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Study on Interference of Optical Coherence Functions by Using Coherence Holographic Interferometry

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ABSTRACT

Interference, where the superposition of two or more waves resulting in a new wave pattern, has been considered as one of the most important and fundamental physical phenomenon. In optics, the statistical properties of light play an important role in determining the outcome of most optical experiments and the cross correlation between the fluctuating fields at different space time points, known as the optical coherence functions is a quantity of great interest. Due to the fact that the wave equations govern propagation of optical coherence, it can be envisaged that the interference phenomena also appear in the format of the optical coherence function. Meanwhile, the study of propagation and superposition of optical coherence function also provides theoretical and experimental foundations for coherence holography. In this paper, we have given the mathematical expression of interference of the optical coherence functions, and have proposed a full-field coherence visualization system for coherence holographic interferometry. The interference of two optical coherence functions has been experimentally investigated which can be regarded as an extension of Young's double-slit interferometer for optical waves. Some interesting phenomena, such as missing class of the coherence function and multiple-coherence function interference are demonstrated for the first time.

Keywords: Interference, Coherence function, Statistical optics, Coherence holography

1. INTRODUCTION

In superposition, waves are combined, and when waves combine they interfere. Since the optical coherence function satisfies the wave equations [1-3], it can be envisaged that the interference phenomena also appear in the format of the optical coherence function. In this paper, we used a coherence holographic interferometer to show how the coherence functions are distributed in the observation plane, while there are two or more extended sources.

2. PRINCIPLE

We will begin our discussion by examining the combination of two spatial incoherence light sources. Fig.1 shows two extended quasi-monochromatic light source I_1 and I_2 , both of them have a same wavelength λ . The natures of the two sources are completely arbitrary. The spatial coherence exists when fluctuations measured at different observation point Q_1 and Q_2 (located by position vector $\vec{u}_1$, $\vec{u}_2$ and $\vec{v}_1$, $\vec{v}_2$) are correlated. Referring to the well-known Van Cittert-Zerinke theorem [4, 5], the coherence function at the two points Q_1 and Q_2 on the observation plane is given by:

$$\tilde{\Gamma}_{total} = FT\{I_1 + I_2\}, \tag{1}$$

where $FT\{\ldots\}$ denotes the Fourier transform.

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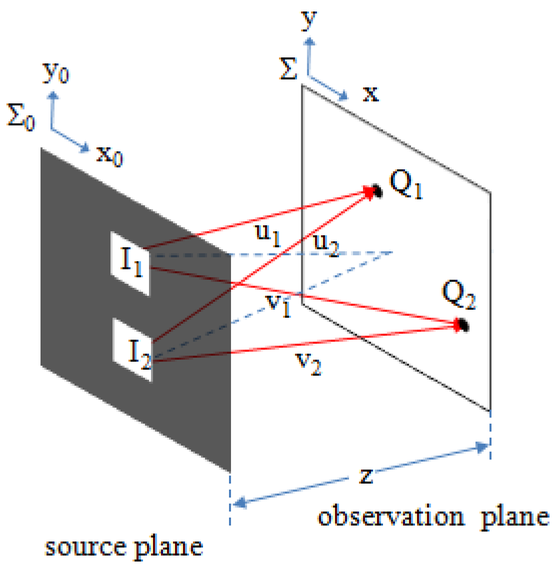

Figure 1. Schematic representation of the interference of the spatial coherence functions.

Let $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ be the coherence function when source I_1 and I_2 exists alone respectively. Here we have :

$$\tilde{\Gamma}_1 = FT\{I_1\}, \quad \tilde{\Gamma}_2 = FT\{I_2\}. \tag{2}$$

According to fundamental principle of Fourier transform:

$$FT\{I_1 + I_2\} = FT\{I_1\} + FT\{I_2\}. \tag{3}$$

On substituting Eq. (1) and Eq. (2) into Eq. (3) it follows that

$$\tilde{\Gamma}_{total} = \tilde{\Gamma}_1 + \tilde{\Gamma}_2. \tag{4}$$

Obviously, the total coherence function is equal to the superposition of all the coherence functions. Therefore the coherence interference phenomenon can be observed when there are two or more extended sources.

