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Investigation of Voltage and Frequency Variation on Induction Motor Core and Copper Losses

Mutasim Nour
School of Engineering and Physical Sciences
Heriot-Watt University Dubai Campus
Dubai, UAE
mutasim.nour@hw.ac.uk

Prabhu Thirugnanam
School of Engineering and Physical Sciences
Heriot-Watt University Dubai Campus
Dubai, UAE
talentedguy12@gmail.com

Abstract—This paper presents a dynamic induction motor (IM) model which incorporates all the power losses. The presented module is entirely built in Simulink to investigate the effect of varying the applied voltage and frequency on IM efficiency for different load applications. The model includes the power losses such as copper losses, core losses, stray load and mechanical. The accurate determination of induction motor efficiency depends on the estimation of all above mentioned power losses which are modeled and presented in this paper. The effect of variation in applied voltage and frequency on induction motor efficiency is investigated at various load conditions and the results are tabulated and evaluated accordingly. The obtained results show that the efficiency of the IM is significantly affected by the voltage and frequency levels especially at low load. Therefore matching the right applied voltage and frequency to the motor terminal based on the load condition will reduce the motor losses and hence increase its efficiency.

Keywords—Induction motor; Power losses; Simulink model; Variable voltage; Variable frequency; Efficiency

I. INTRODUCTION

The role of Induction Motors (IM) is very important in industrial applications because of their robustness, high reliability, low cost and maintenance. Over 60% of total electricity consumption in the industries is through IMs [1]. Significant amount of savings in energy can be obtained by varying the voltage and frequency which perform a crucial role in real time dynamic system[2]. Increase in efficiency and good system response could be obtained by implementing an optimum control of motor drives.[1,3].

Conventionally, IM modeling in various research works were carried out only under ideal conditions without including power losses because of which the actual performance of IM cannot be presented accurately. Efficiency improvement from micro motors to megawatt industrial motors is of great emphasis which depends on accurate estimation of power losses under dynamic conditions [4].

The variation of copper and core losses at various load conditions for a typical IM is shown in Fig. 1 [5,6]. Copper losses are directly proportional to the applied load whereas core losses remain constant irrespective of load and depend on applied voltage and frequency.

From (1) and (2), $V_a$ and $V_β$ are obtained.

\[
V_a = \frac{2}{3} V_a - \frac{1}{3} (V_b + V_c)
\]
$V_\beta = 1/\sqrt{3} (V_\alpha - V_\gamma)$  \hspace{1cm} (2)

B. Power Losses in Induction Motor

The main power losses in IM are classified into copper, core, mechanical and stray losses as briefed in [8]. Copper loss depends on the current flowing through the conductor based on the load size and is directly proportional to the load. It is the sum of stator and rotor copper losses given by (3), (4) and (5) [9, 10].

$$P_{copper} = R_c (I_{\alpha c}^2 + I_{\beta c}^2)$$  \hspace{1cm} (3)

Where, $P_{copper}$ = stator copper loss, $R_c$ = stator resistance, $I_{\alpha c}$, $I_{\beta c}$ = stator currents for $\alpha$ and $\beta$ axis respectively.

$$P_{copper} = R_c (I_{\alpha r}^2 + I_{\beta r}^2)$$  \hspace{1cm} (4)

Where, $P_{copper}$ = rotor copper loss, $R_c$ = rotor resistance, $I_{\alpha r}$, $I_{\beta r}$ = rotor currents for $\alpha$ and $\beta$ axis respectively.

$$P_{copper} = P_{cav} + P_{cur}$$  \hspace{1cm} (5)

The total core losses can be measured by core loss current ($I_{\alpha c}$) flowing through core loss resistance ($R_{c}$) given by (6) [2]. $R_c$ is proportional to angular velocity which is in turn proportional to supply frequency as given in (7) and (8) [11, 12].

$$P_{core} = R_c (I_{\alpha c}^2 + I_{\beta c}^2)$$  \hspace{1cm} (6)

$$R_c = K \omega r^{1.6}$$  \hspace{1cm} (7)

$$\omega r = 2 \pi f$$  \hspace{1cm} (8)

