FEMOS

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APPENDIX

APPENDIX A
PROOF OF LEMMA 1

Note that for any $Q \geq 0, \mu \geq 0, A \geq 0$, we have:

$$\left([Q-\mu]^{+}+A\right)^{2} \leq Q^{2}+A^{2}+\mu^{2}+2Q(A-\mu). \tag{1}$$

Plugging the inequations (1) into the Lyapunov drift function, we have

$$\begin{aligned} L(\boldsymbol{Q}(t+1))-L(\boldsymbol{Q}(t)) \leq \frac{1}{2}\sum_{u\in\mathcal{U}} A_u^2(t)+\frac{1}{2}\sum_{u\in\mathcal{U}}\mu_u^2(t) \\ +\sum_{u\in\mathcal{U}} Q_u^{\text{sum}}(t)[A_u(t)-\mu_u(t)]. \end{aligned} \tag{2}$$

By adding the penalty term $-V\phi(\boldsymbol{\mu}(t))$ on both sides of (2) and rearranging the terms, we have:

$$\begin{aligned} &L(\boldsymbol{Q}(t+1))-L(\boldsymbol{Q}(t))-V\phi(\boldsymbol{\mu}(t)) \\ &\leq \mathcal{K}-\sum_{u\in\mathcal{U}}\left[V+Q_u^{\text{sum}}(t)\right]\mu_u(t)+\sum_{u\in\mathcal{U}} Q_u^{\text{sum}}(t)A_u(t), \end{aligned} \tag{3}$$

where $\mathcal{K}=\frac{|\mathcal{U}|}{2}\left[A_{\max}^2+\mu_{\max}^2\right]$.

Finally, Taking the expectation on both sides of (3) on $\boldsymbol{Q}(t)$, we can prove the *Lemma 1*.

APPENDIX B
PROOF OF LEMMA 2

According to the constraints that $0<\nu_{uh}(t)\leq 1$ when $x_{uh}(t)=1$ and $\nu_{uh}(t)=0$ if $x_{uh}(t)=0$, we know that the UT u will obtain the allocated bandwidth from the FAN h ($\nu_{uh}(t)>0$) if and only if it is associated with the FAN h ($x_{uh}=1$), or else it would be $\nu_{uh}(t)=0$.

Through proof by contradiction, we prove that under the optimal UT-FAN association $\mathbf{X}^{\star}(t)$ and bandwidth allocation $\boldsymbol{\nu}^{\star}(t)$, each FAN $h\in\mathcal{H}$ associates with only one UT in time slot t and allocates the whole bandwidth to the associated UT.

We first assume that under the optimal UT-FAN association $\mathbf{X}^{\star}(t)$, the FAN h associates with more than one UT, i.e., $x_{u_1h}^{\star}=1$, $x_{u_2h}^{\star}=1$, and $x_{uh}^{\star}=0$ where $u\in\mathcal{U}\backslash\{u_1,u_2\}$. Thus we have $\nu_{u_1h}^{\star}(t)>0$, $\nu_{u_2h}^{\star}(t)>0$, $\nu_{u_1h}^{\star}(t)+\nu_{u_2h}^{\star}(t)=1$, and $\nu_{uh}^{\star}(t)=0$ for $u\in\mathcal{U}\backslash\{u_1,u_2\}$.

We further assume that $W_{u_1h}(t)\geq W_{u_2h}(t)$. Let $x'_{u_1h}(t)=1$, $x'_{u_2h}(t)=0$ and $\nu'_{u_1h}(t)=\nu_{u_1h}^{\star}(t)+\nu_{u_2h}^{\star}(t)=1$, $\nu'_{u_2h}(t)=0$, then we have:

$$\begin{aligned} W_{u_1h}(t) &= W_{u_1h}(t)\nu'_{u_1h}(t)x'_{u_1h}(t) \\ &\geq W_{u_1h}(t)\nu_{u_1h}^{\star}(t)x_{u_1h}^{\star}(t)+W_{u_2h}(t)\nu_{u_2h}^{\star}(t)x_{u_1h}^{\star}(t), \end{aligned}$$

