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Abstract

Recent studies show that carry trade returns are predictable and this predictability reflects changes in expected returns. Changes in expected returns may be related to time variation in betas and risk prices. We investigate this issue in carry trades and find clear evidence of time-varying risk prices for the carry factor (HML_{FX}). The results further indicate that time-varying risk prices are more important than time-varying betas for the carry trade asset pricing model. This suggests that investors overreact to changes in economic states.

Keywords: Currency Carry Trades, Exchange Rate, Risk Price, Time-varying Betas, Factor Model, Nonparametric Model, FX market.
JEL codes: C58, E44, F31, G12, G15

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1.   Introduction

Currency carry trades are implemented by borrowing in low interest rate currencies and investing in high interest rate currencies. Asset pricing theory suggests that positive carry returns are compensations for risk. An expanding body of literature tests competing theories in a quest to identify the risk factors that are relevant to carry returns. This literature explores expected returns on risk factors, and these factors are associated with positive or negative factor betas and risk prices as rewards for bearing the risk of theses factors. Using returns on currency portfolios, rather than individual currencies, recent studies have sought to identify both the factor betas and risk prices for carry returns. For example, Lustig and Verdelhan (2007) employ the consumption CAPM, while Burnside et al. (2011) investigate whether the Fama and French (1993) three factor model explains carry returns. Currency carry trade specific factors were also introduced by Lustig et al. (2011) and Menkhoff et al. (2012a). In particular, Lustig et al. (2011) propose a level and a slope factor model, known as their dollar (DOL) and carry (HML_{FX}) factors, and the latter can price cross-sectional currency portfolios, although the dollar does less well. Two questions naturally arise: how do we interpret the level and slope factors? Are factor betas and/or risk prices constant or time varying when modelling carry return risks? One possible solution to factor interpretation and time variation is to introduce forecast variables as proxies to capture changes in economic states and build a conditional factor model. Such a model would provide a mechanism by which risk prices can change over time through changes in the forecast variables.

Studies in the carry trade literature typically use unconditional models to estimate carry factor betas and risk prices, although there are good reasons to suggest that currency risk factors may have time-varying betas and/or risk prices. The broad asset pricing literature mainly focuses on time-varying betas[1] but not time varying risk prices. Factor betas, indeed, are key elements for portfolio risk management, and investors are likely to adjust betas to optimise their portfolio risk level. However, Ferson and Harvey (1991), Evans (1994), and Adrian et al. (2015), amongst others, show that time-varying risk prices play an important role in expected returns for stock and bond markets. It is plausible to assume

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[1]Jagannathan and Wang (1996), Cochrane (1996), Ferson and Harvey (1999), and Lettau and Ludvigson (2001) propose conditional factor models in the stock market and allow time-varying betas to reflect changes in economic states.

that time variation of risk prices is substantial since the representative investor may have time-varying risk aversion in carry trades. Nagel (2013) states that investors are myopic and maximize utility period by period, hence time-varying risk-aversion is required. Given currency carry trades have unwinding risk as pointed out by Brunnermeier et al. (2009), it is reasonable to expect the risk-aversion changes after a market crash. Another reason why risk aversion may vary is due to habits. Verdelhan (2010) proposes a habit model that explains violations of the Uncovered Interest rate Parity (UIP) condition, which is the key mechanism to create positive carry returns.

It is therefore reasonable to expect that time variation is important for the FX market.[2] We would expect portfolio re-adjustments after major economic events. Conditional factor models are more appropriate to managed funds when investors change their positions in response to carry predictability.[3] This predictability reflects time-varying expected returns since investors adjust required returns based on changes in economic states captured by forecast variables. Expected returns are represented by factor betas and risk prices in standard linear factor models. Thus, the time-varying expected returns require time-varying betas and/or risk prices. Christiansen et al. (2011) and Lustig et al. (2011) were early contributions to conditional carry models. Although time-varying betas for carry trades are investigated by Christiansen et al. (2011), time-varying risk prices are not. Lustig et al. (2011) uses rolling regressions to estimate conditional carry models,[4] and although some empirical results are promising, there is no interpretation of the time variation and the conditional results do not fully account for transaction costs. Transaction costs obviously matter to managed funds as they are linked to the frequency of trading dictated by portfolios adjustments. Atanasov and Nitschka (2014), Dobrynskaya (2014), and Lettau et al. (2014) also emphasise conditional carry factor models when estimating time-varying betas, but they do not investigate time-varying risk prices.

This study extends the carry trade literature on two fronts. First, our major innovation to the carry

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[2]The relationship between exchange rates and macro fundamentals is also unstable, consistent with the scape goat theory by Bacchetta and Wincoop (2013). Sarno and Valente (2009), Rossi (2013) and Byrne et al. (2016) among others conduct empirical analysis for exchange rate prediction with time-varying parameters.

[3]Ferson and Harvey (1991) and Ferson and Schadt (1996) emphasise the importance of conditional models for managed investment funds when stock and bond returns are predictable. Studies that suggest carry returns are predictable include Bakshi and Panayotov (2013), Cenedese et al. (2014), and Lu and Jacobsen (2016).

[4]Time-varying risk prices are traditionally explored by rolling regressions as in Ferson and Harvey (1991).

trade literature is the modelling of time variation in both factor betas and risk prices. Several recent studies distinguish between up-side and down-side risk prices but do not investigate time variation in risk prices (Atanasov and Nitschka, 2014; Dobrynskaya, 2014; Lettau et al., 2014). In this paper, we allow for continuous change in risk prices as we explore how they vary over time based on economic states. To this end, we employ econometric methods from Adrian et al. (2015). These methods have successfully been applied to identify time-varying risk prices and factor betas for stock and bond markets. In contrast to rolling regression methods, Adrian et al. (2015) propose a more general approach that incorporates forecast variables with risk factors. The main advantage of this approach relative to the traditional conditional factor model is that it allows for time variation in both risk prices and factor betas. Further, the betas in our model, which are estimated by the non-parametric method of Ang and Kristensen (2012), also fluctuate over time. Non-parametric models are more robust to misspecification, because parametric models tend to overestimate time variations of betas, as pointed out by Ghysels (1998). Factor betas estimated by standard conditional models may be volatile and do not contribute to small pricing errors, while non-parametric models allow smooth changes in betas to improve pricing errors.

Our cross-sectional risk factors are based on Lustig et al. (2011) and Menkhoff et al. (2012a), but our model also includes forecast factors. They allow us to interpret the main drivers of time variation in risk factors. This point is important since Lustig et al.'s (2011) level and slope factors are derived by a data driven approach, and hence interpretation of these factors is an interesting question.

The second contribution of this study is to extend the cross-sectional and forecast literature of carry trades. Forecast variables are widely employed in stock and bond market research (e.g. Ferson and Harvey, 1991, 1999). We employ several forecast factors that include FX market volatility, a commodity price return, and market liquidity. Bakshi and Panayotov (2013) and Cenedese et al. (2014) argue that FX market volatility is related to future carry trade returns. FX market volatility represents uncertainty in FX markets and uncertainty induces unwinding of carry trades. Commodity prices are relevant to carry trades prediction as suggested by Bakshi and Panayotov (2013), because some high interest rate currencies such as the Australian and the New Zealand dollar are commodity exporting currencies.

Ready et al. (2016) propose that commodity exporting countries tend to have higher interest rates because they are more robust to consumption shocks. Moreover, Brunnermeier et al. (2009) indicate that market liquidity, measured by the TED spread, is associated with carry trade returns, since lack of market liquidity causes unwinding of carry trades. In this paper, we connect these forecast factors to cross-sectional risk factors.

To preview our results, we find significant time variation in the risk price of the carry factor (HML_{FX}) and uncertainty in FX markets creates time variation in the risk price of this factor. Time variation of the dollar and the carry risk prices contribute to smaller pricing errors for the asset pricing model, while time variation of factor betas do not. The weak contribution of the time-varying betas implies that change in the factor betas is slow but investors may overreact to shocks in economic states. The importance of time-varying risk prices suggests that predictability is more related to time variation in the risk prices. The commodity price and the market liquidity variables cause a decline in the risk price on the carry factor during the crisis. This result reveals that several forecast variables acted as the driving force behind negative returns when disaster struck.[5] We also find the dollar factor is linked to market liquidity, which is plausible since investors would demand safe assets, such as the U.S. dollar, when market liquidity dries up (flight to safety).

The rest of the paper is organized as follows: Section 2 lays out the econometrics, Section 3 describes the data, Section 4 presents the empirical results, Section 5 presents robustness analyses and Section 6 concludes.

2. Estimation Methodology

This section sets out our empirical methods. To account for the role of time-varying factor betas and/or risk prices for carry returns, we adopt Adrian et al.'s (2015) models. This approach is sufficiently flexible to allow for the following two combinations: constant betas but time-varying risk prices, and time-varying betas and risk prices. These distinctive combinations are important, as our results will

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[5]Empirical evidence also indicates that disaster risk plays an important role for carry trade returns. See, Brunnermeier et al., (2009), Burnside et al. (2011), Farhi et al. (2013), Jurek (2014), and Farhi and Gabaix (2016).

show below.

2.1. Constant Betas and Time-varying Risk Prices

An expected excess return on currency portfolio i, $E[R_i]$, is represented as risk prices lambda, λ, multiplied by factor betas, β_i, using a standard factor pricing model:

$$E[R_i] = \lambda'\beta_i. \tag{1}$$

We use the popular Fama and MacBeth (1973) two-step approach to obtain factor betas and risk prices. Factor betas are obtained by time-series regressions, where the excess return of portfolio i, $R_{i,t+1}$ is regressed on a vector of risk factors, h_{t+1}:

$$R_{i,t+1} = \alpha_i + \beta_i'h_{t+1} + e_{i,t+1} \tag{2}$$

where $e_{i,t+1}$ is an error term. The risk prices, lambda, are estimated by a cross-sectional regression, while substituting all n portfolios' estimated betas $\hat{\beta}_i$ into equation (1).

Basic expected return models assume that both factor betas and risk prices are constant. However, if expected returns change over time to reflect changes in underlying economic states, factor betas and/or risk prices need to vary over time. Adrian et al. (2015) propose a general approach to estimate time-varying betas and risk prices. First, we focus on time-varying risk prices and estimate a model with constant betas but time-varying risk prices. This model is:

$$R_{i,t+1} = \beta_i'\lambda_0 + \beta_i'\Lambda_1 F_t + \beta_i'u_{t+1} + e_{i,t+1} \tag{3}$$

where λ_0 and Λ_1 are risk price parameters, F_t is the vector of forecast factors, and u_{t+1} is the innovations to risk factors. We assume no-arbitrage, which implies $\alpha_i = \beta_i'\lambda_0$. The first two terms in the right hand side of equation (3) are the expected returns, the third term is the component conditionally correlated with the innovations, and the last term represents the pricing errors. There are two key differences between equations (2) and (3). First, the forecast factors, F_t, are introduced to reflect predictability of carry trades. Second, the innovations to the risk factors are employed instead of risk factors, h_{t+1}, since innovation components capture uncertainty in investment opportunities, and hence these components

are linked to risk prices (Campbell, 1996 and Petkova, 2006).

The innovation term u_{t+1} in equation (3) is obtained by a Vector Autoregressive (VAR) approach. We follow Adrian et al. (2015) and assume X_{t+1} is a $K \times 1$ vector of state variables at $t+1$ and contains three types of variables. The first is $X_{1,t+1} \in \mathbb{R}^{K_1}$, which are risk factors only, used to price the cross-section of returns. The second is $X_{2,t+1} \in \mathbb{R}^{K_2}$, which are risk and forecast factors both used to price the cross-section of returns and to forecast the risk factors. Finally, $X_{3,t+1} \in \mathbb{R}^{K_3}$ are forecast factors only. The number of factors is denoted by: $K_C = K_1 + K_2$, $K_F = K_2 + K_3$, and $K = K_1 + K_2 + K_3$ where the subscript C indicates cross-section and the subscript F denotes forecast factors. The VAR dynamics are written as:

$$X_{t+1} = \mu + \Phi X_t + v_{t+1}, \tag{4}$$

where μ and Φ are coefficient vectors, v_{t+1} is the innovations vector and the first K_c columns of v_{t+1} are written as u_{t+1}. Our aim is to obtain the time-varying risk prices $\lambda_0 + \Lambda_1 F_t$ in equation (3). To this end, we need to estimate both the factor betas, β_i, and the risk price parameters, λ_0 and Λ_1. Following Adrian et al. (2015), we employ a three-step approach. In the first step, the VAR system equation (4) is run and $\hat{u}_{t+1}$ is extracted. In the second step, $\hat{u}_{t+1}$ is substituted into equation (3) and the estimated betas, $\hat{\beta}_i$, and the predictive slopes, $\hat{w}_0$ and $\hat{w}_1$, are obtained. The predictive slopes, w_0 and w_1, are:

$$w_0 = \beta_i \lambda_0, w_1 = \beta_i \Lambda_1. \tag{5}$$

Finally, the risk price parameters, $\hat{\lambda}_0$ and $\hat{\Lambda}_1$, are obtained by substituting $\hat{\beta}_i$, $\hat{w}_0$, and $\hat{w}_1$ into equation (5). Adrian et al. (2015) show that these estimated risk price parameters, $\hat{\lambda}_0$ and $\hat{\Lambda}_1$, converge to the limiting normal distribution, and they derive the variance which takes into account estimation uncertainty of the innovations term and factor betas.

As the risk prices are time-varying, they depend upon the forecast factors, F_t. We test whether a sample average of risk prices for given pricing factors, $\bar{\lambda}$, is significantly different from zero. This is obtained as:

$$\bar{\lambda} = \lambda_0 + \Lambda_1 E[F_t]. \tag{6}$$

$\bar{\lambda}$ converges to the limiting normal distribution, as shown by Adrian et al. (2015). We use their closed

form variance to conduct statistical inference.[6] This subsection described the constant beta and time-varying risk price model. In the next section, we allow for time-varying betas.

2.2. Time-varying Betas and Time-varying Risk Prices

We now describe the time-varying beta and risk price model proposed by Adrian et al. (2015). The factor betas (β_i) in equation (3) and the coefficients of the VAR (Φ and μ) in equation (4) follow smooth functions as in Ang and Kristensen (2012). These are given by:

$$\beta_{i,t} = \beta_i(t/T) + o(1), \ \mu_{i,t} = \mu_i(t/T) + o(1), \ \Phi_t = \Phi(t/T) + o(1), \tag{7}$$

where $o(1)$ is a smaller order term and $t = 1, 2, \ldots, T$. These functions are estimated nonparametrically and this approach is more robust to a misspecification problem, as pointed out by Harvey (2001). Moreover the assumption that betas vary at a moderate level is consistent with the findings of Ghysels (1998) in the stock market context.