Where, $P_{core}$ = core losses, $R_c$ = equivalent resistance of core loss, $I_{\alpha c}$, $I_{\beta c}$ = core loss currents for $\alpha$ and $\beta$ axis, $\omega_r$ = angular velocity, $f$ = supply frequency and $K$ = constant assumed as 1. Thus from equations (6-8) it can be clearly noticed that the value of $R_c$ is directly proportional to supply frequency. Also from equations (9) and (10) [2], the values of $I_{\alpha c}$ and $I_{\beta c}$ depend on rotor flux for $\alpha$ axis ($\psi_{\alpha}$) and rotor flux for $\beta$ axis ($\psi_{\beta}$) respectively which depends on supply voltage. Thus by reducing the frequency and voltage applied to IM, the total core losses can be reduced.

$$I_{\alpha c} = (\psi_{\alpha r}/L_a/M) \int [(V_{\alpha a} \cdot R_{\alpha c}) dt - \sigma L_{\alpha \beta c}] / L_{\alpha c}$$  \hspace{1cm} (9)

$$I_{\beta c} = (\psi_{\beta r}/L_a/M) \int [(V_{\beta a} \cdot R_{\beta c}) dt - \sigma L_{\beta \alpha c}] / L_{\beta c}$$  \hspace{1cm} (10)

Where: $L_a$ : rotor inductance, $M$=Lm: constant of mutual induction, $V_{\alpha a}$ : stator voltage for $\alpha$ axis, $R_{\alpha c}$ : stator resistance, $\psi_{\alpha r}$ : stator current for $\alpha$ axis, $\sigma$ : Leakage flux coefficient, $L_c$ : stator inductance, $i_{\beta r}$ : stator current for $\beta$ axis, $V_{\beta a}$ : stator voltage for $\beta$ axis, $L_{\beta \alpha c}$ : rotor leakage inductance.

The overall losses in an IM are the sum of all above mentioned losses as shown in [13] assuming stray and mechanical losses to be negligible [2] given by (11):

$$P_{out} = P_{copper} + P_{core}$$  \hspace{1cm} (11)

C. Induction motor model

In order to simulate the IM in SIMULINK, the motor is represented by (12-15) [12,14] :

$$di_{\alpha a}/dt = (1/L_{\alpha})(V_{\alpha a} - (R_{\alpha} / L_{\alpha}) i_{\alpha a} - (M_{L \alpha}) (d\omega r dt))$$  \hspace{1cm} (12)

$$di_{\beta a}/dt = (1/L_{\beta})(V_{\beta a} - (R_{\beta} / L_{\beta}) i_{\beta a} - (M_{L \beta}) (d\omega r dt))$$  \hspace{1cm} (13)

$$di_{\alpha l}/dt = -(R_{\beta} / L_{\beta}) i_{\beta a} - (M_{L \alpha}) (d\omega r dt)) - \sigma (I_{\beta r} + (M / L_{\alpha})) i_{\beta r}$$  \hspace{1cm} (14)

$$di_{\beta l}/dt = -(R_{\beta} / L_{\beta}) i_{\beta a} - (M / L_{\alpha}) (d\omega r dt)) + \sigma (I_{\alpha r} + (M / L_{\beta})) i_{\alpha r}$$  \hspace{1cm} (15)

Where:

- $i_{\alpha a}$: stator current for $\alpha$ axis, $i_{\beta a}$: stator current for $\beta$ axis, $i_{\alpha l}$: rotor current for $\alpha$ axis, $i_{\beta l}$: rotor current for $\beta$ axis, $V_{\alpha a}$: stator voltage for $\alpha$ axis, $V_{\beta a}$: stator voltage for $\beta$ axis, $R_{\alpha}$: rotor resistance, $L_{\alpha}$: stator inductance, $L_{\beta}$: rotor inductance and $\omega_r$: angular velocity.

