Dynamics and optimization of a new double-axle flexible bogie for a high speed train

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Abstract
The paper is focused on the discussion of a new double-axle flexible bogie for a high-speed train. The main feature of the flexible bogie is that it consists of two sub-bogies connect with diagonal links. Moreover an elastic connection between a carriage and both wheelsets is introduced. These features, helping to increase the bogie flexibility by passing tracks with a low radius of curvature, are studied in this paper numerically. The results demonstrate a great potential of the bogie and ability to travel with no significant oscillations at speed of 432km/h. Numerical optimization of the bogie’s parameters is performed in order to maximize the ride comfort.

Keywords: Two-axle wheelset, high-speed train, train dynamics, vibrations.

1 Introduction

The worldwide trend in last 10-15 years in railway transportation has been pointing towards increasing the axial load and speed of trains, with the latter being a special issue for the passenger trains. The idea of the high-speed passenger transportation is supported by the fact that it is not only environmentally friendly but also because it can compete against the air travel in distances from 500 to 800 km. Many European and Asian countries have regular high-speed train services, usually operating between large cities. In Europe, including the UK, the normal operating speed is currently around 200km/h, whereas in China it is 300km/h (Maglev train systems not considered). In general the term “high-speed” is very broad and related to any speed above 200km/h, although usually it means a travel with 200km/h-250km/h. However, in the future this term will imply an operating speed up to 400km/h. At this speed a train exerts large forces onto the track, which acts back onto the train and along with other forces (aerodynamic, for instance) making train oscillate in all directions. These oscillations, observed in wheelsets, bogies and carriages may lead to passengers’ ride discomfort if not have more severe consequences. Therefore the safety remains the major concern of a high-speed travel.
Bogie may be considered as the most important part of a train from the dynamics point of view, since it carries the load, provides guidance of wheelsets, connecting wheelsets with a carriage and suppose to attenuate adverse vibrations. In last hundred years various bogie designs have been proposed and studied [1]. In general there are two types of worldwide accepted bogies: single-axle or two-axel, where the former has a single wheelset whereas the latter has a double wheelset. Example of a single-axle and two-axle bogies are shown in Figure 1 left and right correspondingly. Since two bogies are usually required for a carriage, a two-axle bogie has lower value of the load per axle. However, the rigid construction of the traditional two-axle bogie creates certain difficulties when curving, limiting the minimal curve radius as well as has some other issues [1],[2].

![Figure 1. Single-axle bogie for Lirex and two-axle bogie.](image)

This paper discusses a novel two-axle bogie that has been developed in Moscow State University of Railway Transportation. The paper is structured as following. The bogie design is presented in section 2, whereas in section 3 governing equations of motion are discussed. Section 4 present results of numerical modelling and optimization and section 5 has conclusions.

## 2 Novel Bogie Design

As it has been mentioned above, the standard two-axle bogie does not suit for high curvature turns, which is essential in some parts of the world. The major benefit of the presented bogie is its design that allows the wheelsets match the radius of the curve, thus decreasing the force of the interaction between wheels and rails [2]. Two (left) and three (right) stages suspension, implemented in the proposed bogie, are presented in Figure 2. Three-stage suspension has an additional connection between the wheelset and carriage.
The primary connection, denoted by number “1” in Figure 2, connects the wheelset and bogie, the secondary connection, denoted by number “2” connects the bogie and carriage, whereas the third connection, denoted by number “3” connects directly the wheelset and carriage. Diagrams presented in Figure 2 demonstrate the concept. It should be mentioned that the idea of connecting two bogies with diagonal linkage itself is not new [1], however other design features make the proposed design rather unique.

