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# THE EFFECTS OF PARAMETER ESTIMATION ON MINIMISING THE IN-CONTROL AVERAGE SAMPLE SIZE FOR THE DOUBLE SAMPLING $\bar{X}$ CHART

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## ABSTRACT

The double sampling (DS)  $\bar{X}$  chart, one of the most widely-used charting methods, is superior for detecting small and moderate shifts in the process mean. In a right skewed run length distribution, the median run length (MRL) provides a more credible representation of the central tendency than the average run length (ARL), as the mean is greater than the median. In this paper, therefore, MRL is used as the performance criterion instead of the traditional ARL. Generally, the performance of the DS  $\bar{X}$  chart is investigated under the assumption of known process parameters. In practice, these parameters are usually estimated from an in-control reference Phase-I dataset. Since the performance of the DS  $\bar{X}$  chart is significantly affected by estimation errors, we study the effects of parameter estimation on the MRL-based DS  $\bar{X}$  chart when the in-control average sample size is minimised. This study reveals that more than 80 samples are required for the MRL-based DS  $\bar{X}$  chart with estimated parameters to perform more favourably than the corresponding chart with known parameters.

## OPSOMMING

Die dubbelmonster  $\bar{X}$ -beheerkaart, wat grootskaals gebruik word, is voortreflik vir die opsporing van geringe en middelmatige verskuiwings van prosesmediaan. By 'n regsskuinse verdeling van looplengte lewer die mediaanlooplengte (MLL) 'n beter grootte waarde as die gemiddelde looplengte. Gewoonlik word die vertoning van die bogenoemde  $\bar{X}$ -beheerkaart bepaal met geskatte prosesparameters. Die skattings word bepaal op die veronderstelling dat die proses onder beheer is. Aangesien die MLL gebaseerde beheerkaartvertoning betekenisvol beïnvloed word deur skattingsfoute, word dit ondersoek uit minimum gemiddelde monstergrootte. Die uiteindelijke resultaat van die navorsing toon dat 80 monsters benodig word vir 'n beter vertoning as die ooreenstemmende beheerkaart vir bekende parameters.

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## 1 INTRODUCTION

Quality is one of the most important decision-making factors for consumers when selecting competing products and services. This is so, whether the consumer is an individual, a retail store, an industrial organisation, or a nation. Statistical process control (SPC) comprises problem-solving approaches that help to achieve quality improvement and minimise production costs. A control chart plays an important role in SPC. It is a simple yet very powerful tool in SPC, enabling a study of the stability of a manufacturing process. Due to its operational simplicity, there has been a significant growth in the use of the Shewhart  $\bar{X}$  chart to monitor processes and improve quality in manufacturing and service industries. The Shewhart  $\bar{X}$  chart is used to detect large process mean shifts in such industries. However, it lacks the sensitivity to detect small and moderate process mean shifts.

Daudin [1] introduced the DS  $\bar{X}$  chart to overcome the drawback of the Shewhart  $\bar{X}$  chart in small and moderate process mean shifts. It is a two-stage Shewhart  $\bar{X}$  chart that incorporates the idea of the double sampling plan. The DS  $\bar{X}$  chart, optimally designed based on the in-control average sample size ( $ASS_0$ ), was provided by Daudin [1]. Inspired by Daudin [1], Irianto & Shinozaki [2] developed an optimisation model to minimise the out-of-control average run length ( $ARL_1$ ). Daudin [1] and Costa [3] revealed that, in some cases, the DS  $\bar{X}$  chart outperforms the Shewhart  $\bar{X}$ , EWMA, CUSUM, variable sampling interval (VSI), and variable sample size (VSS) charts. The DS type control charts therefore continue to attract ongoing research.

Concerning the DS  $\bar{X}$  type control charts, Torng & Lee [4] and Torng et al. [5] studied the DS  $\bar{X}$  and DSVSI (combined DS and VSI)  $\bar{X}$  charts under non-normality. Using a genetic algorithm, Torng et al. [6,7] developed economic design models of the DS  $\bar{X}$  chart for both independent and correlated data. The economic design of the DSVSI  $\bar{X}$  chart was discussed by Lee et al. [8], because this chart is effective in detecting small and moderate mean shifts. Using the Markov chain approach, Costa & Machado [9] compared the performances of the variable parameters (VP)  $\bar{X}$  and DS  $\bar{X}$  charts, in the presence of correlation. The synthetic DS  $\bar{X}$  chart suggested by Khoo et al. [10] significantly improves the detection speed, compared with the synthetic  $\bar{X}$ , DS  $\bar{X}$ , and EWMA  $\bar{X}$  charts - although the detection of small shifts is better accomplished with the EWMA  $\bar{X}$  chart. The most recent literature on the DS  $S$  type charts was studied by Lee et al. [11,12]. Lee et al. [11] found that the DS  $S$  chart performs extraordinarily well in detecting the shifts in the process standard deviation and in reducing the sample size of the destructive testing process. Meanwhile, the DSVSI  $S$  chart was proposed by Lee et al. [12] to improve efficiency in detecting small standard deviation shifts.

