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# Efficient Buyer Groups with Prediction-of-Use Electricity Tariffs

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## Abstract—

Current electricity tariffs do not reflect the real costs that a customer incurs to a supplier, as units are charged at the same rate, regardless of the consumption pattern. In this paper, we propose a prediction-of-use (POU) tariff that better reflects the predictability cost of a customer. Our tariff asks customers to pre-commit to a baseline consumption, and charges them based on both their actual consumption and the deviation from the anticipated baseline. First, we study, from a cooperative game theory perspective, the cost game induced by a single such tariff, and show customers would have an incentive to minimize their risk, by joining together when buying electricity as a grand coalition. Second, we study the efficient (i.e. cost-minimizing) structure of buying groups for the more realistic setting when *multiple*, competing POU tariffs are available. We propose a polynomial time algorithm to compute the efficient buyer groups, and validate our approach experimentally, using a large-scale data set of domestic consumers in the UK.

**Index Terms**—Electricity tariffs, cooperative game theory, coalition formation, collective switching, demand forecasting.

## I. INTRODUCTION

Recent years have seen significant efforts to switch to a more sustainable *smart energy grid*, which incentivises the development of renewable energy resources, as well as lower levels of consumption [1]. However, these efforts have sometimes met with resistance from consumers because they can lead to rises in electricity prices. Given this, we argue, a priority of current research should be to develop tools that empower consumers in interacting with energy providers to obtain the best deals.

Existing electricity tariffs are not well-suited to dealing with these challenges. In most cases, customers are charged at a flat rate, based only on the number of units actually consumed and regardless of their consumption pattern. However, this matches poorly with the structure of the costs that energy suppliers face [2]. Prediction of use is a key issue in power markets, because electricity demand and supply need to be perfectly balanced at all times, unlike most commodities (oil, gas, grain etc.). With other commodities than electricity, exact prediction of the demand by the consumers in each period is not such a

problem because any excess/deficit can be stored/taken from storage in the next time period. But with electricity, grid-scale electrical storage is extremely expensive, hence there is considerable value in accurate prediction of demand. Customers with predictable demand profiles are cheaper to serve than customers with unpredictable consumption patterns. This is because cheaper baseload generation technologies can be effectively planned to service customers with stable demand. In more detail, in most countries, suppliers purchase electricity supplied to their customers through forward contracts, in which they commit to acquire a baseline demand, to be supplied at a later period. These forward purchases can be done either through long term bilateral contracts [3] or through the spot markets (for electricity bought in a shorter time horizon, e.g. a day in advance). Any difference between the baseload and the actual consumption is resolved through the *balancing market*, where prices, especially at peak times, can be much higher than those obtained through forward contracts.

While the focus of new pricing schemes has generally been on time [4], [5], Time-Of-Use (TOU) pricing does not discriminate on the basis of customer demand predictability. Thus, even under these new pricing schemes customers, with stable demand curves are cross-subsidising customers with highly unpredictable demand [6].

This paper addresses these challenges by proposing a tariff that better matches the structure of current electricity markets. Essentially, our proposal involves a new tariff structure, the *prediction-of-use* (POU) tariff, in which customers pre-commit a baseline for their consumption and charges them based on both their actual consumption and the deviation from their anticipated baseline (in the sense that units consumed in excess/short of the anticipated baseline are charged at different marginal rates). Note that tariffs with similar structure are already a reality in a number of markets. For example, Braithwait et al. [7] mention in their business report a real-time tariff offered by several energy suppliers in the US, where customers commit to a self-selected baseline load, and are charged a standard rate for their consumption baseline, a penalty rate for usage in excess of their baseline, and receive return credit for underconsumed units. However, so far, most of these offerings are addressed to large-scale industrial consumers, as individual domestic consumers are typically too small to have significant market power or influence on costs.

Moreover, to date, no existing work has studied the theoretical or experimental properties of such tariffs.

A potential downside of POU tariffs is that they transfer part of the prediction risk faced by the supplier to the consumers. However, one way that customers can collectively act to reduce the risk associated to their consumption imbalance is by joining the electricity tariff as a single virtual consumer (i.e. aggregating their demand). In the energy context, some recent initiatives (e.g., *BigSwitch*, *thePeoplesPower* [8] or *Cheap-EnergyClub* [9]) achieved significant discounts by bringing a large number of consumers together and negotiating a better deal on their behalf with suppliers. Such group buying initiatives are initiated either on a commercial basis (e.g., a start-up or price advice website) or by a community energy group or local authority. Electricity suppliers are interested in this trend because it allows them to capture a substantial market share of savvy consumers, by offering them a customized tariff.

