The sensitivity analysis of the economic and economic statistical designs of the synthetic X chart

The Sensitivity Analysis of the Economic and Economic Statistical Designs of the Synthetic $\bar{X}$ Chart

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Abstract. The economic and economic statistical designs allow the practitioner to implement the control chart in an economically optimal manner. For the economic design, the optimal chart parameters are obtained to minimize the cost, while for the economic statistical design, additional constraints in terms of the average run length is imposed. However, these designs involve the estimation of quite a number of input parameters. Some of these input parameters are difficult to estimate accurately. Thus, a sensitivity analysis is required in order to identify which parameters need to be estimated accurately, and which requires just a rough estimation. This study focuses on the significance of 11 input parameters toward the optimal cost and average run lengths of the synthetic $\bar{X}$ chart. The significant input parameters are identified through a two-level fractional factorial design, which allows interaction effects to be identified. An analysis of variance is performed to obtain the *P*-values by using the Minitab software. The significant input parameters and interactions on the optimal cost and average run lengths are identified based on a 5% significance level. The results of this study show that the input parameters which are significant towards the economic design may not be significant for the economic statistical design, and vice versa. This study also shows that there are quite a number of significant interaction effects, which may mask the significance of the main effects.

Keywords: economic design, economic statistical design, interaction effect, significance of input parameters, synthetic chart.
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INTRODUCTION

Control charts are used to control and monitor one or more variables based on statistical principles. Since the first control chart, the Shewhart $\bar{X}$ chart, was proposed, extensive studies have been done to improve the performance and robustness of control charts.

In order for the chart to show good performance, the control chart parameters have to be selected in an optimal manner. In the existing literature, there are four categories of control chart designs, which are (i) heuristic designs, (ii) statistical designs, (iii) economic designs and (iv) economic-statistical designs.

Heuristic designs depend on rules of thumb for the selection of the chart parameters. For instance, we generally take a sample of size 4 or 5 with a control limit coefficient of 3, while the frequency of sampling usually depends on the preferences of the practitioner. The advantage of the heuristic design is that it is easy to adopt in practice, however this design may not show good statistical performance, and can be expensive to adopt.

In statistical design, chart parameters are chosen in order to optimize the chart's statistical performance [1]. However, the cost of implementing the chart is not considered in a purely statistical design, and this may result in the chart to be expensive to implement [2]. This is especially so when the cost of sampling, false alarms and inability to detect the out-of-control signals are high. Therefore, the economic design should be used when there are significant economic consequences. Economic designs strive to minimize the cost or maximize the profit.

However, economic designs are frequently criticized for their poor statistical properties [3]. This is because when the chart parameters are selected based on a purely economic criterion, the chart may show poor statistical performance. For example, frequent false alarms may occur and this leads to a reduced confidence in the control chart. To overcome the weakness of the economic design, [4] developed the economic-statistical design. In this

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design, statistical constraints are integrated into the economic design. The economic-statistical design improves the economic performance of statistical designs, while improving the statistical performance of the economic design. The economic statistical design is considered to be the most wholesome and robust measure in control chart design, as it consists of both cost and statistical considerations.

A large number of parameters need to be estimated for both economic and economic-statistical designs. Thus, sensitivity analysis is commonly performed to determine which input parameters have a significant effect toward the optimal cost and average run lengths (*ARL*s). This allows us to know which parameters need to be estimated accurately, and which parameters need just a rough estimation.

However, in the current literature, most sensitivity analyses are performed based on a "one-factor-at-a-time" basis. In this approach, only one of the input parameters is changed, while all the other parameters are fixed. This approach ignores possible interaction effects among the input parameters. Thus, the results of the sensitivity analyses may not be conclusive, and may lead to erroneous conclusions regarding the accuracy required in estimating the input parameters.

In the research carried out by [5], a sensitivity analysis is performed on the Shewhart $\bar{X}$ chart through an analysis of variance applied to a two level fractional factorial design. In this approach, the two factor interactions are not ignored. It was found that almost all input parameters have significant effect when interactions between input parameters are taken into account. This contradicts the "one-factor-at-a-time" approach which usually identifies few significant factors.

In this research, we will look at the sensitivity toward the optimal cost and *ARL*s with respect to different values of the input parameters by using analysis of variance for a two-level fractional factorial design. The sensitivity analysis is performed based on the economic and economic statistical designs of the synthetic $\bar{X}$ chart. The results of this project will allow us to identify any significant interaction effects among the input parameters for the synthetic $\bar{X}$ chart.

