Enhanced and reduced natural convection using magnetic fluid in a square cavity

Citation for published version:
Szabo, P & Fruh, W-G 2016, 'Enhanced and reduced natural convection using magnetic fluid in a square cavity', *Proceedings in Applied Mathematics and Mechanics*, vol. 16, no. 1, pp. 651–652.
https://doi.org/10.1002/pamm.201610314

Digital Object Identifier (DOI):
10.1002/pamm.201610314

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
Proceedings in Applied Mathematics and Mechanics

Enhanced and reduced natural convection using magnetic fluid in a square cavity

Peter S. B. Szabo[1,*] and **Wolf-Gerrit Früh**[1**]

[1] School of Engineering and Physical Science, Heriot-Watt University, Riccarton, Edinburgh, EH14 4AS, UK

Natural convection using a magnetic fluid was studied in a square cavity under the influence of a permanent magnet. The aim was to explore the degree by which heat transfer may be controlled, enhanced or reduced, by investigating a set of different distances of a permanent magnet to the cavity. These distances of the magnet were set such that the cavity was in some cases fully dominated by buoyancy or by the magnetic body force and in other cases partly dominated by either of both body forces in different parts of the fluid. The effect on heat transfer was characterised by an averaged Nusselt number, Rayleigh and magnetic Rayleigh number.

1 Introduction

Magnetic fluids are industrial manufactured colloidal suspensions containing about 10^{23} particles per metre cube having a size of about 3 to 15 nm. The particles are coated with a surfactant to prevent agglomeration and sedimentation and are suspended in a carrier fluid such as water, kerosene or mineral-oil [1]. Today magnetic fluids have a wide range of applications including enhancing convection by magnetic forcing [2, 3]. The present study investigates the potential of natural and thermomagnetic convection of a mineral-oil based magnetic fluid.

2 Model formulation

The choice of system to model is a two-dimensional square cavity filled with a magnetic fluid having a side length, L, of 50 mm. The cavity is cooled at the right wall with temperature, $T_c = 290$ K, heated at the left wall with temperature, $T_h = T_c + \Delta T$ and is thermal insulated at all remaining walls. Boundary conditions for velocity are no-slip at all walls. At the top left corner zero pressure constrains was applied. All configuration were studied with the same initial conditions of a uniform distributed temperature of T_c and a stagnant fluid. The magnetic field was performed by a permanent magnet with a remanent flux density of 1.3 Tesla that is placed above the cavity at a certain distance that could vary. The actual magnetic field was calculated by using Maxwell's equations for non-conductive material in static form represented as

$$\nabla \times \mathbf{H} = 0 \qquad\qquad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

where **H** is the magnetic field and **B** the magnetic induction. The magnetic induction is related to the Remanent flux density, $\mathbf{B}_r$ of the permanent magnet and the magnetic field in the following manner $\mathbf{B} = \mu_0\mu_r\mathbf{H} + \mathbf{B_r}$ where μ_0 is the permeability of free space and μ_r the relative material permeability. The magnetization vector, **M**, for a monodispersed, colloidal magnetic fluid may be expressed through Langevin's function $\mathcal{L}(\zeta) = \coth(\zeta) - 1/\zeta$ with $\zeta = mH/(k_\mathrm{B}T)$. Assuming that the magnetic moment always aligns in field direction and including inter-particle interactions the magnetization may be written as

$$\mathbf{M} = \frac{\mathbf{H}}{H}\phi M_\mathrm{d}\,\mathcal{L}(\zeta_\mathrm{p}), \quad \zeta_\mathrm{p} = \frac{m\,(H + M_L/3)}{k_B T} \tag{2}$$

where M_d is the fluids bulk magnetization through volume concentration ϕ, m the magnetic moment of a magnetic particle, H the magnitude of the applied magnetic field, k_B the Boltzmann's constant, T the temperature and $M_\mathrm{L} = \phi M_\mathrm{d}\,\mathcal{L}(\zeta)$ the Langevin magnetization [4]. The continuity and momentum equation for an incompressible magnetic fluid are represented as

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

$$\rho\,(\mathbf{u}\cdot\nabla)\,\mathbf{u} = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\beta\,(T - T_0)\,\mathbf{g} + \mu_0\,(\mathbf{M}\cdot\nabla)\,\mathbf{H} \tag{4}$$

where **u** is the velocity, p the pressure, μ the dynamic viscosity, β the volumetric expansion coefficient and **g** the gravitational acceleration. The second last term on the right side of eq.(4) presents the buoyancy force by using a Boussinesq approximation and the last term is the magnetic body force known as the Kelvin force. The heat equation, solving for temperature, T, is

$$\rho c_p\,(\mathbf{u}\cdot\nabla)\,T = k\nabla^2 T \tag{5}$$

* Corresponding author: e-mail P.Szabo@hw.ac.uk, phone +44 131 451 3539, fax +44 131 451 3129
** E-mail W.G.Fruh@hw.ac.uk, phone +44 131 451 4374, fax +44 131 451 3129

