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# Extra Power Backup for Balancing of Power Grid with Renewables

Sergey Foss

School of Mathematical  
and Computer Sciences  
Heriot-Watt University and  
Institute of Mathematics, Novosibirsk  
Edinburgh, United Kingdom  
s.foss@hw.ac.uk

Georgi Georgiev

Energy Solutions Center  
Varna, Bulgaria  
georgidg@gmail.com

Dmitriy Kim

National Research Laboratory  
of Information and Space Technologies  
Kazakh National Research  
Technical University  
Almaty, Kazakhstan  
kdk26@mail.ru

Andrey Turlikov

Department of Information and Communication Systems  
St.-Petersburg University of Aerospace Instrumentation  
St.-Petersburg, Russia  
turlikov@vu.spb.ru

**Abstract**—We introduce a number of mathematical models for the management of an electrical power system in the presence of renewable generation. We also study the impact of power systems of neighbouring countries on the grid.

## I. INTRODUCTION

The introduction of power stations producing renewable energy leads to the problem of their integration into the existing electricity networks, with taking into account random fluctuations in their power generation. Energy production by renewables has high variability and an irreducible element of uncertainty associated with its prediction. This leads to the need for extra power backup from conventional generation, and for development of new approaches to balancing of the overall systems. There are thus significant control problems related to the stochastic nature of renewable generation – in contrast to the management of traditional energy networks, whose behaviour is in general highly predictable and therefore much more easily manageable.

We consider a power system, which consists of conventional power plants (including that of hydropower stations), renewables (solar and wind power stations), balancing units (electrical power systems of neighbouring countries) and consumers (power loads). Different types of produced energy has different prices, and typically the imported energy (obtained from a balancing unit) is the most expensive. We suppose that the power system is managed by Transmission System Operator (TSO), which can manage conventional power plants and interaction with neighbours. The problem is to define the strategy to manage power grid to meet demand.

Similar models have been discussed in [1]-[5]. An introduction to probabilistic methods for investigation of complex systems may be found in [6].

There are many approaches and recommendations for the integration of renewables into the power grid: geographic diversification of the renewable generation, construction of hybrid plants, combined heat and power production, complex control methods for renewable power generation, technologies for energy storage, increasing technical capabilities of generators, interaction with other energy networks, and reserves management. In this article, we introduce mathematical models for optimal reserve management and interaction with other electrical networks.

## II. THE FORMULATION OF MODELS.

One may represent a power electrical system as a graph with nodes (power loads and generators) and links (power transmission lines). The graph is assumed to be connected (any two nodes are connected by a link or a chain of links).

We assume the model to operate in discrete time. Let  $d_n$  be the total demand at time instant  $n$ , let  $a_n$  be the total power conventional power generation (CP), and  $r_n$  the total renewable power (RP) generation. For simplicity, we assume that hydropower energy is a part of conventional generation. Moreover, we suppose that it is possible to import deficit energy from and export excess energy to a neighbouring grid (call it a balancing unit). Let  $b_n$  be the imported/exported power from/in the balancing unit (BU). We put  $b_n > 0$  for imported power from BU and vice versa  $b_n < 0$  for exported power to BU.

Since demand may generally be well forecasted for a period of time in advance,  $d_n$  may be assumed to be non-random. Renewable power (for instance wind and solar) cannot be exactly predicted, so it is reasonable to assume that  $r_n$  is a random sequence. Then  $a_n$  and  $b_n$  form predictable sequences, their values may be determined at the previous time instant  $n - 1$  given values  $(a_{n-1}, b_{n-1}, r_{n-1}, d_{n-1})$ . Conventional power plants require considerable time to ramp up/down it's power while balancing unit can do it practically instantaneously. We suppose that the power system is centrally managed and TSO is able to choose appropriate  $a_n$  and  $b_n$  to keep the system stable. Thus, the problem for TSO is to meet demand with a high reliability level under minimal costs. Then the problem is reduced to a choice of a control algorithm that counts unpredictability and variability of renewable energy, together with invariability of the consumer's requirements on the quantity and reliability of the energy supply, that has to have the smallest balancing costs.

Suppose that we are interested in studying a power system during a finite time period of length  $N$ , where  $N \geq 0$  is a given integer. The equation of meeting demand during  $N$  time slots can be written as

$$a_n + b_n = d_n - r_n, \quad n = 0, 1, 2, \dots, N. \quad (1)$$

Note that (1) may be modified/generalised in various ways. For example, we may suppose that a proportion of power loads  $dm_n$  may be disconnected by TSO for a certain cost – then we obtain equations

$$a_n + b_n = d_n - dm_n - r_n, \quad n = 0, 1, 2, \dots, N,$$

where  $dm_n$  is a value of disconnected power load, it is upper and lower bounded,  $0 \leq dm_n \leq DM$ , and is a function on  $a_{n-1}, b_{n-1}, r_{n-1}, d_{n-1}, dm_{n-1}$ .

Clearly, the proposed dynamics provides only a “projection” of a much more complex multivariate problem, since a power electrical system includes a large number of elements - consumers (power loads), conventional and renewables generators, and grids of neighbouring countries with their individual characteristics. Thus,  $d_n$ ,  $w_n$ , and  $a_n$  have to be vector-valued, and a mathematical model of the system should necessarily include stochastic vector inequalities of high dimension and periodic time-dependent variables.

Here, for simplicity, we operate with univariate variables and focus on general problem's formulations under general technical restrictions.

First, there are technical restrictions on minimal and maximal values of power, namely:

$$0 \leq \underline{A} \leq a_n \leq \bar{A},$$

$$\underline{B} \leq b_n \leq \bar{B},$$

$$0 \leq d_n \leq D,$$

$$0 \leq r_n \leq R.$$

There are further restrictions on throughput of the electrical network and static and dynamic stability of the power system.