The torque and mechanical model of IM are given by (16) and (17) [11]:

$$T_{em} = 3/2pM(i_{\beta a} - i_{\alpha a} i_{\beta r})$$  \hspace{1cm} (16)

$$(d\omega r / dt) = (p/J) T_{em} - T_m$$  \hspace{1cm} (17)

Where $T_{em}$: electromagnetic torque, $T_m$: load torque, $p$: number of pole pairs and $J$: moment of inertia. Input power, output power, reactive power, apparent power efficiency and power factor are given by (18) to (23) [12]:

$$P_{in} = 3/2(V_{\alpha a} i_{\alpha a} + V_{\beta a} i_{\beta a})$$  \hspace{1cm} (18)

$$Q_p = 3/2(V_{\alpha a}^2 i_{\alpha a} - V_{\alpha a} i_{\beta a})$$  \hspace{1cm} (19)

$$S_p = \sqrt{(P_{in}^2 + Q_p^2)}$$  \hspace{1cm} (20)

$$P_{out} = T_{em} (\omega_r / p)$$  \hspace{1cm} (21)

$$\eta = P_{out} / P_{in}$$  \hspace{1cm} (22)

$$\cos \phi = P_{in} / S_p$$  \hspace{1cm} (23)

Where $P_{in}$: input power, $Q_p$: reactive power, $S_p$: apparent power, $P_{out}$: output power, $\eta$: efficiency and $\cos \phi$: power factor. The IM model without power losses from (1, 2) and (12) to (17) is built in SIMULINK as shown in Fig. 3.

D. Inclusion of power losses in induction motor model

The $\alpha$-$\beta$ model of iron loss in IM is shown in Fig. 5.

Where:

- $R_{\alpha}$: equivalent resistance of core loss, $I_{\alpha c}$: current through $R_{\alpha}$ in $\alpha$ axis, $I_{\beta c}$: current through $R_{\alpha}$ in $\beta$ axis, $I_{\alpha l}$: current through $L_{\alpha}$ in $\alpha$ axis and $I_{\beta l}$: current through $L_{\beta}$ in $\beta$ axis. The formula for iron loss derived from Fig. 4 is given by (6).
Fig. 3. IM model in simulink without power losses.

Fig. 4. α-β model of iron loss in IM.

Fig. 5. Shows the model of total core losses estimated for an induction motor designed based on (6-8).

The overall block diagram of the entire system including various subsystem using all motor equations, power/efficiency, copper losses, rotor flux components, core loss currents and core losses is shown in Fig. 6.

Fig. 6. Block diagram of IM model in Simulink with all power losses and various subsystems.

III. SIMULATION OF INDUCTION MOTOR MODEL

For the purpose of simulation, a three phase AC squirrel cage induction motor with rated power of 18.5 KW (25 HP) is used as a case study with the parameters which are obtained from manufacturer data as given in Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Phase Voltage (V_ph)</td>
<td>220 V</td>
</tr>
<tr>
<td>Rated Current (I_r)</td>
<td>34 A</td>
</tr>
<tr>
<td>Number of pole pairs (p)</td>
<td>2</td>
</tr>
<tr>
<td>Rated speed (rpm)</td>
<td>1500</td>
</tr>
<tr>
<td>Rated frequency (f)</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Rated Torque (T)</td>
<td>125 NM</td>
</tr>
<tr>
<td>Moment of Inertia (J)</td>
<td>0.234 kg.m²</td>
</tr>
<tr>
<td>Stator resistance (R_s)</td>
<td>0.159 Ω</td>
</tr>
<tr>
<td>Rotor resistance (R_r)</td>
<td>0.16 Ω</td>
</tr>
<tr>
<td>Stator Inductance (L_s)</td>
<td>0.05 H</td>
</tr>
<tr>
<td>Rotor Inductance (L_r)</td>
<td>0.051 H</td>
</tr>
<tr>
<td>Mutual Inductance (M_m or L_m)</td>
<td>0.0489 H</td>
</tr>
<tr>
<td>Stator leakage inductance (L_sσ)</td>
<td>0.027 H</td>
</tr>
<tr>
<td>Rotor leakage inductance (L_rσ)</td>
<td>0.027 H</td>
</tr>
<tr>
<td>Leakage flux coefficient (σ)</td>
<td>0.1475</td>
</tr>
</tbody>
</table>