Recently a great deal of research interest has arisen in the area of control charts with estimated parameters. Focusing on the recent works of the  $\bar{X}$  type charts with estimated parameters, Zhang et al. [13] and Khoo et al. [14] evaluated the performances of both the synthetic  $\bar{X}$  and the DS  $\bar{X}$  charts with estimated parameters. They found that there are large differences in the run length properties of the synthetic  $\bar{X}$  and DS  $\bar{X}$  charts between the cases of the known parameters and estimated parameters. Capizzi & Masarotto [15] discussed the effect of parameter estimation on the performance of the combined Shewhart-EWMA (CSEWMA)  $\bar{X}$  chart. For attribute control charts, Testik et al. [16] and Testik [17] investigated the impact of the estimated process mean on the performances of the Poisson EWMA and the Poisson CUSUM charts. All this literature shows that estimation errors affect the in-control cases much more than they do the out-of-control cases. A thorough literature review on the impact of parameter estimation for the properties of the different types of control charts can be found in Jensen et al. [18]. Since the performance of control charts deteriorates significantly because of estimation errors, Jensen et al. [18] pointed out that the effects of parameter estimation need to be investigated for other

types of control charts, such as the DS  $\bar{X}$  chart.

When the process parameters are estimated, multiple performance measures should be included to evaluate the control chart's properties effectively [18]. For control charts with estimated parameters, the existing literature has put too much emphasis on average run length (ARL) and not enough on the median run length (MRL). Criticisms have arisen following the use of the ARL as the sole representative of the run length (see Gan [19], Bischak & Trietsch [20], and Chakraborti [21]). This is because interpretation based on the ARL alone could be complicated yet deceptive, as the shape of the run length distribution changes with the magnitude of the mean shift and the number of Phase-I samples. In the light of this drawback, the MRL is suggested as a supplementary performance measure in this paper; interpretation based on MRL is more readily understood. For example, an in-control MRL ( $MRL_0$ ) of 250 means that 50 per cent of the time a false alarm will occur by the 250<sup>th</sup> sample. Since a more reliable interpretation in assessing a chart's performance could be attained by using the MRL, Golosnoy & Schmid [22], Maravelakis et al. [23], and Khoo et al. [24], to mention a few, have all devoted themselves to the development of MRL-based control charts.

As mentioned before, Daudin [1] and Khoo et al. [14] suggested a procedure to minimise the  $ASS_0$  for the ARL-based DS  $\bar{X}$  chart with both known and estimated parameters. To complement the works of Daudin [1] and Khoo et al. [14], this paper studies the effects of parameter estimation on minimising the  $ASS_0$  for the MRL-based DS  $\bar{X}$  chart. We therefore investigate the MRL and average sample size (ASS) performance of the DS  $\bar{X}$  chart when the optimal parameters that were intended for the known parameters case are used for the estimated parameters case.

The organisation of this paper is as follows: Section 2 reviews the charting procedure of the DS  $\bar{X}$  chart with known and estimated parameters; Section 3 studies the effects of parameter estimation on the MRL-based DS  $\bar{X}$  chart, for minimising the  $ASS_0$ ; and finally, conclusions are drawn in Section 4.

## 2 THE DS $\bar{X}$ CHART

The DS  $\bar{X}$  chart has the favourable properties of the Shewhart  $\bar{X}$  chart. Daudin [1] proposed it to detect small and moderate process mean shifts effectively. The ability of the DS  $\bar{X}$  chart to minimise the average sample size used in process monitoring has made the chart an appealing method for quality practitioners. In this section, we briefly explain the design procedure of the DS  $\bar{X}$  chart defined by Daudin [1].