For conventional generators (that exclude hydro generators), there exist so-called “ramp constraints”:

$$\alpha \leq a_n - a_{n-1} \leq \beta$$

where  $\alpha < 0 < \beta$ .

For a hydro power station, its power  $h_n$  at time  $n$  must be also constrained:

$$0 \leq h_n \leq H,$$

$$\sum_{n=1}^N h_n \leq \bar{H},$$

where  $H$  is a maximal power and constraint  $\bar{H}$  relates to the water level in the reservoir and the water regime.

TSO has to use some policy to meet demand at any time. Suppose, for simplicity, that the power of renewable could only decrease its value at random. We assume that in case of increasing renewable's power the TSO is able to turn off a part of this power.

Therefore the power of renewable decreases rapidly at random time instants, we call them “shocks”.

The management problem for a power electrical system reduces to support stability during these shocks.

The TSO can use one of the following policies to meet demand.

**Policy 1:** The conventional generator produces extra power  $\tilde{a}_n$  backup to be ready to cover the shock at any time. The unused extra power is exported to BU.

**Policy 2:** The only power from BU is used to support network stability during shocks.

**Policy 3:** The combination of CP and BU's power is used to cover power system's shocks. It means that the conventional generator produces extra power backup to cover only part of the shock, the another part is covered by BU.

A choice of the policy depends on the costs of the CP and BU power. As a rule, the cost of conventional power  $c_a \geq 0$  is a constant. The cost of BU's power can vary and depends on agreements between neighbouring grids, which can be quite complicated. In general, the BU's power, as an imported power, is more expensive than CP which is produced inside the own grid. Therefore, TSO try to avoid or minimize use of imported power.

Clearly, an optimal strategy to support stability of the power system depends on specific agreements between neighbouring grids and may vary. We consider here two particular cases.

**Case 1.** Assume that the cost of BU's power is equal to  $c_{bI}$  for imported power and  $c_{bE}$  for exported power. In this case,

the optimal strategy can be obtained by finding a sequence  $\tilde{a}_n$  that satisfies the technical restrictions and minimizes the following balancing cost:

$$\sum_{n=1}^N (c_a \tilde{a}_n + c_{bE}(\tilde{a}_n - r_n)^+ + c_{bI}(r_n - \tilde{a}_n)^+) \quad (2)$$

where

$$(\tilde{a}_n - r_n)^+ = \begin{cases} \tilde{a}_n - r_n, & \text{if } \tilde{a}_n - r_n \geq 0, \\ 0, & \text{if } \tilde{a}_n - r_n < 0. \end{cases}$$

Since the expression in (2) is a random variable, we may instead take its mean:

$$\mathbf{E} \sum_{n=1}^N (c_a \tilde{a}_n + c_{bE}(\tilde{a}_n - r_n)^+ + c_{bI}(r_n - \tilde{a}_n)^+)$$

or, further, consider its upper limiting average

$$\limsup_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{E} (c_a \tilde{a}_n + c_{bE}(\tilde{a}_n - r_n)^+ + c_{bI}(r_n - \tilde{a}_n)^+). \quad (3)$$

Note that the first step in the optimality analysis of (3) is to find conditions for existence of limit

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{E} (c_a \tilde{a}_n + c_{bE}(\tilde{a}_n - r_n)^+ + c_{bI}(r_n - \tilde{a}_n)^+).$$

In particular, if the prices are the same in both directions,  $c_{bE} = c_{bI} = c_b > 0$  then (3) can be rewritten as

$$\limsup_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{E} (c_a \tilde{a}_n + c_b |\tilde{a}_n - r_n|).$$

On the other hand, if the neighbouring grid is much bigger then our power system and is able to accept our extra power for free, then we can assume that  $c_{bE} = 0$  and minimise

$$\limsup_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{E} (c_a \tilde{a}_n + c_{bI}(r_n - \tilde{a}_n)^+).$$

**Case 2.** Assume that the power which we can import from or export to BU is divided into "planned power" and "unexpected power". The planned power is the power that was agreed to be imported/exported in advance whereas the unexpected power is a power that is suddenly needed.

We define  $c_{bIp}$  and  $c_{bIu}$  as the costs of imported planned and unexpected power and  $c_{bEp}$  and  $c_{bEu}$  as the costs of exported planned and unexpected power. In this case, the optimal strategy can be obtained by minimizing

$$\limsup_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{E} (c_a \tilde{a}_n + c_{bEp} \tilde{a}_n^p + c_{bIp} b_n^p + c_{bEu}(\tilde{a}_n^u - r_n)^+ + c_{bIu}(r_n - \tilde{a}_n^u)^+),$$

where  $b_n^p$  is a planned part of imported energy,  $\tilde{a}_n^u$ ,  $\tilde{a}_n^p$  are, correspondingly, unexpected and planned extra power,

$$\tilde{a}_n = \tilde{a}_n^p + \tilde{a}_n^u.$$

If  $c_{bEp} < 0$  and  $c_{bIp} > 0$ ,  $c_{bEu} > 0$ ,  $c_{bIu} > 0$  the optimal strategy is to determine a part of planned extra power for export and a part of unexpected CP to minimize costs for balancing renewables. In this case, the cost of planned exported power can decrease the balancing cost by means of BU.

### III. CONCLUSION

We introduce a mathematical model for power electrical system management in the presence of renewables, with an extra power backup. The optimisation problem reduces to the problem of an optimal reserve control. When there is a possibility of using power from a neighbouring grid, the optimisation problem may depend on a specific agreement on flowing power between grids.

### IV. ACKNOWLEDGMENT

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