This research has wide applications and usefulness in industries. Management would like to know the factors which contribute to the increase in cost in their quality control program, so that improvements can be made on the correct factors. Besides that, by knowing the significant factors towards cost, management will know the factors which need to be estimated accurately, and which requires just a gross estimate to obtain an accurate estimation of the cost of implementing their quality control program.

The remainder of this paper is organized as follows: The next section shows the operation of the synthetic $\bar{X}$ chart, followed by a description of the cost model. The subsequent section illustrates the sensitivity analysis, which shows the significant input parameters and interactions toward the optimal cost and average run lengths of the synthetic $\bar{X}$ chart, for both economic and economic statistical designs. Finally, conclusions are drawn in the last section.

THE SYNTHETIC $\bar{X}$ CHART

The synthetic $\bar{X}$ chart was proposed by [6]. This chart combines the Shewhart $\bar{X}$ chart with that of a conforming run length (*CRL*) chart. The synthetic chart can detect all levels of mean shift better than the standard $\bar{X}$ chart, while compared with the EWMA and joint $\bar{X}$-EWMA charts, the synthetic chart can better detect mean shifts of greater than 0.8 standard deviations. Some of the recent studies on synthetic chart includes [7], who maximized the zero and steady state run lengths; [8], who studied the weighted variance (WV) and scaled weighted variance (SWV) synthetic mean charts for skewed distributions; [9], who proposed a synthetic control chart based on two sample variances for monitoring the covariance matrix of bivariate processes; [10], who developed a generalized synthetic chart; [11], who propose a combination of synthetic $\bar{X}$ and an $\bar{X}$ chart for monitoring the process mean; [12], who studied a synthetic double sampling control chart; [13], who evaluated the performance of the synthetic chart when the process parameters are estimated; [14], who determined the optimal design of the synthetic $\bar{X}$ chart for the process mean based on median run length; [15], who proposed an economically optimum design of a synthetic $\bar{X}$ chart; and [16], who proposed the economic and economic statistical designs of the synthetic $\bar{X}$ chart using loss functions. The combined synthetic $\bar{X}$ and $\bar{X}$ chart proposed by [11] and the synthetic double sampling chart proposed by [12] improves the performance of the synthetic chart by [6], while the other papers study the operation of the synthetic chart under different conditions and different optimization criterion.

The following shows the steps on how to operate the synthetic chart ([6]):

1. Identify the upper and lower control limits, $LCL_{\bar{X}/S}$ and $UCL_{\bar{X}/S}$, of the $\bar{X}/S$ sub-chart and the lower limit, *L* of the *CRL*/*S* sub-chart, where

$$LCL_{\bar{X}/S} = \mu_0 - k\sigma_{\bar{X}} \tag{1a}$$

and

$$UCL_{\bar{X}/S} = \mu_0 + k\sigma_{\bar{X}}, \tag{1b}$$

with μ_0 is the in-control process mean and $\sigma_{\bar{X}}$ is the standard deviation of the sample mean.

2. A random sample of n measurements is determined and the sample mean, $\bar{X}$ is computed for each sampling point.
3. When the sample mean, $\bar{X}$ is located between $LCL_{\bar{X}/S}$ and $UCL_{\bar{X}/S}$, the sample is known as a conforming sample, otherwise the sample is non-conforming.
4. Compute the number of sample means from the initial non-conforming sample (included in the calculation) and the last non-conforming sample (not included in the calculation). This value is considered to be the *CRL* value of the *CRL/S* sub-chart.
5. When the value of *CRL* is greater than *L*, then the process is considered to be in-control. However, if this is not the case, an out-of-control signal will be produced and the assignable cause has to be removed to return the process to an in-control state.

For synthetic chart, the ARL_1 is the average number of sample means needed to signal a shift in the mean. ARL_1 is given by ([6])

$$ARL_1 = \frac{1}{P}\left[\frac{1}{1-(1-P)^L}\right], \tag{2}$$

where $P = 1 - \Phi\left(k - \delta\sqrt{n}\right) + \Phi\left(-k - \delta\sqrt{n}\right)$ and $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution. The in-control *ARL*, ARL_0, can thus be calculated as

$$ARL_0 = \frac{1}{2\Phi(-k)}\left[\frac{1}{1-\left[1-2\Phi(-k)\right]^L}\right]. \tag{3}$$