Item 1 in Figure 3 represents a pneumatic suspension directly connecting the wheelsets and carriage, so that the static load is directly transmitted. This became possible due to the bogie design and is the subject of investigation here. In Figure 3 items 2 represents shock absorbers, connecting the wheelset and carriage and located at an angle to the longitudinal axis of the trolley, whereas item 3 represents springs connecting the bogie and carriage and acting in vertical and transverse directions. Item 4 represents a diagonal linkage which allows two wheelsets turn with respect to each other in yaw when curving, keeping them together. This arrangement provides an opportunity for mitigating hunting oscillations of the carriage and bogies. Mechanical properties of these elements will be discussed in the next section.
Figure 3. The high-speed bogie with the pneumatic suspension.

3 Complete mathematical model with a new bogie

To study the behaviour of the train with the new bogie under various conditions a mathematical model has been created. Figure 4 and Figure 5 demonstrate the rheological elements that have been used for modelling in all directions. Figure 5 and Figure 6 show the front and top view of the model correspondingly. The developed model considers a motion in 3 translational and 3 rotational directions.
The governing equations of motion of the model have been derived based on D’Alembert’s principle. The selected generalized coordinates allowed writing the equations of motion in the following form for translational and rotational motion correspondingly:

Figure 4. The sketch of the carriage, bogie and wheelset (front view)

Figure 5. The sketch of the carriage, bogie and wheelsets (side view)
\[ F_{in} = -m\ddot{y} \]
\[ M_{in}^z = -J\ddot{\phi}_z \]

where \( F_{in} \) are forces with respect to "y" axis, \( M_{in}^z \) are the moments around "z" axis and other directions can be treated similarly. Using a linear stiffness and damping elements one can write forces as:

\[ F_i = s\Delta; \]
\[ F_d = \beta\dot{\Delta}, \]

where \( \Delta \) is the relative deformation of the corresponding elastic element and \( \dot{\Delta} \) is the relative velocity of the corresponding viscous elements. Thus, the set of equations in 3D space can be written as:

**Lateral direction of the wheelset:**

\[ F_{in,w1}^y - F_{b1-w1}^{y-f} - F_{b1-w2}^{y-f} - F_{c-w1}^{y-f} - F_{c-w2}^{y-f} - N_{gr1}^r + F_{cy1}^r + F_{cy2}^r + F_{r-w1}^y - F_{r-w2}^y = 0. \]

**Yaw angle of the wheelset:**

\[ M_{in,w1}^z + b_1 \left( -F_{w1-b1}^{x-f} + F_{w2-b1}^{x-f} \right) + s\left( F_{cr1}^r - F_{cr2}^r \right) M_{gr1}^z = 0. \]

**Roll angle of the wheelset:**

\[ M_{in,w1}^x + b_1 \left( F_{w1-c}^{z-f} - F_{w2-c}^{z-f} \right) + b_1 \left( F_{w1-b1}^{z-f} - F_{w2-b1}^{z-f} \right) + s\left( F_{r-w1}^z - F_{r-w2}^z \right) = 0. \]

**Vertical direction of the wheelset:**

\[ F_{in,w1}^z - F_{w1-b1}^{z-f} - F_{w2-b1}^{z-f} - F_{w1-b1}^{z-f} - F_{r-w1}^z + F_{r-w2}^z = 0. \]

**Lateral direction of the bogie:**

\[ F_{in,b1}^y + F_{b2-b1}^{y-f} + F_{b2-b1}^{y-f} - F_{b1-c}^{y-f} - F_{b2-c}^{y-f} + F_{w1-b1}^{y-f} + F_{w2-b1}^{y-f} = 0. \]

**Yaw angle of the bogie:**

\[ M_{in,b1}^z + a_6 \left( F_{b2-b1}^{y-f} + F_{b2-b1}^{y-f} \right) + a_5 \left( F_{b1-c}^{y-d} + F_{b1-c}^{y-d} \right) + b_1 \left( -F_{b2-b1}^{x-f} + F_{b2-b1}^{x-f} \right) + a_7 \left( F_{w1-b1}^{y-f} - F_{w2-b1}^{y-f} \right) + b_1 \left( F_{b1-w1}^{x-f} - F_{b1-w2}^{x-f} \right) = 0. \]

