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Thermomagnetic convection under alternating magnetic fields

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Thermomagnetic convection within a square cavity filled with a magnetic fluid was studied under alternating magnetic field. The aim was to explore the response of the magnetic fluid towards alternating magnetic fields. A set of frequencies for the sinusoidal variation of the magnetic field intensity was used to study the heat transfer characteristics across the problem geometry. The effect on heat transfer was quantified by an averaged Nusselt number.

1 Introduction

Thermomagnetic convection is based on local variation in magnetisation of a magnetic fluid imposed by a temperature difference in the presence of a external magnetic field [1, 2]. This convection can also be induced by alternating magnetic fields as reported by Kim and Hyun [3]. The present study extends the latter by investigating the response of the fluid and the heat transfer across the system. A set of frequencies starting from 0.2 to 4 Hz, that altered around a mean zero value, with a magnetic field intensity, H_0, of 10^5 A/m imposed as a constant field gradient over the domain, is investigated. As buoyancy was absent the magnetic body force is the volume force that acted on the magnetic fluid and induced thermomagnetic convection. The response of the fluid and the corresponding heat transfer characteristics is investigated in the present study.

2 Model formulation

The problem geometry is a non-dimensional square cavity filled with a magnetic fluid that has the physical properties reported in [4]. The cavity is heated from the left with, $T' = 1$, and cooled at the right with, $T' = 0$, whereas all remaining walls are adiabatic. The velocity boundary conditions are no-slip at all walls. A zero gauge pressure constraints $p'_0 = 0$ was applied at the left top corner of the cavity. All investigated frequencies used the same initial conditions of a uniform distributed temperature $T' = 0$ and a stagnant fluid. The alternating magnetic field, H, within the cavity is calculated via

$$H = \left| H_0 \sin\left(t\, 2\pi f + \frac{\pi}{2}\right) \right| \tag{1}$$

where t is the time. The resulting magnetisation of the magnetic fluid is expressed through Langevin's function $\mathcal{L}(\zeta) = \coth(\zeta) - 1/\zeta$ with $\zeta = mH/(k_{\mathrm{B}}T)$ and may be extended by Pshenichnikov [5] to include particle interactions given by

$$M = \phi M_{\mathrm{d}}\, \mathcal{L}(\zeta_{\mathrm{p}}), \quad \zeta_{\mathrm{p}} = \frac{m\,(H + M_L/3)}{k_B T} \tag{2}$$

where m is the magnetic moment of the magnetic fluid, H the magnitude of the applied magnetic field, k_B the Boltzmann's constant, T the temperature, M_d the fluids bulk magnetisation through volume concentration ϕ and $M_{\mathrm{L}} = \phi M_{\mathrm{d}}\, \mathcal{L}(\zeta)$ the Langevin magnetisation. Transient changes in the magnetisation due to Néel and Brownian relaxation are neglected as they are several orders slower than the imposed frequency. By introducing the following non-dimensional parameters

$$t' = tf, \quad \mathbf{u}' = \frac{\mathbf{u}L}{\nu}, \quad \nabla' = \nabla L, \quad p' = \frac{pL^2}{\rho\nu^2}, \quad T' = \frac{T - T_c}{\Delta T}, \quad \mathrm{Gr_m} = \frac{\mu_0 K \Delta T H L^2}{\rho \nu^2}, \quad H' = \frac{H}{H_0} \tag{3}$$

the continuity, momentum and energy equation for an incompressible magnetic fluid may be expressed dimensionless as

$$\nabla' \cdot \mathbf{u}' = 0 \tag{4}$$

$$\frac{\partial \mathbf{u}'}{\partial t'} + (\mathbf{u}' \cdot \nabla')\,\mathbf{u}' = -\nabla' p' + \nabla'^2 \mathbf{u}' + T'\, \mathrm{Gr_m}\, \mathbf{e}_{\nabla\mathbf{H}} \tag{5}$$

$$\frac{\partial T'}{\partial t'} + (\mathbf{u}' \cdot \nabla')\, T' = \mathrm{Pr}\, \nabla'^2 T' \tag{6}$$

where $\mathbf{u}$ is the velocity, L the characteristic length, ν the kinematic viscosity, p the pressure, ρ the density, T the temperature, Gr_m the magnetic Grashof number, μ_0 the vacuum permeability, $K = -\left(\partial M/\partial H\right)_{\phi,H}$ the pyromagnetic coefficient, $\mathbf{e}_{\nabla\mathbf{H}}$ is