THE COST MODEL

A cost function, constructed by using the general cost function of [17], is used to compute the optimal parameters of the synthetic chart. It is assumed that the process starts in a state of control and follows a normal distribution with mean μ_0 and standard deviation σ. A single assignable cause occurs at a random time and causes a shift in the process mean of a known magnitude $\delta\sigma$ so that the mean value is $\mu_1 = \mu_0 \pm \delta\sigma$. The occurrence of the assignable cause indicates that the process signal as out-of-control. Note that in this paper the assignable cause only results in changes in the mean. The time until the assignable cause occurs is assumed to be exponentially distributed with rate λ. The process stays at the out-of-control level until a lack of control is indicated by the synthetic chart and adjustments are made to bring it back to the in-control level $(\mu = \mu_0)$. Using the approximation of [18] on Lorenzen and Vance's cost model, the following expected cost per unit time, *C* is obtained:

$$\left(\frac{C_0}{\lambda} + C_1 B + \frac{b+cn}{h}\left(\frac{1}{\lambda}+B\right) + \frac{sY}{ARL_0} + W\right) \div \left(\frac{1}{\lambda} + \frac{(1-\gamma_1)sT_0}{ARL_0} + EH\right), \tag{4}$$

where $B = \left(ARL_1 - 0.5\right)h + F,$

$$F = ne + \gamma_1 T_1 + \gamma_2 T_2,$$

$$s = \frac{1}{\lambda h} - 0.5,$$

$$EH = \left(ARL_1 - 0.5\right)h + G,$$

and

$$G = ne + T_1 + T_2.$$

The parameters in Eq. (4) are defined as follows:

ARL_0 In-control average run length

ARL_1 Out-of-control average run length

b	Fixed cost per sample
c	Cost per unit sampled
C	Expected cost per hour
C_0	Expected quality cost per hour while in-control
C_1	Expected quality cost per hour while out-of-control
e	Expected time to sample and interpret one unit
h	Sampling interval
n	Sample size
T_0	Expected search time for a false alarm
T_1	Expected time to find the assignable cause
T_2	Expected time to repair the process
W	Cost of finding and fixing an assignable cause
Y	Cost of false alarm
γ_1	= 1, if production continues during search = 0, if production stops during search
γ_2	= 1, if production continues during repair = 0, if production stops during repair
λ	Process failure rate

By substituting Eqs. (2) and (3) into Eq. (4), we will obtain the cost function for the synthetic chart.

In the economic design, the optimal design parameters are the lower limit for the *CRL / S* sub-chart (L), sample size (n), control limit coefficient for $\bar{X}/S$ sub-chart (k), and sampling interval (h) that minimize the expected cost per hour, C in Eq. (4). In the economic statistical design, the optimal design parameters are L, n, k, and h that minimize the expected cost per hour, subjected to constraints on the ARL_0 and ARL_1. In this paper, we set $ARL_0 \geq 250$ and $ARL_1 \leq 20$. The optimal design parameters are computed based on the method in Yeong *et al.* (2012).

SENSITIVITY ANALYSIS

In this section, the sensitivity analyses of the optimal cost and average run lengths (*ARLs*) with respect to the input parameters are performed through an experimental design approach, where the input parameters play the role of factors, while the optimal cost and *ARLs* are the response variables. The experimental design schedules two levels for each factor. To fully examine the effects of these input parameters, it is necessary to cover a reasonable complete range of these parameters. TABLE 1 shows the low and high levels of the input parameters.

A 2_V^{11-4} factorial design of revolution V with 128 total runs is selected, where two-factor interactions may be estimated. A single experimental run consists of the minimization of Eq. (4), with respect to n, k, L and h when the input parameters are fixed according to the profile of the corresponding factor levels. An analysis of variance is performed at the 5% significance level to identify significant input parameters and interactions on the optimal cost and *ARLs*.

TABLES 2 and 3 show the *P*-values of the significant input parameters in the analysis of variance for the optimal cost and *ARLs* for the economic and economic statistical designs of the synthetic $\bar{X}$ chart. The positive and negative signs inside the brackets represent positive and negative effects of the input parameters. When the *P*-values in TABLES 2 and 3 show a value of 0.000, it indicates that the *P*-value is less than 0.001. The empty entries in TABLES 2 and 3 show that the factors and interactions are not significant, due to having *P*-values of more than 0.05. Based on the main effects, the input parameters which have a positive effect toward the response results in an increase to the response when it increases from the low level to the high level, while the input parameters which have a negative effect toward the response results in a decrease to the response when it increases from the low level to the high level.