The coefficients of the VAR model in equation (4) are estimated by kernel weighted least squares regressions:

$$(\hat{\mu}_{t-1}, \hat{\Phi}_{t-1})' = \left(\sum_{s=1}^{T} K_b((s-t)/T)\tilde{X}_{s-1}\tilde{X}'_{s-1} \right)^{-1} \times \left(\sum_{s=1}^{T} K_b((s-t)/T)\tilde{X}_{s-1}X'_s \right) \tag{8}$$

where $\tilde{X}_{s-1} = (1, X'_{s-1})'$, $K_b(x) = K(x/b)$ for a kernel function $K(\cdot)$. This kernel estimation provides the time-varying coefficients. We choose the Gaussian density used by Ang and Kristensen (2012) and Adrian et al. (2015). The bandwidth denoted by $b \in (0, 1)$ is critical for estimation. A small bandwidth means only data close to t are used. Following Kristensen (2012) and Ang and Kristensen (2012), we employ a plug-in bandwidth method, since they report that cross-validation (CV) procedures show an extremely small bandwidth. We use a different bandwidth for each element of X_s, because each variable has different variation and curvature of the coefficients.

Time-invariant predictive slopes, w_0 and w_1, and factor betas, β_i, in equation (3) are also replaced with time-varying variables. The time-varying predictive slopes and the factor betas are obtained by

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[6]Further detail is described in Adrian et al. (2015) Appendix D.

the following weighted least squares regressions:

$$(\hat{w}_{0,i,t-1}, \hat{w}'_{1,i,t-1}, \hat{\beta}'_{i,t-1}) = \left(\sum_{s=1}^{T} K_h((s-t)/T) z_s^{tv} z_s^{tv'} \right)^{-1}$$
$$\times \left(\sum_{s=1}^{T} K_h((s-t)/T) z_s^{tv} R_{i,s} \right) \tag{9}$$

where $z_s^{tv} = (1, X'_{s-1}, C'_s)'$ and $C_s = (X_{1,s}, X_{2,s})$, and $R_{i,s}$ is the return of portfolio i. Instead of the innovation term, which is employed in the constant beta model, the risk price factor vector, C_s, is used. This change is based on a technical aspect to satisfy uniform convergence.[7] Using the estimation results in equations (8) and (9), the risk price parameters, Λ^{tv}, are obtained as:

$$vec(\hat{\Lambda}^{tv}) = \left(\sum_{t=0}^{T-1} (\hat{F}_t \hat{F}'_t \otimes \hat{B}_t \hat{B}'_t + \rho_T)^{-1} \sum_{t=0}^{T-1} (\hat{F}_t \otimes \hat{B}'_t)(R_{t+1} - \hat{B}_t \hat{u}_{t+1}) \right) \tag{10}$$

where $vec(\cdot)$ is the vectorization operator, $\otimes$ is the Kronecker product, $\hat{B}_t$ is the factor beta matrix that stacks $\beta_{i,t}$, $\hat{F}_t = (1, F'_t)'$, and ρ_T is a positive sequence that satisfies $\rho_T \to 0$. u_{t+1} is obtained by the VAR with the weighted least squares coefficients in equation (8). Adrian et al. (2015) show that Λ^{tv} converges to the limiting normal distribution. When the factor betas are time-varying, the sample average of risk prices in equation (6) is changed to:

$$\bar{\lambda} = \lambda_0 + \Lambda_1 \cdot \lim_{T \to \infty} T^{-1} \sum_{T}^{t=1} E[F_t]. \tag{11}$$

$\bar{\lambda}$ also converges to the limiting normal distribution, as described in the constant beta model. Having set out our empirical method, we introduce the data next.

3. Data

We explain our data in this section and begin with currency portfolio data. We obtain spot and one month forward exchange rates from Datastream. In total 48 currencies are used, and this dataset is similar to that used by Menkhoff et al. (2012a). The base currency is the U.S. dollar, and the dataset extends from November 1983 to December 2013. As data availability for some currencies does not extend back to November 1983, the total number of exchange rates varies during the sample period. We

[7]Lemma D.1. (c) and (d) in Adrian et al. (2015) is derived from the result of Kristensen (2009).

construct six currency portfolios based on the forward discount as in Lustig et al. (2011). We assume that the covered interest rate parity holds and a positive forward discount means that the foreign interest rate is higher than the domestic interest rate (see Akram et al., 2008). We denoted by P1(P6) the lowest (highest) interest rate currency portfolio. Following Lustig et al. (2011), we account for trading costs using bid-ask spreads.[8] Data are pre-treated using the method of Darvas (2009). He uses the previous day's data when there is no difference between bid and ask prices, or when the spread of the forward rates is smaller than that of the spot rates. We also construct 15 developed country currencies, since some high interest rate emerging currencies may have a disproportionate impact on the results. Lustig et al. (2011) and Menkhoff et al. (2012a) employ the same approach and they construct, both all countries', and developed countries' portfolios.

Our risk factors are the dollar (DOL) and carry (HML_{FX}) factors introduced by Lustig et al. (2011). The dollar factor is computed as the average return of the currency portfolios. It acts as a market factor, as in the stock market literature, and the loadings on this factor are almost equal across currency portfolios. This factor is highly correlated with the first principal component of currency portfolio returns. The carry factor is computed as the return spread between high and low interest rate portfolios (P6−P1). This factor determines the cross-sectional return difference across currency portfolios. Lustig et al. (2011) demonstrate that this factor mimics the second principal component of currency portfolio returns.

Next, we turn to the three forecast factors used in this paper. These set out the underlying conditions in the economy. The first is the global FX market volatility analysed by Menkhoff et al. (2012a). It is computed from daily returns for all currencies, and the monthly values are taken as the average of daily values. Let a daily log return of currency j on day τ be $r_{j,\tau} = s_{j,\tau} - s_{j,\tau-1}$, where $s_{j,\tau}$ is the log of the spot exchange rate on day τ. We estimate global FX volatility, $\sigma_{FX,t}$, in month t as:

$$\sigma_{FX,t} = \frac{1}{T_t} \sum_{\tau=1}^{T_t} \sum_{j=1}^{K_\tau} \left(\frac{|r_{j,\tau}|}{K_\tau} \right) \tag{12}$$

where $|r_{j,\tau}|$ is the absolute value of $r_{j,\tau}$, K_τ is the number of currencies on day τ, and T_t is the total number of trading days in month t. We do not take innovations as in Menkhoff et al. (2012a), since our

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[8]We use a bid rate when buying and an ask rate when selling a currency.

time-varying model takes into account innovations in the VAR model. Menkhoff et al. (2012a) use the global FX volatility innovations as a risk factor, but we adopt it as a forecast factor as in Christiansen et al. (2011), Bakshi and Panayotov (2013), and Cenedese et al. (2014). However, we also check in Section 5 whether this factor acts as a risk factor.

The second forecast factor is a commodity price return. Following Bakshi and Panayotov (2013), the Raw Industrials subindex of the CRB Spot Commodity Index is adopted. Monthly returns are used, since our portfolios are constructed at monthly frequency. The third variable is market liquidity for which we use the TED spread. Brunnermeier et al. (2009) show that the TED spread is related to the future return of currency carry trades, and Mancini et al. (2013) and Karnaukh et al. (2015) show that it is strongly related to FX market liquidity. The TED spread is computed as the difference between the three month Eurodollar LIBOR rate and the three month Treasury Bill rate.[9]

4.   Empirical Results

4.1.   Estimated Factor Betas

We begin the presentation of our empirical results with factor betas, which represent exposures on risk factors for each portfolio. Table 1 provides constant beta estimates from equation (3). The beta estimates on the dollar factor (DOL) show that all portfolios have almost the same exposure to this factor, implying that this factor does not account for cross-sectional differences in returns across currency portfolios. In contrast, the estimated betas on the carry factor (HML_{FX}) increase monotonically from P1 to P6. This is evidence that the carry factor is important in pricing the cross-section currency portfolios, and high interest rate currency portfolios are more exposed to this factor. The results for developed countries reported in Panel B show similar patterns.

[Table 1 about here]

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[9]We cannot cover the entire sample period by LIBOR, thus we employ the three-month interbank rate in the U.S. to cover a longer period.

Next, we compare these constant betas with time-varying betas. The time-varying betas are obtained by equation (9). For robustness, 36-month rolling betas are also estimated. The results of betas on the dollar factor are plotted in Figure 1, which clearly shows time variations in the betas. They move around the constant beta estimates, and this is consistent with the results of Adrian et al. (2015) for stock and bond portfolios. The fluctuations of the time-varying betas and rolling betas have a similar pattern, but the time-varying betas are less volatile, because they are obtained by the kernel smoothing method.

[Figure 1 about here]

Figure 2 plots the constant and time-varying betas on the carry factor estimated by equations (3) and (9). Portfolio P1 always has negative, and P6 positive, exposure to the carry risk.[10] We can see that currency portfolios are more sensitive to the carry factor than to the dollar factor. Note that a change in beta does not necessarily lead to a decline in the expected carry return, since returns depend upon both betas and risk prices. If risk prices change over time, this may have an impact on expected returns.

[Figure 2 about here]

4.2. Estimated Risk Price Parameters

Next, we investigate possible relations between the forecast and risk factors. While our results in the previous subsection provide evidence of time-varying betas, risk prices also may vary over time. We investigate which time variation matters most for the carry trade pricing model. To this end, we need to link risk prices and forecast factors, because forecast factors generate time variations of risk prices. Risk price parameters are key elements for the links, since time-varying risk prices are obtained by the product of risk price parameters and forecast factors ($\Lambda_1 F_t$). Note that the risk price parameters are constant while the risk prices vary through changes in the forecast factors.

Table 2 reports estimates of risk price parameters, λ_0 and Λ_1, from equation (5) based on the three forecast factors mentioned above: global FX volatility (VOL_{FX}), commodity price return (CRB), and

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[10]The definition of the carry factor is directly related to P1 and P6, the fluctuations of these portfolios are similar.

market liquidity (TED). Average risk prices $\bar{\lambda}$ from equation (6), and the Wald test results of the null hypothesis that the row is all zeros, are presented to examine our estimation accuracy. We begin on Panel A of Table 2 with the constant beta and time-varying risk price model for all country results. If time-varying risk prices are more important, mistakenly imposing time-varying betas may distort estimated risk price parameters. We find the market liquidity forecast variable is important for the dollar factor, and the FX volatility forecast variable plays a main role in generating carry factor fluctuations. The negative relation between the dollar factor and the market liquidity illustrates that when market liquidity dries up (TED rises), most currencies depreciate against the U.S. dollar. The strong relation between the U.S. dollar and the market liquidity comes from the risk haven characteristic of the U.S. dollar. When currency markets crash, investors shift their allocations from emerging currencies to the U.S. dollar (McCauley and McGuire, 2009). We also observe that the FX market volatility variable is strongly related to the carry factor. High FX volatility leads to low risk price on the carry factor, indicating that market uncertainty induces investors to unwind their carry positions. This result is related to the findings of Bakshi and Panayotov (2013) and Cenedese et al. (2014) who report that volatility in FX markets contains information for future carry trade returns, but our results suggest that volatility generates fluctuations in the risk price. The time series average risk price, $\bar{\lambda}$, on the dollar factor does not differ from zero, while that on the carry factor is statistically significant at the 1% level.

We now turn to the time-varying beta model that reflects investor changes in factor exposure. The results are reported in Panel C of Table 2. The risk price parameters are estimated by equation (10) and the average risk price is obtained by equation (11). We observe a similar pattern in that the market liquidity forecast variable is substantial for the dollar factor and the FX volatility forecast variable is important for the carry factor. Interestingly, most standard errors of the time-varying beta model are smaller than those of the constant beta model. Adrian et al. (2015), who analyse the stock and bond markets, argue that these smaller standard errors are an advantage of the time-varying beta model. Importantly, the average risk price of the dollar is statistically significant at the 5% level and, thus, the dollar factor commands a risk premium. This is direct contrast to the results of the constant beta model and the previous literature, such as Lustig et al. (2011) and Menkhoff et al. (2012a). This finding

highlights the difference between time-varying and constant betas. The time-varying betas generate heterogeneous factor exposures across portfolios and create a statistically significant risk price on the dollar factor in currency portfolios. However we do not obtain heterogeneous factor exposures with constant betas. This is related to the result of Verdelhan (2015) who shows that the dollar factor bears a risk premium. Verdelhan (2015) employs dollar sorted currency portfolios to estimate heterogeneous factor exposures while betas are constant over time. Our time-varying beta model provides heterogeneous factor exposures without adopting dollar sorted currency portfolios.

We repeat the same estimation using developed countries in Table 2. Although the main findings are similar to those of the all countries' results, there are two differences. The market liquidity variable is now related to both the dollar and carry factors, which implies that institutional investors use the currencies of developed countries, and funding constraints may play an important role in the developed countries' sample as reported by Habib and Stracca (2012). Further, the average risk price of the carry factor in developed countries is smaller than that in all countries. In other words, the estimated $\bar{\lambda}$ on the carry factor is smaller in Panels B and D, reflecting that emerging currencies typically have higher interest rates, since emerging countries tend to have relatively high inflation ratios.

In summary, we find the risk price parameter on the market liquidity forecast variable is associated with the dollar factor, and the FX market volatility forecast variable is linked to the carry factor through the risk price parameter. Changes in the market liquidity variable produce time variation in the dollar factor, and changes in the FX market uncertainty variable cause time variation in the carry factor. We find statistically significant relations between risk prices and forecast factors in both the time-varying and the constant beta models. Next, we investigate the pricing errors of these models.

[Table 2 about here]

4.3. Pricing Errors

Having found the factor betas vary over time, and the forecast factors add time variation to the risk prices, we investigate which time variations matter more in terms of the carry pricing model. We do so by examining the pricing errors of our respective models through plots of realized mean returns against

fitted mean returns. Does time variation in the parameters have implications on the size of these errors? Figure 3 provides a visual comparison across our estimation results. The realized mean excess returns of P1 to P6 are on the x-axes and the predicted mean excess returns by a model are on the y-axes. Perfect prediction would imply all six portfolios plotting exactly on the 45-degree line. The upper-left graph displays the result of the time-varying beta and risk price model, and it shows that all predicted returns are close to the 45-degree line. The second portfolio, P2, however, has the largest pricing error, and this model slightly over predicts all portfolio returns. The constant beta and time-varying risk price model of Adrian et al. (2015) shows a better performance in the upper-right graph. The pricing error of P2 is smaller than that of the time-varying beta and time-varying risk price model. In contrast, conventional linear approaches, such as Fama and MacBeth on the bottom-right panel of Figure 3 and Ferson and Harvey (1991) (bottom-left graph), exhibit larger pricing errors. Both these conventional models estimate the betas and the risk prices using 36-month rolling regressions.

[Figure 3 about here]

Accordingly, a carry model with time-varying risk prices dominates alternatives visually, hence we further assess the pricing errors of each portfolio using the Mean Squared Errors (MSE). Table 3 presents the average MSE of each portfolio and the last row displays the average of all portfolios. The time-varying risk price models in columns (a) and (b), exhibit smaller MSEs than those of rolling methods in columns (e) and (f). Comparing time-varying risk price models, we observe that the constant beta and time-varying risk price model has the smallest average MSE, suggesting that time variation in risk prices is more important than time variation in betas in pricing carry trade portfolios. Ghysels (1998) states that misspecification causes overestimation of betas. Investors overreact as they cannot observe a true relationship between changes in economic states and carry returns. This overreaction is plausible for carry trade investors, since these investors know that carry trades contain large downside risk and adjust their portfolio allocations to avoid crashes in FX markets.

