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Cross Country Convergence in a General Lotka-Volterra Model

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This paper uses a general Lotka-Volterra model to estimate convergence for 93 countries over the years 1960 - 2007. It employs an equation with a spatial time lag and common factors. The spatial lag controls for spatial dependence while the common factors control for strong cross sectional dependence. As spatial weights matrices, the share of high skilled migrants, trade shares and foreign direct investments are used. A simultaneous least squares estimator and a dynamic common correlated effects (DCCE) estimator are employed. The DCCE estimator finds conditional convergence. The paper highlights the importance of controlling for both types of cross sectional dependence.

Keywords: Economic growth; Growth Empirics; Spatial Econometrics; Convergence; Lotka-Volterra

JEL classification: O47, C31

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1 Introduction

A key question in the empirics of economic growth is if and what kind of convergence exists. The question goes beyond pure academic relevance and has implications for policy. If countries were converge to a single equilibrium (absolute convergence), then disparities between countries would be a matter of time. Policy could implement measures to increase the speed of convergence and close the gap between poor and rich countries more quickly. If countries converge to their own equilibrium (conditional convergence), policy could work towards closing the difference between equilibria of different countries.

Jones (2015) and Grossman and Helpman (2015) note that the literature agrees that spillovers and interactions between countries exists, however they are neglected by the empirical literature. A related question is, what kind of interactions or dependences between countries exist and how to control for those. Chudik, Pesaran and Tosetti (2011) and Vega and Elhorst (2016) define two forms of dependence between cross sectional units: weak and strong dependence.[1] Spatial econometrics offers methods to account for and estimate the first type, often called spatial dependence. Spillovers and interactions are estimated by adding weighted observations of other units. Strong dependence is treated as an unobserved part of the error term.

The concept of convergence is highly discussed in the literature. Early theoretical contributions base on the Solow Growth model. Seminal contributions towards estimating the rate of convergence date back to Baumol (1986), Mankiw, Romer and Weil (1992) and Barro and Sala-i Martin (1992). Baumol (1986) finds for 16 OECD countries conditional convergence and for a larger set of countries evidence for club convergence. The latter studies find conditional convergence in a cross sectional framework with rates of around 2%. Islam (1995) extends the model to a panel data approach and finds rates of conditional convergence of up to 9%. In a recent paper Barro (2015) estimates rates of convergence between 1.7% and 2.6%, depending on the time frame.[2] All these studies take the underlying growth model seriously in the sense that each country is treated as a single unit and interactions between countries are not considered.

Spatial econometric methods allow the estimation of interactions between countries or spatial dependence. Ertur and Koch (2007, 2011) estimate the rate of convergence in a spatial Solow model and a spatial Schumpeterian growth model. On a regional level, Lopez-Bazo, Vaya and Artis (2004) and Fingleton and López-Bazo (2006) include externalities in their estimation equations in the form of a spatial lag and a spatially weighted initial income. Lopez-Bazo, Vaya and Artis (2004); Ertur and Koch (2007, 2011) use maximum likelihood methods for their estimations. Elhorst, Piras and Arbia (2010) estimate an extended Solow Swan growth model using European regions. They estimate an unconstrained spatial Durbin model using General Methods of Moment (GMM), maximum likelihood and a mixture of both, which allows the inclusion of fixed effects and spatial interaction effects. Arbia and Paelinck (2003a,b) estimate regional convergence for 119 European regions using a Lotka-Volterra approach.

[1] This paper follows Vega and Elhorst (2016) in the notation of cross sectional dependence. Strong cross sectional dependence is labelled as unobserved common factors, while weak cross-sectional dependence is called spatial dependence.

[2] Comprehensive overviews can be found in Islam (2003) and Eberhardt and Teal (2011).

They estimate a difference equation system using OLS, where each region is represented by one equation and use the stability conditions of this system to determine convergence. This approach implies that convergence depends on the parameters of the regions itself and and of parameters of other regions.

While spatial dependence is controlled for by adding spatial lags, strong cross sectional dependence is modelled by a multifactor error structure. The error term contains the strong dependence in the form of an unobserved common factor and a heterogenous factor loading. Therefore, not accounting for strong cross sectional dependence leads to biased OLS regression results. Pesaran (2006) and Chudik and Pesaran (2015a) show that the strong dependence can be factored out if cross sectional means of the dependent and independent variables are added. The estimator was shown to be consistent in a static and dynamic panel model and under a variety of different assumptions on the error term (Pesaran and Tosetti, 2011; Chudik, Pesaran and Tosetti, 2011; Kapetanios, Pesaran and Yamagata, 2011; Chudik and Pesaran, 2015a).[3]

The present paper is based on the work of Arbia and Paelinck (2003a,b), but borrows from the literature on multifactor error structure models. Following Arbia and Paelinck (2003a,b) a general Lotka-Volterra model is used to determine the type of convergence in a set of 93 countries rather than regions. Absolute convergence is tested by estimating the steady states for each country and then testing those for equality. The stability conditions of the Lotka-Volterra model are used in conjunction with a bootstrap to test for conditional convergence. This paper advances the empirical spatial growth literature in the form that both forms of cross sectional dependence are controlled for. Weak cross sectional dependence, or spatial dependence, is controlled for by adding weighted observations from other cross sectional units of the lagged dependent variable. High skilled migration, exports and foreign direct investments (FDIs) are used as spatial weight matrices. Strong cross sectional dependence is taken care of in the form of a multifactor error structure model. The strong dependence in the form of common effects is approximated by cross sectional averages as suggested in Pesaran (2006) and Chudik and Pesaran (2015a).

There are several notable findings. A Lotka-Volterra approach on a classical convergence equation without cross sectional interactions confirms earlier empirical findings in literature. However strong cross sectional dependence remains in these specifications, invalidating the results. Adding cross sectional averages and spatial lags controls sufficiently for cross sectional dependence. In addition, this paper presents evidence for conditional convergence.

The remainder of the paper is structured as follows. The next section introduces a General Lotka-Volterra model, followed by a discussion about convergence. Then a growth model is outlined and the estimation strategy and the empirical equation are discussed. In the following section the empirical results are presented and checked against alternative specifications. The paper closes with a conclusion.

[3]For an overview of the literature on multifactor error structure models see for example Chudik and Pesaran (2015b).

