

# Numerical modeling of fibre-reinforced concrete

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## Abstract

This paper reports the findings of a study into the behaviour of steel-fibre-reinforced concrete (SFRC) using non-linear finite-element analysis and existing experimental data. The overall aim of the research work is to formulate a reappraisal of the way in which stresses, deformations and cracking of such structural elements are predicted at present under both static and dynamic loading and how these predictions can be used to influence design decisions. The literature survey that preceded the work helped identify major shortcoming in the way SFRC mechanical properties are classified and presented at the moment and the lack of a unified approach to selecting a suitable model for general analysis and design purposes. There is also a clear gap in the literature on the application of SFRC constitutive models to study the potential of applying SFRC to enhance the seismic response of a structure and to assess the potential ductility and energy absorption capacity of such composites.

*Keywords:* mechanical properties, steel fibre-reinforced concrete, finite-element analysis

## 1 Introduction

The structural response of SFRC elements is characterised by its tensile strain-softening behaviour. A number of available constitutive models for SFRC have been identified such as those proposed by RILEM, Barros, Lok, Tlemat and others. The main characteristics of the models have been closely studied. Non-linear finite-element analysis was used to calibrate these models and, ultimately, one model has been selected for the subsequent parametric studies on SFRC under seismic conditions. This was achieved by incorporating the models into ABAQUS (smear cracking and brittle cracking) models for concrete and then carrying out comparisons between ABAQUS predictions and existing experimental data on SFRC beams. This paper summarises the first phase of the work which focused on static loading. Further calibration is underway to compare the numerical predictions with the results of full-scale tests on SFRC beams and column-beam joints under cyclic and seismic loading.

## 2 Material models for SFRC

### 2.1 Compressive behaviour

Published work on SFRC suggests that the compressive behaviour of SFRC can be safely assumed to be similar to that of plain concrete (RILEM TC 162-TDF, 2000; 2003; Barros and Tlemat, 1999; Lok and Pei, 1998; Lok and Xiao, 1999). Investigations carried out by Bencardino et al (2008) support this

conclusion as the observed results show that the addition of steel fibres does not significantly affect the compressive strength of concrete. Therefore, in the present work, the compressive behaviour of SFRC is assumed to be similar to the one proposed for plain concrete in Eurocode 2, with an ultimate strain value equal to 0.0035.

## 2.2 Tensile behaviour

To enable the design of SFRC structures, RILEM TC 162-TDF Recommendation (2000; 2003) proposed the stress-strain (i.e.  $\sigma - \varepsilon$ ) diagram shown in Figure 1(a) for FRC under uni-axial tension. In order to determine the values of the parameters defining the  $\sigma - \varepsilon$  model, the corresponding load-deflection curves from beam tests are used. Similarly, based on investigations carried out on SFRC small beam specimens and using fracture energy concepts, Barros and Figueiras (1999; 2001) and Tlemat et al (2006) developed tensile stress-strain diagrams for SFRC as illustrated in Figure 1(b), 1(c) and 1(d), respectively. Figure 1(e) and 1(f) show the generic material models proposed by Lok and Pei (1998) and Lok and Xiao (1999), respectively, and both models may exhibit tension softening or hardening characteristics depending on the amount of fibre considered.

In these models, the residual strength is made up of two components, the steel fibres bridging the crack and the concrete matrix. The parameters of the constitutive models' depicted in Figure 1(a-f) are defined in their respective publications.