[Table 3 about here]

4.4. Interpretation of Time Variation

Given the importance of time variation in risk prices shown in the above results, we now investigate further these dynamics. Figure 4 plots the time evolution of the risk prices of the dollar and the carry factors. The time variation of the dollar price, with 95% confidence intervals, is reported in the upper graph. This is computed as the risk price parameter, λ_0, plus the product of the risk price parameters with the forecast factors, $\Lambda_1 F_t$. The figure shows that the confidence interval is slightly above zero during the middle of the 1990s and 2000s, and there is a substantial drop at the global financial crisis in 2008. The lower graph shows the time variation of the carry factor. The confidence interval is clearly above zero during most periods, and during the crisis, the risk price collapses to -5% per month, which is almost ten times larger in absolute value than that of the average risk price. Increases in FX volatility and market liquidity during the crisis cause the sign of the risk price to flip, because both forecast factors are negatively related to the carry factor, as reported in Table 2. Market uncertainty and lack of liquidity induce unwinding of carry positions and in favor of safer assets such as the U.S. dollar.

[Figure 4 about here]

Figure 5 plots the contribution of each forecast factor. These are obtained by computing the product of the risk price parameter with a particular forecast factor, $\lambda_{1,j} F_{j,t}$, where $\lambda_{1,j}$ is the $(1,j)$ element of the risk price parameter vector and $F_{j,t}$ is the j-th forecast factor. The scale of the y-axes shows the contribution of each forecast factor. The upper three graphs indicate that the main contribution to the dollar factor comes from the market liquidity forecast variable. The lower three graphs show that FX market volatility is the most important contributor to the carry factor. However, we see the market liquidity and the commodity price variables have substantial impact during the crisis. Interestingly, the market liquidity variable is important only when liquidity dries up significantly. This is due to liquidity spirals, as shown by Brunnermeier and Pedersen (2009). All investors demand liquidity and it generates the negative risk price on the carry factor.

[Figure 5 about here]

5. Robustness

5.1. Excluding the Global Financial Crisis

The analyses presented in the previous section show that the global financial crisis affects the estimation of risk prices. For robustness, we repeat the same estimations using data before the crisis only. If these provide different results, we would be able to confirm the importance of the crisis and, consequently, the appropriateness of the time-varying risk price methodology. Table A5 in the Online Appendix presents the estimation results for the pre-crisis period from November 1983 to March 2008. From Panels A and C, we see a weakening relation between the market liquidity variable and the dollar factor, and the average risk prices on the dollar and the carry factors have higher values. For example, using the time-varying beta and risk price model, the average risk price on the carry factor prior to the crisis is 0.50, compared to 0.46 for the full sample period. Surprisingly, Panel D shows that all estimated risk price parameters are insignificant when we analyse the developed country sample. As Farhi and Gabaix (2016) highlight the importance of disaster risk in generating a positive carry trade return, our empirical results consequently show disaster risk is significantly related to carry trade returns. The plots of risk prices in Figure A6 for the pre-crisis sample also display different shapes to those presented in Figure 4 for the full sample. In particular, the variation over time in risk prices is smaller. Overall, these results confirm the existence of time-varying risk prices and the importance of the crisis period.

5.2. FX Volatility Innovation Factor

Also for robustness, we estimate another factor model. Following Menkhoff et al. (2012a), we build a factor model using the dollar and the FX volatility innovation factors. In contrast to Menkhoff et al. (2012a), however, our model allows for time variation in betas and risk prices. To this end, the carry factor is replaced by an FX volatility innovation factor computed by a return-based mimicking portfolio in a similar manner to how the carry factor was constructed. Table 4 reports that the estimation results, together with those of a constant beta and a time-varying beta models.

The basic findings are similar to those reported in Table 2. For example, from Panel C in Table

4, the two forecast variables, FX market volatility and market liquidity, positively impact on the FX volatility innovation factor. This implies that increases in FX market volatility or the TED spread lead to increases in the risk price of the volatility innovation factor. The average risk price on the FX volatility innovation factor is negative, and the risk exposure to this factor is positive for P1 and negative for P6, and hence P1 works as a hedge when FX volatility is high. The Online Appendix (Figure A5) reports the time variation of the volatility risk price. The price is negative for most periods but suddenly jumps during market turmoil. In particular, the most significant jump is observed during the global financial crisis. In summary, we find the time-varying risk price model clearly highlights the time variation of the FX volatility innovation factor.

[Table 4 about here]

5.3. Carry and Momentum Portfolios

As a further robustness exercise we also include momentum strategy currency portfolios as test assets. Lewellen et al. (2010) propose to include portfolios sorted by other characteristics, when test portfolios have a factor structure. Following Menkhoff et al. (2012b), we construct six (five) currency momentum portfolios for the all (developed) countries' sample. The currencies are sorted by one month lagged excess returns, and we take into account transaction costs, as is done thus far.

Table 5 presents the results using carry and momentum currency portfolios. Panel A reports the results of the constant beta and time-varying risk price model. These results correspond to the results of Panel A in Table 2. The main finding, that FX volatility is the main driver of the carry factor, remains the same, and the average risk price of the carry factor has the same magnitude. Table 5 Panels B, C, and D show that the main findings of Table 2 are not affected when momentum portfolios are included. FX volatility is related to the carry factor, and the average price of this factor is positive and statistically significant at least at the 5% level.

[Table 5 about here]

5.4.   Up-side and Down-side Analysis

Given the large negative risk price at the crisis reported in Section 4, we consider the interpretation of this result. Lakonishok et al. (1994) argue that if an investing strategy can be explained by a risk factor, the strategy should have negative returns in recessions. Also, Pettengill et al. (1995) and Hur et al. (2014) report a negative relationship between betas and risk prices in a downside stock market. Our empirical results on carry trades support this relation, because the high interest rate currency portfolio has a positive beta on the carry factor, and the risk price varies over time. To confirm this relation, we split the data into two states as in Pettengill et al. (1995) and Hur et al. (2014). A down market state is defined as one in which the carry factor is negative, and an up market state is defined as one in which the carry factor is positive. The Fama and MacBeth regressions are then run for each state. Table 6 reports the results and column (a) presents the results for all the data as a benchmark, and the risk price on the carry factor is positive. In the results for the separate states, we observe a negative risk price in the down market state in column (b) and a positive risk price in the up market state in column (c). Moreover, the magnitude of the positive risk price is greater than that of the negative risk price, which implies a higher risk price in the up market state. Summarizing the up and down-state results in Table 6, we confirm that the time variation of the risk price on the carry factor is associated with the state of the market.

[Table 6 about here]

6.   Conclusion

If an asset's return is predictable, it means that its expected return changes for investors, as pointed out by Ferson and Harvey (1991) in the context of the stock and bond markets. Time-varying expected returns suggest that factor betas and/or risk prices vary over time, since expected returns depend upon factor betas and risk prices. Ferson and Harvey (1991) and Adrian et al. (2015) decompose the change in expected returns and present evidence that time-varying risk prices are more important than time-varying betas for stock and bond markets.

Motivated by these studies in other asset classes, we explore time-varying betas and risk prices in carry trades. Recently, several studies find carry trades to be predictable. For instance, Bakshi and Panayotov (2013) use commodity prices and Cenedese et al. (2014) adopt FX market volatility to predict carry trades. This predictability motivates the consideration of the time-varying beta and risk price model, since betas and/or risk prices vary over time to reflect changes in the investment environment. We use Adrian et al.'s (2015) approach that allows for time variations of betas and risk prices in the asset pricing model. Further, the time-varying beta and risk price approach explicitly incorporates forecast variables with risk factors, and provides important information for the risk factors. The forecast factors offer an interpretation of the level (dollar) and the slope (carry) factors derived by Lustig et al. (2011).

We find the risk price on the carry factor varies over time, and the global financial crisis has a large impact on this time variation. Menkhoff et al.'s (2012a) FX volatility innovation is a main driver of changes in the risk price of the carry factor. When FX market uncertainty rises, all investors unwind their carry positions simultaneously, and hence carry returns decline. The commodity price and the market liquidity forecast variables also have substantial impact during the crisis. Liquidity plays an important role only when it dries up significantly. This finding could be explained by liquidity spirals during the crisis, as suggested by Brunnermeier and Pedersen (2009). Importantly, our empirical results present evidence that time variations of risk factors dominate time variations of betas in generating small pricing errors. The weak support for time varying betas implies that investors overreact to changes in economic states, since investors know carry trades contain crash risk. Finally, the clear evidence we present of time-variation of risk prices suggests that predictability of carry trades is attributed to changes in the risk prices.

Table 1

Beta Estimate to Risk Factors DOL and HML_{FX}: Constant Beta Model

Panel A: All countries

Portfolio	β_{DOL}	s.e.	β_{HML}	s.e.
P1	1.01***	(0.02)	-0.44***	(0.03)
P2	1.00***	(0.02)	-0.21***	(0.03)
P3	1.00***	(0.02)	-0.05***	(0.03)
P4	0.94***	(0.03)	0.06**	(0.03)
P5	1.03***	(0.03)	0.09**	(0.04)
P6	1.01***	(0.02)	0.56***	(0.03)

Panel B: Developed countries

Portfolio	β_{DOL}	s.e.	β_{HML}	s.e.
P1	1.03***	(0.02)	-0.57***	(0.02)
P2	1.00***	(0.03)	-0.05*	(0.03)
P3	1.00***	(0.02)	0.02	(0.03)
P4	0.93***	(0.03)	0.18***	(0.03)
P5	1.03***	(0.02)	0.43***	(0.02)

Notes: This table presents estimated factor betas from the constant beta model. Factor betas for currency portfolios of carry returns are obtained by equation (3). The risk factors are dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. Asymptotic standard errors are reported in parentheses. The sample period is November 1983 to December 2013. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 2

Risk Price Parameter Estimates on Forecast Factors

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
Constant beta and time-varying risk price model							
Panel A: All countries							
(a)	DOL	0.63	-0.35	0.07	-0.60**	0.18	12.94***
		(0.41)	(0.94)	(0.05)	(0.31)	(0.14)	
	HML_{FX}	2.00***	-3.10***	0.08*	-0.45	0.49***	48.59***
		(0.39)	(0.91)	(0.04)	(0.30)	(0.14)	
Panel B: Developed countries							
(b)	DOL	0.52	0.06	0.08	-0.71**	0.19	11.91**
		(0.47)	(0.94)	(0.05)	(0.34)	(0.16)	
	HML_{FX}	2.02***	-2.65***	0.03	-0.84**	0.34**	29.81***
		(0.47)	(0.95)	(0.05)	(0.34)	(0.17)	
Time-varying beta and time-varying risk price model							
Panel C: All countries							
(c)	DOL	0.72*	-0.43	0.07*	-0.57**	0.25**	18.21***
		(0.37)	(0.86)	(0.04)	(0.28)	(0.11)	
	HML_{FX}	1.98***	-3.01***	0.04	-0.52*	0.46***	41.35***
		(0.39)	(0.88)	(0.04)	(0.29)	(0.11)	
Panel D: Developed countries							
(d)	DOL	0.47	0.30	0.09*	-0.69**	0.27**	17.10***
		(0.43)	(0.86)	(0.05)	(0.31)	(0.12)	
	HML_{FX}	1.98***	-2.64***	0.03	-0.85***	0.30**	32.79***
		(0.43)	(0.85)	(0.04)	(0.31)	(0.12)	

Notes: This table presents risk price parameter estimates on forecast factors, global FX volatility (VOL_{FX}), commodity price (CRB), and market liquidity (TED). The risk price parameters estimates using constant betas are from equation (5) in Panels A and B. The approach to estimate risk price parameters for time-varying betas are equation (10) in Panels C and D. Risk price parameters show relationships between risk and forecast factors, and risk prices are computed as risk price parameters time forecast factors. These methods are from Adrian et al. (2015). The average risk price $\bar{\lambda}$ in Panels A and B is obtained by equation (6) and $\bar{\lambda}$ in Panels C and D is obtained by equation (11). The risk factors are the dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. The test assets of Panels A and C are six forward discount sorted all country currency portfolios and those of Panels B and D are five forward discount sorted developed country currency portfolios. The sample period is November 1983 to December 2013. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3

Mean Squared Pricing Error for Time-varying or Constant Factor Betas and Risk Prices

Panel A						
All countries	(a)	(b)	(c)	(d)	(e)	(f)
Factor betas (β)	Time-varying	Constant	Time-varying	Constant	FH	FM
Risk prices (λ)	Time-varying	Time-varying	Constant	Constant	FH	FM
P1	0.92	0.74	1.05	0.79	1.16	1.05
P2	1.06	0.90	1.14	0.91	1.17	1.14
P3	0.97	0.79	0.98	0.84	1.04	0.98
P4	1.02	0.87	1.16	1.02	1.23	1.16
P5	1.22	1.02	1.31	1.13	1.41	1.31
P6	0.99	0.76	1.49	1.16	1.54	1.49
Average	1.03	0.85	1.19	0.97	1.26	1.19

Panel B						
Developed countries	(a)	(b)	(c)	(d)	(e)	(f)
Factor betas (β)	Time-varying	Constant	Time-varying	Constant	FH	FM
Risk prices (λ)	Time-varying	Time-varying	Constant	Constant	FH	FM
P1	1.28	0.84	1.25	0.89	1.31	1.41
P2	1.38	1.28	1.49	1.32	1.56	1.62
P3	1.24	0.98	1.28	1.09	1.36	1.38
P4	1.36	1.10	1.34	1.24	1.44	1.43
P5	1.17	0.79	1.37	1.10	1.49	1.52
Average	1.29	1.00	1.35	1.13	1.43	1.47

Notes: This table presents the mean squared pricing error across various models. Smaller pricing errors are indicative of better fitting models. Model (b) with time-varying risk prices but constant betas has the smallest pricing errors, indicating the best fit. The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. The risk factors are dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). FH denotes the Ferson and Harvey (1991) procedure using 36 months rolling regressions, and FM denotes the Fama and MacBeth (1973) procedure using 36 months rolling regressions. The sample period is November 1986 to December 2013.

