Efficiency spillovers in Bayesian stochastic frontier models: application to electricity distribution in New Zealand

Citation for published version:
Carvalho, A 2018, 'Efficiency spillovers in Bayesian stochastic frontier models: application to electricity distribution in New Zealand', *Spatial Economic Analysis*, vol. 13, no. 2, pp. 171-190.
https://doi.org/10.1080/17421772.2018.1444280

Digital Object Identifier (DOI):
10.1080/17421772.2018.1444280

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
Spatial Economic Analysis

Efficiency Spillovers in Bayesian Stochastic Frontier Models: Application to electricity distribution in New Zealand

António Carvalho*

Centre for Energy Economics Research and Policy (CEERP)

Heriot-Watt University

Abstract

This paper proposes a Spatial Bayesian Random Effects Stochastic Frontier model, which allows for unobserved heterogeneity and spillovers between firms' efficiencies with an exogenous spatial weights matrix. Proposals for efficiency measurement in the spatial context add to the debate in the literature. The approach shows good small sample performance, which is very relevant for applied researchers, and explores Guided Walk Metropolis as a simple and computationally efficient alternative to classic rejection techniques. The approach is applied to a sample of 28 New Zealand electricity distribution firms between 1996 and 2010, finding spatial dependence with a second-order contiguity matrix.

JEL Codes: C23, C11, Q49

Keywords: Cost Frontier, Cost Efficiency, Spillovers, Spatial Econometrics

07/02/2018

Word Count: 9272 (including all notes, tables, references and captions)

* E-mail: A.Carvalho@hw.ac.uk. Address: Centre for Energy Economics Research and Policy (CEERP), Heriot-Watt University, Edinburgh, United Kingdom EH14 4AS. Telephone: +44 (0)131 451 4578

1. Introduction

Stochastic Frontier (SF) analysis stems from the neoclassical theory of production, where in a production function $f(x_i;\ \beta)$ defines the maximum possible attainable output given the input vector x_i. This production frontier is not reached by every producer even with similar use of inputs. Thus, technical efficiency can be seen as the shortfall of output from its maximum given the inputs. Similar arguments can be made for a cost frontier, where the minimum attainable cost given the inputs is not reached by every firm. Firms in an industry are often subject to benchmarking of their levels of technical or cost efficiency for the construction of relative firm rankings and the measurement of average levels of efficiency. Regulators and firms can then use the findings of this analysis to analyse efficiency gaps between industries in different countries, and the gaps between industry leaders and those lagging behind.

Extensive efforts have been conducted in the literature to build panel data frontier models capable of estimating efficiency and deal with challenges such as unobserved heterogeneity. Most efforts span from the seminal work of Aigner, Lovell and Schmidt (1977), with the True Random Effects (TRE) and True Fixed Effects (TFE) models (Greene, 2005) being particularly influential, as well as easily available in statistical software packages. The literature has expanded to consider multiple concerns, such as heteroskedasticity, nonparametric estimation, distributional assumptions and exogenous determinants of efficiency. An extensive literature review on many of these issues can be found in Parmeter and Kumbhakar (2014). Clearly, most of the aforementioned challenges have been studied in more detail than spatial dependence in frontier models.

The contribution of this paper to the literature is threefold. First, it focuses on the often unexplored issue of spatial dependence and efficiency spillovers between economic agents in frontier models. Spatial dependence in the cross-sections can lead to omitted variable bias, so an autoregressive specification of efficiency is used. The model integrates the unobserved spatial characteristics directly while accounting for unobserved heterogeneity through random effects, with a simple and easily implementable Markov Chain Monte Carlo (MCMC) scheme. The paper also adds to the ongoing debate on the correct formulation of efficiency measurement in spatial models, as proposals are made for both relative and absolute measures under some restrictions. Spatial spillovers of efficiency in an industry can change efficiency results and the regulators perspective on the spread of the impacts of a policy. Secondly, the paper explores small sample performance of the approach, an aspect which is often ignored in the literature but of relevance to applied researchers. Alternative methods in Bayesian econometrics are also investigated, suggesting the use of Guided Walk Metropolis (Gustafson, 1998) to take draws from non-standard distributions. This method is extremely simple to implement and leads to modest but consistent gains versus competing methods across multiple sample sizes. As in other Stochastic Frontier models, performance is encouraging when signal is large relative to noise, in this case even under significant levels of spatial dependence. As performance degrades, the estimate of the spatial parameter estimate shows more bias than the estimate of the parameter of the distribution of efficiency. Simulation results are encouraging for use in empirical analysis in different scenarios. Finally, the proposed approach is applied to a dataset of New Zealand electricity distribution networks, pointing to evidence that there is spatial dependence in the networks' efficiencies, which is an important result to regulators and policy makers. The case of New Zealand is of particular interest as it was an early case of vertical separation in the industry, associated with transparent data reporting of multiple aspects of the operation of firms. The structure of spatial dependence relates to proximity to other networks, where managerial practices can be influenced by the practices of other firms. Some evidence of spillovers exists when a second-order contiguity matrix is used. Although the literature also suggests the measurement of persistent inefficiency which is separated from the Random Effects, such results might be driven by unavoidable operating conditions for which firms should not be penalized.

The rest of the paper is organized as follows. Section 2 discusses the literature review on both spatial Stochastic Frontier modelling and the application of the proposed model to electricity distribution networks. Section 3 presents the proposed model to estimate efficiency and spatial spillovers of efficiency between networks while capturing unobserved heterogeneity. Section 4 discusses performance of the approach in small samples which is of interest to applied researchers, as well as the performance of the proposed Guided Walk Metropolis method versus competing methods. In Section 5 the model is applied to a sample of 28 New Zealand electricity distribution firms between the 1996 and 2010 fiscal years, covering some pitfalls of the approaches in the literature and adding some insight on the debate of efficiency spillovers in the industry. Section 6 concludes.

2. Literature Review

2.1. Spatial Stochastic Frontier

While the efficiency literature usually considers spatial heterogeneity as the differences in efficiency due to location, controlled for by using dummy variables or similar approaches, spatial dependence is the relationship between efficiency in a firm and efficiency in other firms (Areal, Balcombe and Tiffin, 2012). The concepts do not overlap, creating the reasoning for the use of spatial approaches in stochastic frontier modelling. Most approaches are based on Spatial Autoregressive (SAR) or Spatial Error (SEM) models due to seminal influences in the literature such as Anselin (1988). Halleck Vega and Elhorst (2015) suggest the SLX (Spatial Lag of X) model as a starting point if there is no strong theoretical case to indicate which model to use, although such an approach has not been followed in the spatial Stochastic Frontier literature.

Early efforts in the literature are scarce. Druska and Horrace (2004) developed a Generalized Method of Moments (GMM) frontier model with spatial aspects and applied it to rice farms in Indonesia. The authors consider an error term that follows an autoregressive specification to estimate the spatial dependence based on a fixed effects model that follows the methodology of Schmidt and Sickles (1984), meaning that efficiency measures are time-invariant and effects are interpreted as inefficiency which could contain uncontrolled unobserved heterogeneity. This assumption can be unreasonable in many contexts. Schmidt et al. (2009) focus on the unobserved local determinants of inefficiency in farm productivity in the Centre-West of Brazil. In this case, farm inefficiency depends on a parameter that captures unobserved spatial characteristics. The analysis assumes that such characteristics follow a conditional autoregressive prior distribution or a normal distribution.