**Roll angle of the bogie:**
\[ M_{\text{in.b1}} + h_4 \left( -F_{\text{b1-c}}^{y} - F_{\text{b1-c}}^{z} \right) + b_1 \left( -F_{\text{w1-b1}}^{z} + F_{\text{w2-b1}}^{z} \right) + h_5 \left( -F_{\text{b2-b1}}^{z} - F_{\text{b2-b1}}^{z} \right) + b_2 \left( -F_{\text{w1-b1}}^{z} - F_{\text{w2-b1}}^{z} \right) + h_3 \left( -F_{\text{w1-b1}}^{y} - F_{\text{w2-b1}}^{y} \right) = 0. \]

Vertical direction of the bogie:
\[ F_{\text{in.t1}}^{z} + F_{\text{w5-c}}^{y} + F_{\text{w6-c}}^{y} + F_{\text{w7-c}}^{y} + F_{\text{w8-c}}^{y} = 0. \]

Lateral direction of the carriage:
\[ F_{\text{in.c}}^{y} + F_{\text{b1-c}}^{y} + F_{\text{b2-c}}^{y} + F_{\text{b3-c}}^{y} + F_{\text{b4-c}}^{y} + F_{\text{w1-c}}^{y} + F_{\text{w2-c}}^{y} + F_{\text{w3-c}}^{y} + F_{\text{w4-c}}^{y} + F_{\text{w5-c}}^{y} + F_{\text{w6-c}}^{y} + F_{\text{w7-c}}^{y} + F_{\text{w8-c}}^{y} = 0. \]

Yaw angle of the carriage:
\[ M_{\text{in.c}}^{y} + (a_1 + a_4) \left( F_{\text{w1-c}}^{y} + F_{\text{w2-c}}^{y} - F_{\text{w7-c}}^{y} - F_{\text{w8-c}}^{y} \right) + (a_1 - a_4) \left( F_{\text{w3-c}}^{y} + F_{\text{w4-c}}^{y} - F_{\text{w5-c}}^{y} - F_{\text{w6-c}}^{y} \right) + b_1 \left( -F_{\text{c-b1}}^{x} - F_{\text{c-b2}}^{x} + F_{\text{c-b3}}^{x} + F_{\text{c-b4}}^{x} \right) + b_1 \left( -F_{\text{c-b5}}^{x} + F_{\text{c-b6}}^{x} - F_{\text{c-b7}}^{x} + F_{\text{c-b8}}^{x} \right) = 0. \]

Roll angle of the carriage:
\[ M_{\text{in.c}}^{x} + h_1 \left( F_{\text{b1-c}}^{y} + F_{\text{b2-c}}^{y} + F_{\text{b3-c}}^{y} + F_{\text{b4-c}}^{y} \right) + h_2 \left( F_{\text{w1-c}}^{y} + F_{\text{w2-c}}^{y} + F_{\text{w3-c}}^{y} + F_{\text{w4-c}}^{y} \right) + h_2 \left( F_{\text{w5-c}}^{y} + F_{\text{w6-c}}^{y} + F_{\text{w7-c}}^{y} + F_{\text{w8-c}}^{y} \right) + b_1 \left( F_{\text{c-w1}}^{z} - F_{\text{c-w2}}^{z} + F_{\text{c-w3}}^{z} - F_{\text{c-w4}}^{z} \right) + b_1 \left( F_{\text{c-w5}}^{z} - F_{\text{c-w6}}^{z} + F_{\text{c-w7}}^{z} - F_{\text{c-w8}}^{z} \right) - M_{\text{Gc}}^{x} = 0. \]

Vertical direction of the carriage:
\[ F_{\text{in.c}}^{y} + F_{\text{b1-c}}^{z} + F_{\text{b2-c}}^{z} + F_{\text{b3-c}}^{z} + F_{\text{b4-c}}^{z} + F_{\text{c-b1}}^{z} + F_{\text{c-b2}}^{z} + F_{\text{c-b3}}^{z} + F_{\text{c-b4}}^{z} + F_{\text{c-b5}}^{z} + F_{\text{c-b6}}^{z} + F_{\text{c-b7}}^{z} + F_{\text{c-b8}}^{z} = 0. \]