While such initiatives shown the potential impact of group buying, new technologies have the possibility to go much further in allowing suppliers to offer consumers more targeted tariffs based on their predictability, and enabling consumers to form groups which better match their consumption profile. A key technological development is the widespread rollout of smart meters, as well as devices connected to meters. Such devices have the potential not only to record and store consumption data from each consumer with fine granularity (e.g., half-hourly, every 15 min, etc.), but also use this data to provide estimates of their future consumptions, as well as quantify how predictable are those estimates (i.e. the forecasting error distribution) as required for the implementation of a POU tariff scheme. Moreover, these smart metering devices could communicate with cloud-based services to search for the best tariff deals and buying groups a consumer should join, given her consumption profile. In fact, given the large number of residential tariffs available (i.e. almost 100 in the UK), a number of tools such as *USwitch* [10] have appeared to enable consumers to estimate their bills under each tariff, while novel AI solutions such as *AgentSwitch* [11] have proposed autonomous agents to take tariff selection decisions and advise consumers on actions to reduce their electricity bill.

The availability of smart meter data would also go much further in enabling tariffs tailored to reward the predictability of individual consumers. In fact, the introduction of new smart metering standards (such as SMETS-2 in the UK [12]) allows consumers to take control of the sharing of their own private metering data, and to switch smoothly between competing suppliers (or buyers groups) which are offering the best deal.

A key issue in achieving this vision is that a realistic market is likely to have a number of competing POU tariffs, offered by different suppliers, with varying strategies in purchasing electricity. Some suppliers prefer to hedge their purchases through forward contracts, and hence are able to offer a flatter tariff, although at a higher cost, to cover the cost of hedging [2]. Other electricity suppliers, in contrast, may choose to buy on the forward market only the amount of electricity that their customers actually predict would be needed, and have to make up any shortfall on the balancing market, where electricity is more expensive. These different buying strategies may result

in a range of POU tariffs being offered in the market, ranging from flat to more predictive ones. When multiple POU tariffs are available, the game changes. Some consumers, who have less uncertainty about their future consumption may group together to join a more predictive tariff, while others, who have more uncertainty, may prefer to form a different group, under a flatter tariff. The problem of determining the most efficient (i.e. cost-minimizing) group buying structure in this setting is complex, and in this paper we model and provide the first practical solution to this challenge.

To summarize, this work can be seen as having several contributions to the state of the art:

- We propose a new kind of tariff that introduces a *prediction-of-use* component to price electricity and study the cooperative game that our tariff induces among a set of consumers. We show this cost game is concave hence the expected bill is always reduced by grouping and it can be distributed fairly using a number of solution concepts, such as the well-known Shapley value.
- We study efficient group formation in cases where consumers elect to buy electricity as a group, under one or several tariffs. We propose a natural characterization of tariff selection, in terms of the coefficient of variation of each customer, and we discuss its application both when agents join tariffs on their own, or in a group with other agents. Moreover, we formulate a polynomial-time dynamic programming algorithm that computes the optimal structure in the multiple tariff case, and provides high experimental efficiency in practice.
- We test our approach with a large-scale dataset of electricity consumers in the UK. We show, with a set of tariffs ranging from flat to highly predictive, that the tariff choice can differ significantly between the case of individual vs. group buying. More interestingly, we show that in both cases different tariffs may have non-empty market shares. Thus, unlike the single tariff case, the grand coalition is not guaranteed to be the most efficient (i.e. cost minimizing) setting.

The rest of this paper is structured as follows. A review of the literature is provided in Section II. Prediction-of-use tariffs are introduced in Section III, while the coalition formation game is characterised in Section IV. Sections V-VII formulate the multi-tariff model and the characterization of tariff selection, both individually and using group buying. Section VIII presents a dynamic programming algorithm for determining efficient buyer groups and Section IX presents its empirical validation using real consumption data. The paper concludes with a discussion in Section X.

## II. RELATED WORK

In the context of the smart grid, game theory has been extensively used for designing robust smart demand-side management schemes (see, for example, [13] and their list of references). For instance, Mohsenian-Rad *et al.* formulate a noncooperative game for scheduling appliances based on a pricing scheme in which the tariff rate varies with the overall power demand of all customers [14]. Similarly, Ibars *et al.* [15]

propose dynamic consumer tariffs but in the form of a network congestion game. Both works show that the equilibrium of the respective game can be reached in a distributed fashion. Other works study Stackelberg games to model the interaction among multiple utilities (leaders) and multiple consumers (followers) and propose algorithms to solve the resultant bi-level multiobjective optimisation problem [16], [17]. However, all the aforementioned works take a non-cooperative point of view (i.e. no coordination of strategic choices among players). Instead, here we study a cooperative game in which players are provided with incentives to act together as one entity (i.e. coalition, binding agreements) to improve their position in the game. A few studies, including [18], [19], and [20], have presented models considering a cooperative perspective with the formation of coalitions. Baeyens et al. [18] and Robu et al. [19] studied coalitions or cooperatives between multiple renewable energy resources with uncertain and intermittent supply, though it only deals with the production side. On the demand side, Kota *et al.* [20] propose the concept of cooperatives for demand-side management, but they do not study their stability and optimality properties using tools from coalitional game theory.

