Κριτήριο αστοχίας πλακοδοκών σε τέμνουσα

Shear failure criterion for RC T-beams

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Λέξεις κλειδιά: Θεωρεία της τροχιάς της θλιπτικής δύναμης, σχεδιασμός, κριτήριο αστοχίας, πλακοδοκοί

Keywords: compressive force path theory; design; failure criteria; reinforced concrete; T-beams

ΠΕΡΙΛΗΨΗ: Αντικείμενο της παρούσας εργασίας είναι η ανάπτυξη ενός κριτηρίου αστοχίας ικανού να λαμβάνει υπόψη τη συμβολή της πλάκας στην αντοχή σε τέμνουσα πλακοδοκών. Το κριτήριο αυτό αναπτύχθηκε εντός του πλαισίου της μεθόδου της τροχιάς της θλιπτικής δύναμης η οποία έχει βρεθεί να βελτιώνει σημαντικά τις προβλέψεις φέρουσας ικανότητας και τιςλύσεις σχεδιασμού κατασκευών οπλισμένου σκυροδέματος, σε σχέση με αυτές που προκύπτουν από τις ισχύουσες κανονιστικές διατάξεις, ικανοποιώντας ταυτόχρονα τις απαιτήσεις δομοστατικής συμπεριφοράς, κυρίως αυτές για πλαστιμότητα και αντοχή. Η εγκυρότητα του προτεινόμενου κριτηρίου επιβεβαιώθηκε μέσω της σύγκρισης των τιμών υπολογισμού με αντίστοιχες πειραματικές και διαπιστώθηκε ότι οι προβλέψεις του υπερέχουν αυτών που προκύπτουν από τους ισχύοντες κανονισμούς.

ABSTRACT: The paper is concerned with the development of a failure criterion capable of accurately predicting the shear capacity of reinforced concrete T-beams while correctly accounting for the beneficial effect of the increase of the compressive zone due to the presence of flanges. The development of the subject criterion is based on an alternative design method (the compressive force path method) that leads to predictions of reinforced concrete structural behaviour and design solutions considerably different compared to those of the current design codes without however compromising structural performance requirements (mainly associated with ductility and strength). The validity of the proposed failure criterion is verified through a comparative study of the calculated values with their experimentally-established counterparts obtained from an extensive literature survey. Through this comparative study it is demonstrated that the predictions of the proposed criterion provide a closer fit to the available

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experimental data than their counterparts obtained from the design codes considered.

INTRODUCTION

Experimental information, which is used in the work presented herein, shows that reinforced concrete (RC) beams with a T-shaped cross section exhibit values of shear capacity which are higher, often by a significant amount, than those characterising RC beams with a rectangular cross section (Tharmin *et al* 2016). Such behaviour can be attributed to the increase of the beams' compressive zone due to the presence of the flange, the effect of which on shear capacity is not allowed for in current design practice. This is because, in accordance with the simplified beam theory which underlies shear design methods, any increase in shear capacity due to the increase of the width of the compressive zone is, to a large extent, counteracted by the decrease of the shear stresses within the flange as they spread along the flange width (see Fig. 1).

Figure 1: Shear stress distributions for (a) rectangular and (b) T sections

Therefore, there is an inherent difficulty in allowing for the flange's effect on shear capacity without a modification of the concepts underlying shear design methods. And yet, allowing for this effect may lead to a reduction of the amount of transverse reinforcement required to safeguard against shear failure. This may be true, not only when concrete in the presence of transverse reinforcement is considered to contribute to shear capacity (ACI 318 2014), but also when the concrete's contribution is ignored (EC2 2004). In the latter case, if the flange's effect on shear capacity were allowed for, the code criterion for specifying reinforcement may not be fulfilled and, therefore, a nominal amount of transverse reinforcement may be sufficient.

In view of the above, the aim of the present work is the development of a failure criterion which allows for the effect of the compressive zone's size on shear capacity. The work will be based on concepts which underlie the Compressive Force Path (CFP) theory(Kotsovos 2014), since this is the only theory proposed to date which links the causes of an RC beam's failure to the stress conditions in the compressive, rather than the tensile zone. The validity of the proposed criterion will be verified through a comparison of its predictions with experimental data obtained from the literature. The comparison will also include the values predicted by current codes (ACI 318 2014, EC2 2004), as well as an empirical formula, which has been developed so as to allow for the effect of the compressive zone's shape and size on shear capacity and included in the guidance for "design and detailing of concrete structures for fire resistance" of The Institution of Structural Engineers (London), 1978.