Affuso (2010) uses a spatial autoregressive model to evaluate the impact of agricultural extension programmes that have positive effects not only on chosen farmers but also to other farmers due to spatial spillover effects. Pavlyuk (2013) derived Maximum Likelihood (ML) estimators of stochastic frontier models with spatial dependence associated to the dependent variable, the idiosyncratic error and both. This effort paved the way for extended models that surfaced later, but the literature did not elaborate up to that point on unit specific efficiency measurement or other aspects of the model besides the magnitude of the spatial parameters and the variance of error components. Other contributions to the literature include a spatial autoregressive stochastic frontier model for panel data with a specification that allows for time-varying efficiency measurement (Glass, Kenjegalieva and Sickles, 2016). This greatly adds to the literature in terms of calculation of direct and indirect efficiency measures, along with explicit formulas for efficiency measurement and the interpretation of asymmetric efficiency spillovers between units.

Some advances in spatial stochastic frontier modelling have also taken place in the macroeconomic literature. Mastromarco, Serlenga and Shin (2013) use a two-step approach to investigate the channels under which globalisation factors lead to technical efficiency by combining a dynamic stochastic frontier model with a time series approach. Mastromarco, Serlenga and Shin (2016) propose to accommodate both time and cross-sectional dependence by combining the exogenously driven factor-based approach with an endogenous threshold efficiency regime selection mechanism. This is applied to a dataset of 26 OECD countries over the period 1970-2010. Brehm (2013) investigates the link between fiscal decentralization and economic efficiency in China using a frontier model with spatial error correction. The equation that defines technical efficiency is augmented with spatially lagged public expenditures variables as potential sources for regional clustering of efficiency. In this context, spatial interactions are related to the independent variables that are included in the efficiency specification.

However, the use of a spatial structure directly related to the one-sided efficiency component in the literature is sparse but of particular interest. Areal et al. (2012) propose a Bayesian spatial stochastic frontier model with a spatial structure directly specified in the efficiency error component. The model is applied to a sample of 215 dairy farms in England and Wales with data between 2000 and 2005, with the estimation of time-invariant inefficiency in a pooled model. The key contribution of the authors is the direct specification of spatial structure associated to the error components and the inclusion of a parameter that measures the level of spatial dependence. However, there is no exploration of model performance in simulations and no insights into how efficiency is to be measured or interpreted in detail once the model is estimated, focusing instead on the magnitude of the spatial effects. Both issues leave further space for research in the literature. There is also space to explore time-varying efficiency and also random effects to capture unobserved heterogeneity, which is one of the objectives of this paper. An alternative option to deal with heterogeneity is a random coefficient frontier model that allows for different technologies (Tsionas, 2002), which is applied in an energy economics context (cost efficiency of 123 United States electric utility companies in 1970), but in a cross-sectional setting with no model performance analysis using simulated data.

Fusco and Vidoli (2013) present a similar approach to Areal et al. (2012) which differs mostly by using a different distributional assumption (half normal) and Maximum Likelihood (ML) methods. The authors propose to measure the global effect of spatial factors, as well as providing additional insights into the performance of such models, showing a downward bias of the spatial parameter in a simulated dataset. The variance of inefficiency is slightly underestimated. The simulated dataset only has 107 observations and does not investigate alternative signal-to-noise ratios or sample sizes. However, this gives preliminary indications of how such models might underperform in some circumstances. Vidoli et al. (2016) perform a similar analysis with a review of the Italian wine industry and a second stage analysis in which the territorial imbalances (the difference between the proposed spatial model results and the standard SFA model results) are regressed against a set of contextual variables that are generally associated with the presence of a stronger effect. There is a clear tendency of the spatial effect to be significantly lower among large firms. Again, there is no indication of model performance in different sample sizes and signal-to-noise ratios.

Tsionas and Michaelides (2016) propose a latent random effects vector that is specified to follow a Spatial Autoregressive process for panel data. The idiosyncratic component of inefficiency is assumed to be half-normal and the model is estimated using complex Bayesian methods. The authors also consider methods for posterior predictive efficiency measurement, including a Monte Carlo approximation.

A Bayesian approach is adopted here for several reasons outlined in the literature. It allows the use of prior information in the model, such as past information on inefficiency levels. It also provides inference that is conditional on the data without asymptotic approximations, it obeys the likelihood principle and uses MCMC methods which make computations tractable for nearly all parametric models (Tsionas and Michaelides, 2016). A standard non-spatial Bayesian Stochastic Frontier model has been discussed in Koop (2010).

The key challenges that stem from the literature above are twofold. First, there is a lack of detailed simulation studies to assess model performance outside the chosen datasets that the models are applied to, which leads to doubts on how models perform with weak signal-to-noise ratios, differing magnitudes of spatial relationships and different sample sizes (and in the panel data case, for both N and T). Sample sizes are of particular importance to applied researchers as many datasets are relatively small (for example, a reduced number of firms in a specific sector of the economy, over a short time period of available data). Secondly, multiple spatial stochastic frontier models are discussed in the literature without the implications of the increased model complexity for efficiency measurement. This once trivial task becomes more complicated to execute and to interpret in the spatial case. Perhaps for that reason a significant part of the literature focuses on measuring the magnitude of the spatial relationships and the variance of the inefficiency components, instead of individual efficiency scores or average efficiency scores across firms and time. There is also little discussion on the parameter and model restrictions which facilitate interpretation. Those two issues will be addressed in subsequent sections.

2.2. New Zealand Electricity Distribution Networks

In the restructuring of electricity markets across the world, the common pattern has been the privatization of utilities, break up of monopolies and the creation of wholesale markets and incentive regulation (Nillesen and Pollitt, 2011). This pattern was driven by the perspective that the efficiency benefits of competition outweigh the costs of vertical separation.

New Zealand had a typical public generation and transmission monopoly with regional distributors and retailers. The Energy Companies Act of 1992 deregulated the sector and imposed mandatory information disclosures, along with provision for price controls.

However, in the following years a series of problems were identified in the market (Nillesen and Pollitt, 2011). There was a lack of switching in the retail market, no price decreases and a lack of competition in generation. There was also no system to reconcile the distribution of electricity and concerns about price inflation and monopoly rents that could in turn subsidize retail activities and inefficient generation schemes. Liberalization and restructuring of the electricity sector in 1998 (fully concluded by April 1999) forced a vertical separation of the electricity supply industry with respect to ownership (also known as ownership unbundling). The primary motive of ownership unbundling is to prevent any discriminatory behaviour of network owners and facilitate market entry and competition (Nepal, Carvalho and Foster, 2016).