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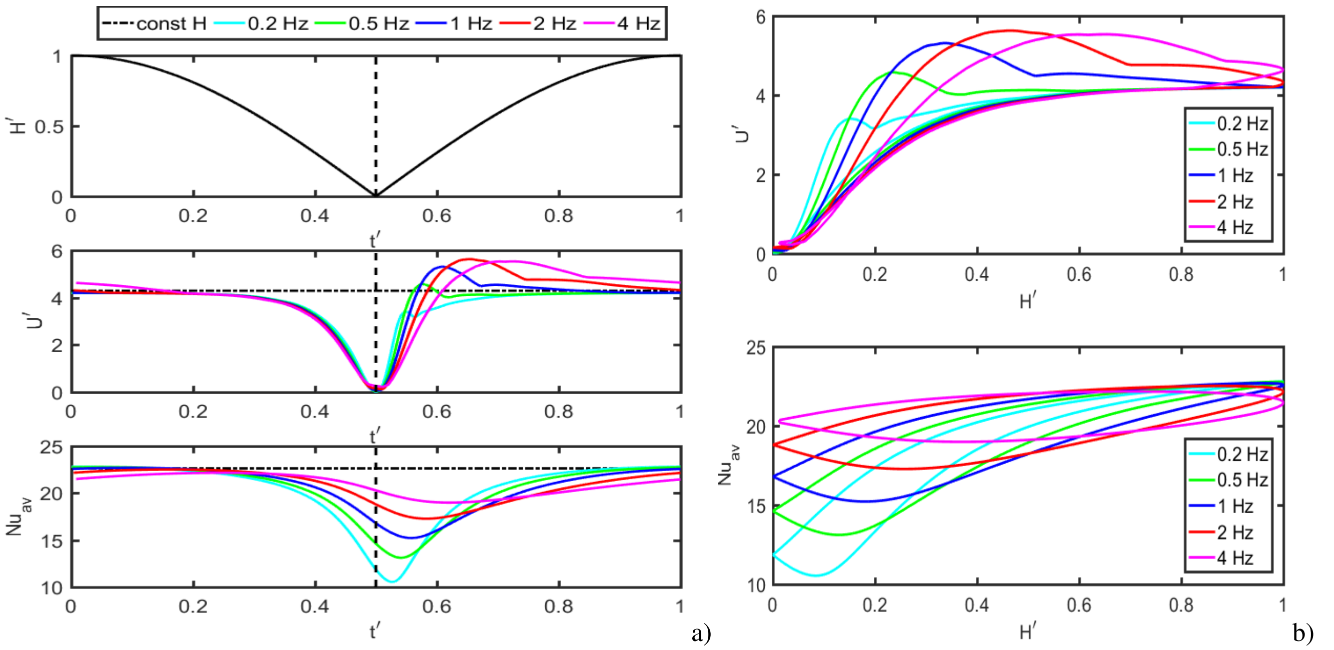

Fig. 1: Non-dimensional magnetic field, H', velocity magnitude, U', and Nusselt number, Nu, plotted versus the time, t', in **a** and the velocity magnitude, U', and Nusselt number, Nu, plotted versus the magnetic magnetic field, H', in **b**.

the unit norm vector in the direction of the magnetic field, $\mathrm{Pr} = \nu/\kappa$ the Prandtl number and κ the thermal diffusivity. The numerical model was solved by using COMSOL Multiphysics, a commercially developed solver that uses the finite-element technique and is validated against de Vahl Davis [6] benchmark solution for natural convection that obtained sufficient accurate results. The heat transfer trough the cavity is quantified by the averaged Nusselt number, Nu_{av}.

3 Results and discussion

Fig. 1 shows the numerical results of the response of the magnetic fluid and the thermo-convective heat transfer quantified by Nu. As observed the magnetic fluid responds to a certain time scale to the alternating magnetic field and may be classified into three time scales. The inertia scale, determined by the fluid's inertial that becomes longer than the time in which the applied body forces change, result in a shift of the curves to the right. However, as the frequency is small, the inertia time scale is marginal and the shift of the curves to the right seen in Fig. 1 is primarily driven by the thermal diffusivity time scale. This time scale is understood by the temporal changes in the spatial temperature distribution via heat conduction when the magnetic force is small or absent. An overshoot is therefore noticed when the magnetic force returns. To re-establish the distorted isotherms to the indicative temperature distribution of thermomagnetic convection, a third time scale, the viscous time scale is introduced and refers to viscous dissipation. In addition, the maximum of Nu for each configuration starts decreasing at higher frequencies and is indicative of a small thermal diffusivity time scale that has a moderate effect on the temperature distribution. In summary, the heat transfer quantified by Nu responds to the alternating magnetic field. The phase lag of velocity and Nu observed, refers therefore to the three scales the inertial, thermal diffusivity and viscous time scale.

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