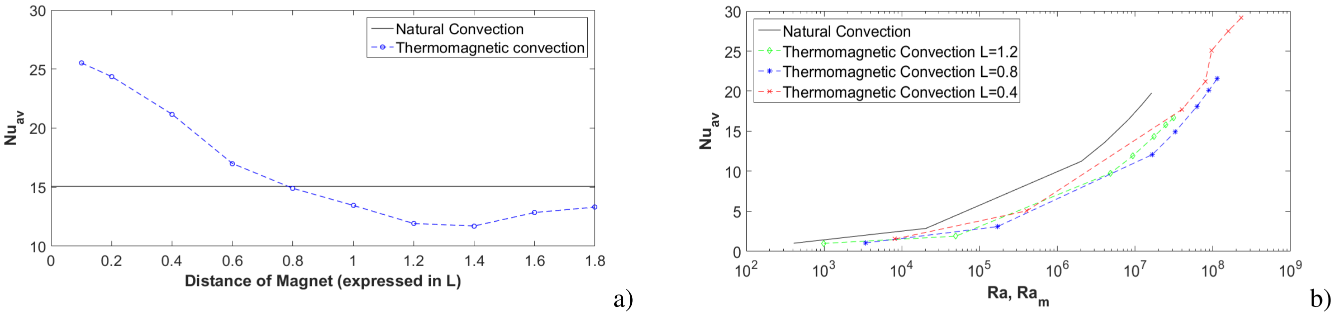

Fig. 1: The averaged Nusselt number plotted as a function of the magnets distance expressed in the cavity length, L in **a** and the averaged Nusselt number plotted as a function of the conventional and magnetic Rayleigh number in **b**.

where c_p is the specific heat capacity and k the thermal conductivity. The heat transfer trough the cavity is quantified by the averaged Nusselt number by integrating the total heat flux, q across a cross-section with the area, A with

$$\mathrm{Nu_{av}} = \frac{L}{k\Delta T}\,\frac{1}{A}\int q\,dA \tag{6}$$

A balance of buoyancy to viscous dissipation is characterised by the Rayleigh number, $\mathrm{Ra} = \left(\beta \Delta T g L^3\right) / \left(\nu\kappa\right)$. An equivalent ratio of magnetic forcing to viscous dissipation is quantified by a magnetic Rayleigh number and developed by replacing the gravity by the magnetic field gradient and the volumetric expansion coefficient by the pyromagnetic coefficient, K, and may be written as $\mathrm{Ra_m} = \left(\mu_0 L^3 K \Delta T |\nabla H|\right) / \left(\nu\kappa\rho\right)$. The numerical solution was based on a finite-element technique developed by COMSOL Multiphysics. The code was validated by comparing our results for natural convection with a comparison exercise of natural convection in a square cavity by Vahl Davis [5]. Since the bounded current in magnetic nano particles is very small, the effect of the fluid's magnetization on the total magnetic field is negligible. Thus, the magnetic field is first calculated separately by solving eq.(1). This is then used in equations (2) and (4) to solve the continuity, momentum and heat equations, (3 - 5) respectively. All results were produced by a linear solver.

3 Results and discussion

The first set of results was performed without the magnet resulting in regular natural convection and was used as a benchmark for further cases where the magnet was present. In the second set, the permanent magnet was initially placed a distance of $1.8L$ above the cavity after which the distance was decreased in steps of $0.2L$ down to $0.2L$. Both results are presented for a temperature difference of 10 K in Fig. 1a where the black line presents natural convection only having a Ra of 4×10^6 and a $\mathrm{Nu_{av}}$ of 15. The blue line with the circles presents thermomagnetic convection as a function of distance of the permanent magnet to the cavity. The graph shows an almost doubled heat transfer when the magnet is very close to the fluid. This then gradually decreases as the distance increases until thermomagnetic and natural convection result in equal heat transfer at a distance of $0.8L$. Beyond this, the Nusselt number decreases further until $1.4L$ to a minimum of around $Nu_{av} = 12$, or 80% of that of natural convection. As the distance of the magnet increases further, the Nusselt number recovers somewhat towards the reference value for natural convection. Fig. 1b provides a comparison of natural and thermomagnetic convection including the difference in distance of the permanent magnet to the cavity indicating that heat transfer by natural convection in the range of $\mathrm{Ra} = 4 \times 10^2 - 10^7$ is as effective as thermomagnetic convection in the range of $\mathrm{Ra_m} = 10^3 - 10^8$. To conclude, the study presented that it is possible by adjusting a certain distance of a permanent magnet to the cavity to active control the fluid flow so that convection can be reduced or enhanced.

Acknowledgements This work was supported by the Konrad-Adenauer Foundation with a stipend to Peter Szabo, the James-Watt scholarship and the Neilson Travel Found by Heriot-Watt University, Edinburgh.

References

[1] R. E. Rosensweig. *Ferrohydrodynamics*. Dover Publications, Dover, 1997.
[2] W.-G. Früh. Heat transfer enhancement by thermomagnetic convection. In P Jansens, A. Stankiewicz, and A. Green, editors, *Sustainable (Bio)Chemical Process Technology*, pages 47–56, Cranfield, 2005. BHR Group Ltd.
[3] W.-G. Früh. Using magnetic fluids to simulate convection in a central force field in the laboratory. *Nonlinear Processes in Geophysics*, 12:877–889, 2005.
[4] A. F. Pshenichnikov and V.V. Mekhononshin. Equilibrium magnetization and microstructure of the system of superparamagnetic interacting particles: numerical simulation. *Journal of Magnetism and Magnetic Materials*, 213:357–369, 1999.
[5] D. de Vahl Davis. Natural convection in a square cavity: A comparison exercise. *Int. J. Numer. Methods in Fluids*, 3:227–248, 1983.