This popular case study has been investigated from multiple perspectives. Nillesen and Pollitt (2011) examine the impact of this policy on electricity prices, quality of service and costs. A Cobb-Douglas cost function is estimated, with a significant effect of the unbundling dummy variable on costs, implying that the policy managed to drive down costs. Nepal, Carvalho and Foster (2016) also find that the unbundling of the industry has contributed to a fall in frequency and duration of outages, but has no effect in reducing distribution losses. The New Zealand case has also proved popular in Stochastic Frontier studies of distribution networks. Ozbugday and Nillesen (2012) estimate a cost frontier function for distribution networks between 1998 and 2010, finding a compound annual growth rate of over 2% using a time-varying decay frontier model. Filippini and Wetzel (2014) estimate a cost frontier with data between 1996 and 2010, with both variable and total cost as dependent variables, suggesting a positive one-off shift in efficiency levels when ownership unbundling is introduced. The authors estimate the model using the Battese and Coelli (1995) approach with and without fixed effects, allowing for the introduction of explanatory variables in the inefficiency component equation. As expected, when the fixed effects are included, the variance of the inefficiency component is reduced, as some persistent inefficiency might be diluted into the captured heterogeneity. Filippini, Greene and Masiero (2017) introduce the measurement of persistent inefficiency to this dataset using the Generalized True Random Effects (GTRE) model, and involving regulation and imperfect information concepts to explain the necessity for the regulator to consider the level of persistent inefficiency in the sample. The model is estimated using the Filippini and Greene (2016) approach with Maximum Simulated Likelihood (MSL) and estimates both time-varying and time-invariant efficiencies. Some discussion about the practicality of the estimation and interpretation of persistent inefficiency in this context will be explored in Section 5.

The use of Bayesian Stochastic Frontier models is extremely scarce in the field of Energy Economics. A rare example is the analysis of technical efficiency of Chinese fossil fuel electricity generation companies (Chen, Barros and Borges, 2015), following the Bayesian stochastic random frontier model approach of Tsionas (2002). This paper contributes further to the energy literature by assessing cost efficiency of distributors in the spatial context, discussed in the light of simulation work that provides information about model performance in comparable small samples. Section 3 discusses the Bayesian modelling approach.

3. Modelling Approach

3.1. Setting up the problem

The model estimates a spatial Bayesian Stochastic Frontier using a novel approach to estimate a time-varying efficiency component with spatial spillovers and a random effect to capture unobserved heterogeneity. The proposed approach differs from Areal et al. (2012) as it proposes a time-varying efficiency measure in a panel data model (instead of time-invariant), as well as including a random effect to account for unobserved heterogeneity. The approach also builds from Feng and Zhang (2012) as it extends their Bayesian Random Effects model to the spatial case.

The following panel data cost frontier with random effects and spatial dependence associated to the efficiency term is outlined, with a balanced panel of N units and T time periods:

$$y_{it} = x_{it}'\beta + u_{it} + v_{it} + \alpha_i \tag{1}$$

$$u_t = \rho W u_t + \tilde{u}_t \tag{2}$$

$$\tilde{u}_{it} \sim Exp(\lambda^{-1}) \tag{3}$$

$$v_{it} \sim N(0, \sigma_v^2) \tag{4}$$

$$\alpha_i \sim N(0, \sigma_\alpha^2) \tag{5}$$

y is a (NT x 1) vector of the dependent variable, which is a firm or unit specific cost (or output, in a production frontier) measure that is log-transformed as recommended in the literature. X is a (NT x K) matrix of exogenous variables of the frontier, with K regressors, including a constant. v is a (NT x 1) vector of traditional idiosyncratic errors of standard linear regression and $\tilde{u}$ is a (NT x 1) vector of one-sided errors that capture inefficiency. α is a (N x 1) vector of time-invariant, zero mean random effects which aim to control for unobserved heterogeneity. Cross-sectional means of regressors can be added if there are concerns about violation of the assumptions of the Random Effects model (Mundlak, 1978). The efficiency component u is decomposed into two parts. The first one is spatial and reflects spillover effects for a given row-standardized, exogenous and known (N x N) spatial weights matrix W. The second component $\tilde{u}$ is idiosyncratic (efficiency that is not influenced by spatial dependence) and is assumed to follow an exponential distribution. ρ measures the intensity of the spatial dependence in efficiencies. ρ, λ^{-1} and W are assumed to be time-invariant for the purposes of this paper, but these assumptions can be relaxed in extensions to this model.

It is also possible that some persistent inefficiency exists. The use of a random effects model opens the way for this research path, unlike fixed effects estimation which make the separation of noise and inefficiency more difficult. However, this adds a new layer of complexity to the model and will not be pursued as it falls out of the scope of this analysis.

Switching the sign of u leads to a production frontier instead of a cost frontier. In either case, equation (1) can be related to a wide range of problems on cost and technical efficiency in firms, industries, sectors or even countries, under a context of unobserved heterogeneity. If α is dropped from the model, u is considered to be time-invariant and observations are pooled, the model is reduced to the approach of Areal et al. (2012). If $\rho = 0$, the model becomes similar to the Bayesian Random Effects Stochastic Frontier model of Feng and Zhang (2012).

3.2. Estimation of the model

A Bayesian approach will be used to estimate the parameters of the model. The following MCMC methodology follows the Bayesian formulation of the True Random Effects model as in Feng and Zhang (2012), with an extension linked to the spatial model of Areal et al. (2012). A cost frontier is outlined below. The priors are as follows, starting with the variance components:

$$p(h_v) \propto \frac{1}{h_v} \; where \; h_v = \frac{1}{\sigma_v^2} > 0 \tag{6}$$

$$p(h_\alpha) \propto \frac{1}{h_\alpha} \; where \; h_\alpha = \frac{1}{\sigma_\alpha^2} > 0 \tag{7}$$

In both cases, h_v and h_α are fully determined by the likelihood function. The prior for β follows the approach of Koop and Steel (2003), where I(.) is an indicator function with value 1 if the argument is true and 0 if the argument is false.

$$p(\beta) \propto I(\beta \in R_j) \tag{8}$$

R_j is the set of permissible β values with j=0 when no monotonicity and curvature constraints must be satisfied and j=1 when those constraints must be satisfied.

The prior for $\tilde{u}_{it}$ follows a special case of the gamma distribution, the exponential distribution with mean λ. To obtain a proper posterior for $\tilde{u}_{it}$, a prior distribution for λ^{-1} is also necessary (Fernández et al., 1997):

$$\mathrm{p}(\tilde{u}_{it}|\lambda^{-1}) \propto \mathrm{f}_G\ (1\,; \lambda^{-1}) \tag{9}$$

$$\mathrm{p}(\lambda^{-1}) \propto \mathrm{f}_G\ (1\,; -ln\tau^*) \tag{10}$$

τ^* is the prior mean of the efficiency distribution, which is defined according to prior information such as past efficiency levels. However, this relates to $\tilde{u}$ instead of u, as the latter is also influenced by both W and ρ. Therefore this hyperparameter defines the prior beliefs about direct efficiencies, excluding those caused by spatial spillovers between neighbours. Efficiency measurement will be explored in further detail in Section 3.3.