Table 4

Risk Price Parameter Estimates on Forecast Factors: FX Volatility Innovations

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
Constant beta and time-varying risk price model							
Panel A: All countries							
(a)	DOL	0.63	-0.36	0.07	-0.60*	0.18	12.54**
		(0.41)	(0.94)	(0.05)	(0.31)	(0.14)	
	ΔVOL_{FX}	-3.37***	4.97***	-0.17**	0.94*	-0.84***	52.12***
		(0.67)	(1.55)	(0.07)	(0.51)	(0.25)	
Panel B: Developed countries							
(b)	DOL	0.52	0.05	0.08	-0.71**	0.19	11.93**
		(0.47)	(0.94)	(0.05)	(0.34)	(0.16)	
	ΔVOL_{FX}	-2.89***	3.34**	-0.09	1.45***	-0.57**	35.59***
		(0.70)	(1.40)	(0.07)	(0.50)	(0.25)	
Time-varying beta and time-varying risk price model							
Panel C: All countries							
(c)	DOL	0.73*	-0.51	0.07	-0.56**	0.24**	17.93***
		(0.38)	(0.88)	(0.04)	(0.28)	(0.12)	
	ΔVOL_{FX}	-3.60***	5.04***	-0.12	1.14**	-0.92***	54.94***
		(0.66)	(1.52)	(0.07)	(0.49)	(0.20)	
Panel D: Developed countries							
(d)	DOL	0.49	0.25	0.08*	-0.68**	0.27**	17.10***
		(0.42)	(0.83)	(0.04)	(0.30)	(0.12)	
	ΔVOL_{FX}	-2.76***	3.20**	-0.09	1.41***	-0.53***	32.79***
		(0.66)	(1.31)	(0.07)	(0.47)	(0.19)	

Notes: This table presents risk price parameter estimates on forecast factors, global FX volatility (VOL_{FX}), commodity price (CRB), and market liquidity (TED). The risk price parameters estimates using constant betas are from equation (5) in Panels A and B. The approach to estimate risk price parameters for time-varying betas are equation (10) in Panels C and D. Risk price parameters show relationships between risk and forecast factors, and risk prices are computed as risk price parameters time forecast factors. These methods are from Adrian et al. (2015). The average risk price $\bar{\lambda}$ in Panels A and B is obtained by equation (6) and $\bar{\lambda}$ in Panels C and D is obtained by equation (11). The risk factors are the dollar (DOL) and the global FX volatility innovations (ΔVOL_{FX}) as in Menkhoff et al. (2012). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. The test assets of Panels A and C are six forward discount sorted all country currency portfolios and those of Panels B and D are five forward discount sorted developed country currency portfolios. The sample period is November 1983 to December 2013. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 5

Risk Price Parameter Estimates on Forecast Factors: Carry and Momentum Portfolios

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
Constant beta and time-varying risk price model							
Panel A: All countries							
(a)	DOL	0.65	-0.44	0.06	-0.59*	0.18	12.32**
		(0.41)	(0.94)	(0.05)	(0.31)	(0.14)	
	HML_{FX}	2.17***	-3.70***	0.08*	-0.40	0.43***	50.59***
		(0.39)	(0.92)	(0.04)	(0.30)	(0.15)	
Panel B: Developed countries							
(c)	DOL	0.52	0.01	0.08	-0.70**	0.18	11.74**
		(0.47)	(0.94)	(0.05)	(0.34)	(0.16)	
	HML_{FX}	1.77***	-2.04**	0.04	-0.88**	0.36**	26.16***
		(0.48)	(0.97)	(0.05)	(0.34)	(0.16)	
Time-varying beta and time-varying risk price model							
Panel C: All countries							
(b)	DOL	0.75**	-0.55	0.07*	-0.55*	0.25**	17.96***
		(0.38)	(0.87)	(0.04)	(0.28)	(0.11)	
	HML_{FX}	1.83***	-2.82***	0.02	-0.63**	0.33***	32.03***
		(0.40)	(0.91)	(0.04)	(0.30)	(0.12)	
Panel D: Developed countries							
(d)	DOL	0.50	0.21	0.09*	-0.68**	0.26**	16.99***
		(0.43)	(0.86)	(0.05)	(0.31)	(0.12)	
	HML_{FX}	1.41***	-1.54*	0.03	-0.75**	0.30**	22.54***
		(0.42)	(0.84)	(0.04)	(0.30)	(0.12)	

Notes: This table presents risk price parameter estimates on forecast factors, global FX volatility (VOL_{FX}), commodity price (CRB), and market liquidity (TED). The risk price parameters estimates using constant betas are from equation (5) in Panels A and B. The approach to estimate risk price parameters for time-varying betas are equation (10) in Panels C and D. Risk price parameters show relationships between risk and forecast factors, and risk prices are computed as risk price parameters time forecast factors. These methods are from Adrian et al. (2015). The average risk price $\bar{\lambda}$ in Panels A and B is obtained by equation (6) and $\bar{\lambda}$ in Panels C and D is obtained by equation (11). The risk factors are the dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. The test assets of Panels A and C are six forward discount and six past one month currency excess return sorted all country currency portfolios and those of Panels B and D are five forward discount and five past one month currency excess return sorted developed country currency portfolios. The sample period is November 1983 to December 2013. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 6

Fama and MacBeth Cross-Sectional Regressions for Down and Up Markets

Panel A: All countries			
	All	Down	Up
	(a)	(b)	(c)
DOL	0.18	0.05	0.26
	(0.12)	(0.22)	(0.14)
HML_{FX}	0.49***	-1.31***	1.65***
	(0.12)	(0.15)	(0.11)
R^2	0.86	0.82	0.90
Panel B: Developed countries			
	All	Down	Up
	(a)	(b)	(c)
DOL	0.19	0.30	0.12
	(0.13)	(0.23)	(0.17)
HML_{FX}	0.33**	-2.04***	1.86***
	(0.12)	(0.19)	(0.11)
R^2	0.53	0.93	0.95

Notes: This table presents risk prices for up and down markets estimated by the Fama and MacBeth (1973) methodology. The market is defined as a down market if $HML_{FX} < 0$, and an up market if $HML_{FX} > 0$ as in Pettengill et al. (1995). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. 'All' denotes both up and down markets. Shanken (1992) standard errors are reported in parentheses. The R^2 is a measure of fit between the sample mean of returns and the predicted mean returns. The sample period is November 1983 to December 2013. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.
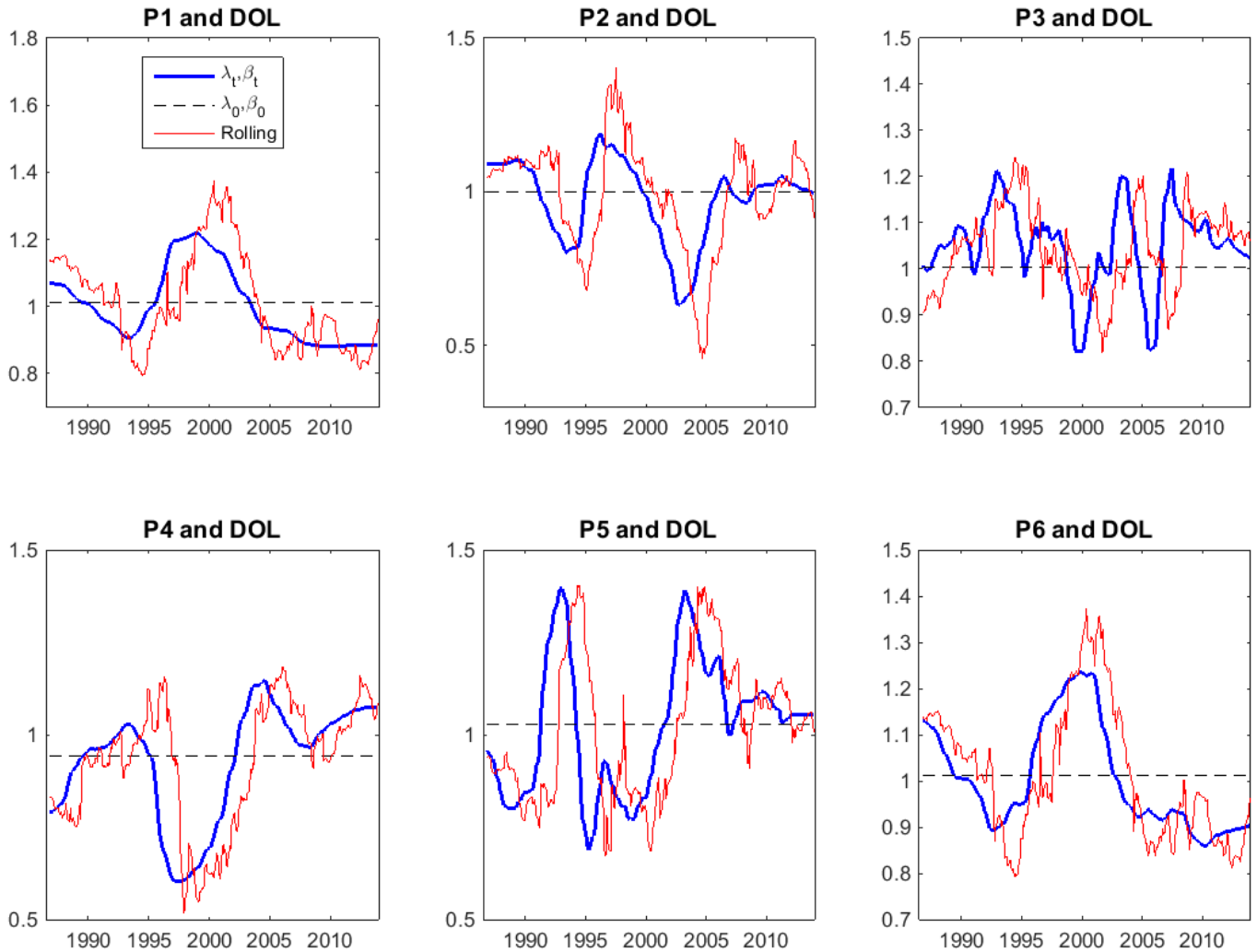
Figure 1.

Comparison of time series portfolio betas on DOL

Notes: This figure provides plots of the estimated time series of betas on the dollar (DOL). λ_t, β_t denotes the time-varying risk price and beta model and the betas are obtained by equation (9) (thick blue line). λ_t, β_0 denotes the constant beta and time-varying risk price model and the betas are obtained by equation (3) (dashed black line). Rolling denotes the 36 months rolling window beta (thin red line).

Figure 2.

Comparison of time series portfolio betas on HML_{FX}

Notes: This figure provides plots of the estimated time series of betas on the return spread between high and low interest rate currency portfolios (HML_{FX}). λ_t, β_t denotes the time-varying risk price and beta model and the betas are obtained by equation (9) (thick blue line). λ_t, β_0 denotes the constant beta and time-varying risk price model and the betas are obtained by equation (3) (dashed black line). Rolling denotes the 36 months rolling window beta (thin red line).
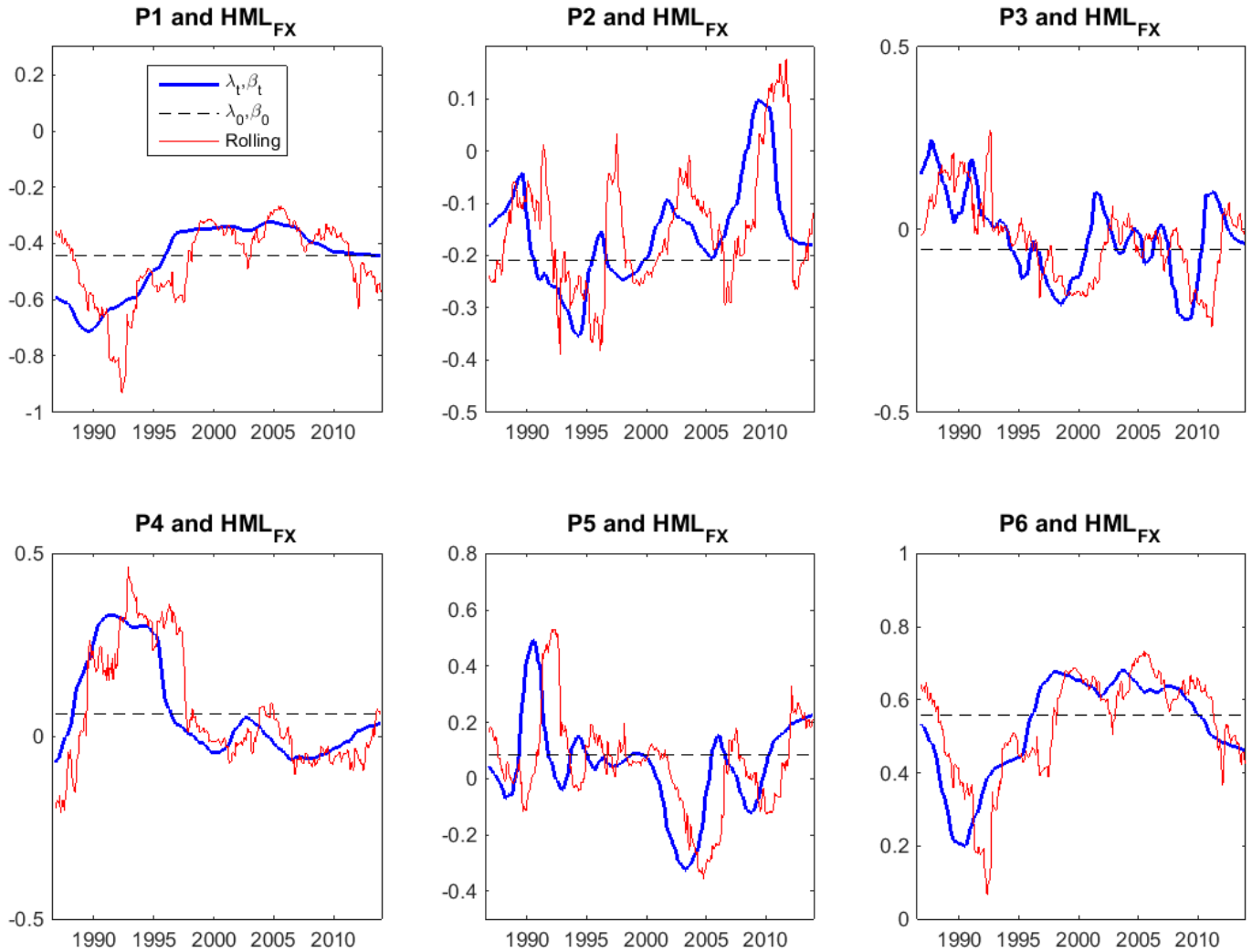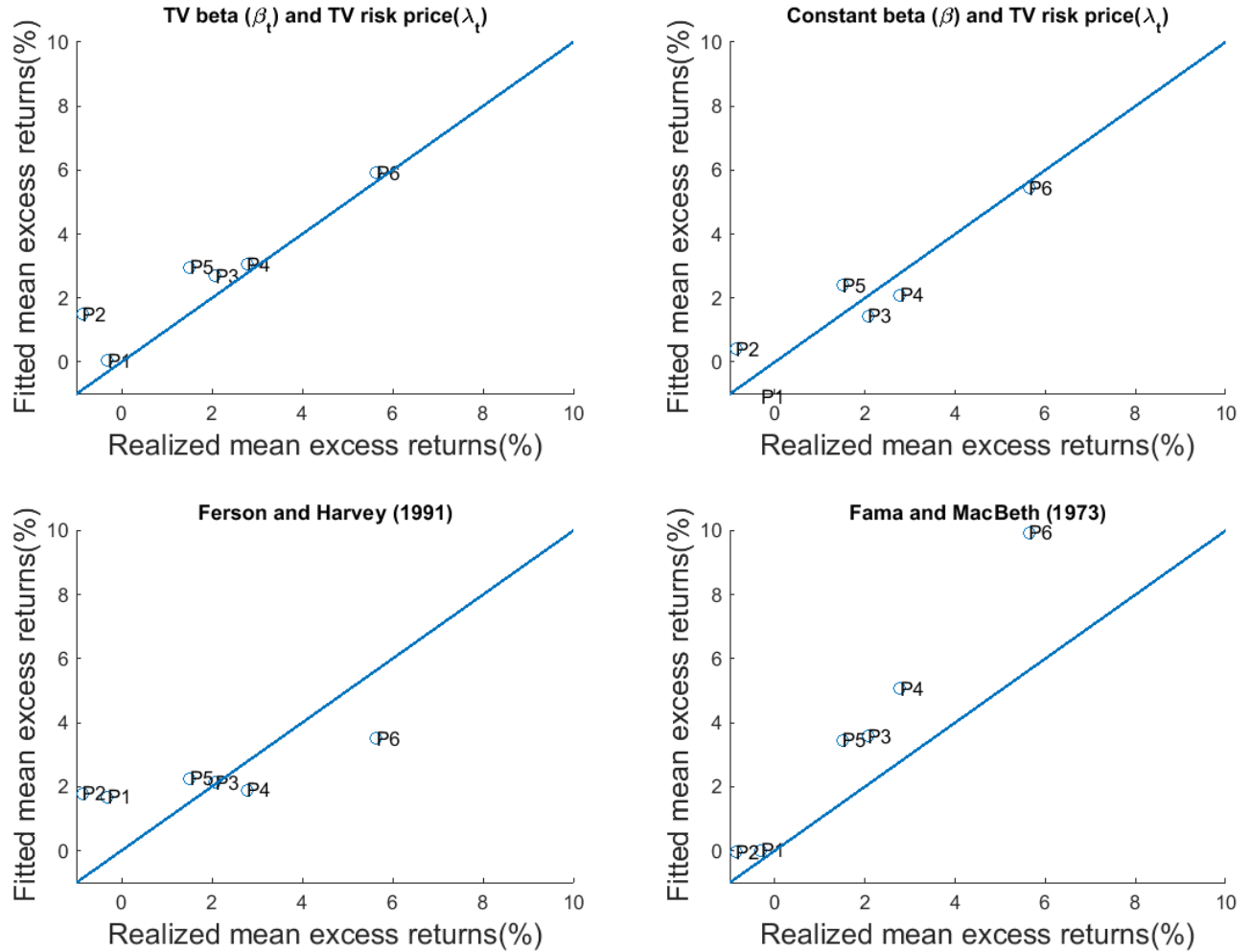
Figure 3.

