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Polarization Imaging of an Edge Object with Partially Coherent Light

Xiaoying Hu^{*a,b}, Ailing Tian^b and Wei Wang^a

^aSchool of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, UK

^bSchool of Optoelectronic Engineering, Xi'an Technological University, Xi'an, Shaanxi, China.
710032

ABSTRACT

Although most scientists and engineers working in the field of image acquisition and processing are well aware of the partially polarized nature of the optical fields used to form images, the effect of coherence on polarization imaging systems seems to have gone largely unnoticed. In this paper, the effects of polarization imaging of an edge object with partially coherent light are investigated theoretically and experimentally. We have extended the use of edge trace analysis in the evaluation of optical system performance and presented theoretical analysis on polarization imaging of an edge object with partially coherent and partially polarized illumination. Some interesting effects, such as the edge ringing and shifting in Stokes vector image construction, have been demonstrated experimentally.

Keywords: polarization image, coherent and incoherent light, edge object

1. INTRODUCTION

The literature contains a number of papers dealing with the image of an edge and the transfer function of a system for an edge object^[1-5]. The use of edge trace analysis in the evaluation of system performance is now well established; high quality man made edges are used for the evaluation of instruments as well as certain photographic systems^[6-8]. The presence of edges, such as roof lines on houses, etc., in aerial photographs enable overall system performance to be established. Instruments for determining the transfer function based on edge trace analysis are reviewed by MURATA^[9]. The basic principles of the technique are easily appreciated.

The image of an edge or a line with partially coherent illumination has been examined by HOPKINS, STEEL, CANALS-FRAU and ROUSSEAU, KINZLY, SKINNER and CONSIDINE^[10-11]. The influence of the state of coherence of the object on image formation in a microscope was clearly recognized by CHARMAN who carried out experiments to show the effect of variations of the condenser aperture and hence the coherence on the image^[12-14]. He compared these results with the theoretical predictions of scalar theory. WATRASIEWICZ examined the effects at high numerical aperture in both polarized and non-polarized light to show the limits of the scalar theory^[15-16]. The effects in scanning microscopy were first discussed by WELFORD and later by KINZLY^[17].

All these authors showed the basic differences in the image of an edge. But they don't consider the polarimetric properties of light. Theoretical results are presented about polarization image of an edge with coherent and incoherent light. Both theoretical and experimental results are presented.

2. NONLINEARITY IN IMAGING WITH PARTIALLY COHERENT LIGHT

The basic quantity in the vector theory of partial coherence is the generalized Stokes parameters defined by

$$\begin{cases} S'_0(x_1, x_2) = \langle \tilde{E}_x(x_1, t)p(x_1)\tilde{E}_x^*(x_2, t)p(x_2) + \tilde{E}_y(x_1, t)p(x_1)\tilde{E}_y^*(x_2, t)p(x_2) \rangle \\ S'_1(x_1, x_2) = \langle \tilde{E}_x(x_1, t)p(x_1)\tilde{E}_x^*(x_2, t)p(x_2) - \tilde{E}_y(x_1, t)p(x_1)\tilde{E}_y^*(x_2, t)p(x_2) \rangle \\ S'_2(x_1, x_2) = \langle \tilde{E}_x(x_1, t)p(x_1)\tilde{E}_y^*(x_2, t)p(x_2) + \tilde{E}_x^*(x_2, t)p(x_2)\tilde{E}_y(x_1, t)p(x_1) \rangle \\ S'_3(x_1, x_2) = \langle i(\tilde{E}_x^*(x_2, t)p(x_2)\tilde{E}_y(x_1, t)p(x_1) - \tilde{E}_x(x_1, t)p(x_1)\tilde{E}_y^*(x_2, t)p(x_2)) \rangle \end{cases} \quad (1)$$

$S_n^t(x_1, x_2)$, $n=0,1,2,3$ is the generalized Stokes parameters in the object plane of an optical imaging system in Eq. (1). The sharp brackets in Eq. (1) indicate a long time average and $E(x,t)$ is the analytic signal associated with the optical disturbance at the point x and time t . A more useful form for application to image analysis is obtained by considering the object to be transilluminated. This is certainly the case in all uses of microscopes, enlargers, and microdensitometers in image evaluation. For transilluminated objects, where $p(x)$ is the amplitude transmittance of the object.

