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## FEMOS

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# APPENDIX

## APPENDIX A PROOF OF LEMMA 1

Note that for any  $Q \geq 0, \mu \geq 0, A \geq 0$ , we have:

$$([Q - \mu]^+ + A)^2 \leq Q^2 + A^2 + \mu^2 + 2Q(A - \mu). \quad (1)$$

Plugging the inequations (1) into the Lyapunov drift function, we have

$$\begin{aligned} L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) &\leq \frac{1}{2} \sum_{u \in \mathcal{U}} A_u^2(t) + \frac{1}{2} \sum_{u \in \mathcal{U}} \mu_u^2(t) \\ &\quad + \sum_{u \in \mathcal{U}} Q_u^{\text{sum}}(t)[A_u(t) - \mu_u(t)]. \end{aligned} \quad (2)$$

By adding the penalty term  $-V\phi(\boldsymbol{\mu}(t))$  on both sides of (2) and rearranging the terms, we have:

$$\begin{aligned} L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) - V\phi(\boldsymbol{\mu}(t)) \\ \leq \mathcal{K} - \sum_{u \in \mathcal{U}} [V + Q_u^{\text{sum}}(t)]\mu_u(t) + \sum_{u \in \mathcal{U}} Q_u^{\text{sum}}(t)A_u(t), \end{aligned} \quad (3)$$

where  $\mathcal{K} = \frac{|\mathcal{U}|}{2} [A_{\max}^2 + \mu_{\max}^2]$ .

Finally, Taking the expectation on both sides of (3) on  $\mathbf{Q}(t)$ , we can prove the *Lemma 1*.

## APPENDIX B PROOF OF LEMMA 2

According to the constraints that  $0 < v_{uh}(t) \leq 1$  when  $x_{uh}(t) = 1$  and  $v_{uh}(t) = 0$  if  $x_{uh}(t) = 0$ , we know that the UT  $u$  will obtain the allocated bandwidth from the FAN  $h$  ( $v_{uh}(t) > 0$ ) if and only if it is associated with the FAN  $h$  ( $x_{uh} = 1$ ), or else it would be  $v_{uh}(t) = 0$ .

Through proof by contradiction, we prove that under the optimal UT-FAN association  $\mathbf{X}^*(t)$  and bandwidth allocation  $\mathbf{v}^*(t)$ , each FAN  $h \in \mathcal{H}$  associates with only one UT in time slot  $t$  and allocates the whole bandwidth to the associated UT.

We first assume that under the optimal UT-FAN association  $\mathbf{X}^*(t)$ , the FAN  $h$  associates with more than one UT, i.e.,  $x_{u_1h}^* = 1, x_{u_2h}^* = 1$ , and  $x_{uh}^* = 0$  where  $u \in \mathcal{U} \setminus \{u_1, u_2\}$ . Thus we have  $v_{u_1h}^*(t) > 0, v_{u_2h}^*(t) > 0, v_{u_1h}^*(t) + v_{u_2h}^*(t) = 1$ , and  $v_{uh}^*(t) = 0$  for  $u \in \mathcal{U} \setminus \{u_1, u_2\}$ .

We further assume that  $W_{u_1h}(t) \geq W_{u_2h}(t)$ . Let  $x'_{u_1h}(t) = 1, x'_{u_2h}(t) = 0$  and  $v'_{u_1h}(t) = v_{u_1h}^*(t) + v_{u_2h}^*(t) = 1, v'_{u_2h}(t) = 0$ , then we have:

$$\begin{aligned} W_{u_1h}(t) &= W_{u_1h}(t)v'_{u_1h}(t)x'_{u_1h}(t) \\ &\geq W_{u_1h}(t)v_{u_1h}^*(t)x_{u_1h}^*(t) + W_{u_2h}(t)v_{u_2h}^*(t)x_{u_1h}^*(t), \end{aligned}$$