In the smart grid community, several works propose new tariff structures, that are better suited to meet emerging challenges of modern electricity grids (e.g. adaptive pricing, flat pricing, peak load pricing, etc). Several studies focus on Time-Of-Use (TOU) tariffs and devise stochastic optimisation methods for their design, to account for various constraints such as elasticity of demand and revenue of the utility company [4], [5]. In particular, Gu *et al.* [21] proposed a set of financial hedging tools to help consumers to deal with the price volatility present in the Use-of-System (UoS) tariffs, where consumers are charged for their use of the distribution network. On the other hand, Li *et al.* [22] considered a stepwise tariff model, in which higher consumption leads to a higher per unit price, and use a genetic algorithm to design its tariff parameters, so as to meet the network constraints. Zedan et al. [23] formulated a tariff scheme based on the average utilised capacity and test this scheme on collected utilities data confirming its viability. Unlike the POU tariffs we study here, these proposals (UoS, TOU etc.) incentivise either lowering consumption or shifting demand to cheaper times, rather than incentivising predictability of future demand.

A separate strand of work focuses on schemes that deal with the interaction between the distribution system operator (DSO) and the aggregators. Specifically, dynamic tariff (DT) methods based on the distribution locational marginal price concept [24] have been proposed for decentralised congestion management within distribution networks [25]–[28]. Under these approaches, it is the DSO who computes the DT tariff and sends it to the aggregators, which autonomously make the optimal energy planning of flexible demands on behalf of the final owners. Huang *et al.* [29] further enhance DT tariffs to also consider network (feeder) reconfiguration and line loss reduction. On the other hand, Steen et Tuan [30] study power based tariffs (PBT), which include, in addition to the typical charge based on the energy consumed by the consumer, a fee based on the customer peak-demand. Further, Saad *et*

*al.* [31] study the use of cooperative games for enabling cooperative energy exchange between microgrids with the aim of reducing power losses. Unlike this paper that focuses on the interaction between individual end-users, these studies take a grid operator perspective with a focus on congestion and capacity management and line loss reduction.

From a conceptual perspective, our work is also related to the newsvendor game problem studied in operations research [32], [33]. However, that literature does not deal with multiple tariffs. Finally, there is a clear connection between our work and the work on optimal coalition structure generation [34] in game theory. However, ours is the first work to consider coalition formation for minimizing group buying risk.

### III. PREDICTION-OF-USE TARIFFS

In this section, we first define the basic properties of POU tariffs following the formalization in [35]. The central idea of a POU tariff is that each customer is asked to provide, in advance, a prediction for her consumption (called *baseline*) during a specific time period. A POU tariff is then defined as a tuple  $(\underline{p}, \underline{p}, \bar{p})$ , where  $\underline{p}$  is the baseline rate (with  $\underline{p} > 0$ ) and  $\underline{p}$ ,  $\bar{p}$  are the penalty rates for underconsumed and overconsumed units respectively ( $\underline{p} > \underline{p} \geq 0$  and  $\bar{p} \geq 0$ ).

Then the payment of a consumer with an ex-ante baseline  $b$  and an actual (ex-post) consumption  $x$  is determined as:

$$\psi(x, b) = \begin{cases} \underline{p} \cdot x + \bar{p} \cdot (x - b) & \text{if } b \leq x \\ \underline{p} \cdot x + \underline{p} \cdot (b - x) & \text{otherwise} \end{cases} \quad (1)$$

This tariff allows us to distinguish between a potentially lower baseline rate, and a higher penalty for the deviation. Thus, the expected payment of a customer under a POU tariff does not depend only on her realized consumption but also on her prediction error.

#### A. POU Tariffs: An Illustrative Example

Next, we illustrate the effect of POU tariffs by means of three examples. The parameters  $(\underline{p}, \bar{p}, \underline{p})$  (listed in Table I (a)) are set to roughly match long-term averages from the UK balancing market<sup>1</sup>.

**Example 1** (Highly predictive). *This tariff gives the lowest baseline rate but severely penalizes any imbalance. It models the costs of a supplier when contracting the baseline predicted by the consumer in the forward market and charging the consumer's imbalance at the balancing market prices. Hence  $\underline{p}$  is set as the price for kWh obtained from the forward market (£0.05);  $\bar{p}$  is set as the difference between the expected buying rate in the imbalance market and the baseline rate (£0.1 – £0.05 = £0.05); and  $\underline{p}$  is set as the difference between the baseline rate and the expected selling rate in the imbalance market (£0.05 – £0.02 = £0.03).*

**Example 2** (Predictive). *This tariff reduces the penalty for imbalances at the cost of increasing the baseline rate. It models a supplier when contracting not only the baseline at the forward market price but also some extra quantity to account for potential imbalances. Although the baseline rate increases slightly (from £0.05 to £0.06) it also reduces the imbalance rates, from £0.03 to £0.02 for underconsumption and from £0.05 to £0.02 for overconsumption.*

<sup>1</sup>Historical balancing prices for every settlement period in a particular day are available at <http://www.elexon.co.uk/reference/credit-pricing/imbalance-pricing>

(a) POU tariffs parameters (in £/kWh)

	$\underline{p}$	$\underline{\bar{p}}$	$\bar{\bar{p}}$
Highly predictive	0.05	0.03	0.05
Predictive	0.06	0.02	0.02
Flat	0.08	0	0

(b) Monthly consumptions, predicted baselines, and payments.

