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A Study on the Run Sum X-bar Control Chart with Unknown Parameters

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Abstract. It is well known that the run sum control chart is a simple and powerful statistical process control tool in the monitoring of the process mean. The implementation of the run sum chart is generally based on the assumption that the process parameters are known. However, since the process parameters are usually unknown in practice, they are estimated from an in-control Phase I data set. In this paper, by means of the Markov chain approach, we investigate the effects of parameter estimation on the performance of the run sum $\bar{X}$ chart with the scores 0, 1, 2 and 4. The results reveal that when the size of the shift and the number of samples from the Phase I process used for the estimation of parameters are both small, the performance of the run sum $\bar{X}$ chart is significantly deteriorated. Moreover, very large sample sizes are required for the chart with estimated parameters to have a favorable performance like the known parameters case. By virtue of this adverse performance, new charting parameters are proposed for practitioners in the design of the run sum $\bar{X}$ chart, based on the weights (0, 1, 2, 4) when parameters are estimated. The suggested parameters give a satisfactory performance even when process parameters are estimated from small number of samples.

Keywords: run sum $\bar{X}$ control chart; estimated parameters; average run length; standard deviation of the run length; Markov chain
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INTRODUCTION

Statistical Process Control (SPC) is a collection of statistical techniques used as a primary tool in reducing the variability of the desired quality characteristics of a product so that the final product of a high quality is produced. Control charts, viewed as the simplest and most crucial tool in SPC, are increasingly adopted in manufacturing and service industries. The traditional Shewhart $\bar{X}$ chart, having a remarkable performance in detecting large mean shifts, was originally developed by Shewhart [1] in the 1920s. Since then, many researchers have been focusing on improving the sensitivity of the Shewhart $\bar{X}$ chart in detecting small and moderate process mean shifts. In order to enhance and sensitize the performance of the Shewhart $\bar{X}$ chart, the run sum chart was developed by Reberts [2] as an intermediate solution.

The run sum chart involves dividing the control chart into regions or zones around the centre line and assigning a score to each region. The zone control chart which is a special case of the run sum chart was proposed by Jaehn [3] and studied further by Davis et al. [4]. Since Davis et al. [4] recommended employing the scores 0, 1, 2 and 4 or equivalently the scores 0, 2, 4 and 8, the run sum $\bar{X}$ chart with the weights (0, 1, 2, 4) is considered and discussed throughout this paper. Champ and Rigdon [5] manifested that the run sum $\bar{X}$ chart is superior to the Shewhart $\bar{X}$ chart with supplementary runs rules. Furthermore, they also claimed that by increasing the number of regions and scores, the run sum $\bar{X}$ chart is competitive with the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts in detecting mean shifts in the process. In light of the effectiveness and simplicity of the run sum chart, several researchers have contributed to this research area, including Ho and Case [6], Davis et al. [7], Aguirre-Torres and Reyes-López [8], Davis and Krehbiel [9], as well as Acosta-Mejia and Pignatiello [10].

The conventional application of the run sum $\bar{X}$ chart for monitoring the process mean is based on the assumption of known parameters. Nevertheless, the process parameters are unknown in many practical applications; hence, they are estimated from a set of in-control data acquired from the Phase I process. Because of the extra variability in the estimators, Quesenberry [11], Chen [12]-[13], Chakraborti [14]-[15], Jones et al. [16]-[17], as well as Capizzi and Masarotto [18], to name a few, showed that parameter estimation affects the performance of a control chart substantially. Therefore, the effect of parameter estimation should be taken into account in designing a control chart. In recent years, considerable attention has been devoted to propose new charting parameters or optimal parameters in devising control charts with

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estimated parameters; for the related literatures, see Jones [19], Castagliola et al. [20], Zhang and Castagliola [21], Zhang et al. [22] and Castagliola and Maravelakis [23].

To the best of the authors' knowledge, no researchers pay attention to the run sum chart with estimated parameters. In this paper, an analytical approach is adopted to study the performance of the run sum $\bar{X}$ chart (0, 1, 2, 4) when the parameters are unknown. Also, the new charting parameters are suggested to solve the variability incurred from parameter estimation. In the following sections, firstly, an introduction on the run sum $\bar{X}$ chart is presented. Next, the Markov chain approach of the run sum $\bar{X}$ chart with known parameters is reviewed. Then the run length properties of the run sum $\bar{X}$ chart with known parameters are compared with that of estimated parameters. Moreover, by taking the number of samples m and sample size n into account, the new limits of the regions are proposed for the run sum $\bar{X}$ chart with unknown parameters. Finally, some concluding remarks are summarized.