Then the expression for the generalized Stokes parameters $S_m^o(x_1, x_2)$, $m=0,1,2,3$ in the image plane of an optical imaging system is

$$S_m^o(x_1, x_2) = \iint S_n^t(\xi_1, \xi_2) p(\xi_1) p^*(\xi_2) h(x_1 - \xi_1) h^*(x_2 - \xi_2) d\xi_1 d\xi_2 \quad m, n = 0, 1, 2, 3 \quad (2)$$

In Eq. (2), $S_m^o(x_1, x_2)$ is the generalized Stokes parameters in the image plane, $p(\xi)$ is the amplitude transmittance of the object, and $h(x - \xi)$ is essentially the spatially stationary amplitude impulse response of the imaging system. However, when the detection process is included in the analysis, the generalized Stokes parameters in the image plane is the quantity of interest. For transilluminated objects, this generalized Stokes parameters is found from Eq. (2) and the definition (1) to be

$$S_m^o(x) = S_m^o(x, x) = \iint S_n^t(\xi_1, \xi_2) p(\xi_1) p^*(\xi_2) h(x - \xi_1) h^*(x - \xi_2) d\xi_1 d\xi_2 \quad m, n = 0, 1, 2, 3 \quad (3)$$

For the important practical case where $S_n^t(\xi_1, \xi_2)$ is spatially stationary, i.e., for

$$S_n^t(\xi_1, \xi_2) = S_n^t(\xi_1 - \xi_2) \quad (4)$$

Eq. (3) can be written

$$S_m^o(x) = \iint S_n^t(\xi_1 - \xi_2) p(\xi_1) p^*(\xi_2) h(x - \xi_1) h^*(x - \xi_2) d\xi_1 d\xi_2 \quad m, n = 0, 1, 2, 3 \quad (5)$$

From Eq. (5) it is clear that, for transilluminated objects, the transition from object intensity $p(\xi_1) p^*(\xi_2)$ to image intensity $S_m^o(x)$ is nonlinear. The significance of this conclusion is that the customary image-evaluation techniques and criteria are not, in general, applicable to such systems. Knowing how such a system images edges does not permit us to describe how it images other objects. Furthermore, the same optical system and object could be expected to yield different polarization image $S_m^o(x)$ if the coherence of the illumination represented by $S_n^t(\xi_1 - \xi_2)$ were varied.

Since systems of this type are inherently nonlinear, it is impossible to characterize them by a transfer function. Because of its usefulness, and because it outlines an analytical technique, it will be worthwhile to go through the transformation in detail. Symmetrical transforms will be employed. The spectrum is given by.

$$\mathbf{S}_m^o(f_x) = \int S_m^o(x) e^{-2\pi i f_x x} dx \quad (6)$$

When Eqs. (5) and (6) are combined, and the integrand regrouped, we find

$$\mathbf{S}_m^o(f_x) = \iint S_n^t(\xi_1 - \xi_2) p(\xi_1) p^*(\xi_2) \left\{ \int h(x - \xi_1) h^*(x - \xi_2) e^{-2\pi i f_x x} dx \right\} d\xi_1 d\xi_2 \quad (7)$$

Take ξ and σ to be conjugate variables in $\bar{\xi}$ and $\bar{\sigma}$, respectively. We can then evaluate the inner integral. Thus,

$$\int h(x - \xi_1) h^*(x - \xi_2) e^{-2\pi i f_x x} dx = \exp(-2\pi i f_x \xi_1) \int \mathbf{H}(f_x - \sigma) \mathbf{H}^*(\sigma) \exp[2\pi i \sigma(\xi_1 - \xi_2)] d\sigma \quad (8)$$