Assume that the observations of the quality characteristic being monitored are independently and identically distributed (iid) normal random variables, with a known in-control mean  $\mu_0$ , and a known in-control standard deviation  $\sigma_0$ . The warning limits of the first-sample stage are expressed as  $\pm L_1$  while  $\pm L$  denotes the control limits of the first-sample stage. The control limits of the combined-sample stage are given by  $\pm L_2$ . Figure 1 gives a graphic view of the DS  $\bar{X}$  chart.

As shown in Figure 1, the DS  $\bar{X}$  chart is partitioned into a few intervals that are denoted by  $I_1 = [-L_1, L_1]$ ,  $I_2 = [-L, -L_1) \cup (L_1, L]$ ,  $I_3 = (-\infty, -L) \cup (L, +\infty)$ , and  $I_4 = [-L_2, L_2]$ . Based on Figure 1, the DS  $\bar{X}$  chart is implemented as follows [1]:

- a. Calculate the sample mean  $\bar{X}_i = \sum_{j=1}^{n_i} X_{i,j} / n_i$  after a first sample of size  $n_i$  is taken. Here,  $X_{i,j}$  with  $j = 1, 2, \dots, n_i$  represents the  $j^{\text{th}}$  observation at the  $i^{\text{th}}$  sampling time of the first sample.

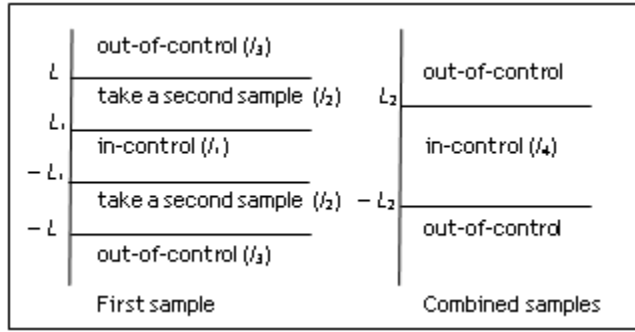


Figure 1: Graphic view of the implementation of the DS  $\bar{X}$  chart

- b. The process is in-control at the first sample if  $Z_{1i} = \left[ (\bar{X}_{1i} - \mu_0) \sqrt{n_1} \right] / \sigma_0 \in I_1$ .
- c. The process is out-of-control at the first sample if  $Z_{1i} \in I_3$ .
- d. Select a second sample with size  $n_2$  if  $Z_{1i} \in I_2$ . Then calculate the sample mean  $\bar{X}_{2i} = \sum_{j=1}^{n_2} X_{2i,j} / n_2$ , where  $X_{2i,j}$  with  $j = 1, 2, \dots, n_2$  refers to the  $j^{\text{th}}$  observation at the  $i^{\text{th}}$  sampling time of the second sample.
- e. The combined sample mean at the  $i^{\text{th}}$  sampling time  $\bar{X}_i = (n_1 \bar{X}_{1i} + n_2 \bar{X}_{2i}) / (n_1 + n_2)$  is calculated.
- f. The process is in-control at the second sample if  $Z_i = \left[ (\bar{X}_i - \mu_0) \sqrt{n_1 + n_2} \right] / \sigma_0 \in I_4$ . Otherwise the process is declared to be out-of-control.

It must be emphasised that at the  $i^{\text{th}}$  sampling time, two possible situations could occur. The first situation occurs when only the first sample of size  $n_1$  needs to be obtained. The second situation takes place when the combined samples of size  $n_1 + n_2$  must be taken.

For the DS  $\bar{X}$  chart with known parameters, let  $P_{ak}$  denote the probability that the process remains in-control at sampling stage  $k$  with  $k \in \{1, 2\}$ . Then the probability that the process is in-control is  $P_a = P_{a1} + P_{a2}$ , where [1]

$$P_{a1} = \Pr(Z_{1i} \in I_1) = \Phi(L_1 + \delta\sqrt{n_1}) - \Phi(-L_1 + \delta\sqrt{n_1}) \quad (1)$$

and

$$P_{a2} = \Pr(Z_i \in I_4 \text{ and } Z_{1i} \in I_2) = \int_{z \in I_2} \left[ \Phi\left(cL_2 + rc\delta - \sqrt{\frac{n_1}{n_2}}z\right) - \Phi\left(-cL_2 + rc\delta - \sqrt{\frac{n_1}{n_2}}z\right) \right] \phi(z) dz. \quad (2)$$