3. EXPERIMENTS AND RESULTS

3.1 Coherence holographic interferometer

The coherence holographic interferometer is a full field spatial coherence visualization system [6]. As shown in Fig.2, the target with two rectangle holes is firstly illuminated by a laser beam and then projected on a rapidly rotating ground glass to serve as spatially incoherent sources. The distance between the two sources is l, and the sizes of the sources are both $a \times a$. The intensity distribution of source I_1 and source I_2 are given by :

$$I_1 = rect(\frac{x_0}{a} + \frac{y_o}{a}) * \delta(x_0 - \frac{l}{2}, y_0). \tag{5}$$

$$I_2 = rect(\frac{x_0}{a} + \frac{y_o}{a}) * \delta(x_0 + \frac{l}{2}, y_0). \tag{6}$$

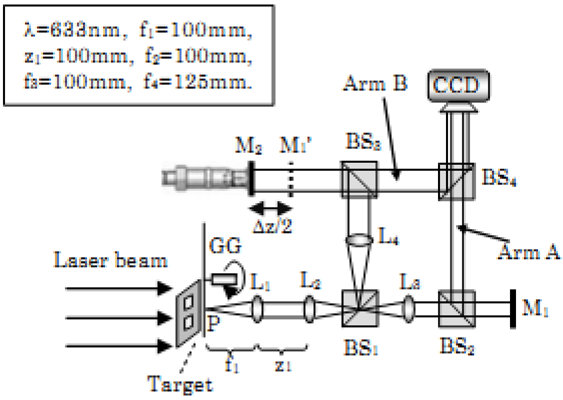

Figure 2. Coherence holographic interferometer.

The full-field coherence function detected by CCD is given by the expression:

$$\tilde{\Gamma}_{total}(x,y,Z) = \frac{\exp(jk\Delta z)}{\lambda^2 f_1^2 M_A M_B} \iint_S [I_1(x_0,y_0) + I_2(x_0,y_0)] \exp[-\frac{2j\pi(M_A - M_B)}{\lambda f_1 M_A M_B}(xx_0 + yy_0)] \exp[\frac{j\pi Z}{\lambda f_1^2}(x_0^2 + y_0^2)] dx_0 dy_0, \quad (7)$$

where, $M_A = f_3/f_2$ is magnification for arm A, $M_B = f_4/f_2$ is magnification for arm B, and $Z(\Delta z) = f_2(M_A - M_B)/(M_A M_B) - (z_2 + f_3 - f_4 + \Delta z)/M_B^2 + z_2/M_A^2$ is synthetic optical path difference (OPD) stemming from the path differences Δz between interfering arm A and arm B.

3.2 Interference patterns in the format of coherence function

Through an appropriate selection of Δz, we will make $Z = 0$, the phase factor in Eq.(7) disappears, leaving an exact Fourier transform relation:

$$\begin{aligned} \tilde{\Gamma}_{total}(x,y,0) &= FT\{rect(\frac{x_0}{a} + \frac{y_o}{a}) * \delta(x_0 - \frac{l}{2}, y_0) + rect(\frac{x_0}{a} + \frac{y_o}{a}) * \delta(x_0 + \frac{l}{2}, y_0)\} \\ &= 2a^2 \cos(\pi l f_x) \sin c(a f_x) \sin c(a f_y), \end{aligned} \quad (8)$$

where $f_x = x(M_A - M_B)/(\lambda f_1 M_A M_B)$, $f_y = y(M_A - M_B)/(\lambda f_1 M_A M_B)$.

Here, we are only focused on x-axis. We find,

$$\tilde{\Gamma}_{total\_x}(x,0,0) = 2a \cos(\pi l f_x) \sin c(a f_x). \quad (9)$$

The $\sin c-$function is contributed by diffraction patterns originating from each rectangle source, and cosine function is interference term. Bright interference fringes occur when $l f_x = m$ (with m any integer); and zeros of envelope $\sin c(a f_x)$ occur when $a f_x = n\pi$ (with n non-zero integer).

As shown in Fig.3, (a) is a interferogram produced by coherence holographic interferometer; (b) is the distribution of coherence function reconstructed from (a) by using Fourier fringe analysis [7]; the dotted line in (c) is the envelope of coherence diffraction pattern from a single rectangular source, and the solid line is interference pattern of the coherence functions from two rectangular sources.