When we insert this in Eq. (7) and regroup, we find

$$\mathbf{S}_m^o(f_x) = \iint \mathbf{H}(f_x - \sigma) \mathbf{H}^*(\sigma) p^*(\xi_2) \left\{ \int S_n^t(\xi_1 - \xi_2) p(\xi_1) \exp[-2\pi i(f_x - \sigma)\xi_1] d\xi_1 \right\} \exp(-2\pi i \sigma \xi_2) d\sigma d\xi_2 \quad (9)$$

Take ζ and γ to be conjugate variables in $\bar{\zeta}$. We can then evaluate the inner integral. Thus,

$$\int S_n^t(\xi_1 - \xi_2) p(\xi_1) \exp[-2\pi i(f_x - \sigma)\xi_1] d\xi_1 = \exp[-2\pi i(f_x - \sigma)\xi_2] \int \mathbf{S}_n^t[f_x - (\sigma + \gamma)] \mathbf{P}(\gamma) \exp(2\pi i \gamma \xi_2) d\gamma \quad (10)$$

When the insert this in Eq. (9) and make a final regrouping, we have

$$\mathbf{S}_m^o(f_x) = \iint \mathbf{S}_n^t[f_x - (\sigma + \gamma)] \mathbf{H}(f_x - \sigma) \mathbf{H}^*(\sigma) \mathbf{P}(\beta) \left\{ \int p^*(\xi_2) \exp[2\pi i \xi_2(f_x - \gamma)] d\xi_2 \right\} d\sigma d\gamma \quad (11)$$

The inner integral can immediately be recognized as a Fourier transform with ξ_2 and $(f_x - \gamma)$ as conjugate variables. Thus, the polarization image spectrum for the one-dimensional case of image formation is given by

$$\mathbf{S}_m^o(f_x) = \int \mathbf{P}(\gamma) \mathbf{P}^*(f_x - \gamma) \left\{ \int \mathbf{S}_n^t[f_x - (\sigma + \gamma)] \mathbf{H}(f_x - \sigma) \mathbf{H}^*(\sigma) d\sigma \right\} d\gamma \quad (12)$$

Where the grouping of terms has been deliberate, to point out the source of the nonlinearity. The inner integral contains the optical-system and illumination characteristics. It is a function of two spatial frequencies and has been referred to as

the "transmission cross coefficient". In only a vague sense does it constitute a transfer function, and is not used as one in Eq. (12). Therefore, for an arbitrary mutual polarization image, the behavior of an optical system is inherently nonlinear. The normal linear multiplicative relation between object spectrum and system transfer function is no longer applicable, and the current technique of optical system analysis through the cascading of component transfer functions is clearly subject to error.

In Eq. (12) the inner integral is characteristic of the instrument and the illumination while the factors $\mathbf{P}(\gamma)$ and $\mathbf{P}^*(f_x - \gamma)$ are determined solely by the object. However, the right side of Eq. (12) is not in the form of a product of object spectrum and transfer function as it would be if the system were linear. The inner integral in Eq. (12) has been referred to as a generalized transfer function, but that nomenclature is rather misleading since the function is not used as a transfer function. A better terminology may be the more cumbersome one introduced by Wolf, i.e., the "transmission cross coefficient."

3. DESCRIPTION OF IMAGING SYSTEM

This section contains a description of an optical imaging system for which the transfer function is to be measured. The mutual intensity of the light incident on the object and the amplitude impulse response of the imaging system, and their Fourier transforms, are found in this section. These are used in the following sections to solve the imaging problem and to determine the apparent transfer functions for edge objects.

For simplicity, the treatment given here is restricted to one-dimensional object variations, slit sources, and slit apertures. An incoherent source in the β plane illuminates an object in the ξ plane. The ξ plane is imaged onto the x plane by an imaging system with exit pupil in the α plane. The distance in image space is normalized by the lateral magnification of the imaging system and the positive direction in image space is opposite to that in object space. This is done to make the ordinate of a given object point equal to that of its corresponding image point.