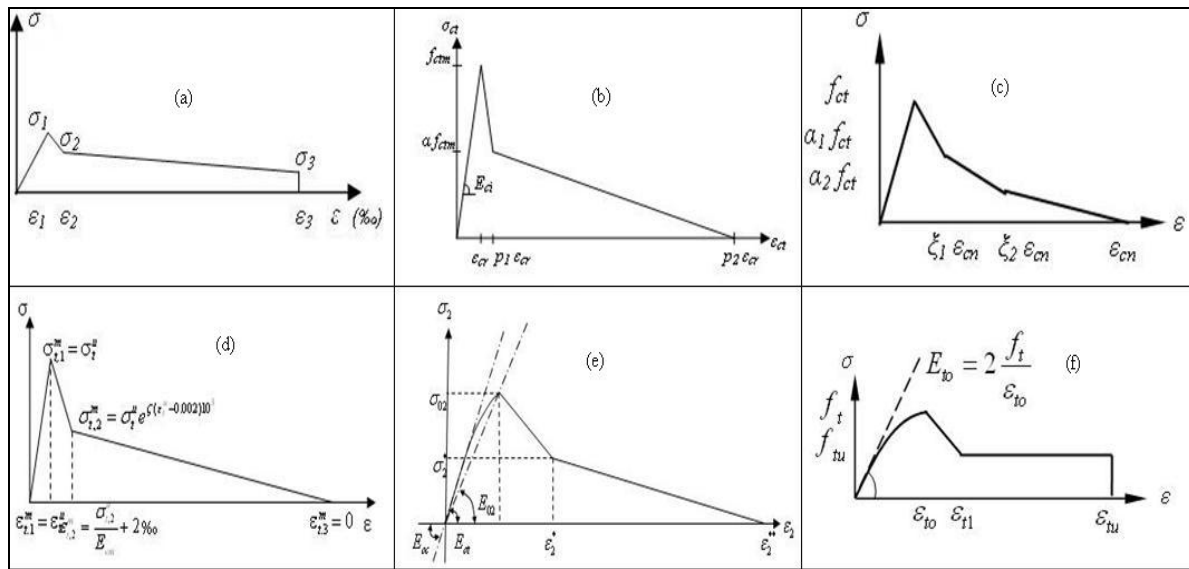


Figure 1. The proposed stress-strain diagrams for SFRC under uni-axial tension: (a) RILEM TC 162-TDF Recommendation (2000; 2003), (b) Barros and Figueiras (1999), (c) Barros and Figueiras (2001), (d) Tlemat et al. (2006), (e) Lok and Pei (1998) and (f) Lok and Xiao (1999)

## 3 ABAQUS constitutive models adopted

In ABAQUS (2007), two main constitutive models are available for non-linear finite-element analysis of plain concrete.

### 3.1 Smearred cracking concrete model

In general, smeared crack (SC) concrete model is designed to investigate the behaviour of concrete subjected to monotonic straining at low confining pressures. The model is capable of modelling all types of concrete structures, and uses “Riks” solution procedure to enhance numerical stability. The

SC model requires material definition in uni-axial stress-strain data. The compressive behaviour of concrete is modelled as elastic-plastic. The effect of steel fibres is modelled using tension stiffening (Figure 2 (a)) to incorporate the constitutive models for SFRC. Additionally, the relationship between compressive and tensile behaviour is specified in the failure ratio values, as shown in Figure 2(b), in order to allow ABAQUS to define the shape of failure surface. Furthermore, a “shear retention factor (SRF)” is introduced to allow for the reduction in shear stiffness of concrete as the crack propagates. The value of SRF obviously relies on mechanisms such as aggregate interlock, fibres bridging the crack .etc. Full shear retention (i.e. no reduction) is the default value assumed by ABAQUS.