BACKGROUND

Shear resistance and force transfer

In accordance with the CFP theory (Kotsovos 2014), the shear resistance of the tensile zone of RC beams becomes negligible, if any, after the formation of flexural and/or inclined cracks. This is because cracking diminishes the shear stiffness of concrete and causes stress redistribution towards the stiff crack-free concrete of the compressive zone; thus, the latter becomes the sole contributor to the beam's shear resistance. Moreover, it has been demonstrated (Kotsovos 2014) that such behaviour is compatible with the experimentally-established behaviour of concrete as a material as regards both its stress-strain behaviour and its cracking mechanism, the latter involving crack extension in the direction of cracking and opening in the orthogonal direction, thus precluding any shear movement of the crack faces that may be resisted by aggregate interlock and dowel action as widely assumed.

In view of the above, internal force transfer is accomplished by the compressive zone through a beam-like action mechanism schematically described in Fig. 2. From the figure, it can be seen that, under the action of the bond force,ΔT, developing at the interface between concrete and flexural steel, concrete cantilevers (such as that indicated in Fig. 2(a)) which form between successive flexural or inclined cracks, subject the compressive zone to a moment ΔM (see Fig. 2(b)) which transfers the shear force V acting on the right-hand side of the portion to which the cantilever is fixed to the left-hand side (see Fig.

2(c)).Moreover, it has been shown that the presence of triaxial compressive-stress conditions developing for purposes of transverse deformation compatibility enables the compressive zone to sustain alone the total action of V(Kotsovos 2014), commonly assumed to be sustained by the beam cross section.

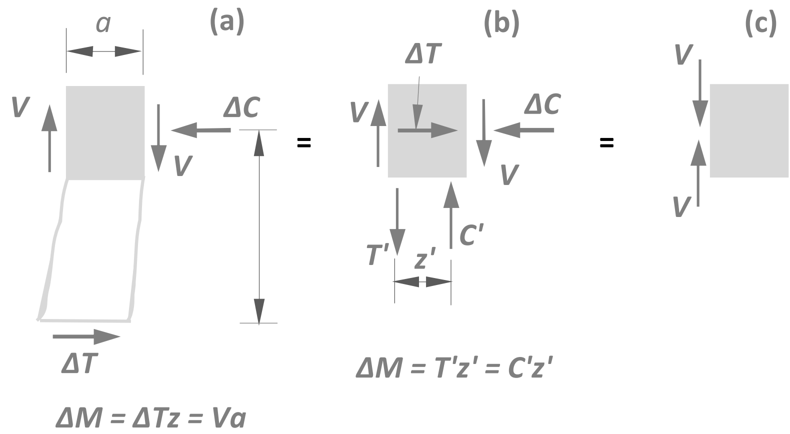

Figure 2: Mechanisms of load transfer through cantilever action

Shear failure and underlying causes

In accordance with the experimental findings of Kani, 1964, RC beams, without transverse reinforcement, with shear span-to depth ratios (a_v/d) ranging between 1 and a value (dependent on the reinforcement ratio ρ) of the order of 5, exhibit load-carrying capacities smaller than that corresponding to flexural capacity (see Fig. 3). The causes of such 'premature' loss of load-carrying capacity are attributed by the CFP theory to longitudinal spitting of the compressive zone which may occur when the transverse component of the compressive stress field referred to at the bottom of the preceding section becomes tensile (Kotsovos 2014). Therefore, transverse reinforcement may be required for preventing such splitting, rather than for improving the shear capacity of the beam's cross section as widely considered.

More specifically, for $1 < a_v/d \leq 2.5$, the deep inclined crack which forms within the shear span penetrates deeply into the compressive zone causing a type of failure which is commonly referred to as shear-compression failure (Kong and Evans 1987). However, it has been shown that this is essentially a flexural type of failure which is brittle in nature in that concrete in the compressive zone fails before yielding of the flexural reinforcement (Kotsovos 2014). A similar type of failure may be suffered by RC beams with $a_v/d > 2.5$ in the region of load points where the large bending moment combines with a large shear force. This is one of the two types of failure characterising beams with $a_v/d > 2.5$ commonly referred to as diagonal-tension failures (Kong and Evans 1987); it takes the form of longitudinal splitting of the compressive zone which gives the impression that it forms extension of a major inclined crack the formation of which invariably precedes failure of this type of beams (Kotsovos 2014). However, the most usual type of failure suffered by such beams is horizontal splitting of the compressive zone occurring at a distance of around 2.5*d* from the nearest support (Kotsovos 2014). As shown in Fig. 4, such splitting occurs independently of any other form of cracking in the region of the compressive zone between the tip of the deepest inclined crack and the extreme compressive fibre of the beam; it is referred to as type II failure at location 1 (location at a distance of 2.5*d* from nearest support), in short type II,1 failure (Kotsovos 2014).