2 A General Lotka-Volterra Model

Lotka-Volterra models are used in mathematical biology to model the evolution of two dependent species and have been developed by Lotka (1920) and Volterra (1926). Samuelson (1971) presented the standard Lotka-Volterra model in an economic context, generalized it to more than two species and derived its equilibrium behavior.[4]

The general two equation Lotka-Volterra model describes the evolution of two species, x and y, with the help of a differential equation system. The population of x depends negatively on the level of the other, y, but grows positively in the absence of y. The population of y depends positively on the population size of specie x, but decreases if the size is zero. The course over time of the two populations can be described by the following two equations:

$$\begin{aligned}\frac{dx}{dt} &= x(a - by)\\ \frac{dy}{dt} &= y(-c + dx)\end{aligned}$$

where $a > 0, b > 0, c > 0, d > 0$. a and c capture the effect the specie has on its own, while b and d represent the interaction between both.

Instead of the two species, assume two countries R and P, with output y_R and y_P. Similar to the population Lotka-Volterra model, one is preying upon the other. Say, the output per capita of country P increases if country R's increases. However the opposite does not hold and country P's output is reduced with the others'. The two interactions can be captured by the terms $-\rho_R w_R$ and $\rho_P w_P$. The system can be then expressed by:

$$\begin{aligned}\frac{dy_R}{dt} &= y_R(a - \rho_R w_R y_P) \Rightarrow \frac{\dot{y}_R}{y_R} = a - \rho_R w_R y_P\\ \frac{dy_P}{dt} &= y_P(-c + \rho_P w_P y_R) \Rightarrow \frac{\dot{y}_P}{y_P} = -c + \rho_P w_P y_R\end{aligned}$$

The two equations can be stacked in matrices:

$$\begin{aligned}\begin{pmatrix} \frac{\dot{y}_R}{y_R} \\ \frac{\dot{y}_P}{y_P} \end{pmatrix} &= \begin{pmatrix} a \\ -c \end{pmatrix} + \begin{pmatrix} -\rho_R w_R y_P \\ \rho_P w_P y_R \end{pmatrix}\\ &= \begin{pmatrix} a \\ -c \end{pmatrix} + \begin{pmatrix} -\rho_R & 0 \\ 0 & \rho_P \end{pmatrix} \begin{pmatrix} 0 & w_R \\ w_P & 0 \end{pmatrix} \begin{pmatrix} y_R \\ y_P \end{pmatrix}\\ &= C + \rho W Y\\ &= C + DY.\end{aligned}$$

Adding time indices gives

$$\Delta Y_t = C + DY_{t-1}. \tag{1}$$

[4] The textbook example of the Lotka-Volterra Model is on the interaction of a predator, y, and a prey, x. In absence of the predator, the population of the prey increases by xa. As the predator needs the prey to survive its population decreases by yc. The larger the population of the predator, more prey are hunted and their population decreases by by. On the other hand, large numbers of prey lead to an increase of the population of the predators by dx.

If the growth rate converges to zero, two solutions can be derived from $Y_t = Y_{t-1} = Y^* = -D^{-1}C$. The first solution is $(y_R, y_P) = (0, 0)$ and in the prevailing context meaningless as it would imply that the income of a country is zero. The second solution is $(y_R, y_P) = ((c/(\rho_P w_P), a/(\rho_R w_R)))$ and has a more meaningful interpretation. If $y_R = y_P$ then the countries converge to the same output per capita. Both solutions are stable if the real parts of the eigenvalues of the community matrix D are negative (Arbia and Paelinck, 2003b; Griffith and Paelinck, 2011).[5] Thus in the case of $y_R \neq y_P$ and stability, both countries would converge to their own equilibrium and conditional convergence occurs.

Using matrix notation, the steady state under convergence would be:

$$\begin{aligned} &\Delta Y_t = 0 \\ &\Rightarrow Y_t = Y_{t-1} = Y^* = -D^{-1}C. \end{aligned}$$

The assumptions on the signs of the interactions $-\rho_R w_R$ and $\rho_P w_P$ can be relaxed, such that $\rho_R w_R$ and $\rho_P w_P$ are positive or negative. Then Lotka-Voterra models can be applied to a more general setting, as for example interactions between countries. In addition, the model is not limited to two countries or equations. In the case of N countries, there would be N difference equations. Still, it would be possible that some countries benefit, while others lose from the interactions. Samuelson (1971) describes such an extension with $n > 2$ predators and prey. Convergence would depend on the properties of the eigenvalues of the community matrix D.

3 Economic Growth Model and Convergence

3.1 Growth Model

Basing on a Cobb-Douglas production function, Barro and Sala-i Martin (1992) derive a closed form solution for a simple growth model of the fashion:

$$\Delta y_{i,t} = \beta_i + \alpha y_{i,t-1} + u_{i,t}, \tag{2}$$

where $y_{i,t}$ is the log of GDP per capita at time period t in country i, β_i is a country specific technology term and $u_{i,t}$ is an iid error term. The two countries converge if the coefficient α on the lagged income per capita level is between -1 and 0 (Mankiw, Romer and Weil, 1992; Islam, 1995). The interpretation is, the higher the income per capita in period t, the lower the growth rate. Therefore each country converges as the growth rate of income per capita declines. Under the assumption that technology is the same for all countries $\beta_i = \beta\ \forall i$, unconditional or absolute convergence occurs if the coefficient α on the lagged income per capita level is between $-1 < \alpha < 0$ (Mankiw, Romer and Weil, 1992; Islam, 1995). Unconditional convergence implies that the gap between countries will be depleted. In the case that β_i is different for countries and $-1 < \alpha < 0$, countries converge to their own steady state, implying conditional convergence. With conditional convergence poor countries do not necessarily catch up and differences

[5]The condition is derived from a Liapunov function, for further details see Arbia and Paelinck (2003b); Griffith and Paelinck (2011). In mathematical biology the matrix D in equation (1) is often called a *community matrix* (Murray, 2002). This paper follows this notation.

in income levels can persist. In both cases, poor countries with a smaller level of GDP grow with a larger growth rate, as the coefficient on lagged GDP per capita is negative.