The incoherent source is taken to be uniform and of finite size such that its intensity is

$$I_s(\beta) = \begin{cases} 1, & |\beta| \leq \beta_0 \\ 0, & |\beta| > \beta_0 \end{cases} \quad (13)$$

where β_0 is a constant. The mutual intensity of the object-plane illumination is found from the well-known van Cittert-Zernike theorem to be given to a good approximation by

$$S_n'(\xi_1 - \xi_2) = \int I_s(\beta) e^{2\pi i \beta (\xi_1 - \xi_2) / \lambda d} d\beta \quad (14)$$

for points ξ_1 and ξ_2 close to the optical axis. Here d is the distance between the incoherent-source plane β and the object plane ξ . When Eq. (13) is used to evaluate Eq. (14), the mutual intensity in the object plane is found to be

$$S_n'(\xi_1 - \xi_2) = \text{const} \sin[2\pi\beta_0(\xi_1 - \xi_2)/\lambda d] \quad (15)$$

The distance from the center to the first zero of the mutual intensity function in Eq. (15) will be referred to as the coherence interval of the object illumination.

The Fourier transform of the mutual intensity function Eq. (15) is

$$\mathbf{S}_n'(f_x) = \int S_n'(\xi_1 - \xi_2) e^{2\pi i f_x (\xi_1 - \xi_2)} d(\xi_1 - \xi_2) = \text{const} \begin{cases} 1, & |f_x| \leq \beta_0 / \lambda d = b \\ 0, & |f_x| > \beta_0 / \lambda d = b \end{cases} \quad (16)$$

Equation (15) serves to define the parameter b .

To emphasize the coherence effects and to minimize the complications arising from aberrations, the restriction to diffraction-limited optics is imposed. The amplitude in the exit pupil of the imaging system due to a point object is taken to be

$$A(\alpha) = \text{const} \begin{cases} 1, & |\alpha| \leq \alpha_0 \\ 0, & |\alpha| > \alpha_0 \end{cases} \quad (17)$$

where α_0 is a constant. Under the usual approximations which characterize Fraunhofer diffraction, the amplitude impulse response corresponding to Eq. (17) is

$$h(x) = \int A(\alpha) e^{2\pi i \alpha x / \lambda s} d\alpha = \text{const} \sin[2\pi\alpha_0 x / \lambda s] \quad (18)$$

where s is the distance from the plane α of the exit pupil to the image plane x . The distance from the center to the first zero of the impulse response function in Eq. (18) will be referred to as the size of the imaging system's diffraction pattern. The Fourier transform of Eq. (18) is

$$\mathbf{H}(f_x) = \int h(x) e^{2\pi i f_x x} dx = \text{const} \begin{cases} 1, & |f_x| \leq \alpha_0 / \lambda d = a \\ 0, & |f_x| > \alpha_0 / \lambda d = a \end{cases} \quad (19)$$

Equation (19) serves to define the parameter a .

4. POLARIZATION IMAGE FOURIER TRANSFORM FOR EDGE OBJECT

The approach used in this section is to carry through the calculation for the case where the edge object is illuminated by partially coherent light. Equation (12) will therefore be used to determine the quantity $\mathbf{S}_m^o(f_x)$. The result of this calculation may not, of course, be interpretable as a Fourier transform of the generalized Stokes parameters in the image plane. that is, it will describe the result of an edge-trace experimental program.

For the present problem, the amplitude transmittance of the edge object is therefore taken to be

$$p(\xi) = \frac{1}{2} + \frac{1}{2} \text{sgn } \xi \quad (20)$$

and

$$\text{sgn } \xi = \xi / |\xi| = \begin{cases} 1, & \xi > 0 \\ -1, & \xi < 0 \end{cases} \quad (21)$$

The Fourier transform of Eq. (20) is

$$\mathbf{P}(f_x) = \frac{1}{2} [\delta(f_x) + iP/\pi f_x] \quad (22)$$

where $\delta(f_x)$ is the Dirac δ function. and P is to be interpreted as the principal value operator if \mathbf{P} occurs in an integration. Equation (12) can be rearranged to have the form

$$\mathbf{S}_m^o(f_x) = \int \left\{ \mathbf{P}(\gamma) \mathbf{P}^*(f_x - \gamma) \mathbf{S}_n'(f_x - \sigma - \gamma) d\gamma \right\} \mathbf{H}(f_x - \sigma) \mathbf{H}^*(\sigma) d\sigma \quad (23)$$