TABLE (1): Low and High Levels for Input Parameters

Parameter	Low	High
b: Fixed cost per sample	0.5	5.0
c: Cost per unit sampled	0.1	1.0
Y: Cost per false alarm	50	500
W: Cost of finding and fixing an assignable cause	25	250
C_0 : Expected quality cost per hour while in-control	100	200
C_1 : Expected quality cost per hour while out-of-control	250	500
e: Expected time to sample and interpret one unit	0.05	0.50
T_1: Expected time to find the assignable cause	0.5	10
T_2: Expected time to repair the process	2	20
λ :Process failure rate	0.01	0.05
δ : Magnitude of shift in multiples of standard deviation	0.5	2.5

For significant interaction effects, the effect of an input parameter towards the response is different at different levels of the input parameter from which it interacts with. Note that we will refer to this parameter as the second factor. For positive interaction effect, a factor which results in an increase in the response at both the high and low levels of the second factor will result in a larger magnitude of increase at the high levels of the second factor. On the other hand, a factor which results in a decrease in the response at both the high and low levels of the second factor will result in a larger magnitude of decrease at the low levels of the second factor. Besides that, the factors may have different effects toward the response at a different level of the second factor. When the interaction is positive, it shows positive effects at the high levels, while when the interaction is negative, it shows negative effects at low levels. Note that for negative effects, the effects are totally opposite of the positive effects.

For the economic design, TABLE 2 shows that c, W, e and λ do not have significant effects toward the optimal cost because the P-values are larger than 0.05, while Y, C_0, C_1 and δ have positive significant effects, and b and T_2 show negative significant effects. For the economic statistical design (TABLE 3), Y and W do not have significant effects toward the optimal cost, while b, c, C_0, C_1, e, T_1 and λ have positive significant effects towards cost, and the input parameters T_2 and δ show negative significant effects towards cost. For significant interaction effects toward cost, for the economic design, there are four positive significant interaction effects ($b*\delta$, $Y*C_0$, $T_1*\delta$ and $\lambda*\delta$) and five significant negative interaction effects ($Y*\delta$, $C_1*\delta$, $e*T_1$, $T_2*\lambda$ and $T_2*\delta$), while for the economic statistical design, there are five significant positive interactions (C_1*e, C_1*T_1, $C_1*\lambda$, $e*\lambda$ and $T_1*\lambda$) and eight significant negative interactions ($c*\delta$, C_0*T_2, $C_0*\lambda$, C_1*T_2, $C_1*\delta$, $e*\delta$, $T_2*\lambda$ and $\lambda*\delta$).

Next, we look at the significant effects toward the in-control average run length (ARL_0). For the economic design (TABLE 2), there are five input parameters which do not have significant effects (W, C_0, C_1, T_1 and T_2), while Y and δ have positive significant effects, and b, c, e and λ show negative significant effects. For the economic statistical design (TABLE 3), W, C_0, C_1, T_1, T_2 and λ do not have significant effects, while Y and δ have positive significant effects, and b, c and e show significant negative effects. For both economic and economic statistical designs, there are six positive interaction effects ($b*c$, $b*T_1$, $c*e$, $Y*\delta$, $C_0*\lambda$ and $e*T_1$) and six negative interaction effects ($b*Y$, $b*\delta$, $c*Y$, $c*\delta$, $Y*e$ and $e*\delta$) toward ARL_0.

Finally, we look at the significant effects toward the out-of-control average run length (ARL_1). For the economic design (TABLE 2), W, C_0, C_1, T_1, T_2 and λ do not have significant effects, while b, c, Y, e and δ show significant effects. For the economic statistical design (TABLE 3), Y, W and T_1 do not have significant effects, while b, c, C_0, C_1, e, T_2, λ and δ show significant effects. In terms of the interaction effects, there are three positive interaction effects ($b*\delta$, $c*\delta$ and $Y*e$) and five negative interaction effects ($b*Y$, $b*e$, $c*T_1$, $Y*\delta$ and $e*\delta$) for the economic design, while there are six positive interaction effects ($b*\delta$, $c*\delta$, C_1*e, C_1*T_2, $e*T_2$and $e*\lambda$) and nine negative interaction effects ($b*C_1$, $b*e$, $b*T_2$, $b*\lambda$, $c*C_0$, $C_1*\delta$, $e*\delta$, $T_2*\delta$ and $\lambda*\delta$) for the economic statistical design.