In the fashion of Gallo and Fingleton (2014) the growth model from equation (2) is extended by a spatial component to account for spatial dependence.[6] To model strong dependence, an unobserved common factor, f_t, and an heterogeneous unit specific factor loading, γ_i, are added. In addition, the coefficient on the lagged dependent variable is allowed to vary across countries:

$$\begin{aligned} \Delta y_{i,t} &= \beta_i + \alpha_i y_{i,t-1} + \rho_i \sum_{i \neq j, j=1}^{N} w_{i,j} y_{j,t-1} + \epsilon_{i,t}, \\ \epsilon_{i,t} &= \gamma_i f_t + u_{i,t} \end{aligned} \tag{3}$$

where ρ_i is the spatial autocorrelation coefficient, $w_{i,j} y_{i,t-1}$ the spatial time lag and $w_{i,j}$ observed spatial weights. The spatial time lag accounts for observed spatial dependence. Growth models like the Solow growth model (Solow, 1956) are a single country model and do not include any interactions or dependencies between countries. As pointed out by De Long and Summers (1991), there is no reason to believe that the growth process of a country is completely isolated from the rest of the world. Countries interact with each other by trade, knowledge transfers and humans migrate and spread their ideas. It might be no coincidence that the first of the New Kaldor Facts laid out by Jones and Romer (2010) emphasizes the importance of dependence between countries. Comin and Hobijn (2010) for example show that the diffusion of new technology intensifies over time. Dependence between countries is modelled in theoretical models such as Krugman (1991) in terms of specialization or Coe and Helpman (1995) which considers the effects of spillover in R&D. In equation (3) the spillovers are captured by the spatial time lag and are therefore global. This means that the dependent variable depends on time lags of the dependent variable of the other cross sectional units. In addition, the spatial weight matrices capture the effect of the neighbours and of neighbours of a higher order as well. Possible channels are trade, allowing for specialization, foreign direct investment which would raise or lower the accumulation of capital or migration in the form of human capital producing more ideas. The common factors, or strong dependence, can be for example common aggregate shocks or time specific fixed effects (Pesaran, 2006; Kuersteiner and Prucha, 2015).

In addition, the coefficients on the lagged dependent variable and the spatial autocorrelation coefficient are allowed to vary across countries. There is no reason to assume parameter homogeneity, as discussed in Durlauf (2001) or Brock and Durlauf (2000). For example, it is unlikely that an increase in lagged GDP has the exact same effect on the growth rate for a country in Africa and the US. The same applies to the spatial autocorrelation coefficient.

3.2 Convergence in spatial growth models

Determining convergence solely on the basis of the coefficient α_i, would miss out the interactions between countries represented by the spatial time lag. Therefore

[6] For an overview of spatial growth models see Fingleton and López-Bazo (2006) and Abreu, De Groot and Florax (2004).

it is necessary to include the spatial time lag into the condition for convergence. Equation (3) can be rewritten in matrix form as:

$$\Delta Y_t = \beta_0 + diag(\alpha Y_{t-1}) + diag(\rho) W Y_{t-1} + \epsilon_t,$$

with

$$Y_t = (y_{1,t}, ..., y_{N,t})' \qquad \epsilon_t = (\epsilon_{1,t}, ..., \epsilon_{N,t})'$$
$$\alpha = (\alpha_1, ..., \alpha_N)' \qquad \rho = (\rho_i, ..., \rho_N)'$$
$$W = \begin{pmatrix} 0 & w_{1,2} & \dots & w_{1,N} \\ w_{2,1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ w_{N,1} & \dots & \dots & 0 \end{pmatrix} \qquad \beta_0 = (\beta_{0,1}, ..., \beta_{0,N})'$$

In the next step $diag(\alpha)$ and $diag(\rho)W$ are added and the matrix system becomes:

$$\Delta Y_t = B + A Y_{t-1} + \epsilon_t, \tag{4}$$

with

$$B = \beta_0$$
$$A = diag(\alpha) + diag(\rho)W = \begin{pmatrix} \alpha_1 & \rho_1 w_{1,2} & \dots & \rho_1 w_{1,N} \\ \rho_2 w_{2,1} & \alpha_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho_N w_{N,1} & \dots & \dots & \alpha_N \end{pmatrix}$$

The last equation is an economic growth representation of the Lotka-Volterra model as presented in equation (1) and used by Arbia and Paelinck (2003a). Similar conditions for convergence as for the Lotka-Volterra model outlined in Section 2 apply here. For stability, the real parts of the eigenvalues of the community matrix A have to be $-2 < \lambda_1, ..., \lambda_N < 0$.[7] In addition equation (3) can be re-written if $y_{i,t} = y_{i,t-1} \equiv y_i^*$ and $\Delta y_{i,t} = 0$ as:

$$y_i^* = -\frac{\beta_{0,i} + \rho_i \sum_{j=1, i \neq j} w_{i,j} y_j^*}{\alpha_i}. \tag{5}$$

Equation (5) illustrates several requirements for convergence. As α_i is expected to be negative, the sum of numerator has to be positive, such that y_i^* is positive as well. If one country does not converge, all other countries do not converge as well, if $\rho_i \neq 0$ and $w_{i,j} \neq 0$. If either $\rho_i = 0$ or $w_{i,j} = 0$, then convergence for the other countries is possible, despite the entire system not being stable. Moreover, a negative ρ_i implies a larger steady state. For unconditional convergence, the entries of matrix $A^{-1}B$ are the same for each country, i.e. $y_i^* = y_j^* = y^*, \ \forall i, j$. Thus the steady state under absolute convergence is:

$$y_i^* = y^* = -\frac{\beta_{0,i}}{\alpha_i + \rho_i \sum_{j=1, i \neq j} w_{i,j}}. \tag{6}$$

[7] The requirement for stability for a discreet dynamic equation is, $|\lambda_i| < 1, \ \forall i$, see Galor (2007). This is the case if the level occurs on both sides, here on the left hand side the growth rate occurs.

4 Heterogeneous Panel Estimators

In the following, two estimators are discussed: the Simultaneous Dynamic Least Square estimator (SDLS) and the DCCE estimator. Both produce consistent estimates of the unit specific coefficients as necessary for the construction of the matrices A and B in equation (4). While DCCE allows to control for both types of cross sectional dependence, SDLS can only handle spatial dependence.