Here,  $\Phi(\cdot)$  represents the cumulative distribution function (cdf) of the standard normal random variable, while the probability density function (pdf) of the standard normal random variable is expressed as  $\phi(\cdot)$ . Also,  $\delta = |\mu_1 - \mu_0| / \sigma_0$  refers to the magnitude of the standardised mean shift having an out-of-control mean  $\mu_1$ ,  $r = \sqrt{n_1 + n_2}$ ,  $c = r / \sqrt{n_2}$ , and  $I_2^c = [-L + \delta\sqrt{n_1}, -L_1 + \delta\sqrt{n_1}] \cup [L_1 + \delta\sqrt{n_1}, L + \delta\sqrt{n_1}]$ .

According to Montgomery [25], the run length (RL) distribution of the Shewhart  $\bar{X}$  chart is geometric. Note that the RL is defined as the number of sample points plotted on a control chart before the chart issues the first out-of-control signal. Since the DS  $\bar{X}$  chart is a

Shewhart type chart, its RL follows the geometric distribution. The cdf of the RL is

$$F_{RL}(\ell) = \Pr(RL \leq \ell) = 1 - P_a^\ell, \quad (3)$$

with  $\ell \in \{1, 2, 3, \dots\}$ . The MRL of the DS  $\bar{X}$  chart can be computed as follows [19]:

$$\Pr(RL \leq MRL - 1) \leq 0.5 \text{ and } \Pr(RL \leq MRL) > 0.5. \quad (4)$$

The ASS at each sampling time is expressed as [1]

$$ASS = n_1 + n_2 \Pr(Z_{1i} \in I_2), \quad (5)$$

where

$$\Pr(Z_{1i} \in I_2) = \Phi(L + \delta\sqrt{n_1}) - \Phi(L_1 + \delta\sqrt{n_1}) + \Phi(-L_1 + \delta\sqrt{n_1}) - \Phi(-L + \delta\sqrt{n_1}). \quad (6)$$

In real-world applications  $\mu_0$  and  $\sigma_0$  are usually unknowns. They are both estimated from an in-control Phase-I dataset, comprising  $i = 1, 2, \dots, m$  samples, each having  $n$  observations. The estimator  $\hat{\mu}_0$  of  $\mu_0$  is equal to

$$\hat{\mu}_0 = \frac{1}{m} \sum_{i=1}^m \bar{X}_i \quad (7)$$

and the estimator  $\hat{\sigma}_0$  of  $\sigma_0$  is

$$\hat{\sigma}_0 = \sqrt{\frac{1}{m(n-1)} \sum_{i=1}^m \sum_{j=1}^n (X_{i,j} - \bar{X}_i)^2} \quad (8)$$

where  $\bar{X}_i = \sum_{j=1}^n X_{i,j} / n$  is the  $i^{\text{th}}$  sample mean.

Khoo et al. [14] recently proposed the ARL-based DS  $\bar{X}$  chart with estimated parameters.

When parameters are estimated, the conditional probability  $\hat{P}_a = \hat{P}_{a1} + \hat{P}_{a2}$  is the probability that the process is declared to be in-control, where [14]

$$\hat{P}_{a1} = \Phi \left[ U \sqrt{\frac{n_1}{mn}} + VL_1 - \delta\sqrt{n_1} \right] - \Phi \left[ U \sqrt{\frac{n_1}{mn}} - VL_1 - \delta\sqrt{n_1} \right] \quad (9)$$

and

$$\hat{P}_{a2} = \int_{z \in I_2} \Pr(\hat{Z}_i \in I_4 | \hat{Z}_{1i} = z, \hat{\mu}_0, \hat{\sigma}_0) f_{\hat{Z}_{1i}}(z | \hat{\mu}_0, \hat{\sigma}_0) dz \quad (10)$$

where

$$\Pr(\hat{Z}_i \in I_4 | \hat{Z}_{1i} = z, \hat{\mu}_0, \hat{\sigma}_0) = \Phi \left[ U \sqrt{\frac{n_2}{mn}} + V \left( \frac{L_2 \sqrt{n_1 + n_2} - z\sqrt{n_1}}{\sqrt{n_2}} \right) - \delta\sqrt{n_2} \right] - \Phi \left[ U \sqrt{\frac{n_2}{mn}} - V \left( \frac{L_2 \sqrt{n_1 + n_2} + z\sqrt{n_1}}{\sqrt{n_2}} \right) - \delta\sqrt{n_2} \right] \quad (11)$$

and

$$f_{\hat{Z}_{1i}}(z | \hat{\mu}_0, \hat{\sigma}_0) = V\phi \left( U \sqrt{\frac{n_1}{mn}} + Vz - \delta\sqrt{n_1} \right) \quad (12)$$