Comparison of cross-sectional pricing models

Notes: This figure displays pricing errors for asset pricing models. The realized mean excess returns $(r_{i,t})$ are on the horizontal line and the mean fitted excess returns are on the vertical line. Both excess returns are annualized returns. The risk factors are dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). The upper-left graph shows the time-varying beta and risk price model, the upper-right graph shows the constant beta and time-varying price model, the lower-left graphs shows the Ferson and Harvey (1991) approach, which uses 36 months rolling regressions, and the lower-right graph shows the Fama and MacBeth (1973) approach, which uses 36 months rolling regressions.

Figure 4.

Time-varying risk prices (λ) of DOL and HML_{FX}

Notes: This figure displays time series risk prices of the dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) with their 95% confidence intervals. The risk price is obtained as the risk price parameter (λ_0) plus the risk price parameters (Λ_1) multiplied by the time forecast factors (F_t), $\lambda = \lambda_0 + \Lambda_1 F_t$. Three forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).

Figure 5

Contribution of forecast factors

Notes: This figure displays the contribution of the three forecast factors with their 95% confidence intervals. The contribution is estimated as the risk price parameter times the forecast factor, $\lambda_{1,j} F_{j,t}$. The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).
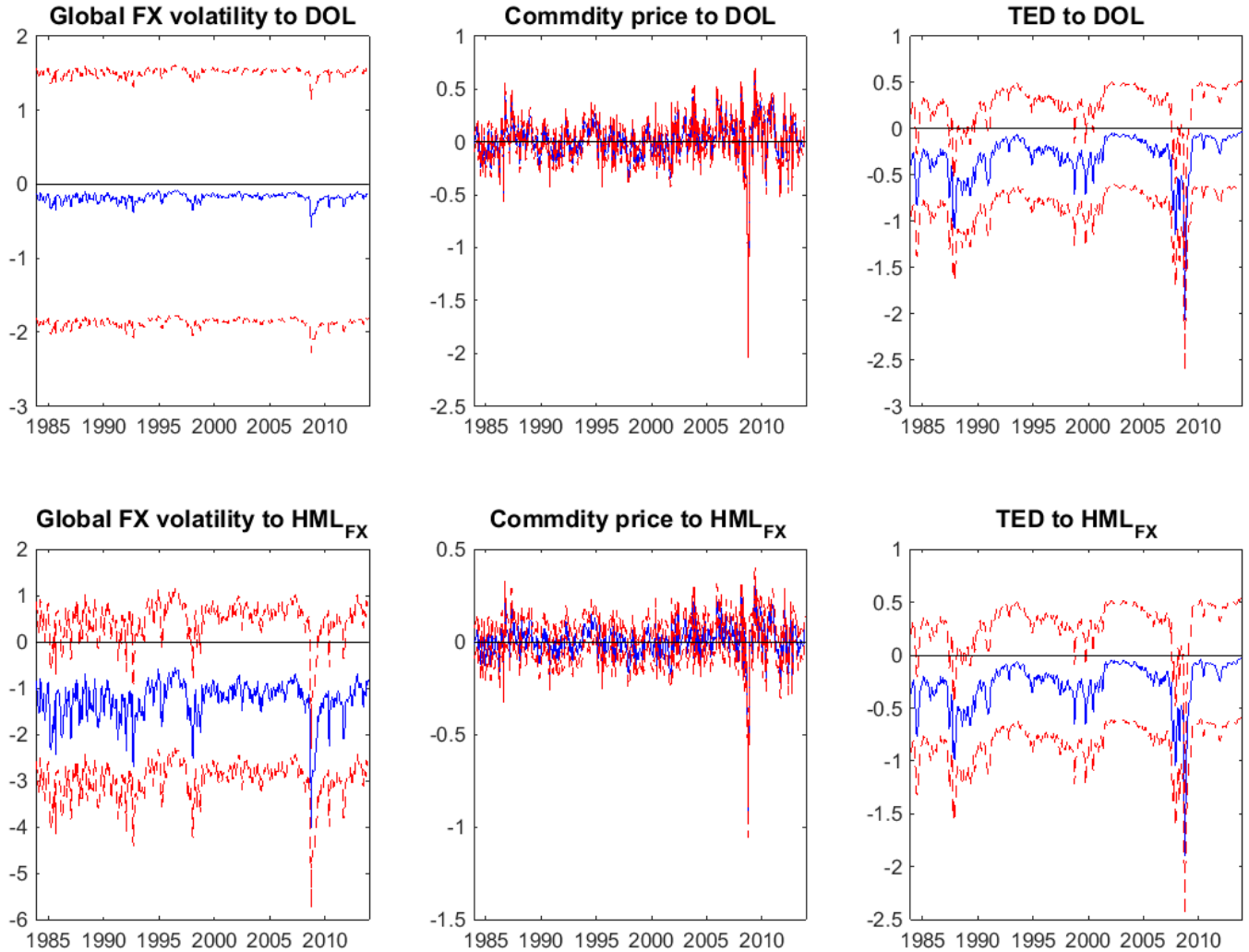
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Time-Varying Risk Price of Currency Carry Trades

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This material provides additional results which are not reported in the main text. Section A shows estimation procedures for the constant beta and time-varying risk price model and the time-varying beta and risk price model. Section B explains the bandwidth estimation process using the time-varying beta and risk price model. Section C proposes another liquidity factor as a forecast factor based on bid-ask spreads. Tables and Figures present robustness.

A. Estimation Process

This section shows an estimation process of the constant beta and time-varying risk price model and the time-varying beta and risk price model as in Adrian et al. (2015). We use the following three steps for the constant beta and time-varying risk price model.

A1. Constant Beta and Time-varying Risk Price Model Estimation

1. Using stack vectors, equation (3) is written as:

$$R = B\lambda_0 \iota_T^{'} + B\Lambda_1 F_- + BU + E, \tag{A-1}$$

where R is the $N \times T$ carry return matrix, ι_T is a $T \times 1$ vector of ones, $F_- = [F_0 \ldots F_T]$ is the $K_F \times T$ forecast factor matrix, U is the $K_C \times T$ innovations term matrix, which is extracted as the first K_c columns of V, where $V = [v_1 \ldots v_T]$, and v_j is the innovation vector of the jth risk factor. E is the $N \times T$ pricing error matrix. B is the $N \times K_C$ factor beta matrix and $B = (\beta_1, \ldots, \beta_N)'$ where β_j is the coefficient obtained by regressing the carry return vector of portfolio i on the innovation vector of the jth risk factor. There are two risk price matrices, and λ_0 is the $K_C \times 1$ and Λ_1 is the $K_C \times K_F$. In the first step, the VAR model in equation (4) is estimated and $\hat{\Sigma}_u = \hat{U}\hat{U}'/T$ and $\hat{\gamma}_{FF} = \tilde{F}_-\tilde{F}'_-/T$ are obtained.

2. Let $A_0 = B\lambda_0$ and $A_1 = B\Lambda_1$, and equation (A-1) can be written as:

$$R = A_0\iota'_T + A_1F_- + BU + E. \tag{A-2}$$

Let $\hat{z}_t = (1, F'_{t-1}, \hat{u}'_t)$, $\hat{Z} = [\iota_T \ F'_- \ \hat{U}']'$, and $\hat{A} = R\hat{Z}'(\hat{Z}\hat{Z})^{-1}$ is estimated by equation (A-2). The heteroskedasticity robust standard error $\hat{\nu}_{rob}$ is obtained as:

$$\hat{\nu}_{rob} = T\left((\hat{Z}\hat{Z}')^{-1} \otimes I_N\right)\left(\sum_{t=1}^{T}\left(\hat{z}_t\hat{z}'_t \otimes \hat{e}_t\hat{e}'_t\right)\right)\left((\hat{Z}\hat{Z}')^{-1} \otimes I_N\right) \tag{A-3}$$

where $\hat{e}_t = R_t - \hat{A}\hat{z}_t$, I_N is the $N \times N$ identity matrix. This heteroskedasticity robust variance estimator is used in Table 1.

3. The risk price parameters, $\hat{\lambda}_0$ and $\hat{\Lambda}_1$, are obtained as:

$$\hat{\lambda}_0 = (\hat{B}'\hat{B})^{-1}\hat{B}'\hat{A}_0, \ \hat{\Lambda}_1 = (\hat{B}'\hat{B})^{-1}\hat{B}'\hat{A}_1. \tag{A-4}$$

The heteroskedasticity robust standard error $\hat{\nu}_{\Lambda,ols}$ is obtained as:

$$\hat{\nu}_{\Lambda,ols} = \left(\hat{\gamma}_{FF}^{-1} \otimes \hat{\Sigma}_u\right) + \eta_\lambda(\hat{B}, \hat{\Lambda})\hat{\nu}_{rob}\eta_\lambda(\hat{B}, \hat{\Lambda})' \tag{A-5}$$

where $\eta_\lambda = \left[\left(I_{(K_F+1)} \otimes (\hat{B}'\hat{B})^{-1} B' \quad - \left(\hat{\Lambda}' \otimes (\hat{B}'\hat{B})^{-1}\hat{B}' \right) \right) \right]$. This heteroskedasticity robust variance estimator is used for the constant beta and time-varying risk price model in Tables 2,4, and 5.

A2. Time-varying Beta and Risk Price Model Estimation

For the time-varying beta and risk price model, the kernel estimation method proposed by Ang and Kristensen (2012) is employed in the steps 1 and 2.

1. Let $\Psi_t = (\mu_t \ \Phi_t)$ and the VAR model in equation (8) is estimated as:

$$\left[\hat{\Psi}_{t-1}\right]_i = \sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) X_{i,s}\tilde{X}'_{s-1} \left(\sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) \tilde{X}_{s-1}\tilde{X}'_{s-1} \right)^{-1} \tag{A-6}$$

where $b = b_i^{sr}$ is the short-run bandwidth as in Ang and Kristensen (2012), $X_{i,s}$ is the ith element of X_s and $\tilde{X}_{s-1} = (1, X'_{s-1})'$. Following Ang and Kristensen (2012), $K(x)$ is the Gaussian density as:

$$K(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{x^2}{2} \right) \tag{A-7}$$

The residual vector $\hat{v}_t$ is obtained as $\hat{v}_t = X_t - \hat{\Psi}_{t-1}\tilde{X}_{t-1}$. We also construct $\hat{\Omega}_{x,t}$ and $\hat{\Sigma}_{v,t}$ using the kernel density:

$$\hat{\Omega}_{x,t} = T^{-1} \sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) \tilde{X}_{s-1}\tilde{X}'_{s-1}, \ \hat{\Sigma}_{v,t} = T^{-1} \sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) \hat{v}_s\hat{v}'_s \tag{A-8}$$

where $b = b_c$ is the average bandwidth across the K equations, which is used by Adrian et al. (2015).

2. Let $A_{i,t} = (\hat{A}_{0,i,t-1}, \hat{A}'_{1,i,t-1}, \hat{\beta}'_{i,t-1})$ and $\hat{A}_{i,t}$ is estimated by equation (A-2) using the short-run bandwidth. Then, $\hat{\Omega}_{f,t}$ and $\hat{\Sigma}_{e,t}$ are obtained by the kernel estimation as:

$$\hat{\Omega}_{F,t} = T^{-1} \sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) \tilde{F}_{s-1}\tilde{F}'_{s-1}, \ \hat{\Sigma}_{e,t} = T^{-1} \sum_{s=1}^{T} K_b\left(\frac{s-t}{T}\right) \hat{e}_s\hat{e}'_s \tag{A-9}$$

where $\hat{e}_i = R_{i,t} - \hat{A}_{i,t-1}z_t^{tv}$ and $b = b_c$ is the average bandwidth that is used by Adrian et al. (2015).