Pitch angle of the carriage:
\[ F_{\text{in.c}}^{y} + F_{\text{b1-c}}^{z} + F_{\text{b2-c}}^{z} + F_{\text{b3-c}}^{z} + F_{\text{b4-c}}^{z} + F_{\text{c-b1}}^{z} + F_{\text{c-b2}}^{z} + F_{\text{c-b3}}^{z} + F_{\text{c-b4}}^{z} + F_{\text{c-b5}}^{z} + F_{\text{c-b6}}^{z} + F_{\text{c-b7}}^{z} + F_{\text{c-b8}}^{z} = 0. \]
The above formulas use the following notation: \( F_{b1-w1}^{x-f} \) – the elastic forces between the bogie and wheelset in “x” direction (even numbers for the right rail, odd for the left rail); \( F_{b-w1}^{z-f} \) – the elastic forces between the bogie and wheelset in “z” direction and \( F_{b1-w1}^{y-f} \) – the elastic forces between the bogie and wheelset in “y” direction.

Flat leaf suspension has non-symmetrical stiffness characteristics because the lower part of it is longer than the upper one resulting in higher force required to move the bogie upwards than downwards. This characteristics can be described by the following equation (\( k =0,1 \)):

\[
F_{b-w}^{z} = \begin{cases} 
  s_z \cdot (1-k) \Delta_{b1}^{z} & \text{if } \hat{\Delta}_{b1}^{z} < 0 \\
  s_z \cdot (1+k) \Delta_{b1}^{z} & \text{if } \hat{\Delta}_{b1}^{z} > 0 
\end{cases}
\]

There are forces acting from the rails onto the wheelset, which can be divided into two groups: elastic forces due to a side contact of the wheels and rails and creep forces due to the contact between two surfaces. \( F_{r-w1}^{y-f} \) – the elastic forces, exerted onto the wheels from the rails due to a side interaction, with the gap equal to 0,007 m, in y direction:

\[
F_{r-w1}^{y-f} = \begin{cases} 
  0 & \text{if } y_{w1} - \eta_r < 0,007 m \\
  s_f y_{w1} & \text{if } y_{w1} - \eta_r \geq 0,007 m 
\end{cases}
\]

This characteristic is presented in Figure 6 left.
The lateral and longitudinal creep forces have been derived based on the Kalker theory with corrections [3],[4]. In the above notation $F_{cr}^{Rx}$ – projection of the creep forces in “x” axis, whereas $F_{cr}^{ Ry}$ – projection of the creep forces in “y” axis. Moreover, the gravity force $N_{gr}$ and moment $M_{gr}$ have been applied to the wheelset:

$$N_{gr} = -2\Pi \cdot \frac{i}{S} \cdot y_w ; \quad M_{gr} = 2\Pi \cdot i \cdot S \cdot \varphi_w^z .$$

Viscous forces between the bogie and carriage in “x” and “z” directions are denoted as $F_{c-b1}^{xd}$ and $F_{b1-c}^{zd}$ correspondingly. The elastic forces between the bogie and carriage in “y” direction have been modelled as piecewise linear function:

$$F_{b-c}^{y-f} = \begin{cases} s_2^y \Delta_{bi-c} & \text{if } |\Delta_{bi-c}| < 0.1 \text{ m}; \\ 3.2 s_2^y \Delta_{bi-c}^2 & \text{if } 0.1 \text{ m} \leq |\Delta_{bi-c}| \leq 0.2 \text{ m}; \\ 50 s_2^y \Delta_{bi-c} & \text{if } 0.2 \text{ m} \leq |\Delta_{bi-c}| \leq 0.25 \text{ m}; \end{cases}$$

which is presented in Figure 6 right.
Diagonal forces in the interacting bogies have been denoted as $F_{b2-b1}^{x-f}$, $F_{b2-b1}^{y-f}$, $F_{b2-b1}^{z-f}$ in “y”, “x” and “z” directions correspondingly. In total a set of 37 ordinary differential equations has been derived and will be studied in next section.