The simulation is performed for the SIMULINK Model of IM shown in Fig. 6, at 25%, 50% and full load conditions under the following defined values of voltages and respective frequencies to estimate the core losses, copper losses and the resulting efficiencies:

- Rated voltage and 30 Hz frequency.
- Rated voltage and 60 Hz frequency.
- Rated voltage and 75 Hz frequency.
- 70% of rated voltage and rated frequency.
- 80% of rated voltage and rated frequency.
- 100% rated voltage and rated frequency.
The voltage and frequency ranges are selected based on allowable voltage and frequency variation explained in [15] as per National Electric Manufacturer’s Association (NEMA).

IV. SIMULATION RESULTS AND OBSERVATIONS

The values of copper losses, core losses and efficiencies are determined at the defined conditions by simulating the IM model and have been tabulated in Tables 2 to 7. The simulation is carried out for a time period of 2 seconds and the IM is loaded with a step input signal (load torque) at the time of 0.5 second.

### A. Evaluation of results at rated voltage and various frequencies at defined load conditions

- **25% load condition**
  
  Table 2 represents this load condition. It can be noticed that copper losses are approximately the same at all frequency ranges. This is because the copper losses are proportional to the load current and almost independent of frequency. Core losses increased at higher frequency (75 Hz) as they are proportional to the supply frequency. Due to the increase in core losses at high frequency (75 Hz), IM efficiency drastically reduced to 10% which would result in very poor performance. Life time of IM would be affected and soon need to be replaced within a short period of time. But when IM is operated at low frequency (30 Hz) and rated frequency (60 Hz), significant increase in the efficiency is noticed due to reduction in core losses. Thus by controlling the applied frequency, core losses can be kept at minimum leading to increase in efficiency.

- **50% load condition**
  
  Table 3 represents this load condition. Even though copper losses are increased due to flow of higher load current in the motor windings, the magnitude of losses is approximately the same on all frequency ranges as they are independent of supply frequency. Core losses are found to be increased at higher frequency (75 Hz) as they are directly proportional to the supply frequency. Due to the increase in core losses at higher frequency, efficiency reduces to 21% which would also result in poor performance. At lower frequency ranges, core losses are found to be reduced resulting in higher efficiencies.

- **100% load condition**
  
  Table 4 represents this load condition in which the full load current flows across the conductor but the magnitude is same at all frequencies. Variation of core losses are same as that of 25% and 50% load conditions as they are dependent on frequency and independent of load. Variation of efficiency at all frequencies is similar to that of 25% load conditions.

- **Recommendation**
  
  As the core losses are found to be increased when IM is operated at high frequency ranges irrespective of any load, it is recommended to operate IM only at low and rated frequency ranges under any load condition which would reduce the losses and lead to significant increase in IM efficiency.

### B. Evaluation of results at rated frequency and various voltages at various defined load conditions

- **25% load condition**
  
  Table 5 represents this load condition. Copper losses are found to be lower on all voltage ranges as the IM is loaded with only 25% of full load. Core losses are found to be increased when the applied voltage is increased to 80% and 100% of rated voltage. This is because the core losses are proportional to the applied voltage even though IM is operated at rated frequency. Due to the increase of core losses at higher

<table>
<thead>
<tr>
<th>Description</th>
<th>70% V</th>
<th>80% V</th>
<th>100% V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pcore (W)</td>
<td>3000</td>
<td>4100</td>
<td>6000</td>
</tr>
<tr>
<td>Pcopper (W)</td>
<td>130</td>
<td>182</td>
<td>200</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>90</td>
<td>75</td>
<td>39</td>
</tr>
</tbody>
</table>

- **50% load condition**
  
  Table 6 represents this load condition in which the full load current flows across the conductor but the magnitude is same at all frequencies. Variation of core losses are same as that of 25% and 50% load conditions as they are dependent on frequency and independent of load. Variation of efficiency at all frequencies is similar to that of 25% load conditions.