Finally, the prior for the spatial parameter ρ is assumed to be an indicator function, where $\frac{1}{r_{min}}$ is the most negative real characteristic root of W:

$$p(\rho) \propto \mathrm{I}(\rho \in [\frac{1}{r_{min}}, 1]) \tag{11}$$

Note that W is considered to be row-standardized, so the lower bound of ρ is -1. Areal et al. (2012) assume a uniform prior distribution for this parameter that is non-negative (between 0 and 1). That assumption stems from the fact that in most applied contexts of the model a positive spillover is expected. Although this is not explicit in the literature, this assumption also facilitates interpretation of efficiency scores. Therefore, that restriction is relaxed for performance analysis in Section 4, but applied in Section 5 to the New Zealand electricity distribution dataset. The conditional likelihood function is given by:

$$p(y|\beta, h_v, h_\alpha, \rho, \tilde{u}, \lambda^{-1}) \propto h_v^{\frac{NT}{2}} \exp[-\frac{h_v}{2}(\tilde{y} - X\beta)'(\tilde{y} - X\beta)] \tag{12}$$

$$\tilde{y} = y - u - \alpha \tag{13}$$

Conditional posteriors for each of the parameters can be seen in Supplemental Data Appendix A, leading to a Gibbs sampler where draws are taken sequentially. This is a simple methodology and easily implementable in standard statistical software packages. The conditional posteriors for efficiencies and the spatial parameter are non-standard distributions, for which the usual method to obtain draws is often Random Walk Metropolis (Metropolis et al., 1953). To improve performance, a Guided Walk Metropolis method is used (Gustafson, 1998), allowing for better performance over a range of sample sizes. This method has been used sparsely in applied research, with one example in the marketing literature (Ansari, Mela and Neslin, 2008) and has not been used in the field of econometrics. The utilized algorithm tuning parameter ensures that the acceptance rate lies between 50% and 60%, an interval which shows good properties in Gustafson (1998). The performance improvements of this approach are explored in Section 4.2. All estimations are conducted using own code in R 3.4.1, available in Online Supplemental Data. To save space, Supplemental Data Appendix A presents a detailed MCMC scheme to estimate the model.

3.3. Efficiency Measurement

Efficiency measurement is an overlooked aspect of the spatial frontier literature. Areal et al. (2012) and other examples in the literature focus on the issue of the magnitude of the spatial relationship and variances of error components, but do not explain how to determine efficiency scores and efficiency rankings given the spatial structure. Similar patterns are seen in the literature, for example in Pavlyuk (2013), as the author discusses multiple spatial models but fails to explain how efficiency scores should be calculated after estimating the model. A simple Monte Carlo approximation is proposed here, inspired by the approach of Glass, Kenjegalieva and Sickles (2016).

It is desirable to measure efficiency in some absolute scale to assert how distant firms are from the frontier in general. This measure can be obtained by rewriting equation (2) in the reduced form $\tilde{u}_t = (I_n - \rho W)^{-1}$, and substituting this expression in equation (1). Next, suppose $\tilde{u}_{it}^{(s)}$ is a draw from the conditional posterior of $\tilde{u}_{it}$ for the s^{th} pass of the MCMC scheme. This leads to the following NT by 1 vector of posterior means of the absolute measure of efficiency,

$$Absolute\ Efficiency = S^{-1} \sum_{s=1}^{S} \frac{\left(I_{NT} - I_T \otimes \rho^{(s)} W\right)^{-1} \exp[-\tilde{u}^{(s)}]}{\frac{1}{(1-\rho^{(s)})}} \tag{14}$$

Provided that the spatial weight matrix is row-normalized and ρW is non-negative. In equation (14), the denominator represents the maximum attainable efficiency scores given the observed level of spatial dependence and full direct efficiency. The denominator is no longer bounded between 0 and 1, but it represents a benchmark for firms or units.

At first, the alternative measure for efficiency $S^{-1}\sum_{s=1}^{S}\exp[-\mathrm{u}^{(s)}]$, which stems from the fact that the variables in stochastic frontier models are log-transformed, appears to be a more attractive and natural solution. However, it produces wrong efficiency estimates because it ignores the spatial multiplier matrix (the Jacobian term of the transformation of u_t to $\tilde{u}_t$). This leads to counterintuitive effects as efficiency tends to zero with increasing spatial dependence and the median and minimum efficiencies are increasingly distant from the most efficient observation. A detailed appendix with a numerical example and further discussion on alternative efficiency measures can be found in Supplemental Data Appendix B.

The measure in (14) can be decomposed into direct and indirect efficiency. Direct efficiency relates to the firms' efficiency that is directly attributable to the firm and is not related to proximity to other firms. The indirect effects matrix summarizes the effects that efficiency levels of the neighbours have on a firm's own efficiency and relates to the difference between total and direct efficiencies. This makes it possible to measure the magnitude and the sign of efficiency exchanges between the units. For example, a firm can "import" or "export" efficiency depending on its location. However, this is not elaborated further as it is not the main topic of the paper.

Finally, relative efficiencies can be calculated as in equation (15):

$$Relative\ Efficiency \ = S^{-1}\sum_{s=1}^{S}\frac{\left(I_{NT}-I_T\otimes \rho^{(s)}\mathrm{W}\right)^{-1}\exp[-\tilde{\mathrm{u}}^{(s)}]}{max[(I_{NT}-I_T\otimes \rho^{(s)}\mathrm{W})^{-1}\exp[-\tilde{\mathrm{u}}^{(s)}]]} \qquad (15)$$

With this proposed measure, the relative distance between the most and the least efficient unit decreases with increasing ρ, preserving the behaviour seen in equation (14). The ratio between the median score and the lowest score in the proposed relative and absolute measures is the same, retaining the original structure. However, the disadvantage of this method is that the most efficient firm is located on the frontier. This is a common pitfall of some SF models, including the Schmidt and Sickles (1984) efficiency estimator. A numerical example of this is also present in Supplemental Data Appendix B.

Note that the non-negative ρW restriction is relatively general and also used by Areal et al. (2012), as long as there is a positive spatial parameter coefficient and the spatial matrix is, for example, an inverse distance matrix or a contiguity matrix. However, there can be cases where there is a strong argument for negative spillovers, which are a theme for future research on how to determine and interpret efficiency measures in spatial models.

4. Model Performance

This section aims to measure performance of the modelling approach. First, performance is evaluated under different simulated data scenarios. Secondly, the performance gains of the introduction of Guided Walk Metropolis are evaluated.

4.1. Performance under different scenarios

The assessment of model performance aims to evaluate small sample performance, as most empirical work relies on relatively small panels. Therefore, two scenarios are created, with two sizes for N: N=100 (10 x 10 square grid) and N=25 (5 x 5 square grid). W is a first-order contiguity matrix in both cases. T varies between 5, 10 and 15. For both scenarios, two exogenous regressors are considered in the Data Generating Process, a constant and a standard normal variable, with both coefficients equal to 1. The spatial parameter ρ is considered to be 0.3 or 0.6 to represent lower and higher levels of dependence in efficiency between the units. The scenarios are as follows:

Scenario 1: $\sigma_v^2 = 0.01$, $\lambda^{-1} = 4$ and $\sigma_\alpha^2 = 0.05$;

Scenario 2: $\sigma_v^2 = 0.01$, $\lambda^{-1} = 10$ and $\sigma_\alpha^2 = 0.05$;

Scenario 1 has a higher signal-to-noise ratio with some unobserved heterogeneity. However, Scenario 2 has a much lower signal-to-noise ratio and keeps the moderate amount of unobserved heterogeneity of Scenario 1. The expectations of worse performance are therefore centred on Scenario 2. Means of $\tilde{u}$ are shown to highlight that the model renders relatively small average bias. The correlations between estimated and true individual $\tilde{u}$ are also displayed as they are crucial for relative firm rankings. All examples have a prior assuming 80% direct efficiency. Changing this prior does not significantly change results. Table 1 presents results for Scenario 1.