4 Modelling and numerical results

In the first phase the task of choosing parameters of the spring suspension, ensuring the compliance with the lateral variability was undertaken, and investigation of the type of oscillations was conducted. Numerical simulations have been conducted using Dormand-Prince (RKDP) method. Due to a large number of calculations required Nelder-Mead optimization method has been implemented. The main reason is the fact that this method allows performing numerical calculations in parallel, significantly reducing the amount of computational time. To select parameters of the spring suspension an optimization method was implemented, where the target function was the intensity of crossings $u_i(t)$ above a certain level used as an indicator of dynamic qualities (DQ) [2],[5]:

$$U = \sum_{i=1}^{m} f_e(u_i) \exp \left\{ -\frac{[u_i]}{2S(u_i)} \right\}$$  \hspace{1cm} (1)

where $S(u_i)$ - mean square displacement and $f_e(u_i)$ - is the effective frequency of the random processes $u_i(t)$, defined as:

$$S(u_i) = \sqrt{\int_{-\infty}^{\infty} G_{u_i}(f) df}$$  \hspace{1cm} (2)

$$f_e(u_i) = \frac{1}{S(u_i)} \sqrt{\int_{-\infty}^{\infty} f^2 G_{u_i}(f) df}$$  \hspace{1cm} (3)

and $G_{u_i}(f)$ is the spectral density of the DQ random process, obtained as a result of the numerical simulations.

The DQ was based on the total vertical and horizontal acceleration of the carriage $z_{k\Sigma}$ и $\bar{y}_{k\Sigma}$ at the points of connections of the bogies, as well as the sum of the vertical and horizontal coefficients of the dynamic load at all suspension points $\bar{z}_{\Sigma,\Sigma}$ и $\bar{y}_{\Sigma,\Sigma}$. Thus, the dynamic quality of the system is defined by (1): the target function and eight quality coefficients. Since it is a very hard optimization problem in 9D space, the procedure of separate optimization has been adopted. First, the suspension optimization of the carriage in the horizontal direction is performed.
Then, with the obtain set of parameters from the first step, the suspension optimization in the vertical direction is conducted. Finally, the optimization in the horizontal direction is performed with the parameters obtained from the second stage.

Let’s consider, for example, the results obtained at the first stage. In this stage a nondimensional parameter of DQ were used \( U_i = \bar{H}(u_i)/[u_i] \) where \( \bar{H}(u_i) \) is the mean value of the maximums of random process \( u_i(t) \) according to Kramer formula, assuming that the distribution of maximums \( f(H) \) of Gaussian stationary random process \( f(u) \), which is described by the double exponent law:

\[
f(H) = \frac{d}{dH} \exp\left\{-f_c(u_i)t_p(u_i)\exp\left[\frac{(H - \bar{H})^2}{S^2(u_i)}\right]\right\}; \tag{4}
\]

\[
\bar{H}(u_i) \cong S(u_i)\left(\sqrt{2\ln f_c(u_i)t_p(u_i)} + \frac{1}{\sqrt{2\ln f_c(u_i)t_p(u_i)}}\right). \tag{5}
\]

where \( t_p(u_i) \) the length of the sample \( u_i(t) \), which was taken 38.1 sec.

To find optimal parameters of the suspension at the first step 56250 cycles of optimization has been conducted. Some numerical results of the optimization are shown in Figure 7. Each circle has three values \( U_1, U_2 \) and \( U_4 \), where its diameter is related to the dynamic coefficient and its colour reflects the value of the cost function. Point 1 in figure 7 corresponds to the optimal values for the suspension. In point 1 values of all parameters are acceptable, i.e. lie within the specified range and the total intensity of excursions is 1.51. Having computed this number, all numerical simulations have been conducted for finding stable motion of the carriage along straight path, turns as well as influence of random vibrations onto the system.