THE RUN SUM $\bar{X}$ CHART

Let Y_{ij} be the j^{th} observation in the i^{th} sample of a Phase II process, where $j = 1, 2, \ldots, n$. Let us assume that the observations Y_{ij} are independently and identically distributed having a normal distribution with the in-control mean μ_0 and the in-control variance $\sigma_0^{\,2}$. Figure 1 illustrates the regions, as well as their associated scores and probabilities for the run

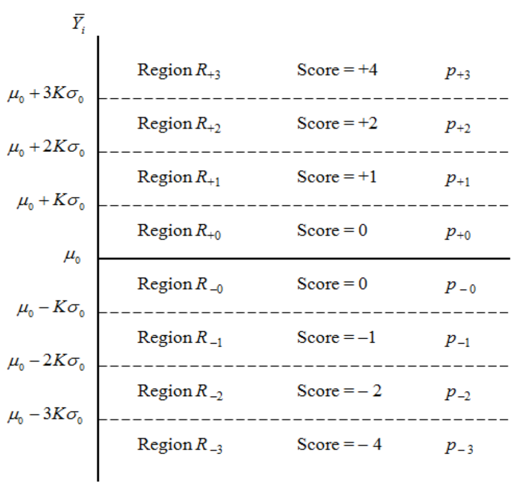

FIGURE 1. Regions, Scores and Probabilities for the Rum Sum $\bar{X}$ Control Chart.

sum $\bar{X}$ chart. In the rest of this paper, we consider four regions above and below a centre line set at μ_0 for the run sum $\bar{X}$ chart, for both cases of known and unknown parameters.

By referring to Fig. 1, ($\mu_0 + K\sigma_0 < \mu_0 + 2K\sigma_0 < \mu_0 + 3K\sigma_0$) and ($\mu_0 - K\sigma_0 > \mu_0 - 2K\sigma_0 > \mu_0 - 3K\sigma_0$) are the limits of the regions above and below the centre line μ_0 , respectively. Here, $K = K'/\sqrt{n}$ where K' is a constant independent of the sample size n. This two-sided run sum $\bar{X}$ chart is denoted by

$$RS_4\left[0,(0),K,(1),2K,(2),3K,(4),+\infty\right] \qquad (1)$$

and

$$RS_{-4}\left[-\infty,(-4),-3K,(-2),-2K,(-1),-K,(0),0\right]. \qquad (2)$$

According to Champ and Rigdon [5], the run sum $\bar{X}$ chart's procedure is outlined as follows:

1. Take a sample of size n and evaluate the sample mean $\bar{Y}_i = \sum_{j=1}^{n} Y_{ij}/n$.
2. Begin the cumulative score at 0.
3. Accumulate the score of the region in which the sample mean $\bar{Y}_i$ falls.
4. If $\bar{Y}_i$ is in the other side of the centre line μ_0 , start the cumulative score anew with the score corresponding to the current $\bar{Y}_i$.
5. If a positive cumulative score of greater than or equal to +4 is obtained, terminate sampling and conclude that the process is out-of-control.
6. If a negative cumulative score of less than or equal to –4 is obtained, the process is deemed as out-of-control.

From a mathematical point of view, the two-sided run sum $\bar{X}$ chart is based on the cumulative sum

$$CS_i = \max\left(U_i, -L_i\right) \qquad (3)$$

where

$$U_i = \begin{cases} U_{i-1} + S_i, & \text{if } \bar{Y}_i \ge \mu_0 \\ 0, & \text{if } \bar{Y}_i < \mu_0 \end{cases} \qquad (4)$$

and

$$L_i = \begin{cases} L_{i-1} + S_i, & \text{if } \bar{Y}_i \le \mu_0 \\ 0, & \text{if } \bar{Y}_i > \mu_0 \end{cases}. \qquad (5)$$

Here, $i = 1, 2, 3, \ldots$ and S_i is the score allocated to the region in which $\bar{Y}_i$ falls. For simplicity, we assume that $U_0 = 0$ and $L_0 = 0$.

In real industrial applications, the in-control mean μ_0 and standard deviation σ_0 of a process are unspecified. In such a case, they are both estimated from an in-control Phase I data set which comprises observations X_{ij}. Here, X_{ij} is the j^{th} Phase I observation in the i^{th} sample, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The estimator $\hat{\mu}_0$ of μ_0 is

$$\hat{\mu}_0 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} \tag{6}$$

and a (biased) estimator $\hat{\sigma}_0$ of σ_0 is

$$\hat{\sigma}_0 = \sqrt{\frac{1}{m(n-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{ij} - \bar{X}_i\right)^2}, \tag{7}$$

where $\bar{X}_i$ is the i^{th} sample mean, i.e.