[Table 1 Here]

In Scenario 1, there is a very high correlation between estimated and true $\tilde{u}$, meaning that relative efficiency rankings should be well preserved. The spatial parameter is also reasonably estimated although there is some downward bias. This performance issue can contaminate (underestimate) estimates of efficiency spillovers. When N is increased from 25 to 100, the correlations between true and estimated values increase, while the λ^{-1} parameter approaches the true value. However, the improvements are small or not noticeable for other parameters of the model, as the performance in this scenario is already very encouraging in small samples. The mean bias of $\tilde{u}_{it}$ is slightly reduced as the magnitude of the Random Effects decreases. Table 2 presents simulation results for Scenario 2.

[Table 2 Here]

In Scenario 2 the signal-to-noise relationship changes considerably as the level of inefficiency decreases, making it harder to effectively separate the error components. Also, note that the size of the unobserved heterogeneity is now, in relative terms, much larger than in Scenario 1. The correlations between true and estimated values are, as expected, lower and more volatile than in the first scenario. The spatial parameter is underestimated further, leading to shrinking of indirect efficiency although the mean of $\tilde{u}$ is estimated correctly. However, given the multiple confounding factors, performance can be classified as encouraging. Curiously, there are two noticeable effects which are also seen in Fusco and Vidoli (2013): a downwards bias in the size of the inefficiencies and a downward bias in ρ. The degradation of performance in this case might be exacerbated by the fact that efficiency is time-varying instead of time-invariant.

In general, there are negligible distortions in the estimate of λ^{-1}, as the degradation of results is mostly seen in ρ and the correlations of true and estimated $\tilde{u}$. Due to the degradation of results in Scenario 2, the effect of panel sizes in differing signal-to-noise ratios is investigated further in Figure 1. Performance degradation is faster in smaller samples as the signal is reduced, which is a widely witnessed result across stochastic frontier models.

[Figure 1 Here]

In general, these findings point for good performance of the estimation procedure, but with some of the standard issues in the literature that are important for the empirical researcher. The model is expected to perform worse with lower signal-to-noise ratios, and that pattern is clearly observed here with ratios being a much more important factor than sample size. In Scenario 2 the correlation between true and estimated $\tilde{u}$ is not converging to high levels with increasing sample sizes as the identification is difficult in a context of multiple confounding factors. However, in this case, performance degradation is more visible in ρ than on the mean of ũ and it might be difficult to identify small amounts of spatial dependence in efficiency between the units, even with relatively large sample sizes.

4.2.The added value of Guided Walk Metropolis

The Guided Walk Metropolis (GWM) algorithm (Gustafson, 1998) presents an alternative to the classic Random Walk Metropolis (RWM) algorithm. The algorithm outperforms RWM in a variety of examples, including a standard normal, a multivariate normal, a bivariate distribution and an exchangeable multivariate normal distribution. In all cases, relative error is reduced. The reasoning for discussion and use of this method in this context is twofold: first, it allows for modest but consistent performance gains against RWM without computational costs, and secondly, those gains are particularly more important in this case, as both ρ and ũ are estimated using rejection techniques in this case. This is a more complex case than in other stochastic frontier models and therefore there is a bigger role of algorithm choices on the results of interest. As pointed by Gustafson (1998) for different examples, the GWM method seems to perform well for a very wide range of acceptance rates as it runs towards the true value faster if the starting value is distant. The algorithm is explained in detail in Supplementary Data Appendix A.

The calculation of effective sample sizes and the use of samplers that provide large and quick effective sizes are not thoroughly considered in the spatial econometrics literature. Wolf, Anselin and Arribas-Bel (2017) compare algorithms in Bayesian Spatial models, finding that effective sizes are often significantly smaller than nominal sizes. This problem also applies here because procedures that move slowly through the parameter space produce highly correlated draws, which leads to loss of information. Therefore, a detailed comparison of effective sample sizes for both ρ and ũ is made for different possible combinations of RWM, GWM and the Slice Sampler (Neal, 2003), which is often very efficient when the parameter is well-behaved and very simple to implement for univariate parameters. The No-U-Turn (NUTS) Sampler (Hoffman and Gelman, 2014) was deemed too numerically inefficient for spatial econometric models in some cases (Wolf, Anselin and Arribas-Bel, 2017). This leads to possible combinations of samplers for ρ and ũ respectively: 1) RWM-RWM, 2) GWM-RWM, 3) RWM-GWM, 4) GWM-GWM and 5) SS-GWM, where the last case uses the Slice Sampler (SS) for the spatial parameter only. The Slice sampler is not used for ũ due to its slow speed in drawing NT latent efficiencies. Performance of Effective Throughput (draws) per second is presented in Figure 2.

[Figure 2 Here]

The GWM method is very competitive as it shows consistent gains across multiple sample sizes, without significantly increasing the computation time due its extreme simplicity. On the other hand, the additional efficiency of the Slice sampler quickly decreases with an increasing sample size. Supplemental Data Appendix C presents extended tables with multiple performance measures across methods. Using N=49 as a benchmark sample size with varying T (5,10 and 15), RWM and GWM methods are compared further in Figure 3. It is clear that GWM allows for gains in bias reduction of mean ũ vs RWM but also some gains in the correlations between true and estimated values.

[Figure 3 Here]

Although the gains of the use of GWM are modest, they are also consistent and easy to implement, and therefore will be used throughout, including in the next section where the method is applied to a dataset of New Zealand electricity distribution networks.

5. Application to cost efficiency of New Zealand electricity distribution networks

5.1. Background

The proposed modelling approach is now applied to the context of electricity distribution. The New Zealand case of liberalization and restructuring of the sector in 1998 was one of the first of its kind, providing unique insights into the effects of such policies to other countries interested in similar policies. After vertical disintegration, the companies have severe limitations on ownership of generation or retail activities. Section 2.2 discusses the history of vertical separation in the sector in New Zealand and contains a series of examples of approaches to measure cost efficiency of operational expenditure in the sector, both before and after the vertical separation policy.

This paper analyses the problem from a spatial perspective, where the industry is defined by a cost function which takes into account time-invariant differences between the distribution networks and also quantifies spatial spillovers of efficiency across distribution networks. These effects could appear due to interactions between managers of different networks and observing the behaviour of competitors. This is a particularly important effect in an industry which has been unbundled and is dealing with a competitive environment in the sector. The application of the model focuses on time-varying efficiency estimation, to assess how levels have evolved since the introduction of the reform, compared to the period preceding it.

5.2. Data and setting up

The dataset is a balanced panel from 1996 to 2010 (annual fiscal years, from April to March) across 28 distribution networks, with a total of 420 observations. The data is mostly sourced from the Economic Insights “NZ EDB Database”, which collects data at the firm level from the Electricity Information Disclosures, required annually by the electricity regulator

(Economic Insights, 2009). The data has been used in Nepal, Carvalho and Foster (2016), and it has been expanded and slightly modified to improve data coverage. The data is augmented for the year 2009 and 2010 by using Disclosures but not extended further due to the Christchurch earthquake which devastated local areas in the (fiscal) year of 2011. All variables are logarithmically transformed as recommended in the literature. Table 3 shows descriptive statistics.

[Table 3 Here]

In the cost frontier model, Variable Cost (VC) or operational expenditure of the firm i at time t is a function of a constant, the energy delivered by the firm (ENERGY), the number of customers (CUSTOMERS), the load factor (LOADFACTOR), the System Average Interruption Duration Index (SAIDI), customer density (CUSTDENSITY), a proxy measure of capital of the network (CAPITAL) and the ratio of overhead to underground power lines of the network (RATIO_UG). Cross-sectional averages of the variables are also added to relax the assumptions of the random effects model, and a quadratic time trend is included. Row-standardized first-order and second-order contiguity spatial weights matrices are used in the estimation of the frontier model. Networks from different islands are never considered as neighbours.

