T_1T_j}$, is simply

$$d_{T_1T_j} = (j-1) \times (s_t + w)$$

For a coil with $N_l$ layers and $N_l$ loops per layer, the total self-inductance can be modeled as

$$L_{\text{self}} = N_l \sum_{i=1}^{N_l} L(R_i, w) + N_l \sum_{i=1}^{N_l} \sum_{j=1}^{N_l} M(R_i, R_j, 0) \times (1 - \delta_{ij})$$

where $\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ otherwise. The first term in (7) is the self-inductance of the $N_l$ layers containing $N_l$ loops each. The second term accounts for the mutual inductance effect of the loops for a single layer. The last term considers the mutual inductance of different loops of different layers with one layer always considered as a reference plane.

In Fig. 4, the performance of $L_{\text{self}}$, considered as an ideal CC and MLHC is benchmarked against the 3D EM simulation as a function of numbers of layers where $N_l = 2$, $w = 400 \mu m$, $D = 8.4$ mm, $s_t = 10 \mu m$ and $s_r = 10 \mu m$. This configuration is representative of biomedical implant applications such as capsule endoscopy [38]. The variation with respect to the EM software results, %variation, is around 12% for $N_l = 2$, decreasing to less than 5% for a larger number of layers. Again, the proposed model shows better performance for a high number of $N_l$ and $N_r$.

### III. Mutual Inductance of Inductively Coupled Coils

In this section, the mutual inductance is modelled for perfectly aligned and misaligned inductively coupled coils. Results are compared with the 3D EM simulation.

#### A. Case of perfectly aligned coils

The mutual inductance, $M$, for two current carrying coils, C1 and C2, can be calculated using the Neumann’s formula [26], [39]

$$M = \frac{\mu}{4\pi} \left[ \frac{R_{C1} R_{C2}}{\sqrt{R_{C1}^2 + R_{C2}^2 + d_t^2}} \right] \int_0^{2\pi} \int_0^{2\pi} \frac{\cos(\varphi_{C1} - \varphi_{C2})}{\left[1 - \gamma \cos(\varphi_{C1} - \varphi_{C2})\right]^2} d\varphi_{C1} d\varphi_{C2}$$

where $d_t$ is the center-to-center distance for the top and bottom layers of C1 and C2, respectively. The point P lies on the $N_l,C_1$th layer in C1 taking the reference layer as the top layer in Fig. 5. In the same way, the point Q lies in the $N_l,C_2$th layer in C2 taking this time the bottom layer as the reference layer. Using (9) and the dot product of the two infinitesimal displacement vectors, $d_{l_1}$ and $d_{l_2}$, (8) can be rewritten as:

$$R_p = \left[ \frac{R_{C1}^2 + R_{C2}^2 + d_t^2}{-2R_{C1} R_{C2} \cos(\varphi_{C1} - \varphi_{C2})} \right]^{\frac{1}{2}}$$

Fig. 4. $L_{\text{self}}$ for different layers $N_l$ of multi-layer helical coil using 3D EM simulation results, ideal (concentrical circles) model and proposed model.

Fig. 5. Configuration of perfectly aligned coils. In this example, coil C1 has 5 loops and 4 layers. Coil C2 has 6 loops and 3 layers. Further, $d_l=d_t$, due to the choice of the points P and Q.

The magnitude, $R_p$, of the vector joining a point P on C1 to a point Q lying on C2, as shown in Fig. 5, is:

$$R_p = \left[ \frac{R_{C1}^2 + R_{C2}^2 + d_t^2}{-2R_{C1} R_{C2} \cos(\varphi_{C1} - \varphi_{C2})} \right]^{\frac{1}{2}}$$

where $\varphi_{C1}$ and $\varphi_{C2}$ are the angular co-ordinates of P on C1 and Q on C2, respectively. $R_{C1}$ and $R_{C2}$ are the radii of the coils C1 of wire diameter $w_{C1}$, and C2 of wire diameter $w_{C2}$, respectively. The relative center-to-center distance, $d_t$, between the coils where these points lie, can be approximated as

$$d_t = d_{l_1} + [(N_l,C_1 - 1) \times (w_{C1} + s_{l,C1})]$$

$$+ [(N_l,C_2 - 1) \times (w_{C2} + s_{l,C2})]$$

![Image](image_url)
where \( \gamma [33] \) is

\[
\gamma = \frac{2R_{C1}R_{C2}}{R_{C1}^2 + R_{C2}^2 + d_2^2}
\]  

(12)

Typical coil configurations used in WPT such as \( R_{C1}=30 \) mm and \( R_{C2}=4.2 \) mm and \( d_1=100 \) mm, provide a value of \( \gamma << 1 \).