Stability of motion along a straight path has been done numerically by integrating the set of nonlinear differential equations [6]. It has been observed that the intensity of oscillations increases with the increase of carriage speed, although the oscillations remain decaying. When the critical speed \( v_{cr} \) is attained the oscillations become sustainable and when \( v > v_{cr} \) amplitude of oscillations will increase until a limit cycle is reached. The critical speed of the bogie with respect to the lateral oscillations in this case was obtained \( v_{cr} = 806 \text{km/h} \). Safety regulations state that the maximum speed a carriage can operate at is defines as \( v_c = v_{cr} / \sqrt{3} = 466 \text{km/h} \).
Figure 7 demonstrates a set of intermediate values of the cost function and DQ at the various stages of the optimization procedure in 5D space.

The proposed bogie can move along curved paths with a relatively small radius of 50m. The study of the bogie motion along a curved path has been based on the theory of relative motion and non-inertial local coordinate systems. The numerical simulations have shown that the bogie keeps all the wheels on the rails, therefore the side forces are equal along both the axes and are below 70kN. This is a great advantage compared to other bogies, which usually have larger load to the outer wheel of the front wheelset.

To show the advantage of the proposed bogie a set of numerical simulations with various radius has been performed. These results have been analysed and an empirical formula connecting the velocity and the radius has been derived:

\[ v_{\text{max}} = 5.29 \sqrt{R} \]

For other standard bogies this number is \( v_{\text{max}} = 4.6 \sqrt{R} \), which indicates that the developed bogie may run along a curve path with a radius 30% lower than that for the typical bogie. This has direct influence onto the cost of building the railways.

To study the influence of the carriage speed onto the DQ coefficients, a set of nonlinear differential equations with random multidimensional stationary processes acting in the horizontal and vertical direction from both rails \([1,2,7]\). The non-stationarity of the input and nonlinearity, presented in the system, including the nonlinearity on Figure 4 and Figure 5 indicate that the output process will be nonstationary.

To obtain some response characteristics for comparison with other responses, it is required to average over a large number of samples, and in this case a number of samples for each speed value was taken \( N=4096 \) with a time step \( t=0.0031 \) sec, resulting in total time \( T=38.1 \) sec. The overall number of points within a sample was
Figure 8, for example, demonstrates several samples generated for the horizontal roughness of the left rail at 20m/s.

Figure 8. Generated roughness of a rail at 20m/s (top left), carriage motion (top right), spectral density of the carriage at 20m/s (down left).

Lateral carriage oscillations $y_k(t)$ can be observed in Figure 9. It can be seen that some samples are very much different in the amplitudes and frequency contents, which supports the fact that the process is nonstationary.

To study the random process $y_k(t_1, t_2)$ it is required to obtain a square matrix. Since the number of $y_k(t_1)$ was $N_p=12288$, the number of samples $y_k(t_2)$ was N=4096, therefore every third value was selected from $y_k(t_1)$ and $\Delta t_1 = 0.0093s$.

Therefore the matrix $y_k(t_1, t_2)$ had 4096x4096, which is equivalent in time to 38.1x38.1 sec. This matrix was used to build other characteristics of the random process, such as a correlation function $R_y(t_1, t_2)$ and spectral density $G_y(f_1, f_2)$:

$$R_y(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y_k(t_1)y_k(t_2)f[y_k(t_1), y_k(t_2), t_1, t_2]dy_k(t_1)dy_k(t_2); \quad (7)$$
\[ G_y(f_1, f_2) = \int_0^{N_p} \int_0^{N_p} R_y(t_1, t_2) e^{-j2\pi f_1 t_1} e^{-j2\pi f_2 t_2} dt_1 dt_2 = . \] (8)