	Demand (in kW)	Baseline (in kW)	Payment (in £)		
			Highly predictive	Predictive	Flat
Bob	300	500	24	22	24
Annie	300	100	35	34	24
John	300	300	15	18	24

TABLE I

EXAMPLE OF (A) THREE DIFFERENT POU TARIFFS AND (B) ACTUAL PAYMENTS OF THREE CUSTOMERS W.R.T THESE TARIFFS.

**Example 3 (Flat).** A flat tariff with  $\bar{p} = £0.08$  per kWh, roughly similar to wholesale costs in the prevalent tariffs in the UK.

Next, we illustrate the computation of actual payments w.r.t. these tariffs. Consider three customers, Bob, Annie and John, that contracted a POU tariff with predicted monthly baselines of 500kW, 100kW and 300kW respectively. Although at the time of demand, all of them realise a consumption of 300kW, their payments may vary because of the different predicted baselines. Table I (b) shows the respective payment of each consumer under each of the three tariffs. Bob is penalized for his 200 units of overconsumption, Annie for her 200 units of underconsumption, whereas John pays his consumption at the baseline rate with no penalties. As we observe in this example, POU tariffs reward more predictable consumers: John always pays the cheapest rate in all tariffs. We also observe that the benefit of John is greater in the highly predictive tariff than in the Predictive or Flat tariffs.

### B. Expected payment under a POU tariff

Following Vinyals *et al.* [35], we assume that the prediction error of a customer with respect to her baseline is given by a normal (i.e. Gaussian) probability distribution  $N(\mu, \sigma)$ . Since this is a prediction error, modeling it as a normal distribution follows a common assumption in statistical modeling [36]. In practice, the availability of large-scale smart metering data, collected over time for each consumer, would make such errors converge over time to a normal distribution. Now, the *optimal baseline* that a customer should report to minimize her expected payment ( $b^*$ ) is characterized as a quantile of her prediction distribution. Formally,

$$b^* = \mu + \sigma \cdot \Phi^{-1}(r^*)$$

where  $\Phi^{-1}(\cdot)$  is the quantile function of the standard normal distribution  $N(0, 1)$  (i.e. independent of the specific  $\mu$  and  $\sigma$  of the consumer) and  $r^* = \frac{\bar{p}}{\underline{p} + \bar{p}}$  is the tariff optimal ratio.

Given this optimal baseline, the expected payment of a customer whose prediction error follows a normal distribution  $N(\mu, \sigma)$  under a POU tariff is:

$$\mathbb{E}[\psi^*(x)] = \underbrace{\mu \underline{p}}_{\text{Consumption term}} - \underbrace{\sigma(\underline{p} + \bar{p}) \int_0^{r^*} \Phi^{-1}(y) dy}_{\text{Penalty term}} \quad (2)$$

In the above equation, we distinguish two terms: a *consumption term* that depends on the expected consumption value of the agent, and a *penalty term* that depends on its standard deviation. We also refer to the factor multiplying  $\sigma$  as the *penalty rate*. Note that the penalty rate depends only on the parameters  $\langle \underline{p}, \underline{\bar{p}}, \bar{\bar{p}} \rangle$  of the tariff being applied.

For the results in the next section, it is useful to show the penalty term is always greater or equal to 0. First, note that by definition  $\sigma \geq 0$ . Thus, for the product in the penalty term of Eq. 2 to be  $\geq 0$ , it remains to show that:

$$(-\underline{p} - \bar{p}) \int_0^{r^*} \Phi^{-1}(y) dy \geq 0 \text{ for all } r^* \in [0, 1] \quad (3)$$

To prove this, notice that since  $\bar{p} \geq 0$  and  $\underline{p} > 0$  then  $(-\underline{p} - \bar{p}) \leq 0$ . Hence for the product in Eq. 3 to be  $\geq 0$ , it remains to show that  $\int_0^{r^*} \Phi^{-1}(y) dy \geq 0$  for all  $r^* \in [0, 1]$ . We will distinguish between two cases:  $r^* \leq \frac{1}{2}$  and  $r^* > \frac{1}{2}$ . First, if  $r^* \leq \frac{1}{2}$  we find directly, using that  $\Phi^{-1}(r^*) \leq 0$  for all  $0 \leq r^* \leq \frac{1}{2}$  that (3) holds. Secondly, suppose  $r^* > \frac{1}{2}$ . Then,  $\int_0^{r^*} \Phi^{-1}(r^*) = \int_0^{\frac{1}{2}} \Phi^{-1}(r^*) + \int_{\frac{1}{2}}^{r^*} \Phi^{-1}(r^*)$  and since  $\Phi^{-1}(r^*) \geq 0$  for all  $\frac{1}{2} \leq r^* \leq 1$  and  $\int_0^{\frac{1}{2}} \Phi^{-1}(r^*) - \int_{\frac{1}{2}}^{r^*} \Phi^{-1}(r^*) \leq 0$  because of the symmetry of the standard normal distribution, hence inequality (3) holds and the penalty term in Eq. 2 is always greater or equal to 0.