The integrand inside (11) can therefore be linearized as a Taylor series up to the 5th order such that

\[
M = \frac{\mu \pi R_{C1}^2 R_{C2}^2}{2(R_{C1}^2 + R_{C2}^2 + d_2^2)^2} \left[ 1 + \frac{15}{32} \gamma^2 + \frac{315}{1024} \gamma^4 \right]
\]  

(13)

For a coil \( C1 \) containing \( T_{C1} \) layers and \( L_{C1} \) loops and a coil \( C2 \) containing \( T_{C2} \) layers and \( L_{C2} \) loops, the total mutual inductance is then

\[
M_{\text{total}} = \sum_{i=1}^{T_{C1}} \sum_{j=1}^{T_{C2}} \sum_{k=1}^{L_{C1}} \sum_{l=1}^{L_{C2}} M(R_{C1:i}, R_{C2:l}, d_{r(i,j)})
\]  

(14)

where \( R_{C1:k} \) and \( R_{C2:l} \) are the radius of the \( k \)th loop of \( C1 \) and the \( l \)th loop of \( C2 \), respectively, and \( d_{r(i,j)} \) is calculated according to (10).

Fig. 6 shows the value of the mutual inductance as a function of \( d_1 \) using the 3D EM simulation and (14). The coil radius for the two coils was calculated using (4). The total numbers of layers and loops were 12 and 1, respectively for \( C1 \), and 10 and 3, respectively for \( C2 \). The outer diameters for coils \( C1 \) and \( C2 \) were taken as 60 mm and 8.4 mm, respectively. The % variation between 3D EM simulation and proposed model is less than 10% for separation distance less than 70 mm and increases to around 14% at \( d_1=100 \) mm.

B. Case of translational misalignment

Fig. 7 shows the configuration between \( C1 \) and \( C2 \) in the case of translational misalignment. \( C2 \) is off-axis by a distance, \( d_2 \), from the axis of the coil \( C1 \). The distance between two arbitrary points \( P \) and \( Q \) lying on \( C1 \) and \( C2 \), respectively, can be written as:

\[
R_{pq} = \sqrt{(2R_{C1} + R_{C2}^2 + d_2^2 - 2R_{C1}R_{C2}\cos(\varphi_{C1} - \varphi_{C2}) - 2d_2R_{C1}\sin\varphi_{C1} + 2d_2R_{C2}\sin\varphi_{C2})^2 + d_r^2 + d_r^2}
\]  

(15)

To calculate the mutual inductance, the following parameters, \( \gamma_a, \gamma_b \) and \( \gamma_c \) are introduced:

\[
\gamma_a = \frac{2R_{C1}R_{C2}}{R_{C1}^2 + R_{C2}^2 + d_2^2 + d_2^2}
\]  

(16)

\[
\gamma_b = \frac{2R_{C1}d_2}{R_{C1}^2 + R_{C2}^2 + d_2^2 + d_2^2}
\]  

(17)

\[
\gamma_c = \frac{2R_{C2}d_2}{R_{C1}^2 + R_{C2}^2 + d_2^2 + d_2^2}
\]  

(18)

Replacing (16), (17), (18) in (15) and using (8):

\[
M = \frac{\mu}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \cos(\varphi_{C1} - \varphi_{C2}) \left[ 1 - \gamma_a \cos(\varphi_{C1} - \varphi_{C2}) - \gamma_b \cos \varphi_{C1} + \gamma_c \cos \varphi_{C2} \right] d\varphi_{C1} d\varphi_{C2}
\]  

(19)

Using a Taylor series expansion to the 4th order and integrating...
\[ M = \frac{\mu \pi R_{C1}^2 R_{C2}^2}{2(R_{C1}^2 + R_{C2}^2 + d_1^2 + d_2^2)^\frac{3}{2}} \left[ 1 - \frac{3}{2} \delta + \frac{15}{32} \gamma_2^2 \left( 1 - \frac{21}{2} \delta \right) + \frac{15}{16} \left( \gamma_2^2 + \gamma_3^2 \right) \left( 1 - \frac{7}{4} \delta \right) \right] \]  

where \[ \delta = \frac{d_2^2}{R_{C1}^2 + R_{C2}^2 + d_1^2 + d_2^2} \]  