which shows that $x'_{u_1h}(t)=1$, $x'_{u_2h}(t)=0$, and $\nu'_{u_1h}(t)=1, \nu'_{u_2h}(t)=0$ would be better solution for $\mathcal{P}_{\text{AR}}$. In this optimal case, the FAN h associates with UT u with the maximum $W_{uh}(t)$ and allocates the whole bandwidth to it, which is contradictory to the prior assumption. Extend this to all the FANs $h\in\mathcal{H}$, in which case the optimal UT-FAN association can be obtained through solving the optimization problem $\mathcal{P}'_{\text{AR}}$, where the constraint $\sum_u x_{uh}(t)\leq M$ is reduced to $\sum_u x_{uh}(t)\leq 1$ for $h\in\mathcal{H}$. Then the optimal bandwidth allocation $\nu_{uh}(t)=1$ if $x_{uh}(t)$, otherwise, $\nu_{uh}(t)=0$.

APPENDIX C
PROOF OF LEMMA 3

Using the definitions in Appendix G, we see that $\mathcal{I}$ is the intersection of two partition matroids. Thus $\mathcal{P}'_{\text{AR}}$ can be reformulated as

$$\max_{\underline{\mathcal{E}}\in\mathcal{I}}\left\{\mathcal{W}_{\underline{\mathcal{E}}}\right\} \Longleftrightarrow \max_{\underline{\mathcal{E}}\in\mathcal{I}}\left\{\sum_{(u,h)\in\underline{\mathcal{E}}} W_{uh}(t)\right\}. \tag{4}$$

Invoking the definition about submodular function given in Appendix G, for all $\underline{\mathcal{E}}\subseteq\underline{\mathcal{E}}'\subseteq\mathcal{E}$, we can deduce that:

$$\mathcal{W}_{\underline{\mathcal{E}}\cup(u,h)}-\mathcal{W}_{\underline{\mathcal{E}}}=\mathcal{W}_{\underline{\mathcal{E}}'\cup(u,h)}-\mathcal{W}_{\underline{\mathcal{E}}'},\ \forall(u,h)\in\mathcal{E}\backslash\underline{\mathcal{E}}', \tag{5}$$

and $\mathcal{W}_{\emptyset}=0$, which proves the desired result that $\mathcal{P}'_{\text{AR}}$ can be formulated as a normalized modular function with two partition matroid constraints.

APPENDIX D
PROOF OF LEMMA 4

In the β-reduced problem, perform a change of variables $\boldsymbol{\mu}'(t)=\boldsymbol{\mu}(t)/\beta$. Using the fact that $\phi(\boldsymbol{\mu}(t))=\sum_{u\in\mathcal{U}}\mu_u(t)$, we have $\phi(\mu_u(t))=\beta\mu'_u(t)$. Thus we prove the result that the β-reduced problem becomes equivalent to the problem $\mathcal{P}1$. Hence, $\boldsymbol{\mu}^{*,\beta}(t)=\beta\boldsymbol{\mu}^{*,0}(t)$.

APPENDIX E
PROOF OF LEMMA 5

Because the FEMOS algorithm comes from the minimization of the upper bound of $\Delta_V(\boldsymbol{Q}(t))$, for any alternative (possibly randomized) imperfect scheduling policy $\mathbf{X}(t)\in\mathcal{A}$ and $\boldsymbol{\nu}(t)\in\mathcal{B}$, we have

$$\begin{aligned} \Delta_V(\boldsymbol{Q}(t)) \leq& \mathcal{K}-V\phi(\boldsymbol{\mu}^{*,\beta}(t)) \\ &+\sum_{u\in\mathcal{U}} Q_u^{\text{sum}}(t)\mathbb{E}\{A_u(t)-\mu_u^{*,\beta}(t)|\boldsymbol{Q}(t)\}, \end{aligned} \tag{6}$$

where $\boldsymbol{\mu}^{*,\beta}(t)=[\mu_1^{*,\beta}(t),\ldots,\mu_{|\mathcal{U}|}^{*,\beta}(t)]$ is from the imperfect scheduling policy.