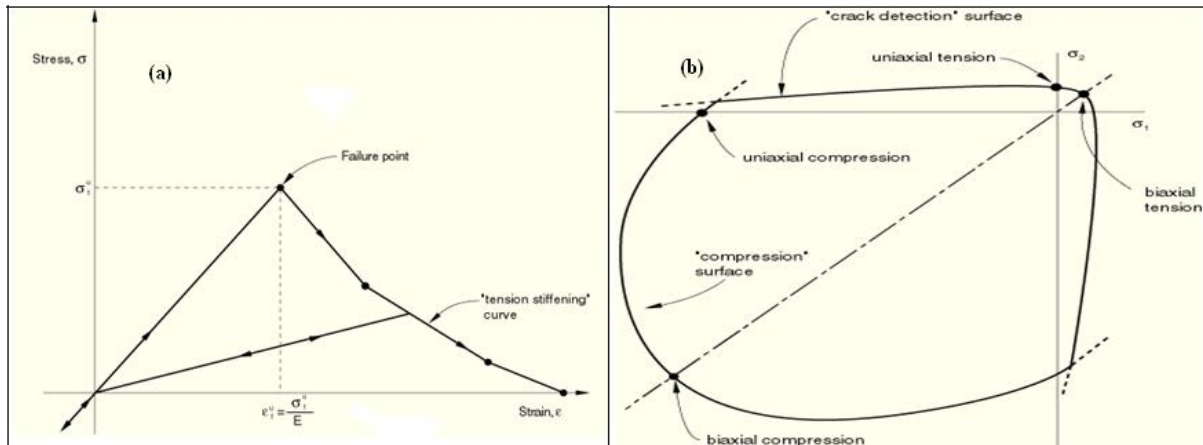


Figure 2. (a) Tension stiffening model, (b) Yield and failure surface in plane stress (Adopted from ABAQUS, 2007)

### 3.2 Brittle cracking concrete model

Brittle cracking (BC) model is designed to simulate tensile-cracking-dominated behaviour, which is applicable to all types of concrete structures. In this model, the compressive behaviour is assumed to be always linear elastic; therefore a linear elastic material model has to be used in compression. The SFRC constitutive model is inserted into post-failure curve to define the tensile behaviour of SFRC structure. Rankine failure criterion is implied in the BC model to detect crack initiation. A crack is formed when the principal tensile stress exceeds the tensile strength of the brittle material. There are two types of failure modes in this criterion, namely Mode I (tension softening/stiffening) and Mode II (shear softening/retention). Crack initiation is based on Mode I failure, whilst both modes effect the post-cracked behaviour. In Mode II failure, the cracked shear modulus is reduced as the crack opens using the shear retention factor discussed earlier.

## 4 Calibration of the numerical models

The calibration was carried out by incorporating the models into ABAQUS (SC and BC models for concrete) and then comparing the predictions with existing experimental data on SFRC specimens. This paper summarises the first phase of the work which focused on static loading. Further calibration is underway to compare the numerical predictions with the results of cyclic and seismic tests.

At the material level, two small simply supported SFRC beam/prism specimens investigated by Tlemat et al. (2006) and Barros et al. (2005) are studied. Tlemat et al. (2006) beam represents the case with high fibre concentration, whilst, Barros et al. (2005) beam represents the case with low content of fibre concentration. At the structural level, Ozcan et al. (2009) and Cho and Kim (2003) SFRC much larger beams were selected which included both longitudinal and transverse reinforcement as well as steel fibres (full details are can be found in respective publications).

Lok and Pei (1998) and Lok and Xiao (1999) proposed models are calibrated for all SFRC beams cases, whilst Tlemat et al (2006) proposed model is only calibrated using their own experimental data. Owing to symmetry, only a quarter of the beam specimen was modelled. Two approaches were used to model the behaviour of the SFRC beam at material level, namely a SC concrete model and a BC model. However, at the structural level, only BC model is applied for the analysis.

The SC concrete model requires the definition of material properties under both compressive and tensile conditions. The tensile behaviour of the proposed constitutive model (Tlemat et al., 2006; Lok and Pei, 1998; Lok and Xiao, 1999) is inserted in the tension softening data section. The tension softening of the concrete after cracking is simulated by a multi-linear descending curve (see Figure 1(d-f)). As for the compressive behaviour, the one proposed for plain concrete in Eurocode 2 is adopted. The analysis is performed using a static “Riks” procedure. The load is applied using a displacement based method (DBM) to minimise convergence problems. A 2D 4-noded bilinear plane stress quadrilateral (CPS4) element type was chosen to model the beam at material level.

For BC concrete model, only tensile behaviour of the SFRC is required for material properties definition (as the compressive behaviour is assumed to be elastic). The proposed constitutive model (Tlemat et al., 2006; Lok and Pei, 1998; Lok and Xiao, 1999) is applied in this section. Shear retention factor and crack opening strain are defined in brittle shear data. A quasi-static analysis is carried out using the dynamic explicit procedure. The load is applied as DBM. The element type chosen for this analysis is a 2D 4-noded bilinear plane stress quadrilateral, reduced integration (CPS4R) for the beam at material level, and an 8-noded linear 3D brick element (C3D8R), reduced integration for the beam at structural level. The steel reinforcement was defined as a 1D 2-node linear truss element (T3D2).