Note that, when \( d_2 = 0 \), which is the case of perfectly aligned coils, the expression of \( M \) from (13) is recovered up to the third order. The total mutual inductance, \( M_{\text{total}} \), can be calculated as \[ M_{\text{total}} = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} M \left( R_{C1; k}, R_{C2; l}, d_{r(ij)}, d_2 \right) \]  

Fig. 8 shows the variation of \( M_{\text{total}} \) between C1 and C2 at \( d_1 = 50 \) mm for \( d_2 \) ranging from 0 to 50 mm using the 3D EM simulation software and (22). The coil parameters are the same as in the case of perfectly aligned coils. The % variation between 3D EM simulation and proposed model is less than 9%. 

**C. Case of angular misalignment**

The configuration of coils suffering from angular misalignment is shown in Fig. 9. Angular misalignment can be due to roll and pitch rotations which are represented in the figure by \( \theta \) and \( \lambda \), respectively. The distance between two arbitrary point lying on C1 and C2 can be written as 

\[ R_p = \left[ \frac{R_{C1}^2 + R_{C2}^2 + d_1^2 - 2 R_{C1} R_{C2} (\cos \varphi_{C1} \cos \varphi_{C2} \cos \theta \cos \lambda + \sin \varphi_{C1} \sin \varphi_{C2})}{(\sin \theta \cos \lambda + \sin \lambda \cos \theta) + 2 R_{C2} \cos^2 \varphi_{C2} \sin \lambda \cos \theta} \right]^{\frac{1}{2}} \]  

The parameters \( \gamma_a, \gamma_b \) and \( \gamma_c \) are introduced as 

\[ \gamma_a = \frac{2 R_{C1} R_{C2}}{R_{C1}^2 + R_{C2}^2 + d_r^2} \]  
\[ \gamma_b = \frac{2 R_{C2} d_r}{R_{C1}^2 + R_{C2}^2 + d_r^2} \]  
\[ \gamma_c = \frac{2 R_{C2}^2}{R_{C1}^2 + R_{C2}^2 + d_r^2} \]  

The values of \( \gamma_a, \gamma_b \) and \( \gamma_c \) are less than unity for wireless power application. Replacing (24), (25) and (26) in (23) the modified \( M \) from (8) can be written as 

\[ M = \frac{\mu}{4 \pi \sqrt{R_{C1}^2 + R_{C2}^2 + d_r^2}} \int_0^{2\pi} \int_0^{2\pi} \frac{(\sin \varphi_{C1} \sin \varphi_{C2} \cos \theta \cos \lambda + \cos \varphi_{C1} \cos \varphi_{C2})}{(\sin \theta \cos \lambda + \sin \lambda \cos \theta) + \gamma_c \cos^2 \varphi_{C2} \sin \theta \cos \sin \lambda \cos \theta} d\varphi_{C1} d\varphi_{C2} \]  

The Taylor expansion of \( M \) can be written as:
\[ M = \frac{\mu \nu R_{c1}^2 R_{c2}^2 \cos \theta \cos \lambda}{2(R_{c1}^2 + R_{c2}^2 + d_L^2)^2} \left[ 1 + \frac{15}{64} \gamma_a^2 + \frac{15}{64} \gamma_b^2 \cos^2 \theta \cos^2 \lambda + \frac{15}{8} \gamma_c^2 (\sin \theta \cos \lambda \sin \lambda \cos \theta) \right] \]  

(28)

\[ M_{\text{total}} = \sum_{i=1}^{T_{c1}} \sum_{j=1}^{T_{c2}} \sum_{k=1}^{L_{c1}} \sum_{l=1}^{L_{c2}} M \left( \frac{R_{c1:k}, R_{c2:l}}{d_{r(l,f)}}, \theta, \lambda \right) \]  

(29)