For the case of \( v=20 \text{m/s} \) a two-dimensional distribution:

\[ f[y_k(t_1), y_k(t_2)] = \frac{1}{2\pi S^2_y} \exp \left\{ -\frac{1}{2S^2_y} \left[ y_k(t_1 \text{ when } t_2 = \text{const}) \right]^2 + \left[ y_k(t_2 \text{ when } t_1 = \text{const}) \right]^2 \right\} . \] (9)

Based on the obtained distribution the autocorrelation function and spectral density have been obtained for the lateral displacement of the carriage and the frequency range was between 0.1Hz and 10 Hz. The spectral density is presented in Figure 8. It can be seen that all the energy at 20m/s is contained within the frequency range of 0.2Hz up to 2.25Hz and 10Hz frequency has never been reached.

The frequency at the maximum corresponds to the frequency of lateral vibrations, whereas the frequencies of smaller local maximums can be approximately described as:

\[ \frac{f_{1,1}}{f_{2,1}} \approx 0.8 , \quad \frac{f_{1,2}}{f_{2,2}} \approx 1.4 , \quad \frac{f_{1,3}}{f_{2,3}} \approx 2.1 \]

Based on the fact that the peaks amplitudes at the diagonal are higher than that everywhere else, one can state that the system responds with ultraharmonic oscillations 2:1 and 3:1. The spectral response of the carriage accelerations has much more complicated structure.

Another set of simulations has been conducted for \( v=120 \text{km/h} \). Figure 9 shows the results for dynamic coefficients connecting the carriage and bogie, whereas Figure 9 shows their spectral representation. The main peak in Figure 9 is observed at 0.75Hz, and there are two minor symmetrical with respect to main diagonal peaks with coordinates (0.7Hz, 4.8Hz).

Much more complex peaks layout can be observed in Figure 9 for the coefficients connecting the bogie and wheelset. The number of small peaks along the edges of the plot correspond to ultraharmonic oscillations with frequencies 2,3,4…n-times higher the main resonance frequency.
Figure 9. Dynamic coefficient connecting the carriage and bogie at 120 m/s (top left), connecting the carriage and bogie (top right), the bogie and wheelset (low left).

Figure 10. Left: Dependence of DQ on the carriage speed: 1 – between the bogie and bushing, 2 – between the carriage and bogie, 3 – between the carriage and bushing, 4- maximum acceleration. Right: Ride coefficient vs train speed.
Having obtained the spectral densities it is possible to calculate the variance, the dominant frequencies and the mean values of peaks depending on the carriage speed. In figure 10 left one can observe that all DQ factors are inside the acceptable range, however one of the peaks is at 20m/s. It can be explained by the fact that the excitation frequency at this speed coincides with that of the carriage leading to the resonance. It also can be seen that at speeds above 80m/s DQ factors increases and at 120m/s approaching their critical values.

The ride smoothness coefficient \( C \) was calculated according to the following formula:

\[
C = \alpha \left[ 6,67 \left( \frac{f_B f_B}{f_H f_H} \right)^2 \int \int q_H^2(f)G_{\tilde{y}}(f)df_1df_2 \right].
\]  

(10)

Figure 10 right shows dependence of the ride smoothness coefficient on the train speed and it can be observed that up to 108m/s the quality of a ride is excellent. Increase in carriage speed results in reducing the ride quality.

5 Conclusions

The paper presents a novel bogie design for a high-speed train. The bogie comprises two subunits connected by diagonal linkages and has a three-stage suspension system, where an extra connection between the carriage and wheelset has been introduced. Another advantage of the double-wheelset bogie is its suitability for tracks with higher curvatures, where regular bogies cannot be used. The numerical simulations of 37 equations of motion, describing the train-bogie-wheel dynamics and its interaction with a track, have been used to find the optimal set of parameters of the bogie. The train vibrations have been excited by surface roughness of the rails that has been modeled as a random process with a given spectral density. Optimal parameters of the bogie has numerically been obtained and enabled the bogie safely travel with no excessive oscillations at speed up to 432km/h.

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