Invoking *Lemma 4* and rearranging right-hand-side of (6), we have:

$$\begin{aligned} \Delta_V(\boldsymbol{Q}(t)) \leq& \mathcal{K}-V\phi(\boldsymbol{\mu}^{*,\beta}(t)) \\ &+\beta\sum_{u\in\mathcal{U}} Q_u^{\text{sum}}(t)\mathbb{E}\{A_u(t)-\mu_u^{*,0}(t)|\boldsymbol{Q}(t)\}. \end{aligned} \tag{7}$$

Plugging in the *slater-type conditions* into the right-hand-side of (7), we have

$$\Delta(\boldsymbol{Q}(t))-V\mathbb{E}\{\phi(\boldsymbol{\mu}(t))|\boldsymbol{Q}(t)\}\leq\mathcal{K}-V\phi_{\epsilon}-\beta\epsilon\sum_{u\in\mathcal{U}} Q_u^{\text{sum}}(t), \tag{8}$$

which proves the desired result.

Appendix F
Proof of Theorem 1

Taking the expectation of (8) and using the law of iterated expectations yields:

$$\begin{aligned}&\mathbb{E}\{L(\boldsymbol{Q}(\tau+1))\}-\mathbb{E}\{L(\boldsymbol{Q}(\tau))\}-V\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}\\&\leq \mathcal{K}-V\phi_{\epsilon}-\beta\epsilon\sum_{u\in\mathcal{U}}\mathbb{E}\{Q_u^{\text{sum}}(\tau)\}.\end{aligned}\tag{9}$$

Summing the above over $\tau\in\{0,1,\cdots,t-1\}$ for some slot $t>0$ and using the law of telescoping sums yields:

$$\begin{aligned}&\mathbb{E}\{L(\boldsymbol{Q}(t))\}-\mathbb{E}\{L(\boldsymbol{Q}(0))\}-V\sum_{\tau=0}^{t-1}\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}\\&\leq \mathcal{K}t-Vt\phi_{\epsilon}-\beta\epsilon\sum_{\tau=0}^{t-1}\sum_{u\in\mathcal{U}}\mathbb{E}\{Q_u^{\text{sum}}(\tau)\}.\end{aligned}\tag{10}$$

Note that $\epsilon>0$, then, dividing (10) by $t\epsilon$, rearranging terms, and using the fact $\mathbb{E}\{L(\boldsymbol{Q}(t))\}>0$ yields:

$$\begin{aligned}\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{u\in\mathcal{U}}\mathbb{E}\{Q_u^{\text{sum}}(\tau)\}\leq&\frac{\mathcal{K}+V\left[\overline{\phi(\boldsymbol{\mu}(t))}-\phi_{\epsilon}\right]}{\beta\epsilon}\\&+\frac{\mathbb{E}\{L(\boldsymbol{Q}(0))\}}{\beta\epsilon t},\end{aligned}\tag{11}$$

where $\overline{\phi(\boldsymbol{\mu}(t))}=\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}$. Note that $\overline{\phi(\boldsymbol{\mu}(t))}\leq\phi^{\text{opt}}$. By taking a lim sup and divide $\sum_u\lambda_u$ on both sides of (11) , we have for all t:

$$\frac{1}{\sum_u\lambda_u}\limsup_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{u}\mathbb{E}\{Q_u^{\text{sum}}(\tau)\}\leq\frac{\mathcal{K}+V(\phi^{\text{opt}}-\phi_{\epsilon})}{\beta\epsilon\sum_u\lambda_u},\tag{12}$$

thus prove the upper bound of $\Lambda_{av}^{\text{FEMOS}}$ in Theorem 1.