In Eq. (21), Eq. (19) has been used to write $\mathbf{H} = \mathbf{H}^*$. Substitution of Eq. (22) in the inner integral of Eq. (23) shows that this inner integral is

$$M(f_x, \sigma) = \int \mathbf{P}(\gamma) \mathbf{P}^*(f_x - \gamma) \mathbf{S}_n'(f_x - \sigma - \gamma) d\gamma = G_1 + G_2 + G_3 + G_4 \quad (24)$$

where

$$G_1 = \frac{1}{4} \int \delta(\gamma) \delta(f_x - \gamma) \mathbf{S}_n'(f_x - \sigma - \gamma) d\gamma = \frac{1}{4} \delta(f_x) \mathbf{S}_n'(f_x - \sigma) \quad (25)$$

$$G_2 = (iP/4\pi) \int \frac{\delta(f_x - \gamma)}{\gamma} \mathbf{S}_n'(f_x - \sigma - \gamma) d\gamma = (iP/4\pi) \mathbf{S}_n'(-\sigma) \quad (26)$$

$$G_3 = (-iP/4\pi) \int \frac{\delta(\gamma)}{f_x - \gamma} \mathbf{S}_n'(f_x - \sigma - \gamma) d\gamma = -(iP/4\pi) \mathbf{S}_n'(f_x - \sigma) \quad (27)$$

$$G_4 = (P/4\pi^2) \int \frac{\mathbf{S}_n'(f_x - \sigma - \gamma)}{\gamma(f_x - \gamma)} d\gamma \quad (28)$$

To perform the integration indicated in Eq.(28), Eq.(16) is used. This yields

$$G_4 = (P/4\pi^2) \int_{u-\sigma+b}^{u-\sigma-b} \frac{d\gamma}{\gamma(f_x - \gamma)} \quad (29)$$

The partial-fraction expansion

$$\frac{1}{\gamma(f_x - \gamma)} = \frac{1}{f_x} \left(\frac{1}{\gamma} \right) + \frac{1}{f_x} \left(\frac{1}{f_x - \gamma} \right) \quad (30)$$

is now used in Eq. (29) so that Eq. (29) becomes

$$G_4 = \left(P/4\pi^2 f_x \right) \int_{f_x-\sigma+b}^{f_x-\sigma-b} \frac{d\gamma}{\gamma} + \int_{f_x-\sigma+b}^{f_x-\sigma-b} \frac{d\gamma}{u-\gamma} \quad (31)$$

$$\begin{aligned} &= \left(1/4\pi^2 f_x \right) \lim_{\varepsilon \rightarrow 0} \left[\int_{f_x-\sigma+b}^{-\varepsilon} \frac{d\gamma}{\gamma} + \int_{\varepsilon}^{f_x-\sigma-b} \frac{d\gamma}{f_x-\gamma} + \int_{f_x-\sigma+b}^{f_x-\varepsilon} \frac{d\gamma}{f_x+\gamma} + \int_{f_x+\varepsilon}^{f_x-\sigma+b} \frac{d\gamma}{f_x-\gamma} \right] \\ &= \left(1/4\pi^2 f_x \right) \ln \left[\frac{(f_x-\sigma-b)(\sigma-b)}{(f_x-\sigma+b)(\sigma+b)} \right] \end{aligned} \quad (32)$$

where the definition of the principal-value operator has been used.