The random variables  $U$  and  $V$  in Equations (9), (11), and (12) are defined as

$$U = (\hat{\mu}_0 - \mu_0) \frac{\sqrt{mn}}{\sigma_0} \quad (13)$$

And

$$V = \frac{\hat{\sigma}_0}{\sigma_0} \quad (14)$$

respectively.

The cdf of RL for the known parameters case can be obtained from Equation (3). However, for the estimated parameters case, the RL distribution no longer follows a geometric distribution [18]. Therefore the cdf of RL for the DS  $\bar{X}$  chart with estimated parameters can be calculated as

$$F_{RL}(\ell) = \int_{-\infty}^{+\infty} \int_0^{+\infty} (1 - \hat{P}_a^\ell) f_U(u) f_V(v) dv du \quad (15)$$

where  $\ell \in \{1, 2, 3, \dots\}$ . Here, the pdf of  $U$  and  $V$ , i.e.  $f_U(u)$  and  $f_V(v)$  are [14]

$$f_U(u) = \phi(u) \quad (16)$$

and

$$f_V(v) = 2v f_\gamma \left( v^2 \left| \frac{m(n-1)}{2}, \frac{2}{m(n-1)} \right. \right) \quad (17)$$

respectively, where  $f_\gamma(\cdot | k, l)$  is the pdf of the gamma distribution with parameters  $k$  and  $l$ . Then the MRL of the DS  $\bar{X}$  chart with estimated parameters can be computed using Equations (4) and (15).

For the DS  $\bar{X}$  chart with estimated parameters, Khoo et al. [14] showed that the ASS can be obtained as

$$ASS = \int_{-\infty}^{+\infty} \int_0^{+\infty} \left[ n_1 + n_2 \cdot \Pr(\hat{Z}_{ii} \in I_2 | \hat{\mu}_0, \hat{\sigma}_0) \right] f_U(u) f_V(v) dv du \quad (18)$$

where

$$\Pr(\hat{Z}_{ii} \in I_2 | \hat{\mu}_0, \hat{\sigma}_0) = \Phi \left[ U \sqrt{\frac{n_1}{mn}} - VL_1 - \delta \sqrt{n_1} \right] - \Phi \left[ U \sqrt{\frac{n_1}{mn}} - VL - \delta \sqrt{n_1} \right] + \Phi \left[ U \sqrt{\frac{n_1}{mn}} + VL - \delta \sqrt{n_1} \right] - \Phi \left[ U \sqrt{\frac{n_1}{mn}} + VL_1 - \delta \sqrt{n_1} \right] \quad (19)$$

### 3 EFFECTS OF PARAMETER ESTIMATION ON MINIMISING $ASS_0$

In this article, we investigate the MRL-based DS  $\bar{X}$  chart's performance for minimising the  $ASS_0$  when process parameters are estimated. In other words, we use the optimal parameters  $(n_1, n_2, L_1, L, L_2)$  to minimise  $ASS_0$ , computed for the known parameters case, to compute the  $MRL_0$ ,  $MRL_1$ ,  $ASS_0$ , and  $ASS_1$  of the chart with estimated parameters, in order to study the chart's performance based on parameter estimation. Here, the subscripts '0' and '1' for MRL and ASS refer to the in-control and out-of-control cases respectively. The optimal parameters  $(n_1, n_2, L_1, L, L_2)$  for the known parameters case are computed employing a nonlinear optimisation algorithm using the Matlab program. The optimisation minimises  $ASS_0$ , subject to  $MRL_0 = \tau$ ,  $MRL_1 = \varepsilon$  and  $1 \leq n_1 < n_{\bar{x}} < n_1 + n_2 \leq n_{\max}$ , where  $n_1 < n_2$ . Note that  $\tau$  and  $\varepsilon$  are the desired  $MRL_0$  and  $MRL_1$  respectively,  $n_{\max}$  is the upper bound of  $n_1 + n_2$ , and  $n_{\bar{x}}$  denotes the sample size of the MRL-based Shewhart  $\bar{X}$  chart with known parameters.