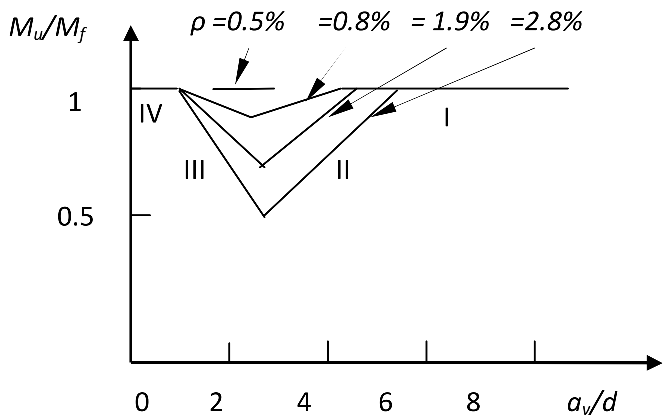

Figure 3: Relation between bending moment corresponding to load-carrying capacity and shear span for various percentages of longitudinal reinforcement with regions I and IV indicating ductile and II and III brittle types of failure.

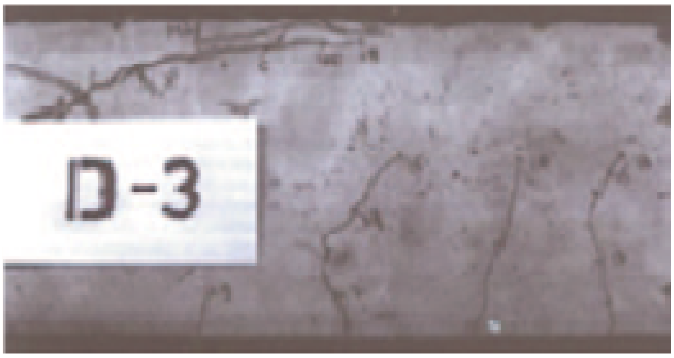

Figure 4: Horizontal splitting of the compressive zone occurring at a distance of around $2.5d$ from nearest support between tip of inclined crack and extreme compressive fibre

Proposed flange contribution to 'shear' capacity

For the flexural types of failure referred to in the preceding section (which are premature in that failure of the compressive zone occurs before yielding of the flexural reinforcement), the effect of the presence of the flange on flexural capacity is allowed for by any of the methods developed to date for calculating flexural capacity. In contrast with these methods, the presence of the flange is ignored when calculating the tensile force $T_{II,1}$ which, in accordance with the CFP theory is sustained by concrete in the region of the tip of the deep inclined crack just before horizontal splitting of the compressive zone. As discussed in what follows, $T_{II,1}$ is obtained from(Kotsovos 2014)

$$T_{II,1} = 0.5 f_t b d \qquad (1)$$

Where b and d are the width and effective depth of the web and f_t the strength of concrete in direct tension.

$T_{II,1}$ (which is numerically equal to the shear force $V_{II,1}$at the cross section at a distance of $2.5d$ from the nearest support (Kotsovos 2014)) is essentially the resultant of the tensile stresses which develop for local equilibrium purposes in the region of the compressive zone where the horizontal flow of the compressive stresses developing on account of bending takes an inclined direction towards the support (see Fig. 5). By invoking the Saint Vennant's principle (Stavridis, 2010), the rapidly diminishing tensile stresses with a peak value of f_t at a distance of $2.5d$ from the nearest support have been replaced with a uniform stress distribution with intensity $0.25f_t$ extending to a distance of d on either side of the location of the peak stress value f_t throughout the cross section's width b (see Fig. 6).Equation

(1) expresses the product of the uniform stress $0.25f_t$ with the area b ($2d$) over which the uniform stress develops, i.e. $T_{II,1} = (0.25\,f_t)\ b\ (2d) = 0.5\,f_t b\ d$.