4.1 Simultaneous Dynamic Least Square

Arbia and Paelinck (2003a,b) employ the SDLS estimator. Therefore the estimator is used in this paper to compare the results with earlier findings in the literature. SDLS minimizes the squared deviations between an observed and an endogenously computed value and is similar to seemingly unrelated regressions (SURE) or a 3-stage least square estimator (3SLS) without endogeneous variables. It is a generalized reduced form estimator, a maximum likelihood estimator if $\epsilon \sim N(0, \sigma^2 I)$ and consistent estimator under homoscedastic errors. For a further description see supplemental data online, section B2, or Griffith and Paelinck (2011, Chapter 11).

4.2 Dynamic Common Correlated Effects

An estimator which accounts for heterogeneous slopes and cross sectional dependence is the Dynamic Common Correlated Effects (DCCE) estimator (Chudik and Pesaran, 2015a). The DCCE estimator is based on the Common Correlated Effects (CCE) estimator developed by Pesaran (2006). The underlying assumption for both estimators is, that the coefficients are randomly distributed around a common mean, i.e. $\beta_i = \beta + v_i,\ v_i \sim IID(0, \Sigma_v)$. Moreover the data generating process includes an unobserved common factor f_t and an heterogeneous unit specific factor loading γ_i, as for example:

$$\begin{aligned} y_{i,t} &= \alpha_i + \lambda_i y_{i,t-1} + \beta_i x_{i,t} + u_{i,t} \\ u_{i,t} &= \gamma_i' f_t + e_{i,t}. \end{aligned} \tag{7}$$

$\gamma_i' f_t$ represents strong cross sectional dependence. If this term is not accounted for in a regression, it becomes part of the error term and renders OLS inconsistent. Pesaran (2006) shows that adding cross sectional averages of the dependent and independent variables in a static panel data model (with $\lambda_i = 0$) factors out cross sectional dependence. The mean group estimates are consistently estimated if $(N, T) \xrightarrow{j} \infty$. If the coefficients of interest are stacked together into the vector $\pi_i = (\alpha_i, \lambda_i, \beta_i)$, then the mean group estimates are:

$$\hat{\pi}_{MG} = \frac{1}{N} \sum_{i=1}^{N} \hat{\pi}_i.$$

For a dynamic panel model ($\lambda \neq 0$) the CCE estimator becomes inconsistent as the lagged dependent variable is no longer strictly exogenous. Chudik and Pesaran (2015a) show that adding p_T lags of the cross sectional averages takes out the strong cross sectional dependence. Similar to the assumptions of the static model, the dynamic model is consistent if $(N, T, p_T) \xrightarrow{j} \infty$, such that

$p_T^3/T \to \varrho_1, 0 < \varrho_1 < \infty$ and $N/T \to \varrho_2, \varrho_2 > 0$. The first condition ensures that the number of lags is restricted to allow a sufficient degree of freedom. Therefore, the floor of $\sqrt[3]{T}$ lags are added, hence $p_T = \left[\sqrt[3]{T}\right]$. The second condition implies that the number of cross sectional units and time periods grows with the same rate. The estimated equation including the cross sectional averages basing on equation (7) becomes:

$$\begin{aligned} y_{i,t} =& \beta_{0,i} + \alpha_i y_{i,t-1} + \beta_i x_{i,t} \\ &+ \sum_{l=0}^{p_T} \left(\vartheta_{i,l,y} \bar{y}_{i,t-l} + \vartheta_{i,l,x} \bar{x}_{i,t-l}\right) + \epsilon_{i,t}, \end{aligned} \tag{8}$$

where a bar denotes the cross sectional averages and p_T is the number of lags. The cross sectional averages approximate the strong unobserved cross sectional dependence between countries. Individual coefficients are estimated for each country and then averaged to obtain the mean group version of the estimator. The coefficients on the cross sectional averages $\vartheta_{i,l,y}$ and $\vartheta_{i,l,x}$ have no meaningful interpretation, because the averages enter solely as further control variables. Therefore they are not reported or discussed in this paper.

4.3 Testing for Weak Cross Sectional Dependence

Following equation (7), the factor f_t is weakly cross sectional dependent if:[8]

$$\lim_{N\to\infty} \sum_{i=1}^{N} |\gamma_i| = \kappa < \infty$$

and strongly cross sectional dependent if:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} |\gamma_i| = \kappa > 0,$$

where κ is a constant. Cross sectional independence is defined by $\gamma_i = 0 \ \forall i$. However cross sectional independence is a restrictive assumption for large panels and only strong cross sectional dependence poses a problem (Pesaran, 2015).

Pesaran (2015) developed a test for weak cross sectional dependence. Under the null, the error terms are weakly cross sectional dependent. Weak cross sectional dependence implies that the average pairwise correlation between units, ρ_{ij}, tends to zero in N. Under the alternative, the error terms are strong cross sectional dependent, meaning they converge to a fixed number if $N \to \infty$. The implication of common factors is that the error term is not iid anymore and therefore OLS becomes inconsistent (Everaert and De Groote, 2016). As the common correlated effects model is essentially an OLS regression, testing for cross sectional dependence becomes indispensable. The CD test statistic is the average over all correlations:

$$CD = \sqrt{\frac{2T}{N\left(N-1\right)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \right),$$

[8] To avoid confusion with the literature, the test is still called a 'test for weak cross-sectional' dependence, rather than a test for spatial dependence.

with

$$\hat{\rho}_{ij} = \hat{\rho}_{j,i} = \frac{\sum_{t=1}^{T} \hat{\epsilon}_{i,t}\hat{\epsilon}_{j,t}}{\left(\sum_{t=1}^{T} \hat{u}_{i,t}^2\right)^{1/2} \left(\sum_{t=1}^{T} \hat{\epsilon}_{j,t}^2\right)^{1/2}},$$

where $\hat{\rho}_{ij}$ is the correlation coefficient and $\hat{\epsilon}_{i,t}$ are the residuals from a regression on equation (8). Under the null, the test statistic is asymptotically normal distributed:

$$CD \sim N(0,1).$$

5 Empirical Equation

Rewriting equation (3) with the cross sectional averages instead of the common factors, the following equation is estimated:[9]

$$\Delta y_{i,t} = \beta_{0,i} + \alpha_i y_{i,t-1} + \sum_{k=1}^{K} \rho_{(k),i} \sum_{j=1,i\neq j}^{N} w_{(k),i,j} y_{j,t-1} + \sum_{l=0}^{p_T} \vartheta_i \bar{y}_{i,t-l} + u_{i,t} \quad (9)$$