When process parameters are known, the DS  $\bar{X}$  chart is designed so that its  $MRL_0$  and  $MRL_1$  (for  $\delta = 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.5$ ) match that of the optimal EWMA  $\bar{X}$  chart with a sample size  $n_{EWMA}$ , where  $n_{EWMA} \in \{1, 3\}$  is considered. At the same time, the DS  $\bar{X}$  chart is also designed to match the  $MRL_0$  and an appropriate  $MRL_1$  of the Shewhart  $\bar{X}$  chart (having  $\pm 2.92362$  standard deviation width), where a suitable  $n_{\bar{x}}$  is chosen so that the Shewhart  $\bar{X}$  chart's  $MRL_1$  is as close as possible to that of the EWMA  $\bar{X}$  chart, for the same shift  $\delta$ .

Due to cost constraints, it has become the norm in industries not to use large sample sizes in process monitoring; either small or moderate sample sizes are preferred. Consequently, we fix  $n_{\max}$  as 15 in this work.

Tables 1 and 2 show the  $MRL_0$ ,  $ASS_0$ ,  $MRL_1$ , and  $ASS_1$  values of the DS  $\bar{X}$  chart for various combinations of  $m$ ,  $n_{\bar{x}}$  (equivalent to the Phase-I sample size,  $n$ ), and  $\delta$  with the optimal parameters  $(n_1, n_2, L_1, L, L_2)$  corresponding to the known parameters ( $m = +\infty$ ) case; and nearly matching the same design as the Shewhart  $\bar{X}$  chart and the optimal EWMA  $\bar{X}$  chart with known parameters. The first and second rows of each cell, for  $m \in \{10, 20, 40, 80, +\infty\}$ , give the  $(MRL_0, ASS_0)$  and  $(MRL_1, ASS_1)$  values, where  $m$  is the number of Phase-I samples.

The accuracy of the results in Tables 1 and 2, computed from the optimisation method using Matlab, has been verified with simulation. It is worth pointing out that in this study,  $MRL_0 = \tau = 200$  and  $MRL_1 = \varepsilon$  are attained for every  $\delta$  with known parameters ( $m = +\infty$ ). The desired  $\varepsilon$  value is designated as the  $MRL_1$  value of the optimal MRL-based EWMA  $\bar{X}$  chart with known parameters for a particular  $\delta$ , where the  $MRL_0$  value of this EWMA  $\bar{X}$  chart is fixed as 200 and  $n_{EWMA} = \{1, 3\}$ . For details about the optimisation procedure of the MRL-based EWMA  $\bar{X}$  chart with known parameters, see Gan [19]. We use the parameters  $(n_1, n_2, L_1, L, L_2)$ , given in columns three to seven of Tables 1 and 2, to compute the MRL and ASS values displayed in columns eight to eleven, for the estimated parameters case. Equations (4) and (15) are used to calculate the MRL values of the chart with estimated parameters, whereas Equation (18) is used to compute the ASS value. For example, if  $\delta = 0.75$  and  $n_{\bar{x}} = 6$ , the optimal parameters  $(n_1, n_2, L_1, L, L_2)$  (1, 11, 1.725, 5.407, 2.305) for the known parameters case can be obtained from Table 2. With these optimal parameters,  $(MRL_0, ASS_0) = (149, 1.977)$  and  $(MRL_1, ASS_1) = (5, 2.934)$ ,  $m = 20$  can be computed from the equations given in Section 2.

An examination of the results in Tables 1 and 2 reveals that, for a fixed value of  $\delta$  and  $n_{\bar{x}} (= n)$ , the  $MRL_0$  value increases and approaches that of the known parameters ( $m = +\infty$ ) case when  $m$  increases. The  $MRL_0$  value is smaller than the desired value of 200, for  $m \in \{10, 20, 40, 80\}$ . This indicates that more frequent false alarms will occur when estimates are used in place of known parameters.

As for the ASS values, it is seen from Tables 1 and 2 that there is no obvious difference between the various values of  $ASS_0$  (and  $ASS_1$ ) for a fixed  $\delta$  and  $n_{\bar{x}} (= n)$  as  $m$  changes, even though both  $ASS_0$  and  $ASS_1$  slowly approach the corresponding values for the known parameters case ( $m = +\infty$ ), as  $m$  increases. Furthermore, it is interesting to highlight that all of the  $ASS_1$  values needed to detect process changes are larger than the corresponding  $ASS_0$  values. This is because the optimisation design minimises  $ASS_0$  and not  $ASS_1$ .