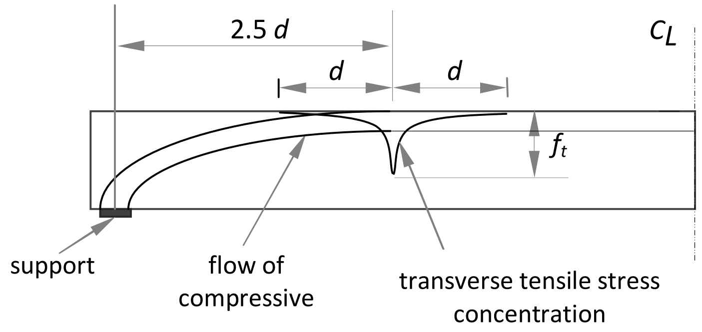

Figure 5: Tensile stress concentration developing in RC beams for local equilibrium purposes at location with a distance of 2.5d from the nearest support

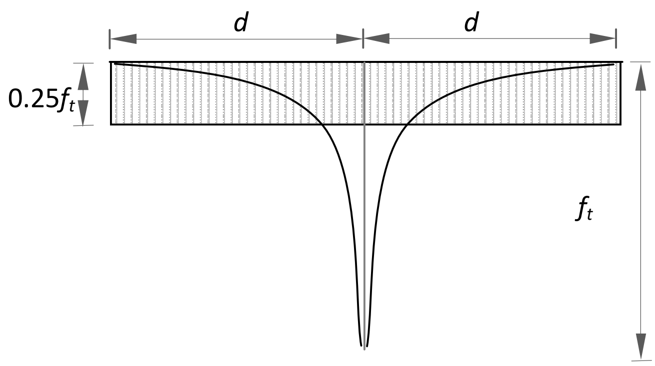

Figure 6:Rapidly diminishing transverse tensile stresses on either side of the location of the peak value of f_t to zero at a distance of d and equivalent uniform stress block with intensity $0.25f_t$

For T-beams, equation (1) may be modified so as to allow for the presence of the flange by following a similar reasoning. By invoking Saint Vennant's principle, the tensile stresses developing in the compressive zone due to change in the direction of the flow of the compressive stresses will be assumed to diminish in the manner indicated in Fig. 6not only in the longitudinal, but also in the transverse direction across the cross-section's width within the flange to a distance of h_f on either side of the web, where h_f is the depth of the flange at its intersection with the web. By adopting a uniformly distributed tensile stress of $0.25f_t$ in the longitudinal direction underlying the derivation of equation (1), the 'true' distribution of the tensile stresses in the flange is replaced with a unifirm stress distribution with intensity 0.25 (0.25 f_t), where 0.25 f_t is (in accordance with equation (1)) the intensity of the uniform stress distribution in the web (see Fig. 7). Then, the additional tensile force sustained by the flange over a length $2d$ (where d is the effective depth of the beam) will be:

$$T_{II,1,f} = 2(0.25)(0.25f_t)h_f(2d) = 0.5^2 f_t h_f d \qquad (2)$$

Therefore, the total force sustained by the T-beam becomes

$$T_{II,1,T} = 0.5f_t b_w d + 0.5^2 f_t h_f d = 0.5f_t\left(b_w + 0.5h_f\right)d \qquad (3)$$

with $T_{II,IT}$ being numerically equal to the shear force $V_{II,IT}$ developing at the same location.

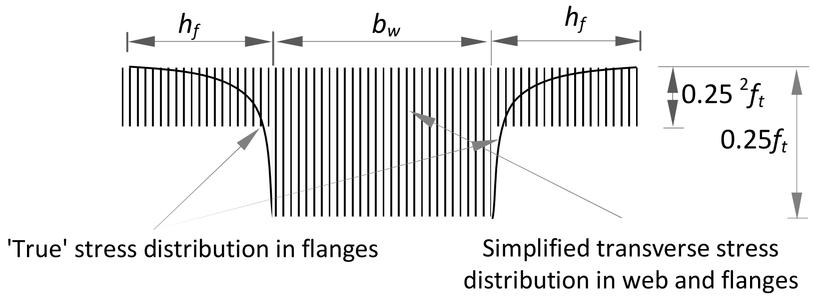

Figure 7:Proposed stress blocks in web and flanges across a t-beam's width

VERIFICATION

Information used

The verification of the ability of equation (3) to produce realistic predictions of the shear force corresponding to failure at the location of T-beams (without transverse reinforcement) at a distance of $2.5d$ from the nearest support has been based on comparing the equation's predictions with the values of shear capacity established from tests on T-beams reported in the literature. The design details of the test specimens are shown in Table 1, which also provides the experimentally-established values of shear capacity.