where $\sum_{k=1}^{K} \rho_{(k),i} \sum_{j=1,i\neq j}^{N} w_{(k),i,j} y_{j,t-1}$ captures spatial dependence and $\sum_{l=0}^{p_T} \vartheta \bar{y}_{i,t-l}$ accounts for common factors. The model can be written in matrix notation:

$$\Delta Y_t = B + AY_{t-1} + u_t,$$

with u_t containing the cross sectional averages. The difference between equation (9) and equation (3) is that the latter allows for multiple spatial interactions, namely K spatial weights matrices, and common factors. Equation (9) follows Arbia and Paelinck (2003a,b) and has several insights. First it is relatively easy to compute. Secondly, no country specific effects have to be considered. The equation can be estimated for each country separately and the constant contains the country specific effects. This procedure avoids removing the country specific effect by transforming the convergence equation into first differences, which would be then exposed to endogeneity.[10] The spatial time lag ensures that the term is exogenous, as neither $y_{i,t}$ nor $y_{i,t-1}$ are included in $y_{j,t-1}$. Endogeneity is not present as long as the errors are not autocorrelated and further explanatory variables are exogenous. Finally it accounts for spatial dependence using the spatial time lag and common factors by the multifactor error structure, respectively the cross sectional averages.

The combination of spatial dependence and common factors recently received attention in the literature and was summarized in Elhorst et al. (2016). Ertur and Musolesi (2016) estimate total factor productivity with a multifactor error structure and spatial dependence in the form of geographical distance. Bailey, Holly and Pesaran (2016) develop a two step estimator to control for spatial dependence and common factors. In the first step the common factors are

[9]Note that the model can be rewritten in levels as $y_{i,t} = \beta_{0,i} + (1+\alpha_i)y_{i,t-1} + \sum_{k=1}^{K} \rho_{(k),i} \sum_{j=1,i\neq j}^{N} w_{(k),i,j} y_{j,t-1} + \epsilon_{i,t}$. The point estimates between the two models do not differ and the dependent variable occurs as a spatial time lag.

[10]For a more detailed discussion on endogeneity in a growth empirical context, see Hauk and Wacziarg (2009); Gallo and Fingleton (2014)

factored out. The weakly cross sectional dependent observations are then used in the second step in a spatial model. Bailey, Holly and Pesaran (2016) apply it to a model of house prices and estimation of the spatial weight matrix. Vega and Elhorst (2016) apply it to a regional unemployment model.

The approach in this paper differs in several ways from the empirical spatial growth models. First of all it takes both sources of cross sectional dependence into account. The common factors capture unobservable shocks which hit all countries at the same time.
Another difference is the inclusion of a spatial time rather than a contemporaneous spatial lag. Lopez-Bazo, Vaya and Artis (2004); Fingleton and López-Bazo (2006); Ertur and Koch (2007); Lesage and Fischer (2008); Elhorst, Piras and Arbia (2010) and Ertur and Koch (2011) use a spatial lag model and emphasise the importance to include a contemporaneous spatial lag. A notable exception in the context of spatial growth models is Ho, Wang and Yu (2013). They follow the approach from Ertur and Koch (2007) and estimate a spatial Solow model extended by a spatial time lag. A contemporaneous spatial lag is unfeasible for the presence of this paper. Estimating a spatial lag model requires methods which do not rely on OLS. The model is rewritten into a closed form solution with the inverse of an identity matrix minus the spatial weight matrix multiplied to the right hand side. Usually this model requires a maximum likelihood estimator (Lee, 2004; Yu, de Jong and Lee, 2008) or GMM estimator (Kelejian and Prucha, 1999) and cannot be used in a multifactor error structure. Thus, adding a contemporaneous spatial lag would rule out the possibility of using the Dynamic Common Correlated Effects estimator to control for common factors. As a result of the spatial lag, the type of endogeneity in this paper differs from the one mainly occurring in the literature. In a spatial lag model with an exogenous spatial weight matrix, reversed causality occurs as the observations of the dependent variable from other cross sectional units are added as explanatory variables. This type of endogeneity in a spatial lag model is usually estimated with an IV or GMM approach (Kelejian and Prucha, 1998, 1999). In this paper the endogeneity stems from the spatial weight matrix and thus the spatial time lag becomes endogenous.
A third difference to the works above and to Barro and Sala-i Martin (1992); Mankiw, Romer and Weil (1992); Islam (1995) is the assumption of slope heterogeneity, as mentioned earlier. In an empirical setting, parameter heterogeneity was discussed in Lee, Pesaran and Smith (1997, 1998); Islam (1998) and Eberhardt and Teal (2011). Especially in a cross country setting the assumption of slope homogeneity is rather strong. Slope heterogeneity is only used in a few papers such as Lee, Pesaran and Smith (1997) and Eberhardt and Teal (2011). It is a prerequisite for the Lotka-Volterra model as the model requires estimates for each cross sectional unit.
Another difference is that the growth model in equation (3) does not include any further covariates, such as physical or human capital or a measure of the size of the population. In the case of a Lotka-Volterra model, further covariates would be required to be stable as well and would complicate the conditions for convergence.

The exclusion of a spatial lag and further covariates are important limitations of the Lotka-Volterra approach. Including those would complicate the estimation method as described above. The focus of this paper is on the Lokta-Volerra approach in a cross country setting with a multifactor error structure, which controls for both types of cross sectional dependence. Therefore the model and the estimation method are kept as simple as possible.

6 Testing for absolute convergence and inference on eigenvalues

In order to state the type of convergence, the conditions for it have to be assessed. The country specific estimates are used to calculate the steady states for each country $\hat{y}_i^*$. Then a Wald Test with the hypothesis $H_0 : y_1^* = y_2^* = ... = y_N^*$ is performed to test for absolute convergence. If the null is rejected, the steady states are different from each other and no absolute convergence is found.

Testing for conditional convergence requires more care, as instead of the steady states, the negativity of the eigenvalues is tested. A vast literature deals with inference of eigenvalues of covariance matrices in principal component analysis. However, in contrast to inference on covariance matrices of principal components, the eigenvalues in this paper are allowed to be negative. Hence the large sample distribution theory as derived for example in Anderson (2003) is not valid.