To illustrate how Tables 1 and 2 assist practitioners to carry out sampling, consider the following example: from these tables, it is obvious that more than 80 Phase-I samples are required to minimise the effects of parameter estimation on the MRL-based DS  $\bar{X}$  chart. When  $\delta = 1.00$  and  $n_{\bar{x}} = 4$  are considered, in the Phase-I analysis a practitioner needs to collect 80 ( $m = 80$ ) Phase-I samples, each having 4 ( $n = n_{\bar{x}} = 4$ ) observations to estimate  $\mu_0$  and  $\sigma_0$ . Then in Phase-II process monitoring, Table 2 suggests the use of parameters  $(n_1, n_2, L_1, L, L_2) = (1, 6, 1.856, 4.963, 2.351)$  to design the DS  $\bar{X}$  chart with estimated parameters. This chart will give  $(MRL_0, ASS_0) = (183, 1.391)$  and  $(MRL_1, ASS_1) = (4, 2.200)$ .



Table 1: (MRL0, ASSO) (first row) and (MRL1, ASS1) (second row) for  $m \in \{10, 20, 40, 80, +\infty\}$  and optimal parameters  $(n_1, n_2, L_1, L, L_2)$  corresponding to the known parameters case, approximately matching a similar design of the Shewhart  $\bar{X}$  and the optimal EWMA  $\bar{X}$  charts (MRL0=200,  $n_{EWMA} = 1$ ) with known parameters.

$n_{EWMA} = 1$											
Shewhart $\bar{X}$ Chart							DS $\bar{X}$ Chart				
$\delta$	$n_{\bar{x}}$	$n_1$	$n_2$	$L_1$	$L$	$L_2$	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
0.50	5	1	10	2.136	4.955	1.961	(124, 1.407) (21, 1.645)	(152, 1.366) (21, 1.598)	(172, 1.346) (21, 1.574)	(185, 1.336) (21, 1.562)	(200, 1.326) (21, 1.551)
0.75	3	1	5	2.132	5.089	2.176	(112, 1.244) (12, 1.530)	(143, 1.204) (12, 1.480)	(166, 1.184) (12, 1.454)	(181, 1.175) (12, 1.441)	(200, 1.165) (12, 1.428)
1.00	3	1	3	2.214	4.952	2.279	(121, 1.124) (8, 1.406)	(150, 1.102) (8, 1.374)	(171, 1.091) (8, 1.357)	(184, 1.086) (8, 1.348)	(200, 1.081) (8, 1.339)
1.25	2	1	2	2.371	4.046	2.283	(100, 1.081) (6, 1.314)	(135, 1.058) (6, 1.292)	(160, 1.046) (4, 1.292)	(178, 1.041) (6, 1.267)	(200, 1.035) (6, 1.257)
1.50	2	1	2	2.443	3.391	2.272	(99, 1.061) (4, 1.295)	(134, 1.045) (4, 1.295)	(160, 1.036) (6, 1.276)	(177, 1.032) (4, 1.290)	(200, 1.028) (4, 1.287)
1.75	2	1	2	2.533	3.048	2.509	(103, 1.038) (3, 1.221)	(137, 1.028) (3, 1.230)	(162, 1.023) (3, 1.235)	(178, 1.021) (3, 1.237)	(200, 1.018) (3, 1.239)
2.00	2	1	2	2.819	3.100	1.638	(108, 1.015) (3, 1.127)	(141, 1.011) (3, 1.134)	(165, 1.008) (3, 1.138)	(180, 1.007) (3, 1.140)	(200, 1.006) (3, 1.142)
2.50	2	1	2	2.923	3.093	0.000	(112, 1.008) (2, 1.097)	(144, 1.006) (2, 1.107)	(167, 1.004) (2, 1.112)	(181, 1.004) (2, 1.116)	(200, 1.003) (2, 1.119)

Table 2: (MRL0, ASSO) (first row) and (MRL1, ASS1) (second row) for  $m \in \{10, 20, 40, 80, +\infty\}$  and optimal parameters  $(n_1, n_2, L_1, L, L_2)$  corresponding to the known parameters case, approximately matching a similar design of the Shewhart  $\bar{X}$  and the optimal EWMA  $\bar{X}$  charts (MRL0=200,  $n_{EWMA} = 3$ ) with known parameters.