When Eqs. (25), (26), (27), and (32) are substituted in Eq. (24), Eq. (24) is substituted in Eq. (23), then the (unnormalized) apparent polarization image Fourier transform, is found to be

$$\begin{aligned} \mathbf{S}_m^o(f_x) &= \frac{1}{4} \int \delta(f_x) \mathbf{S}_n^i(f_x - \sigma) \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma + (iP/4\pi) \int \mathbf{S}_n^i(\sigma) \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma \\ &\quad - (iP/4\pi) \int \mathbf{S}_n^i(f_x - \sigma) \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma + \left(1/4\pi^2 f_x \right) \int \ln \left[\frac{(f_x - \sigma - b)(\sigma - b)}{(f_x - \sigma + b)(\sigma + b)} \right] \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma \end{aligned} \quad (33)$$

In the second integral of Eq. (33), Eq. (16) has been used to write $\mathbf{S}_n^i(\sigma) = \mathbf{S}_n^i(-\sigma)$. From the properties of the Dirac δ function, we see that the first integral in Eq. (33) vanishes. By simple change of variables we can show that the third integral in Eq. (33) is the negative of the second integral. Thus Eq. (34) is

$$\mathbf{S}_m^o(f_x) = (i/2\pi) \int \mathbf{S}_n^i(\sigma) \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma + \left(1/4\pi^2 f_x \right) \int \ln \left[\frac{(f_x - \sigma - b)(\sigma - b)}{(f_x - \sigma + b)(\sigma + b)} \right] \mathbf{H}(f_x - \sigma) \mathbf{H}(\sigma) d\sigma \quad (34)$$

The conditions of incoherent primary source and diffraction-limited optics represented by Eqs. (16) and (19) are now used in Eq. (34). When this is done the integrals in Eq. (34) are evaluated, Eq. (34) becomes

$$\mathbf{S}_m^o(f_x) = \left\{ \begin{array}{l} \left[\begin{array}{ll} \frac{ib}{\pi} & a > b \text{ and } 0 \leq f_x \leq a - b \\ \frac{i}{2\pi}(a + b - f_x) & a > b \text{ and } a - b \leq f_x \leq a + b \\ 0 & a > b \text{ and } a + b \leq f_x \\ \frac{i}{2\pi}(2a - f_x) & a < b \text{ and } 0 \leq f_x \leq 2a \\ 0 & a < b \text{ and } 2a \leq f_x \end{array} \right] + \frac{1}{2\pi^2 f_x} \ln \left[\frac{(a-b)^{(a-b)}(\mu - \sigma + b)^{(f_x - \sigma + b)}}{(a+b)^{(a+b)}(\mu - \sigma - b)^{(f_x - \sigma - b)}} \right] & 0 \leq f_x \leq 2a \\ 0 & 2a \leq f_x \end{array} \right\} \quad (35)$$

Equations (35) express the unnormalized form of the apparent or measured Fourier transform of the generalized Stokes parameters in the image plane obtained by edge-trace analysis for a diffraction-limited imaging system. We notice immediately that this function is complex. Thus, even though the system impulse response (for coherent or incoherent light) is symmetric, the measured apparent Fourier transform will be complex, i.e., it will exhibit nonzero phase shifts. We define the magnitude and phase by the relation

$$\mathbf{S}_m^o(f_x) = |\mathbf{S}_m^o(f_x)| e^{-i\phi(f_x)} \quad (36)$$

In this figure the apparent Fourier transform of the generalized Stokes parameters has been normalized so that in $|\mathbf{S}_m^o(0)| = 1$. From examination of the description of the imaging system in the previous section, particularly Eqs. (15), (16), (17), and (18), it is seen that the parameter is the ratio of the coherence interval of the object illumination to the size of the imaging system's diffraction pattern.

5. POLARIZATION IMAGE OF STEP FUNCTION

There are certain other miscellaneous properties of polarization images formed with coherent light that should be mentioned in any comparison with incoherent images. First, the responses of polarization images under incoherent and coherent systems to sharp edges are notably different. Figure 1 shows the theoretical responses of polarization images under incoherent of a system with a circular pupil to a step function object, while Figure 2 shows the theoretical responses of polarization images under coherent of a system with a circular pupil to a step function object. i.e. an object with amplitude transmittance.