An approximate distribution of the eigenvalues can be derived using the Delta Method. Under the assumption that the eigenvalues are a function of ω, i.e. $\lambda(\omega) = (\lambda_1, ..., \lambda_N)'$, with $\omega = (\alpha, \rho), \omega_i = (\alpha_i, \rho_i)$ and Ω the true value, then $\omega \rightarrow_p \Omega$ and $\sqrt{n}(\omega - \Omega) \rightarrow_d N(0, \Sigma)$.[11] The distribution of the eigenvalues using the delta method is then $\sqrt{(n)}(\lambda(\omega) - \lambda(\Omega)) \rightarrow_d N(0, \Lambda(\Omega)\Sigma\Lambda(\Omega)')$.[12] Then a possible hypothesis would be $H_0 : -2 < \lambda_i < 0 \; \forall i$. However this joint test would involve inequalities under the null hypothesis. As an alternative, the maximum of an Eigenvalue can be tested. The maximum of eigenvalues follows an extreme value distribution. This approach is unfeasible as the moments of the extreme value distribution would rely on a single observation.

As an alternative a bootstrap is carried out, where the cross sectional dimension (N) is fixed and $T \Rightarrow \infty$. Moreover as this is a heterogeneous slope model, the unit specific coefficients ω_i depend on the observed data $X = (X_1, ..., X_T)$, implying that ω is a function of the data, $\omega = \omega(X)$. Hence the eigenvalues are indirectly a function of the observed data as well, $\lambda(\omega(X))$. For the bootstrap the following steps are performed:[13]

1. $X = (X_1, ..., X_T)$ is split into country specific blocks of length B.
2. A sample X^* is drawn with replacement.
3. Equation (9) is estimated, matrix A constructed and the eigenvalues computed.
4. The number of eigenvalues $-2 < \lambda_i < 0$ is stored in T_r.

[11] Additionally the weight matrix W and their weights have to be uncorrelated with ω. $\Lambda(\Omega)$ is the first derivative of the eigenvalues.

[12] See Hayashi (2000) Lemma 2.5.

[13] For an overview of bootstrap methods see Horowitz (2001) and for the block bootstrap Härdle, Horowitz and Kreiss (2003). A non-overlapping block bootstrap is preferred over the more complicated overlapping block bootstrap. Härdle, Horowitz and Kreiss (2003) report that the numerical results of the two approaches are similar. The length for the bootstrap is $B = 10$ periods. Results with smaller and larger blocks are similar and available upon request.

5. Steps 1 - 4 are repeated R times
6. $\bar{T} = \frac{1}{R}\sum_{r=1}^{R} T_r$ is calculated.

Under the null hypothesis all eigenvalues are between -2 and 0, implying convergence, thus $H_0 : \bar{T} = N$, against the alternative $H_0 : \bar{T} < N$.

7 Data

GDP per capita originates from the Penn World Tables 8 (Feenstra, Inklaar and Timmer, 2015) and is transformed into logarithms. The yearly data is restricted to the years 1960 to 2007. Even though the Penn World Tables are available until 2011, data from 2008 onwards is excluded due to the financial crisis.

Spatial Weights Three different row normalised spatial weight matrices are used in this study. The first one is based on migration streams of high skilled workers. The weight matrix is constructed out of data for 2010 from the Institut fr Arbeitsmarkt- und Berufsforschung (IAB, Institute for Employment Research) brain drain dataset (Brücker, Capuano and Marfouk, 2013). The rational is, that high skilled workers contribute to the economic performance of a country and strengthen the ties between countries.[14] A positive spatial autocorrelation coefficient $\rho_{(1)}$ would imply that high skilled migrants are carriers of positive spillover effects.

Mayda (2010) finds using bilateral migration flows from the OECD International Migration Statistics that pull factors (income in destination country) increase the size of emigration rates. Barro and Sala-i Martin (2004) assume a smaller rate of convergence if the regression contains migration. They find a small negative effect of migration on income per capita growth rates.

On a regional level for 27 European regions, Huber and Tondl (2012) find evidence that migration has a positive effect on the income per capita of the destination country, while it negatively effects the country of origin. Ozgen, Nijkamp and Poot (2010) summarize the effect of migration on convergence on a regional level in form of a Meta Study.

The second spatial weights matrix are trade shares between countries, originating from the Direction of Trade Statistics (DOTS) of the IMF. The data ranges from 2009 to 2013 in yearly intervals. As for migration the latest available year is used as the baseline spatial weights matrix. Trade is commonly used as a spatial weight for economic distance (LeSage and Pace, 2008). If two countries trade a lot, spillovers between these two will be larger. A more theoretical rationale is that countries use their comparative advantage in the production of specific goods, which are then traded. Moreover it is suggested in the literature (Grossman and Helpman, 1991; Coe and Helpman, 1995) that international trade is a major channel for diffusion of technology. Both theories strongly suggest an estimated spatial autoregressive coefficient for the trade

[14]Only high skilled migration into OECD countries is taken into account, from OECD countries as well as from non OECD countries. The first reason is that the amount of high skilled migrants into non OECD countries is likely to be very small. Secondly data for streams between non OECD countries is largely unavailable. Alternatively it would be possible to restrict the study to OECD countries, with the cost of losing 3/4 of the observations. A list of the countries can be found in an Online Appendix.

weight matrix larger than zero. In a growth empirical setting, trade as a spatial weights matrix to measure economic distance can be found for example in Ertur and Koch (2011) or Ho, Wang and Yu (2013).

As a third weight matrix foreign direct investments from the Coordinated Direct Investment Survey (CDIS) of the IMF are considered. The data spans the years 1980 until 2010 in 5 year intervals. In line with the other two spatial weights, the latest available year is used as a spatial weight. FDI represents technological diffusion. In a spatial context, it is surveyed in Abreu, De Groot and Florax (2004).

8 Estimations

In the next section regression results are presented. First estimates obtained by the SDLS estimator are discussed and compared to earlier findings in the literature. Then regression results using the Dynamic Common Correlated Effects estimator are analysed.[15]

8.1 Simultaneous Dynamic Least Squares

SDLS results are presented in Table A1. Panel A shows the results without any spatial weights. The p-values from the bootstrap with the hypothesis that all eigenvalues are negative are shown in squared brackets in the column labelled "Eigenval." The p-value for the test of absolute convergence is shown in parenthesis. In the last row of each panel the spatial weight is removed from the computation of the community matrix A and the number of negative eigenvalues and the corresponding p-value of the bootstrap is shown.