TABLE (2): Significant Input Parameters in the Analysis of Variance for the Optimal Cost and *ARLs* for the Synthetic $\bar{X}$ Chart (Economic Design)

Input Parameters	*P*-value (*Cost*)	*P*-value (ARL_0)	*P*-value (ARL_1)
b	0.000 (-)	0.004 (-)	0.000 (-)
c		0.000 (-)	0.012 (-)
Y	0.000 (+)	0.000 (+)	0.000 (+)
C_0	0.000 (+)		
C_1	0.000 (+)		
e		0.001 (-)	0.000 (+)
T_1	0.007 (+)		
T_2	0.000 (-)		
λ		0.038 (-)	
δ	0.000 (+)	0.000 (+)	0.000 (-)
$b*c$		0.017 (+)	
$b*Y$		0.022 (-)	0.024 (-)
$b*e$			0.022 (-)
$b*T_1$		0.021 (+)*	
$b*\delta$	0.000 (+)	0.013 (-)	0.008 (+)
$c*Y$		0.000 (-)	
$c*e$		0.023 (+)	
$c*T_1$			0.043 (-)*
$c*\delta$		0.000 (-)	0.009 (+)
$Y*C_0$	0.045 (+)		
$Y*e$		0.007 (-)	0.000 (+)
$Y*\delta$	0.000 (-)	0.000 (+)	0.000 (-)
$C_0*\lambda$		0.028 (+)*	
$C_1*\delta$	0.032 (-)		
$e*T_1$	0.024 (-)	0.015 (+)*	
$e*\delta$		0.001 (-)	0.003 (-)
$T_1*\delta$	0.005 (+)		
$T_2*\lambda$	0.015 (-)*		
$T_2*\delta$	0.048 (-)		
$\lambda*\delta$	0.033 (+)		

Footnotes:

1. Only significant *P*-values, i.e. *P*-values of less than 0.05, are shown. The empty cells represent input parameters with a *P*-value of more than 0.05.
2. A *P*-value of 0.000 indicates that the *P*-value is less than 0.001.
3. The "+" sign in parentheses represents a positive main or interaction effect, while the "-" sign in parentheses represents a negative main or interaction effect.
4. Interactions marked with "*" are interactions which result in an opposite sign for the first factor at different levels of the second one.

TABLE (3): Significant Input Parameters in the Analysis of Variance for the Optimal Cost and *ARLs* for the Synthetic $\bar{X}$ Chart (Economic Statistical Design)

Input Parameters	*P*-value (*Cost*)	*P*-value (ARL_0)	*P*-value (ARL_1)
b	0.005 (+)	0.009 (-)	0.000 (-)
c	0.001 (+)	0.000 (-)	0.037 (-)
Y		0.000 (+)	
C_0	0.000 (+)		0.022 (-)
C_1	0.000 (+)		0.000 (+)
e	0.000 (+)	0.002 (-)	0.000 (+)
T_1	0.000 (+)		
T_2	0.000 (-)		0.000 (+)
λ	0.000 (+)		0.000 (+)
δ	0.000 (-)	0.000 (+)	0.000 (-)
$b*c$		0.024 (+)	
$b*Y$		0.026 (-)	
$b*C_1$			0.000 (-)
$b*e$			0.000 (-)
$b*T_1$		0.024 (+)*	
$b*T_2$			0.000 (-)*
$b*\lambda$			0.020 (-)
$b*\delta$		0.014 (-)	0.000 (+)
$c*Y$		0.000 (-)	
$c*C_0$			0.001 (-)*
$c*e$		0.025 (+)	
$c*\delta$	0.006 (-)	0.000 (-)	0.020 (+)
$Y*e$		0.007 (-)	
$Y*\delta$		0.000 (+)	
C_0*T_2	0.000 (-)		
$C_0*\lambda$	0.000 (-)*	0.031 (+)*	
C_1*e	0.018 (+)		0.000 (+)
C_1*T_1	0.000 (+)		
C_1*T_2	0.000 (-)		0.000(+)*
$C_1*\lambda$	0.000(+)*		
$C_1*\delta$	0.000 (-)		0.000 (-)
$e*T_1$		0.020(+)*	
$e*T_2$			0.000 (+)*
$e*\lambda$	0.015 (+)		0.000 (+)
$e*\delta$	0.000 (-)	0.002 (-)	0.000 (-)
$T_1*\lambda$	0.000 (+)*		
$T_2*\lambda$	0.000 (-)*		
$T_2*\delta$			0.001 (-)
$\lambda*\delta$	0.000 (-)		0.000 (-)

Footnotes:

1. Only significant *P*-values, i.e. *P*-values of less than 0.05, are shown. The empty cells represent input parameters with a *P*-value of more than 0.05.
2. A *P*-value of 0.000 indicates that the *P*-value is less than 0.001.
3. The "+" sign in parentheses represents a positive main or interaction effect, while the "-" sign in parentheses represents a negative main or interaction effect.
4. Interactions marked with "*" are interactions which result in an opposite sign for the first factor at different levels of the second one.