## 5 Results and discussion

Figure 3 shows ABAQUS numerical predictions as well as the experimental results for the SFRC beams investigated by Tlemat et al. (2006) and Barros et al. (2005). Using both the BC and SC constitutive models to simulate SFRC constitutive models, results in good agreement with experimental data were generated especially for the model proposed by Tlemat et al. (2006). For comparison, the standard deviation (from the experimental data) calculated for each analysis are SC = 0.9896 and BC = 0.9926; SC = 0.9906 and BC = 0.6959; SC = 0.9886 and BC = 0.8305, for Tlemat et al. (2006), Lok and Pei (1998), and Lok and Xiao (1999) proposed model, respectively. This shows that Lok and Pei (1998) and Lok and Xiao (1999) proposed models are practical and able to simulate the SFRC even at high fibre concentrations. For the comparison with Barros et al (2005) SFRC beam experimental data, the constitutive models applied here are Lok and Pei (1998) and Lok and Xiao (1999) and both models produced reasonable predictions.

Considering the standard deviation values, Lok and Xiao (1999) model produced better results than those of Lok and Pei (1998) model (the standard deviation values evaluated are 0.875 and 0.8158 for SC and BC respectively, for the former model and 0.6722 and 0.5331 for SC and BC, respectively, for the latter. Better post-failure numerical predictions for Barros et al. (2005) specimen may be obtained if a higher value of bond stress is adopted to define the tensile stress-strain diagram in both Lok models.

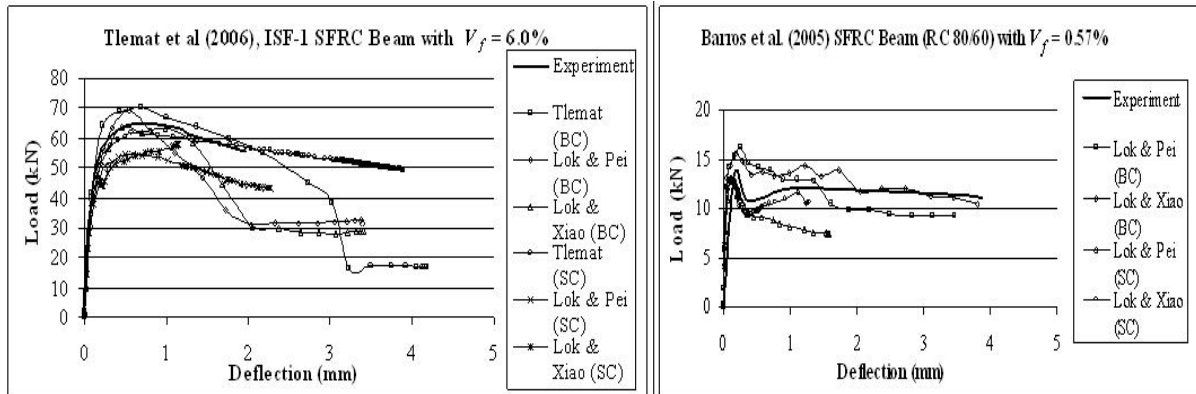


Figure 3. Load-deflection comparison curves for Tlemat et al. (2006) and Barros et al. (2005)

With low content of fibres, the simulation results obtained by Lok and Pei (1998) and Lok and Xiao (1999) models for Ozcan et al (2009) beam (see Figure 4) shows a softening behaviour of post-failure of SFRC structure. The hardening behaviour demonstrated by the SFRC beam in the experiment is probably due to the strength hardening provided by the steel reinforcement. Even so, the curves simulated by the SFRC constitutive models still provide a comparable result, with Lok and Xiao (1999) model results being better (standard deviation = 0.6545) compared with Lok and Pei (1998) model ( standard deviation = 0.49).