$n_{EWMA} = 3$											
Shewhart $\bar{X}$ Chart							DS $\bar{X}$ Chart				
$\delta$	$n_{\bar{x}}$	$n_1$	$n_2$	$L_1$	$L$	$L_2$	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
0.50	8	1	14	1.653	5.292	2.313	(125, 2.468) (10, 3.052)	(153, 2.422) (10, 3.006)	(173, 2.399) (10, 2.983)	(185, 2.387) (10, 2.972)	(200, 2.375) (10, 2.960)
0.75	6	1	11	1.725	5.407	2.305	(120, 2.026) (5, 2.982)	(149, 1.977) (5, 2.934)	(170, 1.953) (5, 2.909)	(184, 1.941) (5, 2.897)	(200, 1.929) (5, 2.885)
1.00	4	1	6	1.856	4.963	2.351	(118, 1.460) (4, 2.273)	(148, 1.420) (4, 2.232)	(169, 1.400) (4, 2.211)	(183, 1.391) (4, 2.200)	(200, 1.381) (4, 2.189)
1.25	3	1	4	1.953	5.210	2.397	(112, 1.276) (3, 2.047)	(143, 1.239) (3, 2.009)	(166, 1.221) (3, 1.988)	(182, 1.212) (3, 1.978)	(200, 1.203) (3, 1.967)
1.50	3	1	3	1.929	5.153	2.497	(116, 1.217) (2, 2.049)	(147, 1.189) (2, 2.028)	(168, 1.175) (2, 2.016)	(182, 1.168) (2, 2.009)	(200, 1.161) (2, 2.003)
1.75	3	1	3	1.618	5.226	2.657	(114, 1.382) (1, 2.664)	(144, 1.350) (1, 2.663)	(167, 1.333) (1, 2.661)	(182, 1.325) (1, 2.660)	(200, 1.317) (1, 2.658)
2.00	3	1	3	1.975	3.469	2.526	(116, 1.191) (1, 2.233)	(146, 1.167) (1, 2.274)	(168, 1.156) (1, 2.296)	(183, 1.149) (1, 2.307)	(200, 1.143) (1, 2.318)
2.50	2	1	2	2.487	2.972	2.889	(105, 1.040) (1, 1.307)	(139, 1.030) (1, 1.336)	(163, 1.025) (1, 1.353)	(179, 1.023) (1, 1.363)	(200, 1.020) (1, 1.374)

## 4 CONCLUSIONS

Since an in-depth knowledge and a good understanding of the control charts used are vital to quality practitioners, the MRL and ASS are selected as alternative measures to evaluate the performance of the DS  $\bar{X}$  chart with estimated parameters. In this paper, we found that adopting estimates instead of known parameters significantly decreases the MRL<sub>0</sub> values, especially when the number of in-control samples  $m$  and sample size  $n$ , used in the Phase-I process, are small. This suggests a higher false alarm rate for the chart with estimated parameters. It is undesirable to have excessive false alarms: unnecessary interruptions in the process will occur, and time will be wasted searching for nonexistent assignable causes. The effects of parameter estimation on the DS  $\bar{X}$  chart should therefore not be ignored. There are also some deviations in the ASS values when parameters are estimated, compared with when they are known.

From the results, it is clear that  $m > 80$  Phase-I samples are required for the DS  $\bar{X}$  chart with estimated parameters to perform satisfactorily, compared with the chart with known parameters. With this recommended number of Phase-I samples,  $m$  can only be implemented for a process with a high production volume. When only a small amount of Phase-I data is available, the optimal parameters that are specially computed for the chart with known parameters are unsuitable for use in the chart with estimated parameters. Therefore the optimal design of the MRL-based DS  $\bar{X}$  chart with estimated parameters for minimising ASS<sub>0</sub> is a crucial research topic to be pursued. These research findings will be presented in a future paper.

It is obvious from this paper that the performance of the DS  $\bar{X}$  chart is adversely affected by parameter estimation. To address this, an extension of this study would be to investigate the impact of parameter estimation on various types of DS charts, such as the triple sampling (TS)  $\bar{X}$ , the DS variable sampling interval  $\bar{X}$ , and synthetic DS  $\bar{X}$  charts. This is because these control charts offer better statistical efficiency than the DS  $\bar{X}$  chart.

It would therefore be worthwhile to develop new methods of parameter estimation for these control charts in the future.

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