## IV. SINGLE-TARIFF PREDICTION-OF-USE GAME

The previous section studied the POU tariff from the perspective of an individual consumer. When applied to a set of consumers, this tariff induces a cooperative game, as consumers can form coalitions to reduce their expected payments.

Consider a set of  $N$  customers that joined the group-tariff scheme. Given a subset of customers  $S \subseteq N$ , let  $x_S = \sum_{i \in S} x_i$  be its aggregate error in demand prediction. We define the corresponding prediction-of-use (POU) game  $G = \langle N, c \rangle$ , where  $c: 2^N \rightarrow \mathfrak{R}$  as:

$$c(S) = \mathbb{E}[\psi^*(x_S)] \text{ for all } S \subseteq N. \quad (4)$$

where  $\mathbb{E}[\psi^*(x_S)]$  is the expected joint payment for deviation demand prediction  $x_S$  defined as in Eq. 2.

An important question raised by this grouping is whether it is beneficial for agents to form a coalition (i.e. to share the risk associated to their consumption imbalance) and whether the coalition that forms is core-stable (i.e. there is no individual agent or subset of agents that have an incentive to break away to form their own group).

In this section, we answer this question by showing that the game induced by a single POU tariff is concave. Concavity in cost games (corresponding to convexity in utility games) can be defined as follows:

**Definition 1.** A cost game  $\langle N, c \rangle$  is concave if  $c(T \cup \{i\}) - c(T) \leq c(S \cup \{i\}) - c(S)$  for all  $i \in N$  and all  $S \subset T \subseteq N \setminus \{i\}$ .

Hence, in concave games the marginal contribution of an agent to any coalition is greater than its marginal contribution to a larger coalition. Thus, concavity implies that it is always beneficial (i.e. reduces cost) to group agents together, meaning

the grand coalition is efficient. Moreover, concavity means that any payment scheme that is a linear combination of marginal contributions (including the *Shapley* value) is in the anticore.

**Theorem 1.** *Assuming independent normal distributions for consumers' prediction errors,  $x_i \sim N(\mu_i, \sigma_i)$  for all  $i \in N$ , the POU game  $(N, c)$  is concave.*

*Proof.* We need to prove that the following function (associated with the concavity condition given in Definition 1) is always negative, i.e.

$$\text{con}_{i,S,T} = c(T \cup \{i\}) - c(T) - (c(S \cup \{i\}) - c(S)) \leq 0 \quad (5)$$

for all  $i \in N$  and all  $S \subset T \subseteq N \setminus \{i\}$ .

Simplifying using Eq. 2, we obtain:

$$\begin{aligned} \text{con}_{i,S,T} = & \mathbb{P}(\mu_{T \cup \{i\}} - \mu_T - \mu_{S \cup \{i\}} + \mu_S) + \\ & (-\underline{p} - \bar{p})(\sigma_{T \cup \{i\}} - \sigma_T - \sigma_{S \cup \{i\}} + \sigma_S) \int_0^{r^*} \Phi^{-1}(y) dy \end{aligned}$$

First, the difference in consumption term cancels out because  $\mu_{T \cup \{i\}} - \mu_T = \mu_i = \mu_{S \cup \{i\}} - \mu_S$  (the mean of a sum of two normal distributions is the sum of the means of their individual distributions). For the second term, we know  $(-\underline{p} - \bar{p}) \int_0^{r^*} \Phi^{-1}(y) dy \geq 0$  from Eq. 3. Finally, the following has been shown in [33] to hold for  $\forall S \subset T \subseteq N \setminus \{i\}$ :  $\sigma_{T \cup \{i\}} - \sigma_T \leq \sigma_{S \cup \{i\}} - \sigma_S$ . Thus it follows that  $\sigma_{T \cup \{i\}} - \sigma_T - (\sigma_{S \cup \{i\}} - \sigma_S) \leq 0$  and therefore Eq. 5 holds.  $\square$

Establishing the POU game is concave allows us to use coalitional game theory principles to define ways to divide costs between the agents in such a game in a fair way. In more detail, given a cost game, there are several ways to divide the cost of the game (i.e. assuming transferable cost) among its members. A vector  $\varphi = \{\varphi_1, \dots, \varphi_n\}$  that assigns some cost to each agent  $i \in N$  is called an *allocation*. We denote  $\sum_{i \in S} \varphi_i$  as  $\varphi(S)$ . An allocation is an *imputation* for  $N$ , if it is *efficient* ( $\varphi(N) = c(N)$ ) and *individually rational* ( $\varphi(\{i\}) \leq c(\{i\}) \quad \forall i \in N$ ). The most important allocation concept aiming at stability in cost games is the anticore (corresponding to the core in utility games).