Now we consider the policy $\mathbf{X}(t)\in\mathcal{A}$ and $\boldsymbol{\nu}(t)\in\mathcal{B}$ which achieve the optimal solution $\phi_{\beta}^{\text{opt}}$ to the $\beta-$reduced problem $\mathcal{P}1$. Then we have

$$\mathbb{E}\{L(\boldsymbol{Q}(\tau+1))\}-\mathbb{E}\{L(\boldsymbol{Q}(\tau))\}-V\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}\leq\mathcal{K}-V\phi_{\beta}^{\text{opt}}.\tag{13}$$

Summing the above over $\tau\in\{0,1,\cdots,t-1\}$ for some slot $t>0$ and using the law of telescoping sums yields:

$$\mathbb{E}\{L(\boldsymbol{Q}(t))\}-\mathbb{E}\{L(\boldsymbol{Q}(0))\}-V\sum_{\tau=0}^{t-1}\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}\leq\mathcal{K}t-V\phi_{\beta}^{\text{opt}}t.\tag{14}$$

Dividing by tV, and rearranging terms, we have:

$$\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}\geq\phi_{\beta}^{\text{opt}}-\frac{\mathcal{K}}{V}-\frac{\mathbb{E}\{L(\boldsymbol{Q}(0))\}}{Vt}.\tag{15}$$

Taking the lim inf as $t\to\infty$ of both side of (15) proves the lower bound of ϕ_{av}^{FEMOS} in *Theorem 1*.

Appendix G
Basic Definitions

Submodular functions: Let $\mathcal{E}$ be a finite ground set. We define its power set (the set containing all the subsets of $\mathcal{E}$) as $2^{\mathcal{E}}$. Then a real-valued function f defined on the subsets of $\mathcal{E}$, $f:2^{\mathcal{E}}\to\mathbb{R}$ is called a ***submodular*** function if and only if

$$f(\underline{\mathcal{E}}\cup a)-f(\underline{\mathcal{E}})\geq f(\underline{\mathcal{E}'}\cup a)-f(\underline{\mathcal{E}'}),\forall\underline{\mathcal{E}}\subseteq\underline{\mathcal{E}'}\subseteq\mathcal{E}\&a\in\mathcal{E}\backslash\underline{\mathcal{E}'}.\tag{16}$$

A real-valued function $f:2^{\mathcal{E}}\to\mathbb{R}$ is called a modular function if and only if the equality holds in (16). The expression $f:2^{\mathcal{E}}\to\mathbb{R}$ is also called normalized function if $f(\emptyset)=0$, where $\emptyset$ denotes an empty set.

Matroids: A *matroid* $\mathcal{S}$ is a tuple $\mathcal{S}=(\mathcal{E},\mathcal{I})$, $\mathcal{I}\subseteq2^{\mathcal{E}}$ such that:

1) $\mathcal{I}$ is nonempty, in particular, $\emptyset\in\mathcal{I}$.
2) $\mathcal{I}$ is downward closed: i.e., if $Y\in\mathcal{I}$ and $X\subseteq Y$, then $X\in\mathcal{I}$.
3) If $X,Y\in\mathcal{I}$, and $|X|<|Y|$, then $\exists y\in Y\backslash X$ such that $X\cup\{y\}\in\mathcal{I}$.

Partition matroid: The matroid $\mathcal{S}=(\mathcal{E},\underline{\mathcal{I}})$ is also a *Partition matroid* when the ground set $\mathcal{E}$ is partitioned into (disjoint) sets $\mathcal{E}_1,\mathcal{E}_2,\ldots,\mathcal{E}_l$ and $\mathcal{I}=\{\mathbf{X}\subseteq\mathcal{E}:|\mathbf{X}\cap\mathcal{E}_i|\leq k_i$ for all $i=1,\ldots,l$, for some given parameters $k_1,k_2,\ldots,k_l$.