The regression results are not in favor of conditional or absolute convergence, as the hypothesis of all eigenvalues being negative and all steady states being the same are rejected. However the mean group estimate of the coefficient on the lagged dependent variable is negative and significant. Therefore in a classical sense, conditional convergence is found.
Adding spatial weights changes the overall picture. All values for α and the real parts of the eigenvalues of A are negative. In the case of migration and exports as spatial weights, conditional convergence is found as all eigenvalues are negative. This evidence is supported by the bootstrap. Moreover the spatial autocorrelation coefficient using exports as a weight is significant at a level of 10% . This result is in line with the results from Arbia and Paelinck (2003a,b), who find convergence once spatial interactions are accounted for. The last panel includes all three spatial weight matrices. The bootstrap just rejectes the hypothesis of conditional convergence. However not all eigenvalues are negative and none of the spatial weights are significant.

As the last column indicates, the hypothesis of spatial dependence in the residuals is rejected for the case without spatial lags and if the FDI migration weight matrix and all three weight matrices are used. This implies that the residuals are not iid and OLS is rendered inconsistent. Therefore, as a next step, the Dynamic Common Correlated Effects estimator is used to control

[15]The estimations are performed in Stata using the `xtdcce2` package (Ditzen, 2017), version 1.32 from July 2017.

for unobserved cross sectional dependence together with spatial interactions to account for observed dependence.

8.2 Dynamic Common Correlated Effects - Mean Group

Table A2 displays results using the common correlated effects estimator. Cross sectional averages of the independent variable and $\left[\sqrt[2]{47}\right] = 3$ lags of it are added in Panels B - F.

Panel A shows results without any cross sectional averages or spatial weights. The mean of the coefficient on the lagged dependent variable, α_i is negative and highly significant. However only 83 eigenvalues are between -2 and 0, indicating no conditional convergence. The bootstrapped p-value of the test that all eigenvalues are between -2 and 0 is 0.988, thus supporting the hypothesis of negative eigenvalues. The hypothesis of absolute convergence is rejected. The p-value of the CD test in the final column rejects the hypothesis of weak cross sectional dependence, ruling the results inconsistent.

If cross sectional averages are added (Panel B), the overall results appear to be similar and the hypothesis of weak cross sectional dependence in the error terms is still rejected. However, the value of the test statistic decreases from 31.55 to 22.02. Only one country has an Eigenvalue not between -1 and 0, implying that most countries show a tendency towards convergence.

In the regressions in Panel C to Panel E one spatial interaction is included. The spatial autocorrelation coefficient for migration and FDI is significant and positive in both cases. This implies that the growth rate increases for countries which have strong ties via migration or FDI. For migration the result is a contradiction to earlier findings as in Barro and Sala-i Martin (2004). The last row on the panel shows the number of eigenvalues between -2 and 0 if the spatial weight is removed from matrix A. In all cases excluding the spatial interaction does not change the number of negative eigenvalues. The coefficient on Exports remains insignificant. The hypothesis of weak cross sectional dependence cannot be rejected in all three cases. This result shows the importance of accounting for both types of cross sectional dependence in a growth regression.

Panel F includes all three spatial weights. Out of the spatial interactions only FDI has a significant positive effect. The number of eigenvalues between -2 and 0 is stable around 88, the bootstrapped p-values imply that the hypothesis of conditional convergence can not be rejected. None of the regressions show any evidence for absolute convergence as the p-values are 0.

The estimated convergence equation is similar to the models estimated by Baumol (1986) and Barro and Sala-i Martin (1992). Baumol finds for a larger set of countries club convergence, but for a smaller absolute convergence. While the former fits loosely into conditional convergence, the latter is clearly contradicted by the results. Barro and Sala-i Martin find conditional convergence for 98 countries. In a more recent study Barro (2015) finds conditional convergence in large panel as well. Therefore the results show that a Lotka-Volterra approach can be placed into the existing literature.

8.3 Robustness Checks

Several robustness checks were carried out and lead to similar conclusions, confirming conditional convergence but no evidence for absolute convergence. To save space, detailed results and discussions are reported in the supplemental data online, section B4.

If the spatial autocorrelation coefficients are constrained to be equal across units ($\rho_{(k),i} = \rho_k \forall k, i$), none of the spatial autocorrelation coefficients are significant and the number of negative eigenvalues remains constant.

Under the assumption that the spatial weights are endogenous an instrumental variable approach following Qu and Lee (2015) is carried out. The three spatial weight matrices are instrumented using distance and distance squared. The overall picture remains the same: FDI and to a lesser extent Exports are the main drivers of spatial interactions. While FDI clearly improves growth rates, Exports have a dampening effect. As in the other specifications, evidence for conditional convergence is found.

In a further robustness check, the number of lags of the cross sectional averages is reduced to 1. As expected this leads to lower p-values of the CD test statistic, which underlines the importance of an appropriate number of lags to control for cross sectional dependence. If the US is modelled as a common factor in style of Chudik and Pesaran (2013), none of the spatial autocorrelation coefficients is significant. This underlines the importance of the US for spatial dependence in terms of foreign direct investments, trade and migration.

9 Conclusion

This paper studies convergence in a cross country framework using a general Lotka-Volterra Model to detect the type of convergence. Two types of cross sectional dependence are accounted for. Common factors are incorporated by adding cross sectional averages of the dependent and independent variables. As a second type, spatial dependence is accounted for by using a spatial time lag with three different weight matrices. The model is estimated with two different types of estimators: a Simultaneous Dynamic Least Square estimator and the Common Correlated Effects Estimator, in the mean group as well as in the pooled version.

The paper extends the current literature manifold. First of all the approach by Arbia and Paelinck (2003a,b) is advanced by taking both types of cross sectional dependence into account. The conditions for absolute and conditional convergence are tested directly and by using a bootstrap method. The type of convergence is identified using a difference equation system, rather than a single parameter. This is a further novelty in cross country regressions. Finally the use of dynamic common correlated effects in combination with spatial weights to account for observed as well as unobserved (global) factors in a growth empirical setting is new to the best of our knowledge. Using this rather simple estimation technique comes at the cost of one important limitation. A spatial time lag is used rather than a more general spatial lag model.