**Definition 2.** *The anticore of a cost game  $(N, c)$  is composed of all imputations of  $N$  such that  $\varphi(S) \leq c(S) \quad \forall S \subset N$ .*

Note that  $\varphi$  is in the anticore of a cost game iff no coalition  $S \subset N$  can improve upon  $\varphi$  (i.e. has no cross-subsidies).

Analogous to the anticore, the most prominent solution aiming at fairness is the *Shapley* value. The *Shapley* value is based on the intuition that the payment that each agent receives should be proportional to its contribution. To formally define the *Shapley* value, we need first to define the notion of marginal contribution.

Let  $\Pi_N$  denote the set of all orderings of agents  $N$ . Given an ordering  $\pi \in \Pi_N$ , let  $S_{< i}^\pi$  be the set of predecessors of  $i$  in  $\pi$ .

**Definition 3.** *The marginal contribution of an agent  $i$  w.r.t. an ordering  $\pi$  is defined as  $\Delta^\pi(i) = c(S_{< i}^\pi \cup \{i\}) - c(S_{< i}^\pi)$ .*

Then, the *Shapley* value of an agent  $i$ , is simply the average of all possible marginal contributions, that is w.r.t all orderings of  $N$ . Formally, the *Shapley* allocation scheme assigns:

$$\varphi(i) = \frac{1}{n!} \sum_{\pi \in \Pi_N} \Delta^\pi(i) \quad \forall i \in N \quad (6)$$

Recall that in Theorem 1 we have shown that the POU game is a concave cost game. These properties follow immediately from the concavity result in Theorem 1 and standard cooperative game theory (c.f. [37]).

**Corollary 1.** *The POU game with independent normal consumption prediction deviations in demand prediction is sub-additive and totally balanced.*

**Corollary 2.** *The Shapley value payments are in the anticore of the POU game with independent normal consumption prediction deviations.*

Moreover if the game is convex (respectively concave, for cost games), for any ordering of the players a core-stable allocation can be constructed in polynomial time by assigning to each agent its marginal contribution with respect to the ordering. For a convex/concave game, all such marginal payments are in the core/anticore. The key limitation of the above results is that they refer to a single POU tariff. Next, we examine the much more challenging case in which consumers can choose from several POU tariffs, competing in the same market.

## V. THE MULTI-TARIFF MODEL

Let  $\Gamma = \{\tau_1, \dots, \tau_M\}$  be a set of POU tariffs offered in the market by competing suppliers. For each tariff  $\tau_k \in \Gamma$  let  $\underline{p}_k, \bar{p}_k, \bar{p}_k$  be the baseline, underconsumption and overconsumption rates, respectively, and  $r_k^*$  the optimal ratio.

The main factor driving the decision of a customer to join a tariff (on her own or through a group-buying scheme) is her *expected payment*. As observed in Eq. 2, the expected payment of a customer in a POU tariff depends on her particular prediction error. Thus, for some customers it is likely to be more beneficial to take on higher deviation rates in order to obtain lower baseline rates. Conversely, others may be willing to pay higher baseline rates to avoid deviation penalties as much as possible. Formally, finding the tariff  $\tau^*$  that minimizes the expected payment of a customer with a prediction error that follows a normal distribution  $N(\mu, \sigma)$ , involves solving the following minimization problem:

$$\tau^* = \arg \min_{\tau_k \in \Gamma} \mathbb{E}[\psi_k^*(x)] \quad (7)$$

where  $\mathbb{E}[\psi_k^*(x)]$  is the expected payment of the customer under tariff  $\tau_k$  defined as in Eq. 2.

To tackle this minimization problem, we will first define the conditions under which a customer will pay less in a tariff than in another. Formally, a customer with a prediction error that follows  $N(\mu, \sigma)$  expects to pay strictly less in some tariff  $\tau_k$  than in tariff  $\tau_l$  if and only if the following inequality holds:

$$\underline{p}_k \mu - \sigma(\underline{p}_k + \bar{p}_k) \int_0^{r_k^*} \Phi^{-1}(y) dy < \underline{p}_l \mu - \sigma(\underline{p}_l + \bar{p}_l) \int_0^{r_l^*} \Phi^{-1}(y) dy$$

Recall that  $r_i^*, r_k^*$  and  $\Phi^{-1}(\cdot)$  are parameters independent of  $\mu$  and  $\sigma$ . By rearranging terms we obtain:

$$\Delta_{k,l}^P \sigma < \Delta_{l,k}^B \mu, \quad (8)$$

where  $\Delta_{l,k}^B = p_l - p_k$  is the difference in baseline rate between tariff  $\tau_l$  and  $\tau_k$  (where  $\Delta_{k,l}^B = -\Delta_{l,k}^B$ ) and

$$\Delta_{k,l}^P = -(p_k + \bar{p}_k) \int_0^{r_k^*} \Phi^{-1}(y) dy + (p_l + \bar{p}_l) \int_0^{r_l^*} \Phi^{-1}(y) dy$$

is the difference in penalty rate between  $\tau_k$  and  $\tau_l$ .