which shows that  $x'_{u_1h}(t) = 1, x'_{u_2h}(t) = 0$ , and  $v'_{u_1h}(t) = 1, v'_{u_2h}(t) = 0$  would be better solution for  $\mathcal{P}_{\text{AR}}$ . In this optimal case, the FAN  $h$  associates with UT  $u$  with the maximum  $W_{uh}(t)$  and allocates the whole bandwidth to it, which is contradictory to the prior assumption. Extend this to all the FANs  $h \in \mathcal{H}$ , in which case the optimal UT-FAN association can be obtained through solving the optimization

problem  $\mathcal{P}'_{\text{AR}}$ , where the constraint  $\sum_u x_{uh}(t) \leq M$  is reduced to  $\sum_u x_{uh}(t) \leq 1$  for  $h \in \mathcal{H}$ . Then the optimal bandwidth allocation  $v_{uh}(t) = 1$  if  $x_{uh}(t)$ , otherwise,  $v_{uh}(t) = 0$ .

## APPENDIX C PROOF OF LEMMA 3

Using the definitions in Appendix G, we see that  $\mathcal{I}$  is the intersection of two partition matroids. Thus  $\mathcal{P}'_{\text{AR}}$  can be reformulated as

$$\max_{\underline{\mathcal{E}} \in \mathcal{I}} \{ \mathcal{W}_{\underline{\mathcal{E}}} \} \iff \max_{\underline{\mathcal{E}} \in \mathcal{I}} \left\{ \sum_{(u,h) \in \underline{\mathcal{E}}} W_{uh}(t) \right\}. \quad (4)$$

Invoking the definition about submodular function given in Appendix G, for all  $\underline{\mathcal{E}} \subseteq \underline{\mathcal{E}}' \subseteq \mathcal{E}$ , we can deduce that:

$$\mathcal{W}_{\underline{\mathcal{E}} \cup (u,h)} - \mathcal{W}_{\underline{\mathcal{E}}} = \mathcal{W}_{\underline{\mathcal{E}}' \cup (u,h)} - \mathcal{W}_{\underline{\mathcal{E}}'}, \quad \forall (u,h) \in \mathcal{E} \setminus \underline{\mathcal{E}}', \quad (5)$$

and  $\mathcal{W}_{\emptyset} = 0$ , which proves the desired result that  $\mathcal{P}'_{\text{AR}}$  can be formulated as a normalized modular function with two partition matroid constraints.

## APPENDIX D PROOF OF LEMMA 4

In the  $\beta$ -reduced problem, perform a change of variables  $\boldsymbol{\mu}'(t) = \boldsymbol{\mu}(t)/\beta$ . Using the fact that  $\phi(\boldsymbol{\mu}(t)) = \sum_{u \in \mathcal{U}} \mu_u(t)$ , we have  $\phi(\mu_u(t)) = \beta \mu'_u(t)$ . Thus we prove the result that the  $\beta$ -reduced problem becomes equivalent to the problem  $\mathcal{P}1$ . Hence,  $\boldsymbol{\mu}^{*\beta}(t) = \beta \boldsymbol{\mu}^{*0}(t)$ .

## APPENDIX E PROOF OF LEMMA 5

Because the FEMOS algorithm comes from the minimization of the upper bound of  $\Delta_V(\mathbf{Q}(t))$ , for any alternative (possibly randomized) imperfect scheduling policy  $\mathbf{X}(t) \in \mathcal{A}$  and  $\mathbf{v}(t) \in \mathcal{B}$ , we have

$$\begin{aligned} \Delta_V(\mathbf{Q}(t)) &\leq \mathcal{K} - V\phi(\boldsymbol{\mu}^{*\beta}(t)) \\ &\quad + \sum_{u \in \mathcal{U}} Q_u^{\text{sum}}(t) \mathbb{E}\{A_u(t) - \mu_u^{*\beta}(t) | \mathbf{Q}(t)\}, \end{aligned} \quad (6)$$

where  $\boldsymbol{\mu}^{*\beta}(t) = [\mu_1^{*\beta}(t), \dots, \mu_{|\mathcal{U}|}^{*\beta}(t)]$  is from the imperfect scheduling policy.