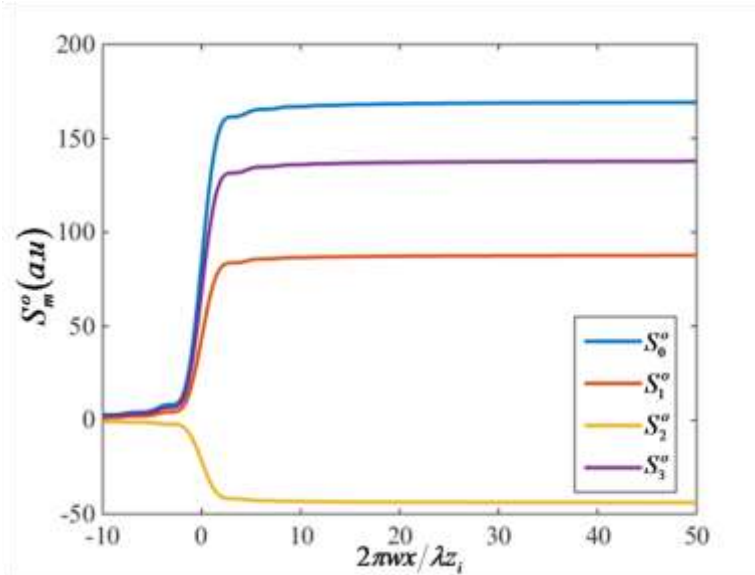


Fig.1 Polarization images of a step in incoherent light.

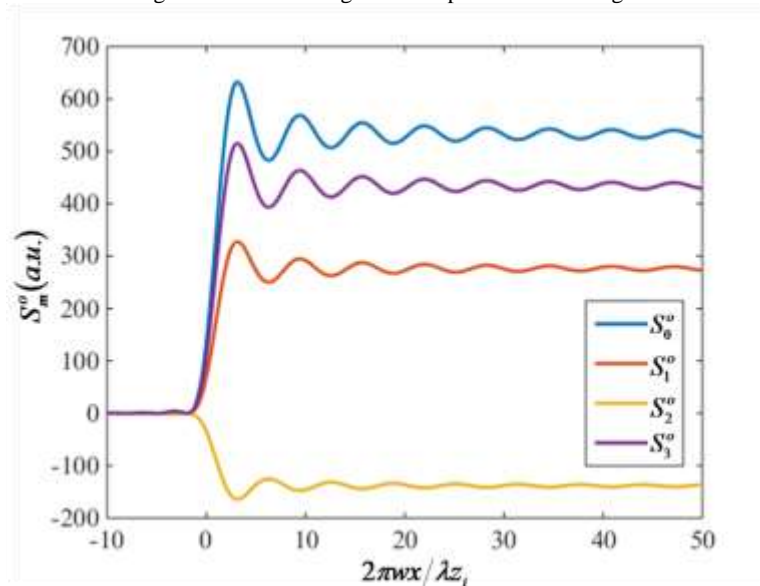


Fig.2 Polarization images of a step in coherent and incoherent light.

The coherent system is seen to exhibit rather pronounced "ringing". This property is analogous to the ringing that occurs in video amplifier circuits with transfer functions that fall too abruptly with frequency. The coherent system has a transfer function with sharp discontinuities, while the falloff of the OTF is much more gradual. Another important property of the coherent image is that it crosses the location of the actual edge with only 1/4 of its asymptotic value of intensity, whereas the incoherent image crosses with a value of 1/2 of its asymptotic value. If we were to assume that the

actual location of the edge is at the position where the intensity reaches half its asymptotic value, we would arrive at a correct estimate of the position of the edge in the incoherent case, but in the coherent case we would err in the direction of the bright side of the edge. This fact can be important, for example, in estimating the widths of lines on integrated circuit masks.

The edge shift and ringing are quite apparent. The amount of the shift depends upon the method used for choosing the position of the edge image. The contrast of the edge obviously plays an important role in the nature of the image. The ringing now appears on either side of the edge and the edge shift decreases with decreasing contrast. A result for an edge is shown in Fig.1 polarization images of a system is under incoherent light and Fig.2 polarization images of a system is under coherent light. The ringing is quite apparent in the latter figure.

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