**Definition 4.** A tariff  $\tau_l$  is said to be dominated in expectation iff there exists another tariff  $\tau_k \in \Gamma$  such that the expected payment under  $\tau_k$  is lower than those in  $\tau_l$  irrespective of the  $\mu$  and  $\sigma$  of the expected prediction error.

In terms of the above notation, a tariff  $\tau_l$  is dominated in expectation by  $\tau_k$  if  $\Delta_{k,l}^B < 0$  and  $\Delta_{k,l}^P < 0$ .

Note that a stronger notion of *ex-post* dominance can also be defined, i.e. if  $p_k < p_l$ ,  $\underline{p}_k < \underline{p}_l$ ,  $\bar{p}_k < \bar{p}_l$ , then a consumer will pay less under tariff  $\tau_k$ , regardless of the realised consumption. However, here we are interested in dominance “in expectation”, because the choice of tariff occurs in expectation.

When faced with the ex-ante choice, no risk-neutral consumer will choose a dominated tariff, hence such a tariff gets no market share and w.l.o.g. we can omit it from further analysis. Also w.l.o.g., we can omit duplicate tariffs, i.e. tariffs which have the same baseline and penalty rates. Henceforth, when referring to  $\Gamma$ , we consider eliminated from it all tariffs that are dominated in expectation or duplicates. Then, for any pair of remaining tariffs  $\tau_k, \tau_l \in \Gamma$  it must hold that: if  $p_k < p_l$  then  $\Delta_{k,l}^P \geq 0$ . Now, we can define the relation of *flatness* among tariffs.

**Definition 5.** Given two tariffs  $\tau_k, \tau_l \in \Gamma$  we say that  $\tau_k$  is less flat than  $\tau_l$ , denoted as  $\tau_k \prec^f \tau_l$ , if  $\tau_k$  has a lower baseline rate and a higher penalty rate than  $\tau_l$  (formally, if  $\Delta_{k,l}^B < 0$  and  $\Delta_{k,l}^P > 0$ ).

Observe that when applied to a set of tariffs  $\Gamma$ , after we eliminate the dominated and duplicate tariffs, the flatness relationship defines a total ordering among remaining tariffs in  $\Gamma$  (i.e. it is transitive and symmetric).

Now, given two tariffs  $\tau_k, \tau_l \in \Gamma$  such that  $\tau_k \prec^f \tau_l$  we can rearrange terms in Eq. 8 to obtain:

$$\frac{\sigma}{\mu} < \left[ \lambda_{k,l} = \frac{\Delta_{l,k}^B}{\Delta_{k,l}^P} \right] \quad (9)$$

Note that the left side of Eq. 9 is known as the *coefficient of variation* (i.e. the ratio between the standard deviation and the mean). The parameter on the right side,  $\lambda_{k,l}$ , is denoted as the *inter-tariff threshold* between  $\tau_k$  and  $\tau_l$ .<sup>2</sup> The following corollary follows directly from previous derivations:

**Corollary 3.** Given two tariffs  $\tau_k \prec^f \tau_l$ , there exists a unique threshold  $\lambda_{k,l} \in \mathbb{R}$  such that the expected payment of any customer is less in tariff  $\tau_k$  than in tariff  $\tau_l$  iff  $\frac{\sigma}{\mu} < \lambda_{k,l}$ . Otherwise her expected payment is lower in tariff  $\tau_l$ .

<sup>2</sup>Notice that the inter-tariff threshold depends only on the parameters  $p, \bar{p}, \underline{p}$  of tariffs  $\tau_k$  and  $\tau_l$ .

## VI. MARKET SEGMENTATION WITH INDIVIDUAL CHOICE

We characterize the partition of the pool of customers in the case where each customer chooses to join a POU tariff based only on its own expected consumption.

Given Corollary 3, for some non-dominated, duplicate-free set of tariffs  $\Gamma$  we can define a set of  $\frac{|\Gamma|(|\Gamma|-1)}{2}$  inter-tariff thresholds, one for each pair of tariffs  $\tau_k, \tau_l \in \Gamma$  such that  $\tau_k \prec^f \tau_l$ . Arranging these thresholds over the coefficient of variation axis, they define a total of  $\frac{|\Gamma|(|\Gamma|-1)}{2} + 1$  intervals ( $\frac{|\Gamma|(|\Gamma|-1)}{2} - 1$  between thresholds plus two at the axis ends). We label these thresholds in ascending order  $\lambda^s$  where  $s$  is the position of the threshold in the ordering.