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}. \tag{8}$$

THE RUN LENGTH PROPERTIES OF THE RUN SUM $\bar{X}$ CHART WITH KNOWN PARAMETERS

Without loss of generality, let us assume that the in-control mean μ_0 and the standard deviation σ_0 are known. The probability p_{+2} displayed in Fig. 1 can be procured as follows:

$$\begin{aligned} p_{+2} &= P\left(\bar{Y}_i \in R_{+2}\right) \\ &= P\left(\mu_0 + 2K\sigma_0 < \bar{Y}_i \le \mu_0 + 3K\sigma_0\right) \\ &= \Phi\left[(3K - \delta)\sqrt{n}\right] - \Phi\left[(2K - \delta)\sqrt{n}\right], \end{aligned} \tag{9}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution. Here, $\delta = |\mu_0 - \mu_1| / \sigma_0$ is the magnitude of the standardized mean shift with the out-of-control mean μ_1. The probabilities $p_{\pm k}$ for $k = \{0, 1, 2, 3\}$ can be obtained by a similar method as shown in Equation (9).

By applying the methods shown in Champ and Rigdon [5], the run sum $\bar{X}$ chart can be modeled as a discrete-time Markov chain with h+1 states, where the states 1, …, h are transient, whereas the state h+1 is an absorbing state. Note that h is the number of possible distinct values for the ordered pair (U_i, L_i) assuming an in-control process. For the run sum $\bar{X}$ chart with the weights (0, 1, 2, 4), the value of h is 7 and the state space Ω is denoted as $\Omega = \{-3, -2, -1, 0, 1, 2, 3, A\}$, where $A = \{\ldots, -5, -4, 4, 5, \ldots\}$ is an absorbing state. Therefore, the transition probability $\mathbf{P}$ has the following structure:

$$\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{r} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{pmatrix}, \tag{10}$$

where $\mathbf{Q}$ is a (7 × 7) matrix of transient probabilities, i.e.

$$\mathbf{Q} = \begin{pmatrix} p_{-0}+p_{+0} & 0 & p_{-2} & p_{-1} & p_{+1} & p_{+2} & 0 \\ p_{+0} & p_{-0} & 0 & 0 & p_{+1} & p_{+2} & 0 \\ p_{+0} & p_{-1} & p_{-0} & 0 & p_{+1} & p_{+2} & 0 \\ p_{+0} & p_{-2} & p_{-1} & p_{-0} & p_{+1} & p_{+2} & 0 \\ p_{-0} & 0 & p_{-2} & p_{-1} & p_{+0} & p_{+1} & p_{+2} \\ p_{-0} & 0 & p_{-2} & p_{-1} & 0 & p_{+0} & p_{+1} \\ p_{-0} & 0 & p_{-2} & p_{-1} & 0 & 0 & p_{+0} \end{pmatrix}. \tag{11}$$

Here, the vector $\mathbf{0}^{\mathrm{T}}$ in Equation (10) is $\mathbf{0}^{\mathrm{T}} = (0, 0, \ldots, 0)$. Also, the (7 × 1) vector $\mathbf{r}$ fulfills $\mathbf{r} = \mathbf{1} - \mathbf{Q}\cdot\mathbf{1}$ with $\mathbf{1}^{\mathrm{T}} = (1, 1, \ldots, 1)$, that is, the row probabilities must sum to 1. Meanwhile, $p_{\pm k}$ with $k = \{0, 1, 2\}$ is the probability as illustrated in Fig. 1 and is obtained as shown in Equation (9).

The probability density function (p.d.f) and the cumulative distribution function (c.d.f) of the run length of the run sum $\bar{X}$ chart can be acquired by employing the same approach as demonstrated in Zhang and Castagliola [21]. Similarly, refer to [21] for the equations of the average run length (*ARL*) and standard deviation of the run length (*SDRL*) by means of the Markov chain.

THE PERFORMANCE OF THE RUN SUM $\bar{X}$ CHART WITH KNOWN VERSUS ESTIMATED PARAMETERS

To gain an insight into the effect of parameter estimation on the performance of the run sum $\bar{X}$ chart, for both cases of known and unknown parameters, the *ARL* and *SDRL* are used as the evaluation criteria. When the process is in-control, the *ARL* and *SDRL* are denoted as ARL_0 and $SDRL_0$, respectively. On the contrary, ARL_1 and $SDRL_1$

represent the out-of-control *ARL* and *SDRL*, respectively.