Table 1: Design characteristics of RC T-beam specimens without shear reinforcement

No	Specimens	Materials & Geometry										V_{EXP}
		f_c	b_w	h_f	b_f	d	ρ	f_y	ρ'	f_y'	a_v/d	
		(MPa)	(mm)	(mm)	(mm)	(mm)	(%)	(MPa)	(%)	(Mpa)	-	(kN)
Bousselham A. &Chaallal O.												
1	SB-S0-0L	25.0	152	102	508	350	3.76	650	1.13	650	3.0	81.3
Ferguson P.M.& Thompson J.N.												
2	A1	29.7	102	38	432	210	4.78	276	-	-	3.4	29.1
3	A2	27.3	102	38	432	210	4.78	276	-	-	3.4	27.0
4	A3	35.1	102	38	432	210	4.78	276	-	-	3.4	33.6
5	A4	34.9	102	38	432	210	4.78	276	-	-	3.4	31.6
6	A5	45.4	102	38	432	210	4.78	276	-	-	3.4	33.9
7	A6	38.7	102	38	432	210	4.78	276	-	-	3.4	35.6
8	D1	31.3	178	38	432	210	2.73	276	-	-	3.4	48.7
9	D2	29.6	178	38	432	210	2.73	276	-	-	3.4	52.1
10	N1	20.7	108	38	483	178	2.97	276	-	-	4.0	23.8
11	N2	20.6	108	38	483	178	2.97	276	-	-	4.0	23.9
12	N3	17.5	108	38	483	178	2.97	276	-	-	4.0	21.5
Kotsovos et al												
13	III	40.4	50	50	200	240	5.23	540	-	-	3.3	37.0
Panda et al												
14	S0-0L	46.2	100	60	250	225	2.79	500	0.89	503	3.3	50.0
Placas& Regan												
15	T2	28.1	152	76	610	254	1.46	621	-	-	3.4	54.7
16	T18	28.4	152	76	610	254	4.16	621	-	-	3.6	74.7
Sahoo et al												
17	TB0.00_2.5	23.2	150	50	300	217	1.85	500	0.96	500	2.5	43.5
18	TB0.00_3.0	23.2	150	50	300	217	1.85	500	0.96	500	3.0	40.0
Thamrin et al												
19	T-01E	32.0	125	70	250	219	0.97	550	-	-	3.7	36.6
20	T-02E	32.0	125	70	250	219	1.45	550	-	-	3.7	38.5
21	T-03E	32.0	125	70	250	212	2.50	550	-	-	3.8	47.5
22	R-01E	32.0	125	0	0	219	0.97	550	-	-	3.7	32.6
23	R-02E	32.0	125	0	0	219	1.45	550	-	-	3.7	37.0
24	R-03E	32.0	125	0	0	212	2.50	550	-	-	3.8	37.6
Wehr K. E.												
25	SS-I	27.7	152	76	914	279	1.33	1118	-	-	3.88	44.5
26	SS-II	31.4	152	76	762	279	1.33	1118	-	-	3.88	50.9
27	SS-III	29.6	152	76	610	279	1.33	1118	-	-	3.88	44.5
28	SS-IV	24.0	152	0	152	279	1.33	1118	-	-	3.88	48.7

The values of shear capacity predicted by the proposed formula are shown in Table 2. For comparison purposes, the table also includes the values predicted by the formulae adopted not only by ACI 318 (2014)and EC2 (2004), which do not allow for the contribution of flanges on shear capacity, but also by the formula proposed by the Institution of Structural Engineers (London), 1978, which allows for the contribution of the flange. All values of the calculated shear capacity are presented in a normalized form by dividing them with their experimentally-established counterparts.

Table 2: Calculated and experimentally-established values of shear capacity of RC T-beams