- **Recommendation**
  
  As the core losses are found to be increased when IM is operated at high frequency ranges irrespective of any load, it is recommended to operate IM only at low and rated frequency ranges under any load condition which would reduce the losses and lead to significant increase in IM efficiency.

- **25% load condition**
  
  Table 7 represents this load condition. Copper losses are found to be lower on all voltage ranges as the IM is loaded with only 25% of full load. Core losses are found to be increased when the applied voltage is increased to 80% and 100% of rated voltage. This is because the core losses are proportional to the applied voltage even though IM is operated at rated frequency. Due to the increase of core losses at higher

- **Recommendation**
  
  As the core losses are found to be increased when IM is operated at high frequency ranges irrespective of any load, it is recommended to operate IM only at low and rated frequency ranges under any load condition which would reduce the losses and lead to significant increase in IM efficiency.

<table>
<thead>
<tr>
<th>Description</th>
<th>70% V</th>
<th>80% V</th>
<th>100% V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pcore (W)</td>
<td>3100</td>
<td>5000</td>
<td>6100</td>
</tr>
<tr>
<td>Pcopper (W)</td>
<td>1800</td>
<td>3800</td>
<td>5800</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>96</td>
<td>95</td>
<td>94.5</td>
</tr>
</tbody>
</table>
voltage, the efficiency is reduced to 39% at 100% rated voltage resulting in poor performance. But when applied voltage is reduced to 80% and 70% of rated voltage, IM efficiency is increased to 75% and 90% respectively due to reduction in core losses.

- **50% load condition**
  
  Table 6 represents this load condition. Copper losses are found to be increased at 80% and 100% rated voltage conditions as they are dependent on load current which in turn dependant on applied voltage according to Ohm’s law. Core losses are found to be increased at higher voltages as they are directly proportional to the applied voltage. Due to the increase in core and copper losses at higher voltage, efficiency reduces to 89%. At lower voltage ranges, core losses are reduced resulting in higher efficiencies.

- **100% load condition:**
  
  Table 7 represents this load condition. Copper losses are increased even at lower voltage ranges as full load current flow through motor windings and tend to increase further when applied voltage is increased to 80% and 100% rated voltage as they are proportional to load current which depends applied voltage. Variation of core losses are same as that of 25% and 50% load conditions as they are dependent on applied voltage and independent of load. Efficiency is slightly increased to 96% when the applied voltage is reduced to 70% of rated voltage at this load condition. This is due to the decrease in core and copper losses at low voltage ranges.

- **Recommendation**
  
  As the core losses are found to be increased when IM is operated at higher voltages irrespective of any load, it is recommended to operate IM only at lower and medium voltage ranges especially at low load conditions which would reduce the losses and lead to significant increase in IM efficiency. Also since copper losses are found to increase when applied voltage is increased, low voltage operations are suggested to reduce the losses to minimum at full load.

V. CONCLUSION

In this paper, the mathematical model of 3 phase IM with all power losses is built in Simulink and the effect of variable voltage and frequency on IM efficiency is investigated at various load conditions. The IM model developed shows better response both at steady state and dynamic conditions with inclusion of all power losses in both conditions. The estimation of power losses at various voltages and frequencies defined has a good accuracy when compared with manufacturer data. The behavior of copper and core losses is in line with the typical load loss characteristics of IM as shown in Fig. 1. Thus it can be summarized that the efficiency of IM can be improved at low load conditions under reduced voltage and frequencies for a short period of time due to which significant amount of energy savings can be achieved. Also the lifetime of IM would be extended due to the lower stress on the motor windings due to lesser losses. It is also worth highlighting that the presented model is closer to the behavior characteristics of IM, which can be widely used in studying induction motors behaviors under different conditions.

The IM model developed so far is based on open loop control method. Closed loop control commonly known as vector or field oriented control (FOC) can be applied to the developed IM model where the actual torque and flux values are compared with the reference values in a proportional integral (PI) controller and the required value of flux and torque determined can be fed to the induction motor. The employment of FOC is currently undergoing as a second phase of this research work.

REFERENCES