The performances of the run sum $\bar{X}$ chart with the weights (0, 1, 2, 4), in terms of *ARL*s and *SDRL*s are summarized in Table 1. In particular, different combinations of the sample size $n = \{5, 8\}$ and the number of samples $m = \{10, 20, 40, 80, +\infty\}$ are considered in Table 1. Note that $m = +\infty$ corresponds to the case of known parameters, whereas $m = \{10, 20, 40, 80\}$ correspond to the case of estimated parameters. The process mean shifts $\delta = \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0\}$ are displayed in the first column. Here, $\delta = 0$ indicates an in-control process; while, $\delta > 0$ implies an out-of-control process. For all the cases shown in Table 1, the region limits are obtained in order to achieve the intended ARL_0 of 370.4 for the case of known parameters. It must be emphasized that the results in columns two to five are calculated based on the constant *K* corresponding to the case of known parameters ($m = +\infty$). Also, the *ARL*s and *SDRL*s in column six are computed by means of the Markov chain approach described in the previous section. Meanwhile, the *ARL*s and *SDRL*s in columns two to five are evaluated from the equations and Markov chain method for the case of estimated parameters, which are not detailed here. For each pair of *m* and δ, the entry in the first row corresponds to the *ARL* and that in the second row corresponds to the *SDRL*.

TABLE 1. *ARL* and *SDRL* of the Run Sum $\bar{X}$ Chart with the Weights (0, 1, 2, 4) for $n = \{5, 8\}$, $m = \{10, 20, 40, 80, +\infty\}$ and $\delta = \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0\}$ when the Constant, *K* Corresponds to the Case of Known Parameters

	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
δ	$n = 5,\ K = 0.5371$				
0.0	292.44	294.73	312.29	331.39	370.40
	763.21	485.35	403.61	375.32	366.84
0.2	155.68	116.89	93.21	80.39	68.55
	505.04	249.14	149.33	102.27	64.72
0.4	33.82	21.64	18.07	16.77	15.71
	143.54	39.33	20.09	15.41	12.29
0.6	9.15	7.75	7.32	7.14	6.97
	20.54	6.87	5.17	4.59	4.12
0.8	4.75	4.52	4.42	4.38	4.34
	3.74	2.59	2.27	2.14	2.01
1.0	3.27	3.21	3.18	3.17	3.16
	1.75	1.49	1.38	1.33	1.29
1.5	1.82	1.80	1.79	1.79	1.79
	0.84	0.79	0.77	0.76	0.75
2.0	1.23	1.22	1.21	1.20	1.20
	0.46	0.44	0.43	0.42	0.42
δ	$n = 8,\ K = 0.4246$				
0.0	248.13	272.10	299.90	324.57	370.40
	466.41	382.74	356.91	351.86	366.84
0.2	96.33	70.50	55.49	48.54	42.71
	252.53	139.98	82.01	56.74	38.94
0.4	14.86	11.43	10.40	9.99	9.63
	39.21	13.23	8.82	7.51	6.51
0.6	5.25	4.94	4.81	4.76	4.71
	4.49	2.98	2.58	2.42	2.27
0.8	3.22	3.16	3.13	3.12	3.11
	1.66	1.43	1.34	1.30	1.26
1.0	2.33	2.30	2.30	2.29	2.29
	1.07	0.99	0.96	0.95	0.93
1.5	1.30	1.29	1.28	1.28	1.28
	0.52	0.50	0.49	0.49	0.48
2.0	1.03	1.02	1.02	1.02	1.02
	0.17	0.16	0.15	0.14	0.14