3. The risk price parameters $\hat{\Lambda}^{tv}$ are estimated by equation (9). The variance estimators $\hat{v}_{\Lambda,1}^{tv}$ and $\hat{v}_{\Lambda,2}^{tv}$ are constructed as:

$$
\begin{aligned}
\hat{v}_{\Lambda,1} = \ \ & T \ \left[\sum_{t=1}^{T}(\hat{\Omega}_{f,t} \otimes \hat{B}'_{t-1}\hat{B}_{t-1}) \right]^{-1} \\
& \times \ \left[\sum_{t=1}^{T}((\hat{\Omega}_{f,t}\hat{\Lambda}^{tv}D'_{B}\hat{\Omega}_{z,t}^{-1}D_{B}\hat{\Lambda}^{tv}\hat{\Omega}_{f,t} + \hat{\Omega}_{f,t}) \otimes \hat{B}'_{t-1}\hat{\Sigma}_{e,t}\hat{B}_{t-1}) \right] \\
& \times \ \left[\sum_{t=1}^{T}(\hat{\Omega}_{f,t} \otimes \hat{B}'_{t-1}\hat{B}_{t-1}) \right]^{-1} \qquad\qquad\qquad\text{(A-10)}
\end{aligned}
$$

where

$$
\hat{\Omega}_{z,t} = T^{-1}\sum_{s=1}^{T}K_{b}\left(\frac{s-t}{T}\right)z_{s}^{tv}z_{s}^{tv'} \qquad\qquad\qquad\text{(A-11)}
$$

and $D_{B} = (A_{t})^{-1}B_{t}$,

$$
\begin{aligned}
\hat{v}_{\Lambda,2} = \ \ & T \ \left[\sum_{t=1}^{T}(\hat{\Omega}_{f,t} \otimes \hat{B}'_{t-1}\hat{B}_{t-1}) \right]^{-1} \\
& \times \ \left[\sum_{t=1}^{T}(\hat{\Omega}_{f,t} \otimes \hat{B}'_{t-1}\hat{B}_{t-1}\hat{\Sigma}_{u,t}\hat{B}'_{t-1}\hat{B}_{t-1}) \right] \\
& \times \ \left[\sum_{t=1}^{T}(\hat{\Omega}_{f,t} \otimes \hat{B}'_{t-1}\hat{B}_{t-1}) \right]^{-1}. \qquad\qquad\qquad\text{(A-12)}
\end{aligned}
$$

Finally, $\hat{v}_{\Lambda} = \hat{v}_{\Lambda,1} + \hat{v}_{\Lambda,2}$ is obtained and this heteroskedasticity robust variance estimator is used for the risk price parameters of the time-varying beta and risk price model in Tables 2,4, and 5.

B. Bandwidth Estimation

A bandwidth is obtained by the plug-in method proposed by Ang and Kristensen (2009), and Kristensen (2012). This method is a two step approach and the first-pass bandwidth is estimated by imposing assumptions on unknown variables. Assuming that $\Omega_{x,t} = \Omega_{x}$ and $\Sigma_{v,t} = \Sigma_{v}$ are constant, and and $\Phi_{i,t} = a_{0,i} + a_{1,i}t + \cdots + a_{p,i}t^{p}$ is a polynomial of order $p \geq 2$.. $\breve{\Omega}_{x}$, $\breve{\Sigma}_{v}$, and $\breve{\Phi}_{i,t}$ are estimated

by parametric least squares. Following Ang and Kristensen (2012), we choose the polynomial order of degree as 6. For each i, we compute $\breve{W}_i$ and $\breve{M}_i$ as:

$$\breve{W}_i = \frac{\kappa_2}{T}\breve{\Omega}_x^{-1} \otimes \breve{\Sigma}_v, \quad \breve{M}_i = \frac{1}{T}\sum_{t=1}^{T}||\breve{\Phi}_{i,t}^{(2)}||^2 \tag{A-13}$$

where $\kappa_2 = 0.2821$ for the Gaussian kernel, $||\cdot||$ is the Euclidean norm, and $\breve{\Phi}_{i,t}^{(2)} = 2\breve{a}_{2,i} + 6\breve{a}_{3,i}t + \cdots + p(p-1)\breve{a}_{p,i}t^{p-2}$. The first-pass bandwidth $\breve{b}_i$ is obtained using these estimates:

$$\breve{b}_i = \left[\frac{\breve{W}_i}{\breve{M}_i}\right]^{1/5} \times T^{-1/5} \tag{A-14}$$

Next, we estimate $\hat{\mu}_{i,t}$, $\hat{\Phi}_{i,t}$, $\hat{\Omega}_{x,t}$, and $\hat{\Sigma}_{v,t}$ using the first-pass bandwidth, $\breve{b}_i$. Then, $\hat{W}_i$ and $\hat{M}_i$ are computed as:

$$\hat{W}_i = \frac{\kappa_2}{T}\hat{\Omega}_{x,t}^{-1} \otimes \hat{\Sigma}_{v,t}, \quad \hat{M}_i = \frac{1}{T}\sum_{t=1}^{T}||\hat{\Phi}_{i,t}^{(2)}||^2. \tag{A-15}$$

Applying the same step of the first-pass, the second-pass bandwidth $\hat{b}_i$ is obtained as:

$$\hat{b}_i = \left[\frac{\hat{W}_i}{\hat{M}_i}\right]^{1/5} \times T^{-1/5}. \tag{A-16}$$

C. Global Bid-ask Spreads

We use the global bid-ask spreads as a forecast factor to capture FX market liquidity, instead of the TED spread. $BAS_{FX,t}$ is the global bid-ask spreads as in Menkhoff et al. (2012a). We use a same similar approach to $VOL_{FX,t}$ and the global FX bid-ask spread measure, $\psi_{FX,t}$, in month t is obtained as:

$$\psi_{FX,t} = \frac{1}{T_t}\sum_{\tau=1}^{T_t}\sum_{j=1}^{K_\tau}\left(\frac{\psi_{j,\tau}}{K_\tau}\right) \tag{A-17}$$

where $\psi_{j,\tau}$ is the bid ask spread measure of spot exchange rate j at day τ.

Table A1

Risk Price Parameter Estimates: Constant Beta Model

Panel A: All countries

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
(a)	DOL	0.77**	-1.41			0.18	4.93**
		(0.38)	(0.87)			(0.12)	
	HML_{FX}	2.20***	-4.11***			0.49***	41.83***
		(0.37)	(0.85)			(0.13)	
(b)	DOL	0.17		0.10*		0.18	7.86**
		(0.12)		(0.04)		(0.13)	
	HML_{FX}	0.46***		0.15***		0.49***	31.18***
		(0.12)		(0.04)		(0.13)	
(c)	DOL	0.59***			-0.78***	0.18	9.73***
		(0.19)			(0.29)	(0.14)	
	HML_{FX}	0.99***			-0.98***	0.49***	28.88***
		(0.29)			(0.28)	(0.14)	

Panel B: Developed countries

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
(d)	DOL	0.59	-0.85			0.19	2.85
		(0.45)	(0.91)			(0.14)	
	HML_{FX}	1.95***	-3.42***			0.34**	20.75***
		(0.45)	(0.90)			(0.15)	
(e)	DOL	0.17		0.11**		0.19	7.29**
		(0.13)		(0.05)		(0.14)	
	HML_{FX}	0.32**		0.11**		0.34**	11.83***
		(0.14)		(0.05)		(0.15)	
(f)	DOL	0.64***			-0.87***	0.19	9.35**
		(0.21)			(0.32)	(0.16)	
	HML_{FX}	0.94***			-1.15***	0.34***	19.43***
		(0.32)			(0.32)	(0.16)	

Notes: This table presents risk price parameters estimated by the constant beta and time-varying risk price model as in Adrian et al. (2015). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. The risk price parameters are obtained by equation (5) and the average risk price $\bar{\lambda}$ is obtained by equation (6). The risk factors are average U.S. dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A2

Risk Price Parameter Estimates: Time-varying Beta Model

Panel A: All countries

				Forecast Factors			
	Risk Factor	λ_0	VOL_{FX}	CRB	TED	$\bar{\lambda}$	Wald
(a)	DOL	0.85**	-1.49*			0.22	7.05**
		(0.36)	(0.81)			(0.12)	
	HML_{FX}	2.07***	-3.81***			0.48***	38.25***
		(0.36)	(0.82)			(0.12)	
(b)	DOL	0.19*		0.11***		0.21*	12.27***
		(0.11)		(0.04)		(0.11)	
	HML_{FX}	0.43***		0.12***		0.45***	22.86***
		(0.12)		(0.04)		(0.12)	
(c)	DOL	0.63***			-0.78***	0.23**	13.28***
		(0.17)			(0.26)	(0.11)	
	HML_{FX}	0.86***			-0.86***	0.41***	23.80***
		(0.18)			(0.26)	(0.11)	

Panel B: Developed countries

				Forecast Factors			
	Risk Factor	λ_0	VOL_{FX}	CRB	TED	$\bar{\lambda}$	Wald
(d)	DOL	0.52	-0.75			0.17	2.37
		(0.43)	(0.85)			(0.13)	
	HML_{FX}	1.83***	-3.25***			0.30**	20.31***
		(0.41)	(0.83)			(0.13)	
(e)	DOL	0.20		0.12***		0.22*	11.80***
		(0.13)		(0.04)		(0.13)	
	HML_{FX}	0.29**		0.08*		0.30**	8.73**
		(0.13)		(0.04)		(0.13)	
(f)	DOL	0.71***			-0.88***	0.25**	13.22***
		(0.20)			(0.29)	(0.12)	
	HML_{FX}	0.85***			-1.02***	0.31**	13.90***
		(0.23)			(0.34)	(0.14)	

Notes: This table presents risk price parameters estimated by the time-varying beta and risk price model as in Adrian et al. (2015). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. The risk price parameters are obtained by equation (10) and the average risk price $\bar{\lambda}$ is obtained by equation (11). The risk factors are average U.S. dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A3

Risk Price Parameter Estimates: Constant Beta Model and Global Bid-ask Spreads

Panel A: All countries

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	BAS_{FX}	$\bar{\lambda}$	Wald
(a)	DOL_{FX}	0.35			-1.46	0.18	2.47
		(0.37)			(3.10)	(0.12)	
	HML_{FX}	1.39***			-8.08***	0.49***	28.87***
		(0.36)			(3.03)	(0.13)	
(b)	DOL_{FX}	0.43	-1.05	0.09*	1.58	0.18	8.88*
		(0.44)	(1.06)	(0.04)	(3.59)	(0.13)	
	HML_{FX}	1.95***	-3.34***	0.10**	-0.73	0.49***	36.38***
		(0.42)	(1.02)	(0.04)	(3.41)	(0.14)	

Panel B: Developed countries

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	BAS_{FX}	$\bar{\lambda}$	Wald
(c)	DOL_{FX}	0.03			2.15	0.19	2.17
		(0.37)			(4.56)	(0.16)	
	HML_{FX}	0.74**			-5.35	0.34**	7.59**
		(0.38)			(4.64)	(0.15)	
(d)	DOL_{FX}	0.08	-0.25	0.11**	2.79	0.19	11.93**
		(0.60)	(0.93)	(0.05)	(4.55)	(0.15)	
	HML_{FX}	2.28***	-3.18***	0.06	-6.03	0.34**	35.59***
		(0.60)	(0.94)	(0.05)	(4.56)	(0.17)	

Notes: This table presents risk price parameters estimated by the constant beta and time-varying risk price model as in Adrian et al. (2015). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. The risk price parameters are obtained by equation (5) and the average risk price $\bar{\lambda}$ is obtained by equation (6). The risk factors are average U.S. dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and BAS_{FX} is the global bid-ask spreads as in Menkhoff et al. (2012a). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A4

Risk Price Parameter Estimates: Time-Varying Beta Model and Global Bid-ask Spreads

Panel A: All countries

				Forecast Factors			
	Risk Factor	λ_0	VOL_{FX}	CRB	BAS_{FX}	$\bar{\lambda}$	Wald
(a)	DOL	0.40			-1.69	0.21*	3.53
		(0.36)			(2.98)	(0.12)	
	HML_{FX}	1.22***			-6.21**	0.53***	25.33***
		(0.35)			(2.90)	(0.11)	
(b)	DOL	0.46	-1.24	0.09**	2.61	0.25**	14.12***
		(0.43)	(0.98)	(0.04)	(3.49)	(0.12)	
	HML_{FX}	2.01***	-3.31***	0.07*	-1.69	0.46***	36.38***
		(0.42)	(0.96)	(0.04)	(3.41)	(0.14)	

Panel B: Developed countries

				Forecast Factors			
	Risk Factor	λ_0	VOL_{FX}	CRB	BAS_{FX}	$\bar{\lambda}$	Wald
(c)	DOL	0.08			1.37	0.18	2.25
		(0.34)			(4.16)	(0.13)	
	HML_{FX}	0.71*			-5.08	0.33**	7.15**
		(0.36)			(4.43)	(0.15)	
(d)	DOL	0.14	-0.22	0.10**	2.24	0.22*	11.28**
		(0.51)	(0.79)	(0.04)	(3.81)	(0.11)	
	HML_{FX}	2.19***	-3.28***	0.05	-4.63	0.30**	17.92***
		(0.61)	(0.95)	(0.05)	(4.55)	(0.14)	

Notes: This table presents risk price parameters estimated by the time-varying beta and risk price model as in Adrian et al. (2015). The test assets of Panel A are six forward discount sorted all country currency portfolios and those of Panel B are five forward discount sorted developed country currency portfolios. The risk price parameters are obtained by equation (10) and the average risk price $\bar{\lambda}$ is obtained by equation (11). The risk factors are average U.S. dollar (DOL_{FX}) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and BAS_{FX} is the global bid-ask spreads as in Menkhoff et al. (2012a). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Table A5

Risk Price Parameters Estimates excluding the Crisis

	Risk Factor	λ_0	VOL_{FX}	Forecast Factors CRB	TED	$\bar{\lambda}$	Wald
Constant beta and time-varying risk price model							
Panel A: All countries							
(a)	DOL	0.12	0.51	0.00	-0.19	0.23	3.50
		(0.49)	(1.10)	(0.06)	(0.37)	(0.13)	
	HML_{FX}	2.04***	-2.89***	0.07	-0.62	0.55***	24.02***
		(0.50)	(1.12)	(0.06)	(0.37)	(0.15)	
Panel B: Developed countries							
(b)	DOL	-0.14	1.09	-0.01	-0.20	0.23	3.58
		(0.62)	(0.86)	(0.06)	(0.40)	(0.15)	
	HML_{FX}	1.81***	-2.16*	0.01	-0.71*	0.46***	12.32**
		(0.62)	(1.26)	(0.06)	(0.39)	(0.15)	
Time-varying beta and time-varying risk price model							
Panel C: All countries and time-varying beta model							
(c)	DOL	0.19	0.39	-0.01	-0.11	0.28**	5.88
		(0.44)	(1.00)	(0.05)	(0.34)	(0.12)	
	HML_{FX}	1.97***	-2.85***	0.06	-0.60	0.50***	22.95***
		(0.48)	(1.08)	(0.06)	(0.37)	(0.13)	
Panel D: Developed countries							
(d)	DOL	-0.16	1.18	-0.04	-0.18	0.25*	5.08
		(0.62)	(1.25)	(0.06)	(0.39)	(0.14)	
	HML_{FX}	1.89	-3.81	-0.10	0.03	0.18	0.39
		(3.28)	(6.62)	(0.33)	(2.07)	(0.74)	