No	Specimen	V_{EXP}	V_{EC2}	V_{EC2}/V_{EXP}	V_{ACI}	V_{ACI}/V_{EXP}	$V_{IStructE}$	$V_{IStructE}/V_{EXP}$	$V_{proposed}$	$V_{proposed}/V_{EXP}$
		(kN)	**(kN)**	-	**(kN)**	-	**(kN)**	-	**(kN)**	-
1	SB-S0-0L	81.3	51.0	0.628	45.2	0.556	159.3	1.960	70.8	0.871
2	A1	29.1	26.4	0.907	19.7	0.677	47.3	1.627	29.7	1.021
3	A2	27.0	25.6	0.948	18.9	0.700	46.7	1.731	27.4	1.015
4	A3	33.6	27.9	0.830	21.4	0.637	48.4	1.440	34.4	1.024
5	A4	31.6	27.8	0.880	21.4	0.677	48.4	1.531	34.2	1.082
6	A5	33.9	30.4	0.897	24.4	0.720	49.7	1.467	42.7	1.260
7	A6	35.6	28.8	0.809	22.5	0.632	48.9	1.375	37.4	1.051
8	D1	48.7	38.9	0.799	35.4	0.727	60.7	1.246	50.8	1.043
9	D2	52.1	38.2	0.733	34.5	0.662	60.2	1.156	48.3	0.927
10	N1	23.8	18.2	0.765	14.9	0.626	32.2	1.355	23.4	0.983
11	N2	23.9	18.2	0.762	14.8	0.619	32.2	1.348	23.3	0.975
12	N3	21.5	17.2	0.800	13.7	0.637	31.5	1.465	19.8	0.921
13	III	37.0	16.4	0.443	13.0	0.351	40.1	1.083	32.8	0.886
14	S0-0L	50.0	26.5	0.530	26.0	0.520	54.1	1.082	41.0	0.820
15	T2	54.7	30.2	0.552	34.9	0.638	77.4	1.415	53.9	0.985
16	T18	74.7	43.0	0.576	35.1	0.470	114.3	1.530	54.5	0.730
17	TB0.00_2.5	43.5	26.8	0.616	26.7	0.614	64.3	1.479	46.3	1.064
18	TB0.00_3.0	40.0	26.8	0.670	26.7	0.668	59.9	1.498	44.1	1.103
19	T-01E	36.6	20.2	0.552	26.3	0.719	44.1	1.206	41.1	1.123
20	T-02E	38.5	23.1	0.600	26.3	0.683	48.3	1.254	44.0	1.143
21	T-03E	47.5	27.0	0.568	25.5	0.537	53.6	1.129	42.6	0.897
22	R-01E	32.6	20.2	0.620	26.3	0.807	34.3	1.052	34.3	1.052
23	R-02E	37.0	23.1	0.624	26.3	0.711	39.4	1.065	34.3	0.927
24	R-03E	37.6	27.0	0.718	25.5	0.678	44.3	1.178	33.3	0.886
25	SS-I	44.5	31.4	0.706	38.1	0.857	74.9	1.684	58.6	1.317
26	SS-II	50.9	32.8	0.644	40.6	0.797	74.7	1.466	65.7	1.291
27	SS-III	44.5	32.1	0.722	39.4	0.886	73.1	1.644	62.2	1.399
28	SS-IV	48.7	30.0	0.616	35.5	0.729	53.7	1.103	51.1	1.050
	AVR			**0.697**		**0.662**		**1.377**		**1.030**
	STD			**0.129**		**0.112**		**0.232**		**0.153**

From Table 2, it can be seen that the values obtained from the proposed formula correlate very closely with the experimental findings. The deviation of the predicted values from their experimentally-obtained counterparts is on average only 3%. In contrast with the proposed formula, those adopted by ACI 318 and EC2 underestimate shear capacity by over 30%. As discussed in the introduction, such an underestimation is expected, since the concepts underlying the derivation of the code formulae do not allow for the effect of the beam's flange on shear capacity. On the other hand, the formula proposed by IStructE, which allows for the effect of the flange, is found to overestimate shear capacity by an amount over 30% on average. The cause for this overestimation may be attributed to the data used for the calibration of this formula. These data were primarily obtained from tests on beams with transverse reinforcement designed so as to prevent shear failure.

CONCLUSIONS

The work described in the paper has been developed with the context of the CFP method. Within this context it has been possible:

- to identify the type of failure for which current failure criteria do not allow for the effect of the flanges on the load-carrying capacity of RC T-beams;
- to complement an existing simple failure criterion developed for RC beam/column elements with a rectangular cross section so as to extend its range of application to the case of RC T-beams;
- to verify the validity of the proposed failure criterion through a comparative study of the calculated values of shear capacity with their experimentally-established counterparts obtained from the literature after an extensive survey.

The implementation of the proposed failure criteria in practice is expected to lead to a reduction of the amount of transverse reinforcement required for safeguarding against shear failure. This is of particular importance in bridge girders with a T-shaped or box cross section in which there is a smooth transition from the web width to the flange.

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