Invoking *Lemma 4* and rearranging right-hand-side of (6), we have:

$$\begin{aligned} \Delta_V(\mathbf{Q}(t)) &\leq \mathcal{K} - V\phi(\boldsymbol{\mu}^{*\beta}(t)) \\ &\quad + \beta \sum_{u \in \mathcal{U}} Q_u^{\text{sum}}(t) \mathbb{E}\{A_u(t) - \mu_u^{*0}(t) | \mathbf{Q}(t)\}. \end{aligned} \quad (7)$$

Plugging in the *slater-type conditions* into the right-hand-side of (7), we have

$$\Delta(\mathbf{Q}(t)) - V\mathbb{E}\{\phi(\boldsymbol{\mu}(t)) | \mathbf{Q}(t)\} \leq \mathcal{K} - V\phi_{\epsilon} - \beta\epsilon \sum_{u \in \mathcal{U}} Q_u^{\text{sum}}(t), \quad (8)$$

which proves the desired result.

APPENDIX F  
PROOF OF THEOREM 1

Taking the expectation of (8) and using the law of iterated expectations yields:

$$\begin{aligned} & \mathbb{E}\{L(\mathbf{Q}(\tau+1))\} - \mathbb{E}\{L(\mathbf{Q}(\tau))\} - V\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\} \\ & \leq \mathcal{K} - V\phi_\epsilon - \beta\epsilon \sum_{u \in \mathcal{U}} \mathbb{E}\{Q_u^{\text{sum}}(\tau)\}. \end{aligned} \quad (9)$$

Summing the above over  $\tau \in \{0, 1, \dots, t-1\}$  for some slot  $t > 0$  and using the law of telescoping sums yields:

$$\begin{aligned} & \mathbb{E}\{L(\mathbf{Q}(t))\} - \mathbb{E}\{L(\mathbf{Q}(0))\} - V \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\} \\ & \leq \mathcal{K}t - Vt\phi_\epsilon - \beta\epsilon \sum_{\tau=0}^{t-1} \sum_{u \in \mathcal{U}} \mathbb{E}\{Q_u^{\text{sum}}(\tau)\}. \end{aligned} \quad (10)$$

Note that  $\epsilon > 0$ , then, dividing (10) by  $t\epsilon$ , rearranging terms, and using the fact  $\mathbb{E}\{L(\mathbf{Q}(t))\} > 0$  yields:

$$\begin{aligned} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{u \in \mathcal{U}} \mathbb{E}\{Q_u^{\text{sum}}(\tau)\} & \leq \frac{\mathcal{K} + V[\overline{\phi(\boldsymbol{\mu}(t))} - \phi_\epsilon]}{\beta\epsilon} \\ & \quad + \frac{\mathbb{E}\{L(\mathbf{Q}(0))\}}{\beta\epsilon t}, \end{aligned} \quad (11)$$

where  $\overline{\phi(\boldsymbol{\mu}(t))} = \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\}$ . Note that  $\overline{\phi(\boldsymbol{\mu}(t))} \leq \phi^{\text{opt}}$ . By taking a lim sup and divide  $\sum_u \lambda_u$  on both sides of (11), we have for all  $t$ :

$$\frac{1}{\sum_u \lambda_u} \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_u \mathbb{E}\{Q_u^{\text{sum}}(\tau)\} \leq \frac{\mathcal{K} + V(\phi^{\text{opt}} - \phi_\epsilon)}{\beta\epsilon \sum_u \lambda_u}, \quad (12)$$

thus prove the upper bound of  $\Lambda_{av}^{\text{FEMOS}}$  in Theorem 1.