5.3. Results and Discussion

Both specifications show evidence of convergence according to the Geweke diagnostic (Geweke, 1992). Although some coefficients contain zero in the credible interval due to uncertainty in parameter estimates, coefficients follow the expected sign from economic theory, as seen in Table 4.

[Table 4 Here]

Operating expenditure is expected to increase with the amount of energy delivered and the number of customers served. Units with a higher load factor are expected to use their line investment better, leading to lower operating costs. Longer average interruptions of service imply higher costs, due to maintenance and emergency fixes, and higher customer density implies more customers concentrated in a given area, and less service to isolated customers, which can lead to lower operating costs. Higher capital stock should also lead to lower operating expenditure, holding all else constant. Although the relative share of overhead power lines in the network might affect the cost, that relationship is not captured here. Both time trend coefficients are significant although with differing signs. Two of the seven cross-sectional mean regressors included to relax assumptions of the random effects model are significant, justifying their inclusion. The estimated signal-to-noise ratio is close to 1, as in Scenario 2 of the simulations, which points to possible difficulties in the identification of the spatial parameter. However, there are strong signs of the presence of a positive spatial parameter when a second-order contiguity matrix is used, as seen in Figure 4. Simulations conducted in Section 4 point that it is likely that there is some downward bias in the estimated spatial parameter in this case.

[Figure 4 Here]

One of the key assumptions for interpretation of the model is $\rho \geq 0$, to represent positive spillovers of efficiency between the distribution networks. Although there is no intuition for negative spillovers in this case, the assumption is relaxed to assess if severe changes can cast doubt on their consistency. For the first-order contiguity matrix, the mean of ρ is 0.101. In the case of the second-order contiguity matrix, it becomes 0.216, very close to the unrestricted mean of 0.231. Medians are extremely close between restricted and unrestricted models. Comparative density plots can be found in Supplemental Data Appendix D. To compare models with different spatial weights matrices and restrictions on the spatial parameter, the Deviance Information Criterion (DIC) is used (Spiegelhalter et al., 2002). DIC allows for easy comparison of models without the calculation of marginal likelihoods. Results from multiple models, including a non-spatial model (with the restriction $\rho = 0$) are presented in Table 5.

[Table 5 Here]

DIC supports the use a spatial model, particularly with a second-order contiguity spatial weights matrix and a $\rho \geq 0$ restriction which helps with the interpretation of results.

There are data revisions in Operating Expenditure between the NZ EDB database and the Disclosures leading to a possible break in the data in 2008 and 2009. The nine firms with significant upward revisions in variable cost in 2008 appear to have an impact on the time-varying average efficiency scores for 2008, but not for 2009. Regarding outages, 2007 was the most problematic year, but that is not reflected in efficiency scores. 2008 and 2009 are years with above average levels of SAIDI.

[Figure 5 Here]

The results in Figure 5 show an improvement in estimated average efficiency scores after the introduction of ownership unbundling. This is in line with the diagnosed problems which could lead to inefficiencies in the market before 1998. The levels of efficiency after 1999 are sustained and do not fall back to pre-unbundling levels. However, the large uncertainty around estimates does not allow to make statements on statistical significance. This is a very common issue with Stochastic Frontier modelling and a key reasons why applied research in the field, be it spatial or not, often does not present measures of uncertainty.

There is strong evidence of positive skewness of the random effects, pointing that there could be a reasonable amount of persistent inefficiency, as investigated by Filippini et al. (2017). However, there are two reasons why estimation of persistent inefficiency is not attempted here. Firstly, the identification of all components of the model would be complex, given the small panel size. Secondly, there are some considerations to have in mind when estimating the model. Even if extreme positive skewness of the random effects in the cost function exists, its meaning might not immediately point for the separation of a time-invariant convolution. These firms operate in extremely different geographical locations, with different contexts of supply in flat or mountainous terrain, and regions that persistently experience different issues with weather, vegetation and wild life. It is extremely difficult to account for these factors, and it is possible that an attempt to estimate a more detailed model would simply capture persistent extreme geographical and climatic circumstances of supply instead of meaningful managerial practices and cost efficiency issues which are of interest to a regulator. The firms with the most positive random effects in the results above are Buller Electricity and Marlborough Lines. In a scenario of a large signal-to-noise ratio of persistent inefficiency and low or negligible influence of a zero-mean random effect, these are likely to be the firms diagnosed as being more inefficient. These firms are located in the northern half of the South Island. Both firms contain National Parks inside their operational areas and also operate in colder than average and considerably more mountainous areas than other distributors in New Zealand. These are underlying conditions of the terrain in which the distributors operate and can hardly be attributed to cost inefficiency for regulation purposes just because they are not accounted for in the regression. A regulator could then take these findings and penalize the firms or perceive them as operating far away from best practices. The authors quantify the inefficiency from a Generalized True Random Effects model (GTRE) in this context, but do not show individual firm rankings, making it impossible to assess if their results also point in this direction. The estimated size of the random effects σ_α is very similar to these results (0.154 vs 0.189), so the aforementioned pitfalls of the analysis might apply.

6. Conclusion

This paper proposes a strengthening of the links between the Spatial Econometrics and Stochastic Frontier literature with a Bayesian approach that places spatial dependence in the efficiency component while also accounting for unobserved heterogeneity through the inclusion of random effects. A simple and easily implementable MCMC is outlined, allowing for flexible priors and estimation of efficiency assuming an exponential distribution. Efficiency measurement formulas in absolute and relative scales are proposed, adding to the ongoing debate in the literature. The paper also contributes to the applied Bayesian econometrics literature by highlighting the unexplored Guided Walk Metropolis method to take draws from non-standard distributions, a method which allows for modest performance gains without hurting computational speed due to its extreme simplicity. This is an overlooked and attractive alternative to other methods that are well established. The model shows strong performance under a variety of scenarios and sample sizes, while suffering from identification problems when signal-to-noise ratios are increasingly low, a usual drawback in stochastic frontier models. The modelling approach is then applied to the context of cost efficiency of New Zealand electricity distribution, a well-known case in the energy regulation literature. A spatial relationship in cost efficiency between firms is detected when using a second-order contiguity matrix and is defended by model selection criteria. This suggests some level of interaction between firms in the industry. An argument is made for not considering persistent inefficiency in this empirical case, but it is a path for future research.

References

Affuso, E., 2010. Spatial Autoregressive Stochastic Frontier Analysis: An Application to an Impact Evaluation Study. *SSRN Electronic Journal*. doi:10.2139/ssrn.1740382

Aigner, D., Lovell, C.K., Schmidt, P., 1977. Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics,* 6, 21–37.