The comparison results of the SFRC beam investigated by Cho and Kim (2003) are shown in Figure 4. Both models proposed by Lok and Pei (1998) and Lok and Xiao (1999), produced good results when compared with the experimental data. The standard deviations evaluated are 0.9562 and 0.9373 for Lok and Pei (1998) and Lok and Xiao (1999) models, respectively. It is interesting to observe the softening (Ozcan et al., 2009) and hardening (Cho and Kim, 2003) behaviour simulated by the models, depicting both low and high fibre contents. This indicates the capability of the proposed models in modelling the behaviour of SFRC structures.

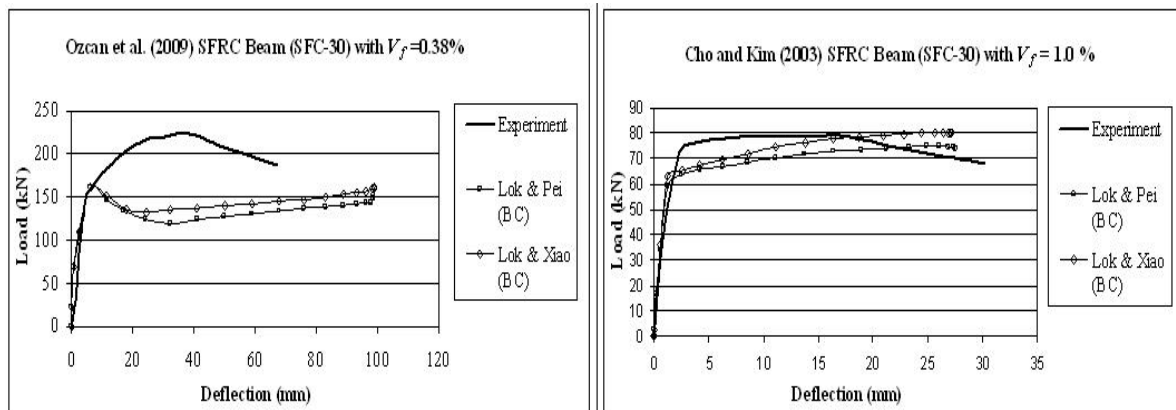


Figure 4. Load-deflection curves for Ozcan et al. (2009) and Cho and Kim (2003)

## 6 Conclusion and recommendation

Three material models were calibrated and discussed in this report, specifically Tlemat et al (2006), Lok and Pei (1998) and Lok and Xiao (1999) models. Tlemat et al (2006) model shows good agreement when calibrated using its own experimental data. The material models proposed by Lok

and Pei (1998) and Lok and Xiao (1999), even though sometimes underestimate the load-deflection curves, still demonstrate reasonable agreements with the experimental results during comparison (no over-estimation of the load carrying capacity was observed, which indicates that the results are on the safe side). The discrepancies in the results are possibly caused by the low fibre concentration (fibre content range tested was 0.38 – 0.57 %) and/or the average value ( $\tau_d = 3.57 \text{ N/mm}^2$ ) used to define the bond stress (Swamy et al. 1981, as cited in Lok and Pei, 1998).

Lok and Pei (1998) and Lok and Xiao (1999) proposed models adopt a similar concept in defining the post-cracking tensile behaviour. It might be time consuming to apply both models in the analysis as these models only have a slight different in defining the characteristic points of the tensile behaviour (refer Figure 1). Therefore, the model proposed by Lok and Xiao (1998) is suggested as a sufficient model to demonstrate the tensile post-cracking behaviour of SFRC based on the following reasons. First, the model is versatile as it allows for definition of different values for fibre content in volume ( $v_f$ ), aspect ratio ( $L/d$ ) and bond stress ( $\tau_d$ ). Secondly, the randomness distribution of the steel fibres is considered in the model (orientation factor). Thirdly, the shape of the tensile stress-strain diagram graph is not conservative as it is similar to the pattern proposed by RILEM TC 162-TDF Recommendation (2000; 2003). Fourthly, the model is capable of exhibiting tension softening and hardening depending on the magnitude of the variables (e.g. fibre concentration, bond stress ..etc). Finally, through the calibration work carried out, good agreement with existing experimental data was obtained and all predictions are on the safe side (i.e. the load carrying capacity is not over-estimated).

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