Now we consider the policy  $\mathbf{X}(t) \in \mathcal{A}$  and  $\mathbf{v}(t) \in \mathcal{B}$  which achieve the optimal solution  $\phi_\beta^{\text{opt}}$  to the  $\beta$ -reduced problem  $\mathcal{P}1$ . Then we have

$$\mathbb{E}\{L(\mathbf{Q}(\tau+1))\} - \mathbb{E}\{L(\mathbf{Q}(\tau))\} - V\mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\} \leq \mathcal{K} - V\phi_\beta^{\text{opt}}. \quad (13)$$

Summing the above over  $\tau \in \{0, 1, \dots, t-1\}$  for some slot  $t > 0$  and using the law of telescoping sums yields:

$$\mathbb{E}\{L(\mathbf{Q}(t))\} - \mathbb{E}\{L(\mathbf{Q}(0))\} - V \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\} \leq \mathcal{K}t - V\phi_\beta^{\text{opt}}t. \quad (14)$$

Dividing by  $tV$ , and rearranging terms, we have:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(\boldsymbol{\mu}(\tau))\} \geq \phi_\beta^{\text{opt}} - \frac{\mathcal{K}}{V} - \frac{\mathbb{E}\{L(\mathbf{Q}(0))\}}{Vt}. \quad (15)$$

Taking the lim inf as  $t \rightarrow \infty$  of both side of (15) proves the lower bound of  $\phi_{av}^{\text{FEMOS}}$  in Theorem 1.

APPENDIX G  
BASIC DEFINITIONS

**Submodular functions:** Let  $\mathcal{E}$  be a finite ground set. We define its power set (the set containing all the subsets of  $\mathcal{E}$ ) as  $2^\mathcal{E}$ . Then a real-valued function  $f$  defined on the subsets of  $\mathcal{E}$ ,  $f : 2^\mathcal{E} \rightarrow \mathbb{R}$  is called a **submodular** function if and only if  $f(\underline{\mathcal{E}} \cup a) - f(\underline{\mathcal{E}}) \geq f(\underline{\mathcal{E}}' \cup a) - f(\underline{\mathcal{E}}')$ ,  $\forall \underline{\mathcal{E}} \subseteq \underline{\mathcal{E}}' \subseteq \mathcal{E} \& a \in \mathcal{E} \setminus \underline{\mathcal{E}}'$ . (16)

A real-valued function  $f : 2^\mathcal{E} \rightarrow \mathbb{R}$  is called a modular function if and only if the equality holds in (16). The expression  $f : 2^\mathcal{E} \rightarrow \mathbb{R}$  is also called normalized function if  $f(\emptyset) = 0$ , where  $\emptyset$  denotes an empty set.

**Matroids:** A *matroid*  $\mathcal{S}$  is a tuple  $\mathcal{S} = (\mathcal{E}, \mathcal{I})$ ,  $\mathcal{I} \subseteq 2^\mathcal{E}$  such that:

- 1)  $\mathcal{I}$  is nonempty, in particular,  $\emptyset \in \mathcal{I}$ .
- 2)  $\mathcal{I}$  is downward closed: i.e., if  $Y \in \mathcal{I}$  and  $X \subseteq Y$ , then  $X \in \mathcal{I}$ .
- 3) If  $X, Y \in \mathcal{I}$ , and  $|X| < |Y|$ , then  $\exists y \in Y \setminus X$  such that  $X \cup \{y\} \in \mathcal{I}$ .

**Partition matroid:** The matroid  $\mathcal{S} = (\mathcal{E}, \mathcal{I})$  is also a *Partition matroid* when the ground set  $\mathcal{E}$  is partitioned into (disjoint) sets  $\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_l$  and  $\mathcal{I} = \{\mathbf{X} \subseteq \mathcal{E} : |\mathbf{X} \cap \mathcal{E}_i| \leq k_i \text{ for all } i = 1, \dots, l, \text{ for some given parameters } k_1, k_2, \dots, k_l\}$ .