Fig. 10 shows the changes of \( M_{\text{total}} \) for \( d_1=50 \text{mm} \) as a function of \( \theta \) (with \( \lambda \) fixed at \( 45^\circ \)) using 3D EM simulation and (29) using the same coil parameters as in the previous section. Allowing independent variations of \( \theta \) and \( \lambda \), the maximum % variation with respect to the simulation model is also less than 7%. The changes of \( M_{\text{total}} \) as a function of \( \theta \) and \( d_1 \) (for \( \lambda=45^\circ \)) using (29) are provided in Fig.11. A 3D plot using the 3D EM simulation provides similar results. The minimum and maximum differences between the two surfaces occur at \( \theta=60^\circ \) and \( d_1=40 \text{mm} \) (4% variation) and \( \theta=80^\circ \) and \( d_1=100 \text{mm} \) (13% variation), respectively.

\[ R_p = \left[ \begin{array}{c} R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2 - 2R_{c1}R_{c2}(\cos \varphi_{c1} \cos \varphi_{c2} \cos \varphi \cos \lambda + \sin \varphi_{c1} \sin \varphi_{c2}) \\ -2d_1 R_{c2} \cos \varphi_{c2} (\sin \theta \cos \lambda + \sin \lambda \cos \theta) - 2d_2 R_{c1} \cos \varphi_{c1} + 2d_2 R_{c2} \cos \varphi_{c2} \cos \theta \\ \cos \varphi + 2R_{c2}^2 \cos^2 \varphi_{c2} \sin \lambda \cos \lambda \sin \lambda \cos \theta \end{array} \right] \]  

(30)

For simplification, the parameters \( \gamma_a, \gamma_b, \gamma_c, \gamma_d \) and \( \gamma_e \) are introduced as

\[ \gamma_a = \frac{2R_{c1} R_{c2}}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]  

(31)

\[ \gamma_b = \frac{2R_{c1} d_2}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]  

(32)

\[ \gamma_c = \frac{2R_{c2} d_2}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]  

(33)

\[ \gamma_d = \frac{2R_{c2} d_1}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]  

(34)

\[ \gamma_e = \frac{2R_{c2}^2}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]  

(35)

These parameters are smaller than unity for the wireless power transfer coils. Replacing (31), (32), (33), (34) and (35) in (30) the modified \( M \) from (8) can be written as
\[ M = \frac{\mu}{4\pi} \left( \frac{R_{c1}R_{c2}}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \right)^2 \int_0^{2\pi} \int_0^{2\pi} \frac{\left( \sin\varphi_{c1}\sin\varphi_{c2}\cos\theta\cos\lambda + \cos\varphi_{c1}\cos\varphi_{c2} \right)}{1 - \gamma_a (\cos\varphi_{c1}\cos\varphi_{c2}\cos\theta\cos\lambda + \sin\varphi_{c1}\sin\varphi_{c2}) - \gamma_b \cos\varphi_{c1} + \gamma_c \cos\varphi_{c2}\cos\lambda - \gamma_d \cos\varphi_{c2} (\sin\varphi_{c1} + \sin\lambda\cos\theta) + \gamma_e \cos^2\varphi_{c2}\sin\theta\cos\lambda\sin\lambda\cos\theta} \, d\varphi_{c1} \, d\varphi_{c2} \]

Finally, the Taylor expansion of \( M \) is

\[ M = \frac{\mu R_{c1}^2 R_{c2} \cos\varphi_{c1}}{2(R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2)^{3/2}} \left[ 1 + \frac{15}{64} \left( \gamma_a^2 + 4\gamma_b^2 \right) + \frac{15}{64} \left( \gamma_c^2 + 4\gamma_d^2 \right) \cos^2\theta\cos^2\lambda + \frac{15}{8} \gamma_e^2 (\sin\theta\cos\lambda\sin\lambda\cos\theta) \right] \]

where

\[ \delta_a = \frac{d_1^2}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]

\[ \delta_b = \frac{d_2^2}{R_{c1}^2 + R_{c2}^2 + d_1^2 + d_2^2} \]

Furthermore, \( M_{total} \) can be expressed as:

\[ M_{total} = \sum_{i=1}^{T} \sum_{j=1}^{T} \sum_{k=1}^{L_1} \sum_{l=1}^{L_2} \left( R_{c1,k} R_{c2,l} \right) \]

\[ d_{(i,j), d_2, \theta, \lambda} \]

and

\[ |Z| = \left| \frac{V_2}{I_1} \right| = \left| \frac{-V_2 R_F}{V_F} \right| \]

where \( f \) is the frequency of the input signal.
right vertical axis.