CONCLUSIONS

The economic and economic-statistical designs involve the estimation of various input parameters. Thus, it is important to know which of the input parameters have a significant effect on the optimal design, so that accurate estimations are done on these input parameters, while only an approximate estimation is required for the input parameters which do not have significant effect on the optimal design. In this paper, this is done by performing sensitivity analyses using fractional factorial design. The sensitivity analysis is performed on both the economic and economic-statistical designs of the synthetic $\bar{X}$ chart. The sensitivity analysis is used to determine which input parameters have a significant effect towards the optimal cost and average run lengths (*ARLs*).

The results of this study show that an input parameter can have a significant effect in the economic design, but it may not be significant in the economic-statistical design, and vice versa. For example, in the economic design *c*, *W*,

e and λ do not have significant effects on the optimal cost, while for the economic-statistical design, only Y and W are not significant.

Besides the main effects, there are also quite a number of significant interaction effects on both the economic and economic-statistical designs. Furthermore, there are input parameters which are not significant as a main effect, but are significant as an interaction effect. For example, for the economic design, e and λ do not show significant effects as the main factors, but are significant as interaction effects. The significant interactions are $e* T_1$ and $T_2*\lambda$ which show negative interaction effects and $\lambda*\delta$ which show positive interaction effects toward the economically optimal cost. In general, when the interaction is large, the corresponding main effects have little practical meaning. For example, the main effect of e is very small, and we are tempted to conclude that is no effect due to e. However, when we study the effect of e at different levels of factor T_1, we see that this is not the case. Therefore, a significant interaction effect will often mask the significance of the main effect. In the presence of significant interaction, the experiment must usually examine the levels of one factor, e, with levels of the other factors fixed to draw conclusions about the main effect of e. As a result, this shows that the interaction effects cannot be ignored.

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REFERENCES

1. W.H. Woodall, *The Statistician* **34**, 155-160 (1985).
2. M. Raghavachari and J. Surtihadi, *International Journal of Production Research* **32**, 2287-2302 (1994).
3. W.H. Woodall, *Technometrics* **28**, 408-409 (1986).
4. E.M. Saniga, *Technometrics* **31**, 313-320 (1989).
5. C. Mortarino, *Quality and Reliability Engineering International* **26**, 17-26 (2010).
6. Z. Wu and T.A. Spedding, *Journal of Quality Technology* **32**, 32-38 (2000).
7. F. Aparisi and M.A. de Luna, *Computers and Operations Research* **38**, 3204-3214 (2009).
8. P. Castagliola and M.B.C. Khoo, *Communications in Statistics-Simulation and Computation* **38**, 1659-1674 (2009).
9. M.A.G. Machado, A.F.B. Costa and M.A. Rahim, *Quality and Reliability Engineering International* **25**, 595-606 (2009).
10. S.M. Scariano and M.E. Calzada, *Sequential Analysis* **28**, 54-68 (2009).
11. Z. Wu, Y.J. Ou, P. Castagliola and M.B.C. Khoo, *International Journal of Production Research* **48**, 7423-7436 (2010).
12. M.B.C. Khoo, H.C. Lee, Z. Wu, C.H. Chen and P. Castagliola, *IIE Transactions* **43**, 23-38 (2011).
13. Y. Zhang, P. Castagliola, Z. Wu and M.B.C. Khoo, *IIE Transactions* **43**, 676-687 (2011).
14. M.B.C. Khoo, V.H. Wong, Z. Wu and P. Castagliola, *IIE Transactions* **44**,765–779 (2012).
15. W.C. Yeong, M.B.C. Khoo, Z. Wu and P. Castagliola, *Quality and Reliability Engineering International* **28**, 725-741 (2012).
16. W.C. Yeong, M.B.C. Khoo, M.H. Lee and M.A. Rahim, *European Journal of Operational Research* **228**, 571-581 (2013).
17. T.J. Lorenzen and L.C. Vance, *Technometrics* **28**, 3-10 (1986).
18. K.J. Chung, *International Journal of Production Research* **20**, 1239-1246 (1990).