Ansari, A., Mela, C.F., Neslin, S.A., 2008. Customer Channel Migration. *Journal of Marketing Research,* 45, 60–76. doi:10.1509/jmkr.45.1.60

Anselin, L., 1988. *Spatial Econometrics: Methods and Models*. Springer Netherlands, Dordrecht. Retrieved from http://dx.doi.org/10.1007/978-94-015-7799-1

Areal, F.J., Balcombe, K., Tiffin, R., 2012. Integrating spatial dependence into Stochastic Frontier Analysis: Integrating spatial dependence into SFA. *Australian Journal of Agricultural and Resource Economics,* 56, 521–541. doi:10.1111/j.1467-8489.2012.00597.x

Battese, G.E., Coelli, T.J., 1995. A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Economics,* 20, 325–332. doi:10.1007/BF01205442

Brehm, S., 2013. Fiscal Incentives, Public Spending, and Productivity – County-Level Evidence from a Chinese Province. *World Development,* 46, 92–103. doi:10.1016/j.worlddev.2013.01.029

Chen, Z., Barros, C.P., Borges, M.R., 2015. A Bayesian stochastic frontier analysis of Chinese fossil-fuel electricity generation companies. *Energy Economics*, 48, 136–144. doi:10.1016/j.eneco.2014.12.020

Druska, V., Horrace, W.C., 2004. Generalized Moments Estimation for Spatial Panel Data: Indonesian Rice Farming. *American Journal of Agricultural Economics*, 86, 185–198. doi:10.1111/j.0092-5853.2004.00571.x

Economic Insights, 2009. Economic Insights NZ EDB Database. Downloaded from: http://www.comcom.govt.nz/assets/Imported-from-old-site/industryregulation/Electricity/PriceQualityPaths/ContentFiles/Documents/comcom-economicinsightedbdatabaseandanalysisdatafiles-aug2009.zip

Feng, G., Zhang, X., 2012. Productivity and efficiency at large and community banks in the US: A Bayesian true random effects stochastic distance frontier analysis. *Journal of Banking & Finance*, 36, 1883–1895. doi:10.1016/j.jbankfin.2012.02.008

Fernández, C., Osiewalski, J., Steel, M.F.J., 1997. On the use of panel data in stochastic frontier models with improper priors. *Journal of Econometrics*, 79, 169–193. doi:10.1016/S0304-4076(97)88050-5

Filippini, M., Greene, W., 2016. Persistent and transient productive inefficiency: a maximum simulated likelihood approach. *Journal of Productivity Analysis*, 45, 187–196. doi:10.1007/s11123-015-0446-y

Filippini, M., Greene, W., Masiero, G., 2017. Persistent and transient productive inefficiency in electricity distribution. *Energy Economics*, Available online 5 December 2017, ISSN 0140-9883, https://doi.org/10.1016/j.eneco.2017.11.016.

Filippini, M., Wetzel, H., 2014. The impact of ownership unbundling on cost efficiency: Empirical evidence from the New Zealand electricity distribution sector. *Energy Economics*, 45, 412–418. doi:10.1016/j.eneco.2014.08.002

Fusco, E., Vidoli, F., 2013. Spatial stochastic frontier models: controlling spatial global and local heterogeneity. *International Review of Applied Economics*, 27, 679–694. doi:10.1080/02692171.2013.804493

Geweke, J., 1992. Evaluating the accuracy of sampling-based approaches to calculating posterior moments. *Bayesian Statistics*, 4, 169–173.

Glass, A.J., Kenjegalieva, K., Sickles, R.C., 2016. A spatial autoregressive stochastic frontier model for panel data with asymmetric efficiency spillovers. *Journal of Econometrics*, 190, 289–300. doi:10.1016/j.jeconom.2015.06.011

Greene, W., 2005. Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics*, 126, 269–303. doi:10.1016/j.jeconom.2004.05.003

Gustafson, P., 1998. A guided walk Metropolis algorithm. *Statistics and Computing*, 8, 357–364. doi:10.1023/A:1008880707168

Halleck Vega, S., Elhorst, J.P., 2015. The SLX Model. *Journal of Regional Science*, 55, 339–363. doi:10.1111/jors.12188

Hoffman, M. D., and A. Gelman. (2014). The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo. *Journal of Machine Learning Research.* 15, 1593–623.

Koop, G., 2010. *Bayesian econometrics*, Reprinted with corr., [Nachdr.]. ed. Wiley, Chichester.

Koop, G., Steel, M.F.J., 2003. Bayesian Analysis of Stochastic Frontier Models, in: Baltagi, B.H. (Ed.), A Companion to Theoretical Econometrics. Blackwell Publishing Ltd, Malden, MA, USA, pp. 520–537. doi:10.1002/9780470996249.ch25

Mastromarco, C., Serlenga, L., Shin, Y., 2016. Modelling Technical Efficiency in Cross Sectionally Dependent Stochastic Frontier Panels. *Journal of Applied Econometrics*, 31, 281–297. doi:10.1002/jae.2439

Mastromarco, C., Serlenga, L., Shin, Y., 2013. Globalisation and technological convergence in the EU. *Journal of Productivity Analysis*, 40, 15–29. doi:10.1007/s11123-012-0308-9

Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., Teller, E., 1953. Equation of State Calculations by Fast Computing Machines. *The Journal of Chemical Physics*, 21, 1087. doi:10.1063/1.1699114

Mundlak, Y., 1978. On the Pooling of Time Series and Cross Section Data. *Econometrica*, 46, 69. doi:10.2307/1913646

Neal, R. (2003). Slice Sampling. *The Annals of Statistics*, 31, 705–741.

Nepal, R., Carvalho, A., Foster, J., 2016. Revisiting electricity liberalization and quality of service: empirical evidence from New Zealand. *Applied Economics*, 48, 2309–2320. doi:10.1080/00036846.2015.1119789

Nillesen, P.H.L., Pollitt, M.G., 2011. Ownership Unbundling in Electricity Distribution: Empirical Evidence from New Zealand. *Review of Industrial Organization*, 38, 61–93. doi:10.1007/s11151-010-9273-5

Ozbugday, F.C., Nillesen, P.H.L., 2012. An Efficiency Analysis of New Zealand Electricity Distributors over Time, in: Barker, J.R., Walters, R. (Eds.), New Zealand and Australia in Focus: Economics, the Environment and Issues in Health Care. Nova Science Publishers, Hauppauge, N.Y, pp. 61–88.

Parmeter, C. and Kumbhakar, S., 2014. Efficiency Analysis: A Primer on Recent Advances. *Foundations and Trends in Econometrics*, 7, 191-385. doi: 10.1561/0800000023

Pavlyuk, D., 2013. Distinguishing Between Spatial Heterogeneity and Inefficiency: Spatial Stochastic Frontier Analysis of European Airports. *Transport and Telecommunication*, 14. doi:10.2478/ttj-2013-0004

Schmidt, A.M., Moreira, A.R.B., Helfand, S.M., Fonseca, T.C.O., 2009. Spatial stochastic frontier models: accounting for unobserved local determinants of inefficiency. *Journal of Productivity Analysis*, 31, 101–112. doi:10.1007/s11123-008-0122-6

Schmidt, P., Sickles, R.C., 1984. Production Frontiers and Panel Data. *Journal of Business & Economic Statistics*, 2, 367–374. doi:10.1080/07350015.1984.10509410

Spiegelhalter, D. J., Best, N. G., Carlin, B. P. and Van Der Linde, A., 2002. Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society*, Series B (Statistical Methodology), 64, 583–639. doi:10.1111/1467-9868.00353

Tsionas, E.G., 2002. Stochastic frontier models with random coefficients. *Journal of Applied Econometrics*, 17, 127–147. doi:10.1002/jae.637