There are several notable findings. A Lotka-Volterra approach on a classical convergence equation without spatial interactions confirms earlier empirical findings in literature. Conditional convergence appears to be more frequent if both types of cross sectional dependence are controlled for. This allows the conclusion that interactions between countries contribute towards convergence. Secondly this argument underlines the importance of considering spatial dependencies between countries in growth empirics. Especially FDI and Migration appear to play a strong role in connecting countries and somewhat surprisingly to a lesser extent trade. Finally, the non rejection of weak cross sectional dependence, if controlled by either spatial interactions or cross sectional averages, is a strong argument to account for those. To put it succinctly, a growth regression without any interactions between countries is likely to be inconsistent.

There are some possible extensions to the estimation approach of this paper. In a growth empirical setting, applying the model to estimate the rate of convergence and the question of club convergence is of interest. Especially the matrix environment of the Lotka-Volterra model has the potential to determine convergence clubs. Three further applications of the Lotka-Volterra model in combination with the cross sectional dependence literature are possible. First, the spatial Solow model developed by Ertur and Koch (2007) with physical and human capital and population growth as explanatory variables can be estimated adding cross sectional averages to control for strong dependence. Secondly, a contemporaneous spatial lag can be added. This would make the model more general, but the estimation procedure would be more complicated. Finally, the two step estimation procedure proposed by Bailey, Holly and Pesaran (2016) and the GMM procedure developed by Kuersteiner and Prucha (2015) can be applied to the settings above. Further work is required to encompass the two strands into a Lotka-Volterra model, which is beyond the scope of this paper and left for further research.

A1 Appendix

A1.1 Tables

Statistic	α_i	$\rho_{1,i}$	$\rho_{2,i}$	$\rho_{3,i}$	$\beta_{0,i}$	Eigenval.	CD Test, p-val
Panel A: No weights							
Mean	-0.0344***				0.0344***	-0.0344	5.6036
SE	0.0026				0.0098	0.0249	0.0000
$\# < 0^{\dagger}$	84				63	84 [0.0000] (0.00)	
Panel C: Migration Weight Matrix							
Mean	-0.6365***	0.1218			0.5114	-0.6365	-1.2999
SE	0.0116	0.1042			0.5900	0.1065	0.1936
$\# < 0^{\dagger}$	93	7			9	93 [1.0000] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$		93 [1.0000]					
Panel D: FDI Weight Matrix							
Mean	-0.0936***		-0.1097		0.2313	-0.0936	6.5453
SE	0.0132		0.1870		0.2483	0.1282	0.0000
$\# < 0^{\dagger}$	75		35		74	76 [0.0000] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$			75 [0.0000]				
Panel E: Exports Weight Matrix							
Mean	-0.9752***			0.0213*	0.9525	-0.9752	0.2940
SE	0.0194			0.0149	0.9261	0.1874	0.7688
$\# < 0^{\dagger}$	93			17	33	93 [1.0000] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$				93 [1.0000]			
Panel F: Migration, Exports and FDI Matrices							
Mean	-0.6964***	-0.4819	0.6428	-0.3328	0.8427	-0.6964	12.0440
SE	0.0386	0.7105	0.6906	0.3071	0.7986	0.3704	0.0000
$\# < 0^{\dagger}$	90	12	46	27	47	90 [0.0540] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$		90 [0.0540]	90 [0.0000]	91 [0.0000]			

Table A1: Simultaneous Dynamic Least Squares Estimation
Row *Mean* displays the mean group estimates, i.e. $\alpha = \frac{1}{N}\sum_{i=1}^{N}\alpha_i$. The last row in each panel counts how often the individual coefficients (eg.: $\alpha_i \forall i$) are smaller than zero. The three steps are 50 times repeated to ensure convergence of the coefficients. For a more detailed discussion see the online appendix. Significance levels: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Statistic	α_i	$\rho_{1,i}$	$\rho_{2,i}$	$\rho_{3,i}$	$\beta_{0,i}$	Eigenval.	CD Test, p-val
Panel A: No weights							
Mean	-0.0466***				0.3882***	-0.0466	31.5479
SE	0.0054				0.0384	0.0520	0.0000
$\# < 0^{\dagger}$	20				0	83 [0.9880] (0.00)	
Panel B: No weights but with cross section averages (CCE)							
Mean	-0.1435***				0.0020	-0.1435	22.0185
SE	0.0115				0.2081	0.1111	0.0000
$\# < 0^{\dagger}$	33				10	92 [1.0000] (0.00)	
Panel C: Migration Weight Matrix and CCE							
Mean	-0.1513***	0.0311**			0.0862	-0.1513	0.5497
SE	0.0124	0.0154			0.1760	0.1197	0.5825
$\# < 0^{\dagger}$	32	0			12	89 [1.0000] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$		89 [1.0000]					
Panel D: FDI Weight Matrix and CCE							
Mean	-0.1863***		0.1002***		0.2277	-0.1863	0.0243
SE	0.0152		0.0401		0.2252	0.1521	0.9806
$\# < 0^{\dagger}$	36		5		8	88 [0.9960] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$			88 [0.9960]				
Panel E: Exports Weight Matrix and CCE							
Mean	-0.1851***			-0.0054	0.2258	-0.1851	0.9099
SE	0.0142			0.0533	0.2127	0.1377	0.3629
$\# < 0^{\dagger}$	35			13	9	91 [0.9980] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$				91 [0.9980]			
Panel F: Migration, Exports and FDI Matrices and CCE							
Mean	-0.2300***	-0.0127	0.0887*	0.0273	0.1816	-0.2300	0.1731
SE	0.0170	0.0385	0.0590	0.0796	0.2912	0.1725	0.8626
$\# < 0^{\dagger}$	41	0	8	13	6	88 [0.9940] (0.00)	
$\# -2 < \lambda_i < 0^{\dagger\dagger}$		89 [0.9900]	86 [0.9800]	88 [0.9920]			

Table A2: Dynamic Common Correlated Effects Estimation, $p_T = 3$

Row *Mean* displays the mean group estimates, i.e. $\alpha = \frac{1}{N}\sum_{i=1}^{N}\alpha_i$ for α and β_0, and row *SE* its standard error. Row $\# < 0$ counts how often the individual coefficients are significantly smaller than zero at a level of 5% $H_0 : \alpha_i < 0$. † Bootstrapped values for the test of conditional convergence are in squared brackets, p-value for test of absolute convergence ($H_0 : y_1^* = ... = y_N^*$) in parenthesis. †† Spatial Weight removed. For a more detailed discussion see section 4. Significance levels: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

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