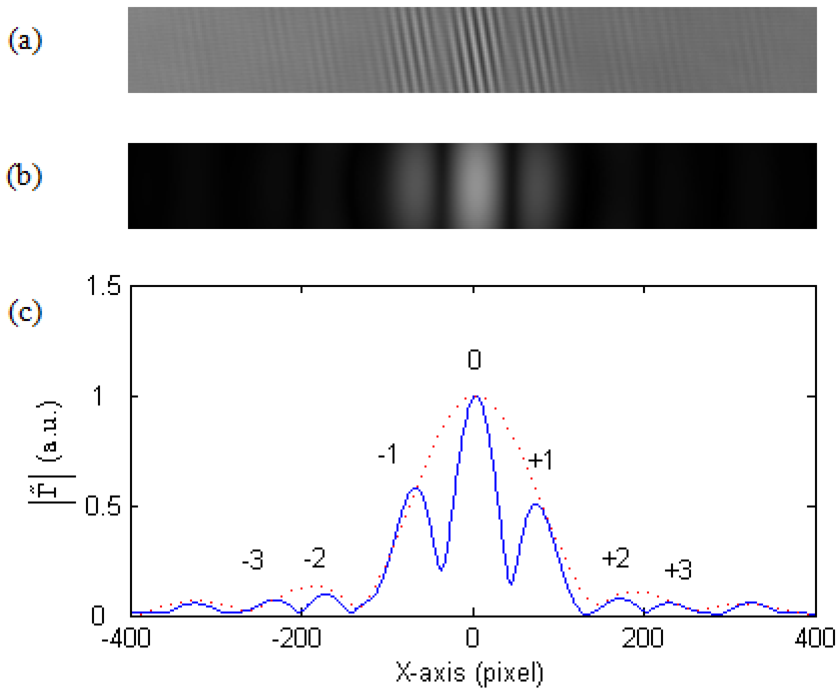

Figure 3. Coherence function from two rectangular sources. (a) Interferogram obtained at $Z=0$. (b) Reconstructed coherence function. (c) Curves of normalized $|\tilde{\Gamma}(x,0,0)|$, where the dotted line is the coherence function from a single rectangular source and the solid line is the coherence function from two rectangular sources.

In addition, let's recall Eq.(9). Let $l:a=\beta$, and β is an integer, interference term maximum coinciding with zero of envelope occur at the position where order number equal to multiple of β. This is called missing order phenomenon. In our experiment, the spacing ratio of hole pitch and hole size is $3:1$. As seen in fig.4, (a) is a interferogram; (b) is the reconstructed coherence function; the dotted line in (c) is the envelope of the diffraction pattern from a single rectangular source, and the solid line is the interference pattern, we can clearly see that the third class bright fringe missing.

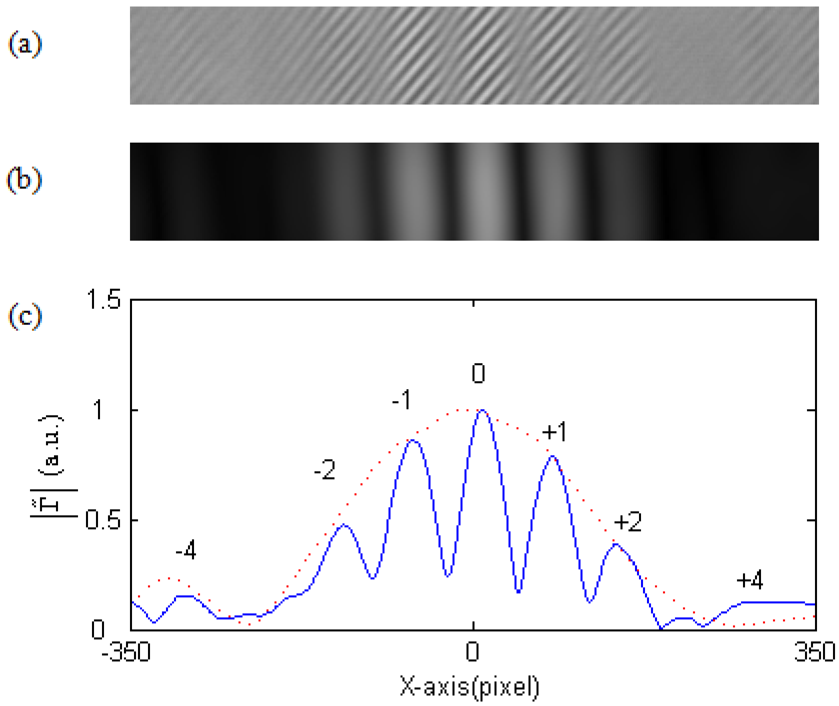

Figure 4. Coherence function from two rectangular sources, with spacing ratio of hole pitch and hole size is $3:1$. (a) Interferogram obtained at $Z=0$. (b) Reconstructed coherence function. (c) Curves of normalized $|\tilde{\Gamma}(x,0,0)|$, where the dotted line is the coherence function from a single rectangular source and the solid line is the coherence function from two rectangular sources. Missing orders occur at ± 3.