Tsionas, E.G., Michaelides, P.G., 2016. A Spatial Stochastic Frontier Model with Spillovers: Evidence for Italian Regions. *Scottish Journal of Political Economy*, 63, 243–257. doi:10.1111/sjpe.12081

Vidoli, F., Cardillo, C., Fusco, E., Canello,J. , 2016. Spatial nonstationarity in the stochastic frontier model: An application to the Italian wine industry. *Regional Science and Urban Economics*, 61, 153-164, ISSN 0166-0462, doi:10.1016/j.regsciurbeco.2016.10.003

Wolf, L. J., Anselin, L. and Arribas-Bel, D. (2017), Stochastic Efficiency of Bayesian Markov Chain Monte Carlo in Spatial Econometric Models: An Empirical Comparison of Exact Sampling Methods. *Geographical Analysis*. doi:10.1111/gean.12135

Tables

Table 1. Simulation results for Scenario 1

	N=25 / T=10		N=25 / T=10		N=100 / T=10		N=100 / T=10	
Means	Est.	True	Est.	True	Est.	True	Est.	True
$\tilde{u}_{it}$	0.241	0.249	0.224	0.249	0.242	0.249	0.228	0.249
λ^{-1}	4.226	4	4.596	4	4.144	4	4.406	4
σ_α^2	0.049	0.05	0.049	0.05	0.051	0.05	0.051	0.05
σ_v^2	0.013	0.01	0.024	0.01	0.013	0.01	0.021	0.01
ρ	0.267	0.3	0.579	0.6	0.279	0.3	0.584	0.6
β_0	1.020	1	1.076	1	1.015	1	1.067	1
β_1	0.999	1	0.999	1	1.000	1	0.999	1
Mean Correlation between est. and true $\tilde{u}_{it}$	0.922		0.917		0.924		0.921	
Worst Correlation between est. and true $\tilde{u}_{it}$ across repetitions	0.853		0.849		0.893		0.888	

Table 2. Simulation results for Scenario 2

	N=25 / T=10		N=25 / T=10		N=100 / T=10		N=100 / T=10	
Means	Est.	True	Est.	True	Est.	True	Est.	True
$\tilde{u}_{it}$	0.098	0.100	0.097	0.100	0.099	0.100	0.098	0.100
λ^{-1}	10.851	10	11.024	10	10.264	10	10.390	10
σ_{α}^{2}	0.050	0.05	0.050	0.05	0.051	0.05	0.051	0.05
σ_{v}^{2}	0.011	0.01	0.013	0.01	0.011	0.01	0.012	0.01
ρ	0.183	0.3	0.472	0.6	0.207	0.3	0.494	0.6
β_0	1.016	1	1.054	1	1.013	1	1.050	1
β_1	0.999	1	0.999	1	1.000	1	1.000	1
Mean Correlation between est. and true $\tilde{u}_{it}$	0.739		0.750		0.739		0.749	
Worst Correlation between est. and true $\tilde{u}_{it}$ across repetitions	0.567		0.579		0.664		0.687	

Table 3. Description of dependent variable and explanatory variables

Variable	Description	Mean	Std. Dev.
VC	Total Operational Expenditure, deflated by the OECD energy consumer price index for New Zealand (base year=2005)[1], in millions of New Zealand dollars	10.685	16.001
ENERGY	Energy delivered in million KWh, calculated as the energy entering the network minus losses	0.00086	0.00153
CUSTOMERS	Number of customers	56090.11	99070.12
LOADFACTOR	Amount of electricity entering the system divided by the maximum demand multiplied by the total number of hours in the year	62.637	6.717
SAIDI	Average total duration of interruptions experienced by the customer	238.063	202.922
CUSTDENSITY	Length of business unit lines in km per each customer of the business unit	11.163	7.603
CAPITAL	Maximum system demand in KW (proxy for capital stock)	169.067	305.432
RATIO_UG	Ratio of overhead vs underground power lines	19.907	22.111

[1] Different deflation procedures exist in the literature. Nillesen and Pollitt (2011) deflate cost by a PPP index, while Filippini and Wetzel (2014) deflate cost by the OECD consumer price index for New Zealand. Note that in this case the price index was rebuilt using the average of quarterly data corresponding to the New Zealand fiscal year, finishing 31st of March each year, instead of the OECD calendar year data. Choice of deflators has little impact on time-varying behaviour of estimated average efficiency.

Table 4. Cost Frontier results. Note: Credible interval between 2.5% and 97.5% percentiles in [brackets], significant coefficients in bold. Cross-sectional averages and constant omitted.

	First-order contiguity matrix	Second-order contiguity matrix
ρ	**0.132** [0.001 ; 0.305]	**0.231** [0.035 ; 0.429]
β_{ENERGY}	0.099 [-0.132 ; 0.333]	0.102 [-0.126 ; 0.333]
$\beta_{\mathrm{CUSTOMERS}}$	**0.846** [0.625 ; 1.066]	**0.836** [0.609 ; 1.061]
$\beta_{\mathrm{LOADFACTOR}}$	-0.201 [-0.476 ; 0.089]	-0.194 [-0.467 ; 0.094]
β_{SAIDI}	**0.059** [0.024 ; 0.094]	**0.058** [0.023 ; 0.093]
$\beta_{\mathrm{CUSTDENSITY}}$	-0.193 [-0.434 ; 0.062]	-0.195 [-0.441 ; 0.056]
β_{CAPITAL}	**-0.133** [-0.227 ; -0.041]	**-0.127** [-0.221 ; -0.033]
$\beta_{\mathrm{RATIO\_UG}}$	0.008 [-0.101 ; 0.119]	0.013 [-0.094 ; 0.123]
$\beta_{\mathrm{TIME\ TREND}}$	**-0.089** [-0.110 ; -0.068]	**-0.086** [-0.108 ; -0.063]
$\beta_{\mathrm{TIME\ TREND^2}}$	**0.004** [0.003 ; 0.005]	**0.004** [0.003 ; 0.005]
$\sigma_\alpha^2 = \frac{1}{h_w}$	0.026 [0.013 ; 0.050]	0.026 [0.013 ; 0.049]
$\sigma_v^2 = \frac{1}{h_v}$	0.016 [0.011 ; 0.020]	0.015 [0.011 ; 0.020]
λ^{-1}	7.591 [6.149 ; 9.463]	7.535 [6.137 ; 9.234]
Signal-to-noise ratio	1.085 [0.767 ; 1.489]	1.105 [0.797 ; 1.495]

Table 5. DIC for different models

Model	DIC
Standard (Non-Spatial) SFA model	-163.29
First-order contiguity matrix - Unrestricted	-172.18
First-order contiguity matrix - Restricted	-176.39
Second-order contiguity matrix - Unrestricted	-198.08
Second-order contiguity matrix - Restricted	-202.81

Figures

Figure 1. Correlations between true and estimated values of $\tilde{u}$ and estimated spatial parameter for Scenarios 1 and 2 with different panel sizes.

Figure 2. Effective Throughput per second of competing sampling methods.

Figure 3. GWM vs RWM comparison in Scenario 1. Correlations between true and estimated values and % mean bias of the idiosyncratic efficiency component with N=49.

Figure 4. Density plots for spatial parameters with different spatial weights matrices

Figure 5. Time-varying average efficiency (second-order contiguity matrix, with 90% Credible Interval)