Comparing the results in Table 1, we notice that adopting estimates in place of known parameters reduces the ARL_0 and increases the ARL_1 of the chart. The actual *SDRL*s are higher than the analogous case with known parameters, indicating higher probabilities in the tails of the run length distribution. The actual ARL_0 is smaller than the expected value of 370.4, suggesting a significant increase in the false alarm rate when estimates are used. As expected, the sensitivity of the chart with estimated parameters in detecting a process shift is reduced when the traditional region limits desired for the case of known parameters are used. For a fixed ARL_0, *n* and *K*, the smaller the values of *m* and δ, the larger the difference between the values of ARL_1 and $SDRL_1$ corresponding to the cases of known and estimated parameters. Also, as *m* increases, the difference decreases. For example, if $n = 5$, $\delta = 0.2$ and $K = 0.5371$, the ARL_1 and $SDRL_1$ calculated are 155.68 and 505.04, respectively when $m = 10$, as opposed to 68.55 and 64.72 when $m = +\infty$. When *m* increases to 80, we have $ARL_1 = 80.39$ and $SDRL_1 = 102.27$. This situation becomes better when $n = 8$. In general, when the process mean shifts are large ($\delta \geq 1$), the effect of estimation error becomes negligible even with small reference sample *m* and sample size *n*. For small and moderate shifts ($\delta \leq 0.8$), more than 80 samples are required to minimize the effect of estimation for the region limits of the run sum $\bar{X}$ chart.

Figure 2 graphically exhibits a comparison of the c.d.f functions of the run length distribution, for the cases of estimated parameters ($m = \{10, 20, 40, 80\}$) drawn in dotted line and the case of known parameters ($m = +\infty$) drawn in solid line, for $n = 5$, $\delta = 0.8$ and $K = 0.5371$. Figure 2 provides evidence of a significantly deteriorated chart's performance as a result of parameter estimation. Indisputably, Fig. 2 confirms our previous conclusion that, as the number of samples *m* increases, the run length distribution of the run sum $\bar{X}$ chart with estimated parameters tends to that of the theoretical case. Clearly, at least 80

Phase I samples are needed to yield a favorable performance as expected.

In real-world applications, the suggested large sample size is economically impractical especially for a low production process. In addition, using a control chart with parameters estimated from a large Phase I sample m will give rise to another problem. It is time consuming and undetected parameter shifts may occur while collecting the necessary data. Undeniably, it is worthwhile to compute the alternative constants K dedicated to each pair of m and n. These new constants K are adjusted to meet the desired ARL_0 of 370.4 for each case shown in Table 2. Here, Table 2 delineates the ARL_1 and $SDRL_1$ of the run sum $\bar{X}$ chart with estimated parameters based on the weights (0, 1, 2, 4), along with their respective constant K for $ARL_0 =$ 370.4, as well as different combinations of n, m and δ. For instance, when $n = 5$, $m = 20$ and $\delta = 0.4$, the value of the new constant K is 0.5518 and the corresponding ARL_1 is 24.02 and $SDRL_1$ is 47.22. The case of $m = +\infty$ is given as reference.