3.3 Interference of Multi-coherence function

It is interesting to re-examine Eq. (8) and Eq. (9). Consider a source plane which consists of more than two structures. Let's assume that the number of structures is N and all of these structures have a same rectangular shape, with width a and separation l. Then, we have

$$\tilde{\Gamma}_{total\_x}(x,0,0) = FT\{rect(\frac{x_0}{a})*\delta(x_0) + rect(\frac{x_0}{a})*\delta[x_0+l] + \cdots + rect(\frac{x_0}{a})*\delta[x_0+(N-1)l]\}$$
$$\propto a\,\mathrm{sin}c(af_x)\frac{\sin(N\pi l f_x)}{\sin(\pi l f_x)} \qquad (10)$$

It is evident from the preceding expression that the interference pattern is also enveloped by a $\sin c-$ function, the principle maxima occur when $lf_x = m$ (where the integer m refer to order number), and the secondary maxima occur when $lf_x = h/2N$ (h is an odd-integer).

Figure 5(a) illustrates the interferogram when $N=4$. After reconstruction, we got the coherence function, shown in fig.5(b). For simplicity, we provided the curve graph in fig.5(c). This curve will attribute changes of the coherence function. Strong constructive interference occurs at the principle maxima, with weaker secondary maxima in between. Between two adjacent principle maxima there are $N-1$ points of minima intensity. The two minima on either side of a principle maximum are separated by twice the distance of the others. And the secondary maxima are much smaller than the principle maxima.

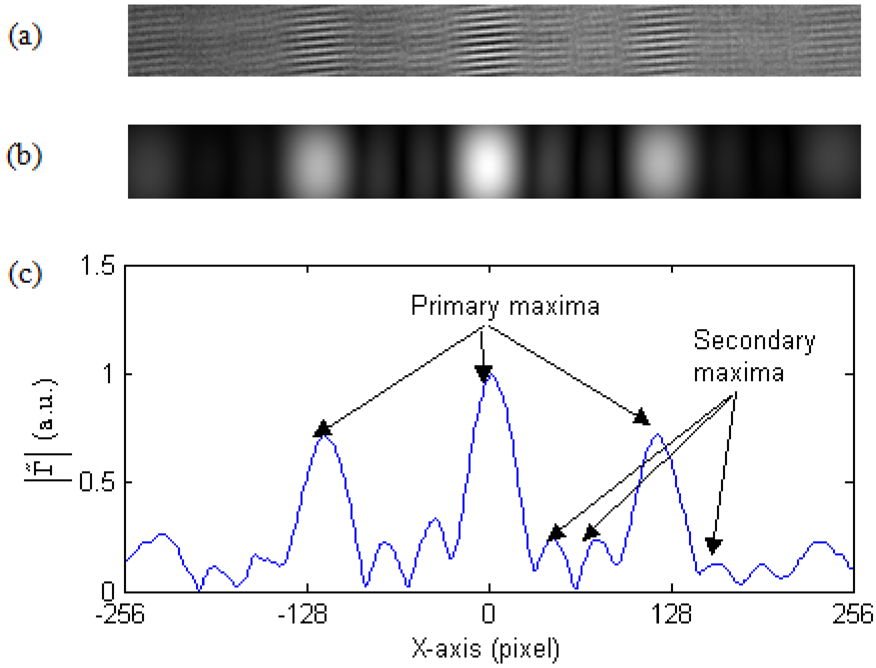

Figure 5. Interference of the spatial coherence function from four rectangular sources. (a) Interferograms obtained at $Z=0$. (b) Reconstructed coherence function. (c) Curves of normalized $\left|\hat{\Gamma}(x,0,0)\right|$.

3.4 The spatial evolution of the interference of coherence functions

According to the principle of independently propagating of light, light waves will continue to their original transmission regardless their overlapping. In other words, the propagation characters of the wave (propagation direction, speed, amplitude, phase, and polarization state) will not be affected the characters of the other wave. In the overlapping area, light vibration must obey the principle of wave superimposing. In vacuum, the principle of independently propagating of light and the principle of superposition is always established. These principles should also apply to coherence functions.

Consider the interferometer illustrated in fig.2. If the mirror M_2 is moved from the position required for equal path length in the two arms of the radial sheering interferometer, an optical path difference is introduced between the two interfering beams. As the mirror M_2 moves, the visibility of the observed fringes is changing. In the following fig.6 the coherence function detected from two rectangular sources varying with propagation is introduced. (a) shows the coherence function form source I_1. (b) shows the coherence function from source I_2 . And (c) shows the interference of these two coherence functions. The interference fringes can only be observed within superposition area (a triangle region wrapped by the dotted line in fig.6(c)) , and fringes spacing is fixed. This superposition area is determined by both the size of each single source and the distance between two sources.