Notes: This table presents risk price parameters estimated by the constant beta and time-varying risk price model and the time-varying beta and risk price model as in Adrian et al. (2015). Data extend to November 1983 to March 2008 to exclude the effect of the global financial crisis. The test assets of Panels A and C are six forward discount sorted all country currency portfolios and those of Panels B and D are five forward discount sorted developed country currency portfolios. The risk price parameters in Panels A and B are obtained by equation (5) and the average risk price $\bar{\lambda}$ in Panels A and C are obtained by equation (6). The risk price parameters in Panels C and D are obtained by equation (10) and the average risk price $\bar{\lambda}$ in Panels C and D is obtained by equation (11). The risk factors are the average U.S. dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) as in Lustig et al. (2011). The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED). Wald indicates the Wald test statistic of the null hypothesis is that the associated row is all zero. Heteroskedasticity robust standard errors are reported in parentheses. Asterisk *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.
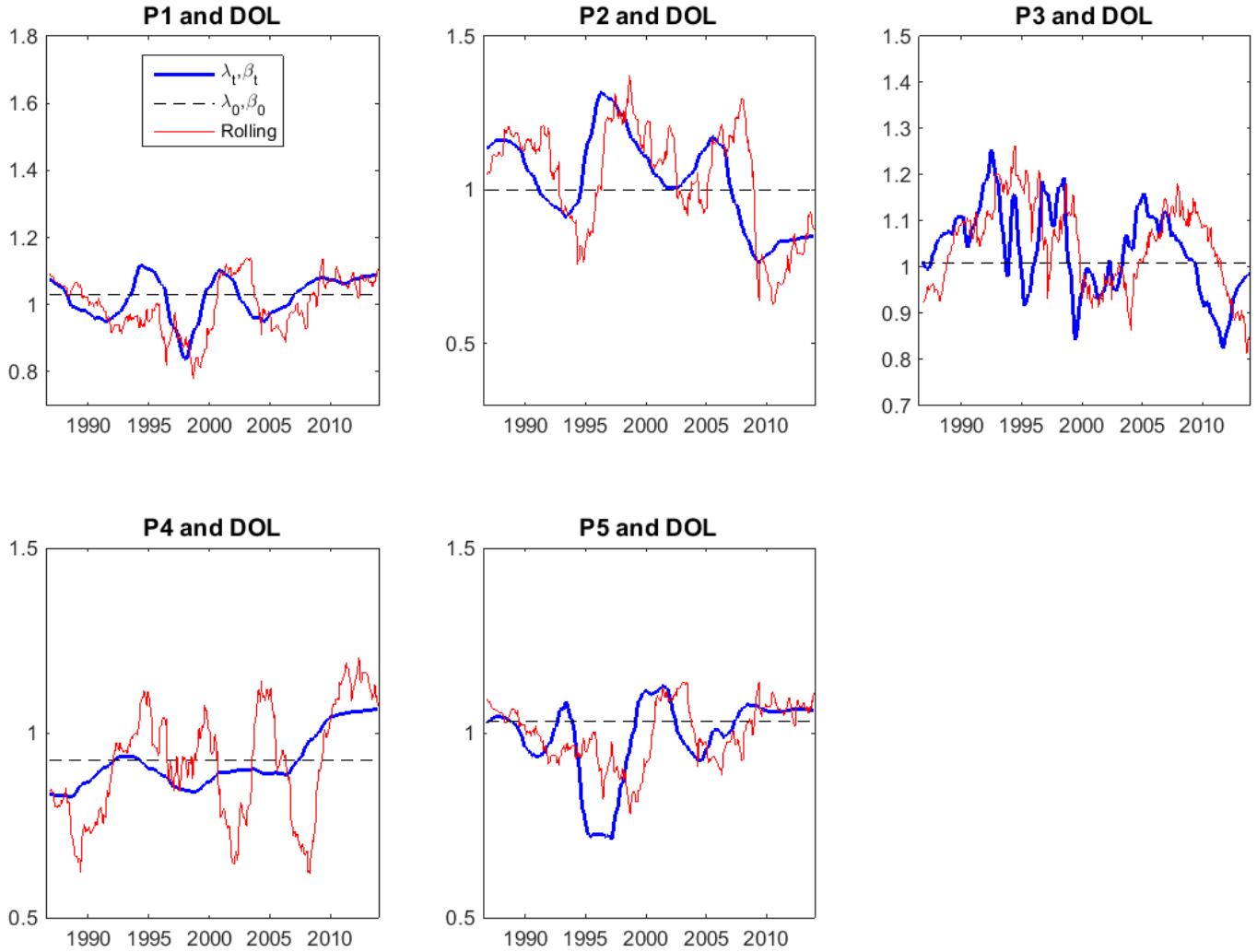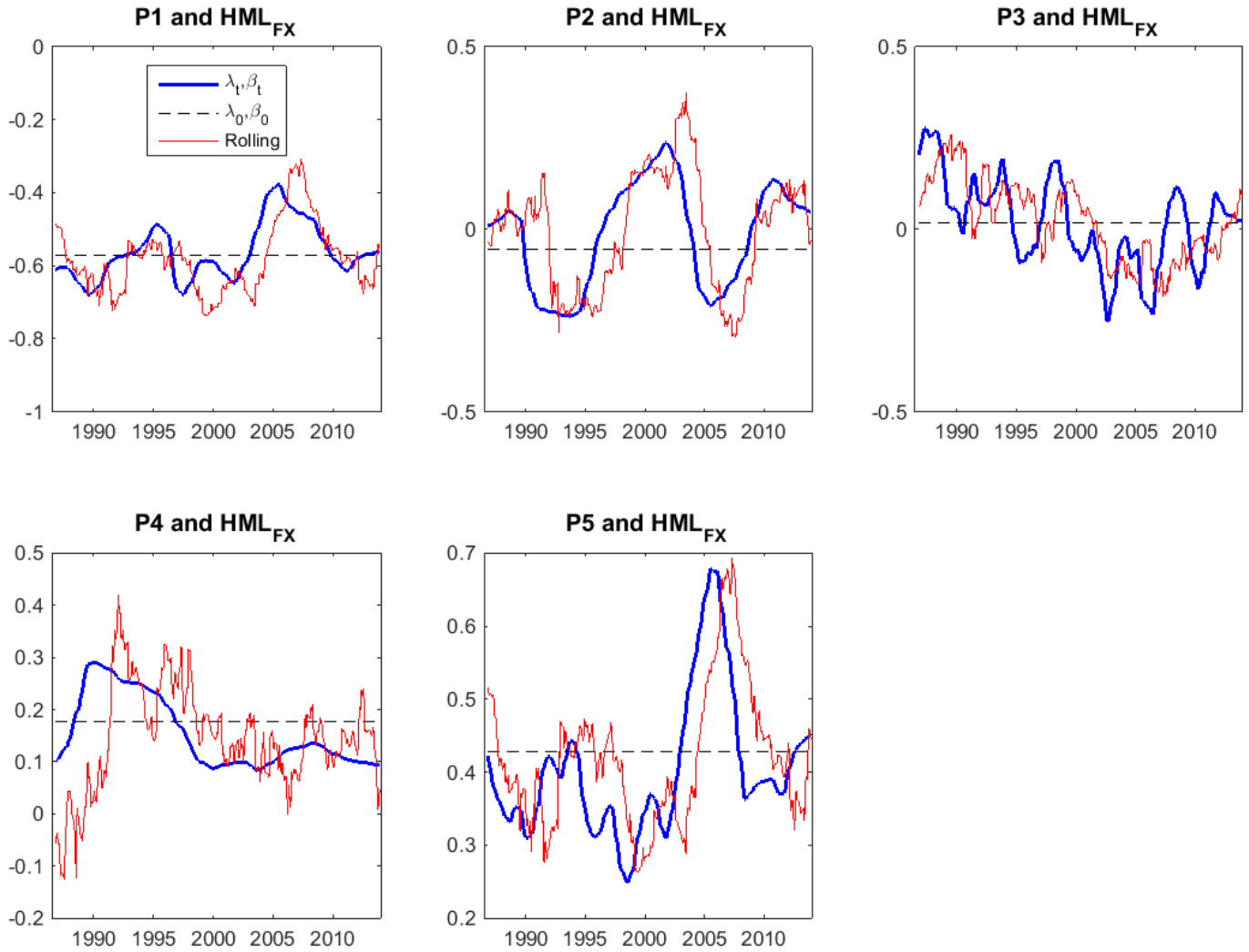
Figure A1.

Comparison of time series portfolio betas on DOL in developed countries

Notes: This figure provides plots of the estimated time series of betas on the dollar (DOL) in developed countries. λ_t, β_t denotes the time-varying beta and risk price model and the betas are obtained by equation (9) (thick blue line). λ_t, β_0 denotes the constant beta and time-varying risk price model and the betas are obtained by equation (3) (dashed black line). Rolling denotes the 36 months rolling window beta (thin red line). The time-varying betas are estimated by the kernel regression approach as in Adrian et al. (2015).

Figure A2.

Comparison of time series portfolio betas on HML_{FX} in developed countries

Notes: This figure provides plots of the estimated time series of betas on the return spread between high and low interest rate currency portfolios (HML_{FX}) in developed countries. λ_t, β_t denotes the time-varying beta and risk price model and the betas are obtained by equation (9) (thick blue line). λ_t, β_0 denotes the constant beta and time-varying risk price model and the betas are obtained by equation (3) (dashed red line). Rolling denotes the 36 months rolling window beta (thin black line).
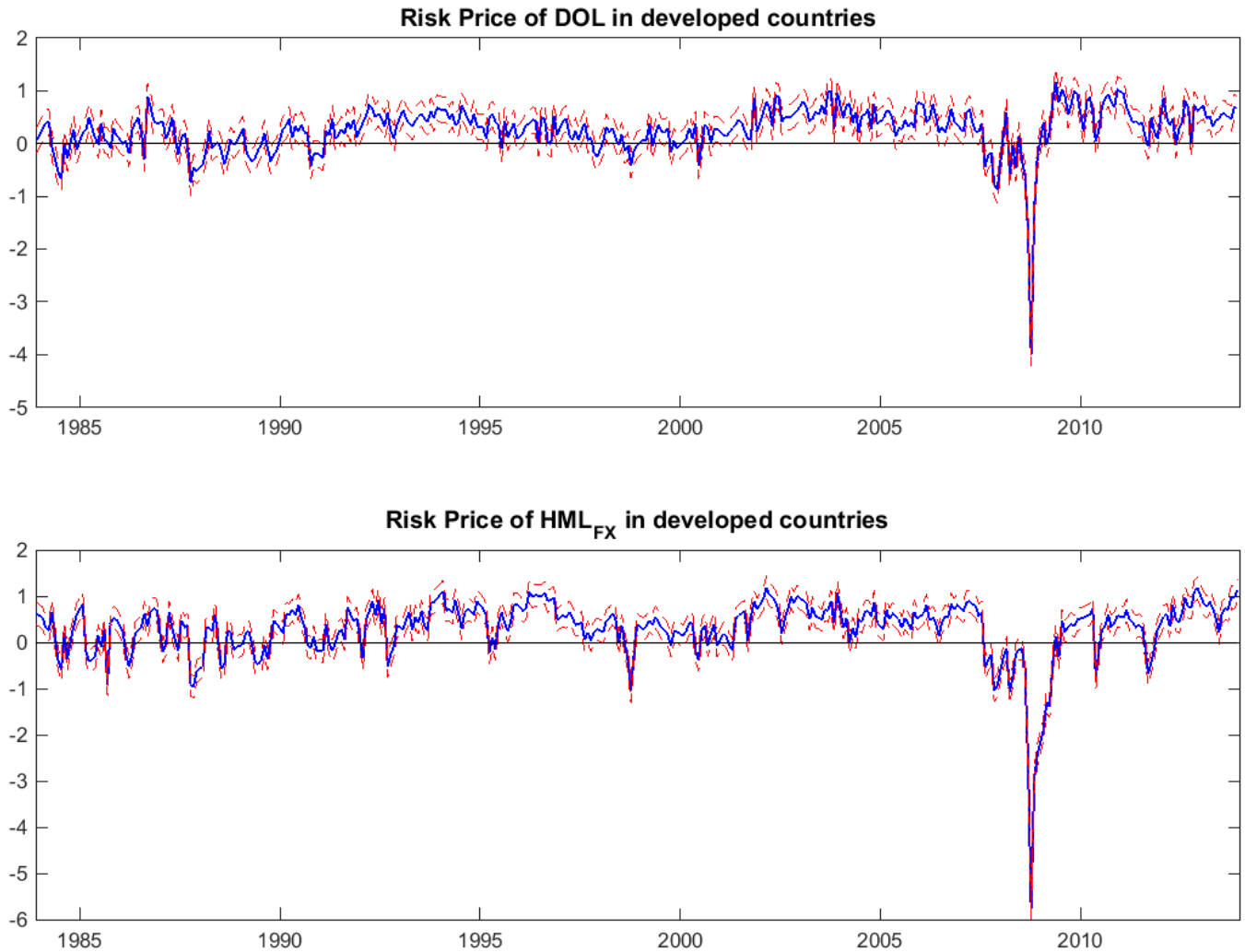
Figure A3.

Time-varying risk prices (λ) of DOL in HML_{FX} in developed countries

Notes: This figure displays time series risk prices of the dollar (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}) with their 95% confidence intervals in developed countries. The risk prices are obtained as $\lambda = \lambda_0 + \Lambda_1 F_t$. Three forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).
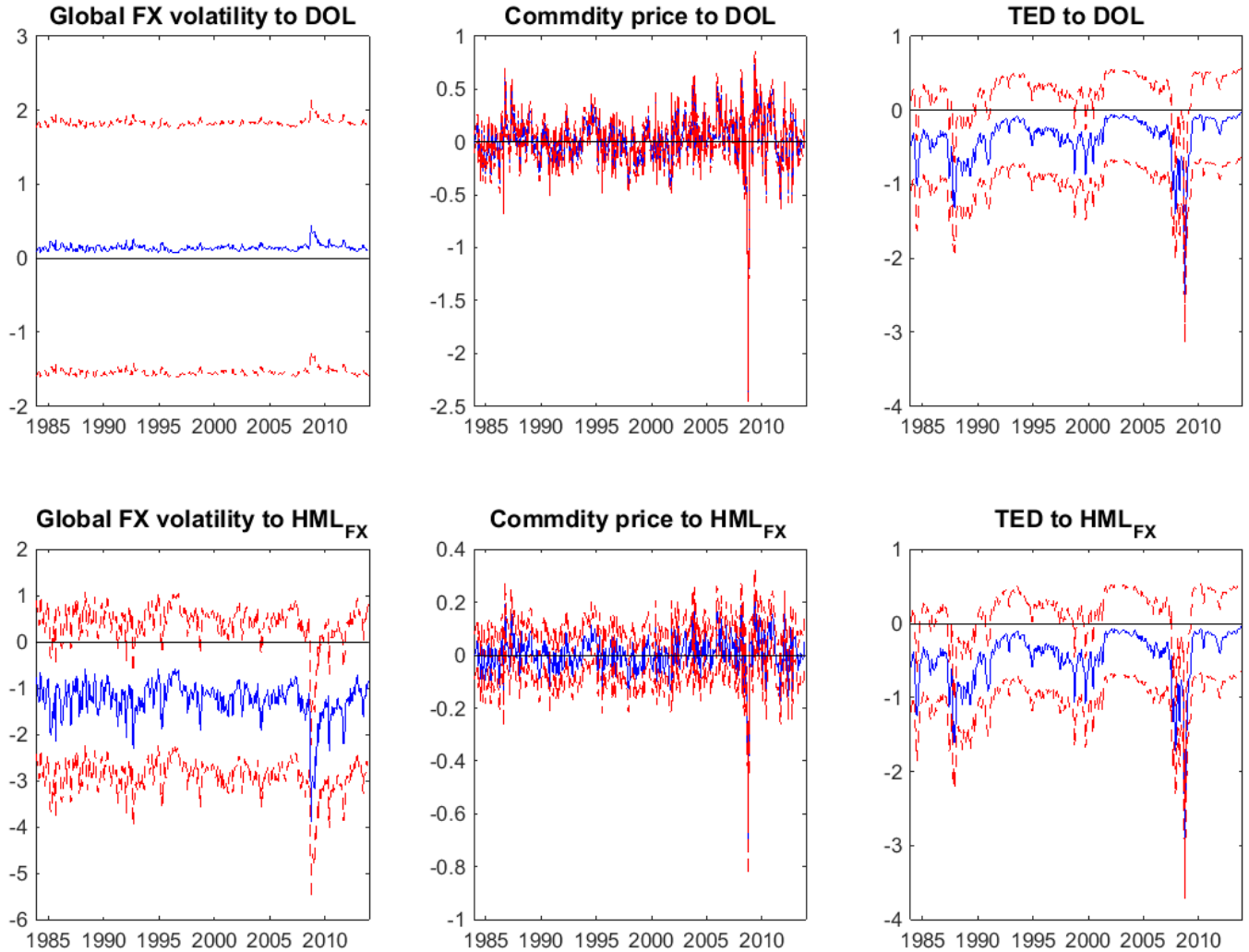
Figure A4.