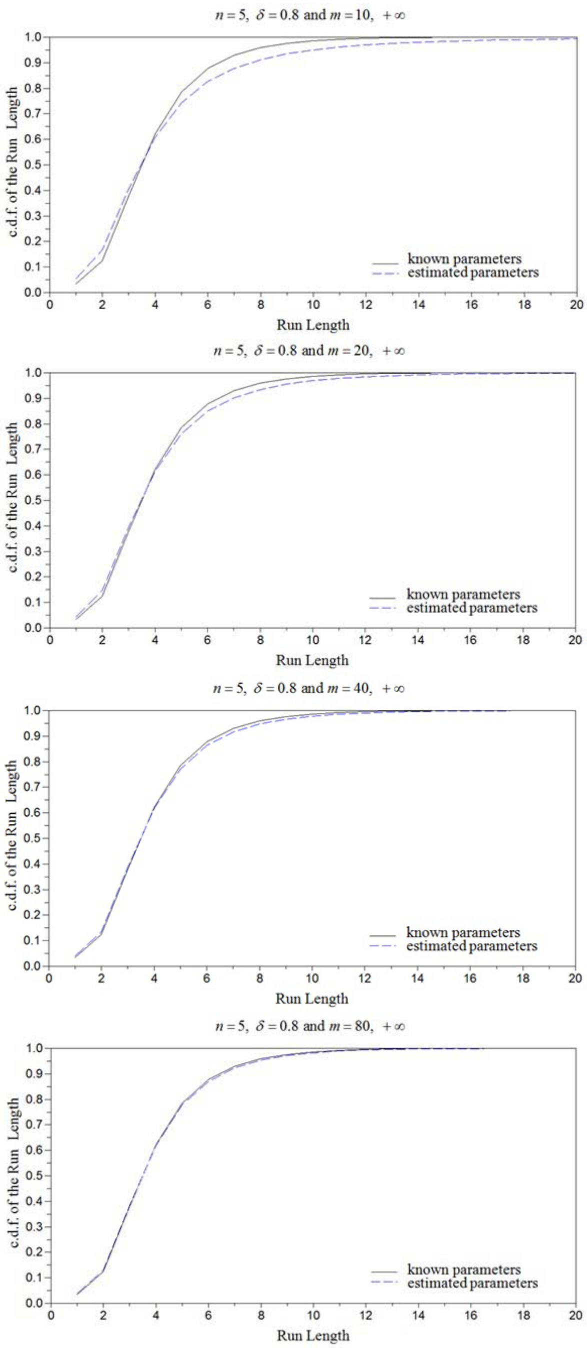

FIGURE 2. c.d.f. of the Run Length Distribution of the Run Sum $\bar{X}$ Chart with the Weights (0, 1, 2, 4) for $n = 5$, $m = \{10, 20, 40, 80, +\infty\}$, $\delta = 0.8$ and $K = 0.5371$.

TABLE 2. K, ARL_1 and $SDRL_1$ of the Run Sum $\bar{X}$ Chart with Estimated Parameters Based on the Weights (0, 1, 2, 4) when $ARL_0 = 370.4$, $n = \{5, 8\}$, $m = \{10, 20, 40, 80, +\infty\}$ and $\delta = \{0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0\}$.

	$n = 5$				
K	0.5514	0.5518	0.5482	0.5443	0.5371
δ	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
0.2	193.48	141.60	105.84	86.65	68.55
	684.30	320.27	175.43	111.80	64.72
0.4	39.28	24.02	19.30	17.46	15.71
	190.21	47.22	22.00	16.19	12.29
0.6	9.82	8.20	7.61	7.32	6.97
	25.55	7.49	5.43	4.73	4.12
0.8	4.97	4.71	4.56	4.47	4.34
	4.04	2.72	2.34	2.17	2.01
1.0	3.40	3.33	3.27	3.23	3.16
	1.81	1.53	1.41	1.35	1.29
1.5	1.88	1.87	1.85	1.82	1.79
	0.86	0.81	0.79	0.77	0.75
2.0	1.26	1.25	1.23	1.22	1.20
	0.49	0.47	0.45	0.44	0.42

	$n = 8$				
K	0.4451	0.4407	0.4356	0.4314	0.4246
δ	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
0.2	136.23	88.72	63.44	52.33	42.71
	401.96	190.88	97.65	62.08	38.94
0.4	17.72	12.63	11.05	10.36	9.63
	57.76	15.59	9.55	7.85	6.51
0.6	5.70	5.24	5.01	4.88	4.71
	5.26	3.20	2.70	2.48	2.27
0.8	3.44	3.33	3.25	3.19	3.11
	1.76	1.49	1.37	1.32	1.26
1.0	2.48	2.43	2.38	2.34	2.29
	1.10	1.02	0.97	0.95	0.93
1.5	1.37	1.34	1.32	1.30	1.28
	0.57	0.54	0.52	0.50	0.48
2.0	1.04	1.03	1.03	1.02	1.02
	0.20	0.18	0.16	0.15	0.14

At the first glance from Table 2, the increasing false alarm rate is solved by the introduction of the new constant *K*, where it establishes a wider region limits for the run sum $\bar{X}$ chart with estimated parameters. However, as expected, when estimates are used, the ARL_1s are larger than the corresponding case of known parameters. It is noticeable that as *m* increases, the values of *K*, ARL_1 and $SDRL_1$ generally converge towards the case of known parameters. Although this new proposed constant *K* slightly decreases the sensitivity of the chart in detecting process changes, the improvement shown in ARL_0 outweighs the costs of taking more Phase I observations. A reasonable number of Phase I samples *m* used in practice is 10 or 20. Therefore, the results in Table 2 provide practitioners with the charting parameter *K* for selected pairs of *m* and *n*, which are particularly useful when large samples are unavailable.

CONCLUSION

This paper clearly demonstrates that the performances of the run sum $\bar{X}$ chart with the weights (0, 1, 2, 4), in terms of the c.d.f. of the run length distribution, *ARL* and *SDRL* are seriously affected by parameter estimation. For the case with estimated parameters, a large number of samples *m* ($m > 80$) is necessary, in order to have a chart's performance similar to the case with known parameters. Since the use of a large number of samples is infeasible in a real situation, an interesting contribution of this study is to provide the new constant *K*, taking *m* and *n* into account. Unequivocally, the proposed constant *K* alleviates the problems of the need to use a large number of samples in the estimation of parameters and the high false alarm rates incurred if a small number of samples is used to estimate the process parameters.

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