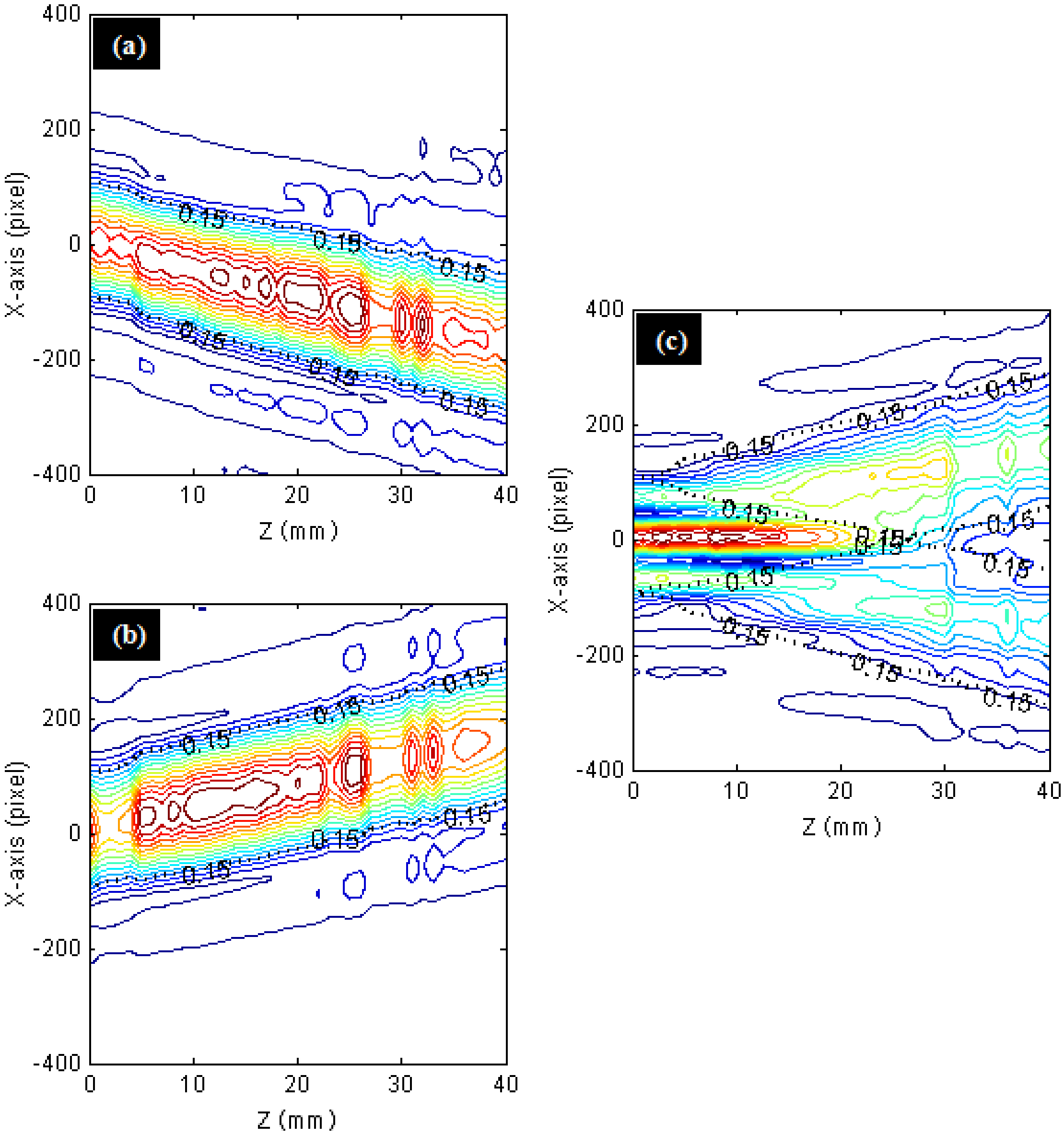

Figure 6. (a) Contour map of the coherence function originates from source I_1. (b) contour map of the coherence function originates from source I_2. (c) contour map of the interference of coherence functions from the two sources. Dotted contour lines with value of 0.15.

4. CONCLUSIONS

We introduced various phenomena of interference of coherence functions. Interference fringes in the format of coherence functions occurs in the region where two or more coherence patterns are superposed. In the superposition area, interference fringes are parallel. The interference patterns of coherence functions show some similar characters with optical interference, such as: variation of intensity distribution, lack of class, principal maxima and secondary maxima in multi-beam system.

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