The measured result is compared with the simulation and the proposed model. The %variation of the model and the 3D EM simulation with respect to measured results is presented on the right vertical axis and show the %variation is less than 7% for both the proposed model and 3D EM simulation.

Fig. 15. Measured $L_{self}$ values for multi-layer, single loop coils and comparison with the proposed model and 3D EM simulation results. The percentage of variation of the values of the proposed model and 3D EM simulation with respect to the measured results is indicated on the right vertical axis.

Fig. 15 shows the measured values of $L_{self}$ for the multi-layer single loop case presented in Table I as the number of layers vary. The %variation of the experimental results with respect to both the proposed and 3D EM simulation models is less than 6%.

Fig. 16 presents the experimental and simulation results of $L_{self}$ for the multi-loop, multi-layer ($N_t=5$) coil as the number of loops is varied from 1 to 5. The variation for both the proposed and 3D EM simulation models with respect to experimental values is less than 8%.

Fig. 16. Measured $L_{self}$ values as a function of the number of loops for the multi-loop ($N_t=5$) WWC and comparison with the proposed model and 3D EM simulation results. The percentage of variation for the proposed model and 3D EM simulation with respect to the measured results is indicated on the right vertical axis.

Fig. 17 shows the measurement results in comparison with (14) and 3D EM simulation for perfectly aligned single layer primary and secondary coils. The %variation occurring with respect to the proposed model is less than 3.5% for single layer coils. Similar plot of multi-layer coils demonstrates less than 10% variation for the proposed model.

Fig. 17. Measurement of $M_{total}$ at different $d_1$ and comparison with proposed model and 3D EM simulation results for perfectly aligned single layer primary and secondary coils. The %variation of the proposed model and 3D EM simulation with respect to measured results are presented in the right vertical axis.

Fig. 18 shows the magnetic field distribution of perfectly aligned single layer primary and secondary coils. In Fig. 18 (a), $d_1=10$ mm, the secondary coil is strongly exposed to the magnetic field radiation of the primary coil due to smaller center-to-center distance. Figs. 18 (b) and (c) demonstrate lower magnetic field intensity in the secondary coil due to the larger separation distance ($d_1=35$ and 50 mm, respectively) of the primary coil. This confirms the reduction of $M_{total}$ due to the increase in $d_1$. 

Fig. 18. Magnetic field distribution for perfectly aligned single layer primary and secondary coils. (a) $d_1=10$ mm, (b) $d_1=35$ mm, (c) $d_1=50$ mm.
Fig. 18. Magnetic field distribution for perfectly aligned coils. (a) $d_1=10$ mm. (b) $d_1=35$ mm. (c) $d_1=50$ mm.

Fig. 19 presents the measured and 3D EM simulated results for different translational misalignment distances, $d_2$, for $d_1=10$ mm for single layer primary and secondary coils. The variation in the proposed model remains less than 3%. The same type of measurement for multi-layer coils can show a %variation of slightly over 6%.

Figs. 20 (a) and (b) show the magnetic field distribution for $d_2=10$ and 30 mm, respectively, for $d_1=10$ mm. The field intensity in the secondary coil due to primary coil is lower in both cases compared to Fig. 18 (a), causing thereby a reduction of the $M_{\text{total}}$.

Fig. 21 shows the measured, calculated (29) and 3D EM simulated results of angular misalignment for single-layer primary and secondary coils. The roll rotation angle, $\theta$ is changed for a constant pitch rotation angle $\lambda=30^\circ$. The %variation remains less than 6.5% for the proposed model and 3D EM simulation for single layer coils. In the case of multi-layer coils, the percentage variation is around 8%.

Fig. 21. Measurement of $M_{\text{total}}$ for single layer primary and secondary coils at different $\theta$ for $\lambda=30^\circ$ and $d_1=35$ mm. The proposed model and 3D EM simulation results are also compared. The %variation of the proposed model with respect to measured results are presented in the right vertical axis.
Further, Fig. 22 shows the measured, calculated (29) and 3D EM simulated results of varying \( \theta \) with respect to constant \( \lambda=45^\circ \) and \( d_1=35 \) mm. The observed % variation is less than 7% for the proposed model and 3D EM simulation for single layer coils. In the case of similar plot of multi-layer coil, this percentage variation is raised up to 10%.