Contribution of forecast factors in developed countries

Notes: This figure displays the contribution of the three forecast factors with their 95% confidence intervals in developed countries. The contribution is estimated as $\Lambda_{1,j}F_{j,t}$. The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).
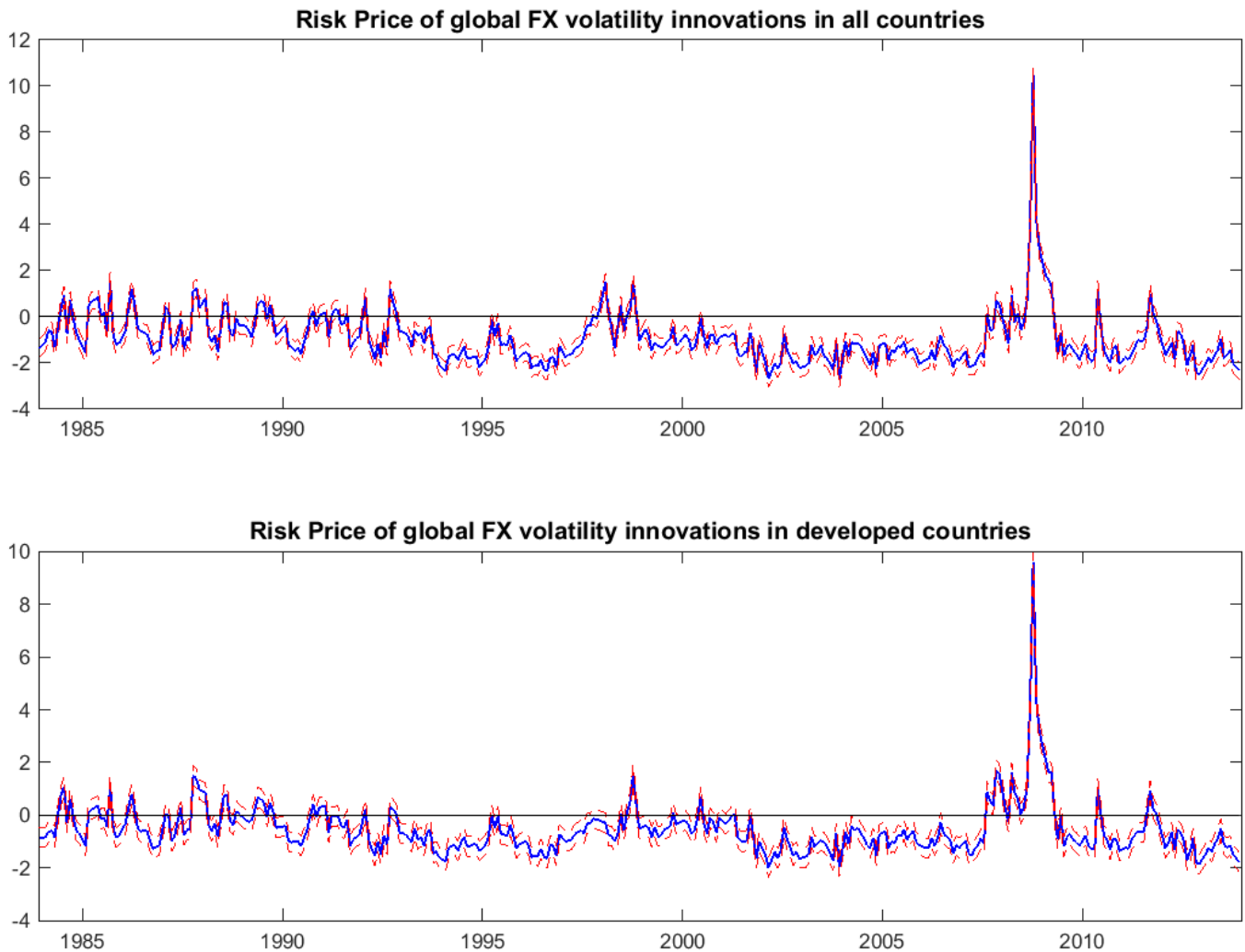
Figure A5.

Time-varying risk price (λ) of ΔVOL_{FX}

Notes: This figure displays time series risk price of the global FX volatility innovations (ΔVOL_{FX}) with its 95% confidence interval. Risk price parameters are obtained by the time-varying beta and risk price model. The risk prices are obtained as $\lambda = \lambda_0 + \Lambda_1 F_t$. The test assets of the upper figure are six forward discount sorted all country currency portfolios and those of the lower figure are five forward discount sorted developed country currency portfolios. The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).
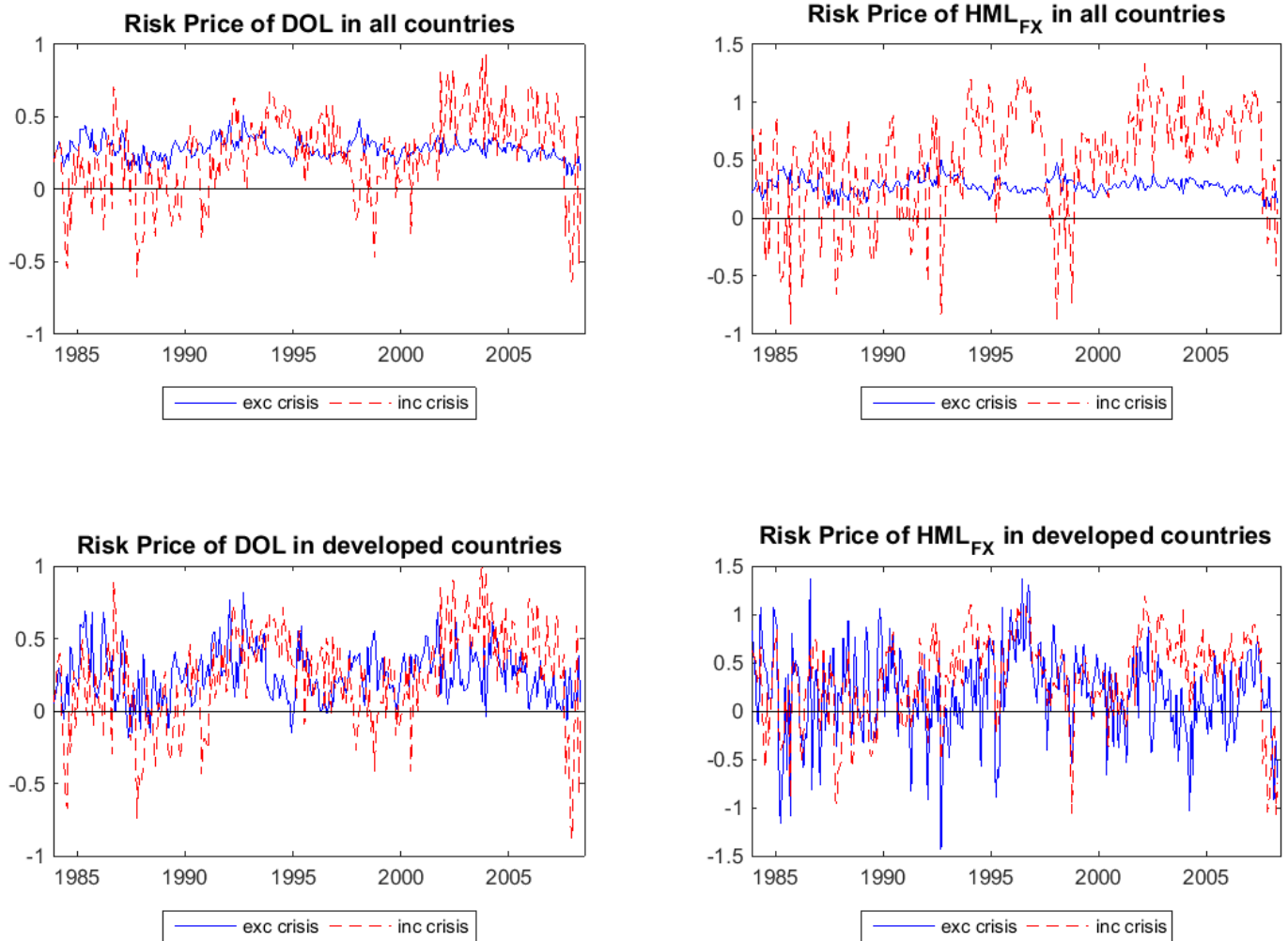
Figure A6.

Time-varying risk price comparison

Notes: This figure displays time series risk price of the dollar risk (DOL) and the return spread between high and low interest rate currency portfolios (HML_{FX}). Risk price parameters are obtained by the time-varying beta and risk price model. exc crisis denotes the estimation results using data which cover November 1983 to March 2008. inc crisis denotes the estimation results using data which cover November 1983 to December 2013. The forecast factors are global FX volatility (VOL_{FX}) as in Menkhoff et al. (2012a), CRB Raw industrial material subindex return (CRB) as in Bakshi and Panayotov (2013), and TED spread (TED).
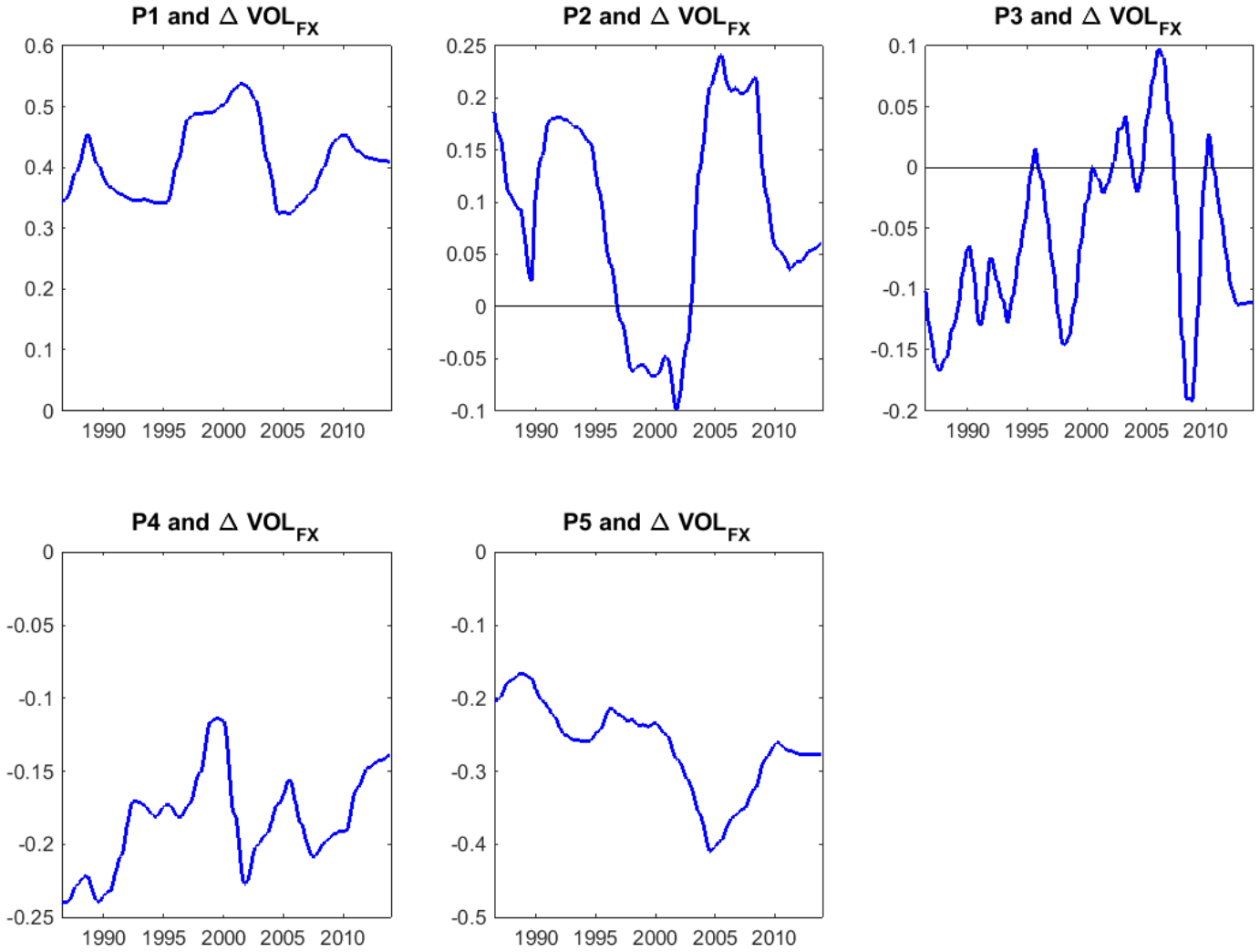
Figure A7.

Time-varying betas on ΔVOL_{FX}

Notes: This figure provides plots of the estimated time series of betas on ΔVOL_{FX} which is the global FX volatility innovation factor as in Menkhoff et al. (2012a). The time-varying betas are obtained by equation (9). The test assets are six forward discount sorted all country currency portfolios.

Figure A8.

Time-varying betas on ΔVOL_{FX} in developed countries

Notes: This figure provides plots of the estimated time series of betas on ΔVOL_{FX} which is the global FX volatility innovation factor as in Menkhoff et al. (2012a). The time-varying betas are obtained by equation (9). The test assets are six forward discount sorted all country currency portfolios.