Fig. 22. Measurement of \( M_{\text{total}} \) for single layer primary and secondary coils at different \( \theta \) for \( \lambda=45^\circ \) and \( d_1=35 \) mm. The proposed model and 3D EM simulation results are also compared. The % variation of the proposed model with respect to measured results are presented in the right vertical axis.

In Fig. 23, the measured, calculated (29) and 3D EM simulated results are illustrated for varying \( \theta \) with respect to constant \( \lambda=60^\circ \) and \( d_1=35 \) mm. The % variation is close to 5% for the proposed model and 3D EM simulation for single layer coils. This percentage variation is less than 11% for the similar plot of multi-layer coil.

Fig. 23. Measurement of \( M_{\text{total}} \) for single layer primary and secondary coils at different \( \theta \) for \( \lambda=60^\circ \) and \( d_1=35 \) mm. The proposed model and 3D EM simulation results are also compared. The % variation of the proposed model with respect to measured results are presented in the right vertical axis.

Figs. 24 (a), (b), (c) and (d) show the magnetic field distribution for \( \theta=\lambda=30^\circ \), \( \theta=\lambda=45^\circ \), \( \theta=\lambda=60^\circ \) and \( \theta=\lambda=90^\circ \), respectively, for \( d_1=35 \) mm. Compared to Fig. 18 (b) the exposure of the magnetic radiation from primary coil reduces with the gradually increasing \( \theta \) and \( \lambda \) of the secondary coil.

\[ M_{\text{total}} \] reduces therefore significantly. In case of \( \theta=\lambda=90^\circ \), the field vectors of primary coil are in parallel with the position of the secondary coil bringing \( M_{\text{total}} \) close to zero.

### Table II

<table>
<thead>
<tr>
<th>Configuration of the coil(s)</th>
<th>Simulation Time (3D EM)</th>
<th>Simulation Time (Proposed Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{self}} ), single layer</td>
<td>10 min</td>
<td>( 12 \times 10^{-3} ) s</td>
</tr>
<tr>
<td>( L_{\text{self}} ), single loop multi-layer</td>
<td>15 min</td>
<td>( 15 \times 10^{-3} ) s</td>
</tr>
<tr>
<td>( L_{\text{self}} ), multi-loop multi-layer</td>
<td>45 min</td>
<td>( 18 \times 10^{-3} ) s</td>
</tr>
<tr>
<td>( M_{\text{total}} ), perfectly aligned (single layer secondary coil)</td>
<td>6 hr 30 min</td>
<td>2.3 s</td>
</tr>
<tr>
<td>( M_{\text{total}} ), translational misaligned (single layer secondary coil)</td>
<td>5 hr 45 min</td>
<td>2.1 s</td>
</tr>
<tr>
<td>( M_{\text{total}} ), angularly misaligned (single layer secondary coil)</td>
<td>5 hr 30 min</td>
<td>2.4 s</td>
</tr>
<tr>
<td>( M_{\text{total}} ), incorporated misaligned (single layer secondary coil)</td>
<td>12 hr 40 min</td>
<td>5 s</td>
</tr>
</tbody>
</table>
are measured and compared with the proposed prediction models. Results demonstrate an excellent agreement in the accuracy of the proposed models. The comparison of the required computational time also proves the efficiency of the proposed models. Therefore, the self- and mutual inductance models presented in this paper can be considered as an excellent candidate for the design and optimization of sophisticated near field wireless power transfer systems.

VI. COMPUTATIONAL EFFICIENCY

Table II provides a comparison of the computational time required for the proposed analytical model (MATLAB™) and the 3D EM software (MAXWELL 3D™) using parameters for the coils manufactured for this study. The simulation was performed on an Intel (R) Xeon (R) CPU E5-2640 with processor speed of 2.5 GHz and 128 GB of RAM (HP Z820 workstation). The comparison confirms that a significant amount of simulation time can be saved using the models proposed in this paper.

VII. CONCLUSIONS

This paper presents an accurate modeling method of the self-inductance, and compact solutions for the calculation of the mutual inductance for near field wireless power transfer systems. The detailed mathematical derivation of inductances is presented. The proposed models have been compared with the 3D EM simulation results for numerical validation and experimentally validated. This method shows excellent prediction capability of self-inductance for all types of WWC coils (single and multi-layer). Additionally, it provides a cost-effective calculation for the determination of the mutual inductance values at any separation distance and misalignment cases. Self- and mutual inductance of different assembled coils

REFERENCES


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