**Lemma 1.** Given a set of tariffs  $\Gamma$ , for all  $s = 1 \dots \frac{|\Gamma|(|\Gamma|-1)}{2} + 1$ , the expected payment of any customer with a coefficient of variation within the range  $(\lambda^{s-1}, \lambda^s)$  is minimized under the same tariff.

*Proof.* Towards a contradiction, consider two consumers  $i, j \in N$  with  $\frac{\sigma_i}{\mu_i}, \frac{\sigma_j}{\mu_j} \in (\lambda^{s-1}, \lambda^s)$ , where  $i$  joins tariff  $\tau_k$ , while  $j$  joins  $\tau_l$  ( $k \neq l$ ). But this implies the existence of another threshold  $\lambda_{k,l} \in (\lambda^{s-1}, \lambda^s)$ , which is not possible as the set of all inter-tariffs thresholds is determined.  $\square$

Thus, for each interval, there is one tariff that dominates all the others, for all customers with a  $\frac{\sigma}{\mu} \in (\lambda^{s-1}, \lambda^s)$ . However, it may not be necessary to keep all these thresholds to characterize the partition because some of the neighbouring intervals will merge, as one tariff dominates in two or more adjoining intervals. Formally, we denote by the set  $\Gamma_{ND}$  the subset of tariffs in  $\Gamma$  that dominate in at least one interval.

It remains to be shown that, if a tariff  $\tau_k$  in  $\Gamma_{ND}$  dominates in a set of intervals, all these intervals must be adjoining (i.e. consecutive). Again, we prove this by contradiction. Consider a tariff  $\tau_k$  in  $\Gamma_{ND}$  such that it dominates in two non-consecutive intervals. Then it means that there is at least one interval in the middle in which another tariff  $\tau_l$ ,  $\tau_l \neq \tau_k$ , dominates. But this would lead to a contradiction with Corollary 3, which states the switch from  $\tau_k$  to  $\tau_l$  is defined by a unique point  $\lambda_{k,l}$ .

## VII. MARKET SEGMENTATION WITH GROUP BUYING

In this section, we extend our analysis to the case when customers group together to purchase electricity as a group, under any tariff in set  $\Gamma$ . We define the corresponding multiple POU tariff cost game  $G = \langle N, c \rangle$ , where  $c : 2^N \rightarrow \mathfrak{R}$ :

$$c(S, \Gamma) = \min_{\tau_k \in \Gamma} \mathbb{E}[\psi_k^*(x_S)] \text{ for all } S \subseteq N. \quad (10)$$

We take a cooperative game theory approach to this problem, studying how to determine which is the optimal buying group structure that minimizes the total cost to all customers (where “cost” is defined as total expected payments under the selected tariffs). In general, coalition structure generation involves finding the exhaustive disjoint partition of customers into groups  $\tilde{S} = \{S_1, \dots, S_m\}$  such that the total cost,  $\sum_{k=1}^m c(S_k, \Gamma)$ , is minimized. However, as stated by the next lemma, in our case at most one group per tariff will form.

**Lemma 2.** *The optimal structure of a pool of customers under a set of tariffs  $\Gamma$  can be expressed in terms of  $|\Gamma|$  groups,  $\{S_1, \dots, S_{|\Gamma|}\}$ , being for all  $k = 1 \dots |\Gamma|$   $S_k$  a set of customers (possibly empty) that will group under tariff  $\tau_k \in \Gamma$ .*

*Proof.* Consider that the optimal structure contains two groups  $S, S'$  that join the same tariff  $\tau_k$ . This leads to a contradiction because, given the concavity (and subadditive) results proved in [35] for the single tariff game, customers choosing the same tariff reduce their joint expected payment by grouping ( $c(S \cup S', \tau_k) < c(S, \tau_k) + c(S', \tau_k)$ ).  $\square$

Henceforth, we can use the notation  $S_k = \vec{S}(k) \subseteq N$  to refer to consumers assigned to  $\tau_k$  in the structure  $\vec{S}$ .

Unfortunately, despite this bound on the maximum number of groups, the group buying structure generation problem is still a complex one because the optimal tariff group of each customer depends on the set of customers that already joined the group. Hence, in order to tackle this problem, we first single out a tractable grouping case for which the order property based on the coefficient of variation holds.

#### A. Optimal Group Structure with Equal $\sigma$ 's

As stated by the next lemma, for the restricted case with equal standard deviation error prediction, the grouping follows the same order as the individual choice case.

**Lemma 3.** *Consider the case when all customers have the same standard deviation error prediction,  $\sigma$  ( $\sigma_1 = \dots = \sigma_n = \sigma$ ). Then, for any two customers  $\forall i, j \in N$  with  $\frac{\sigma_i}{\mu_i} < \frac{\sigma_j}{\mu_j}$ , if the optimal group structure assigns  $j$  to  $S_k$ , corresponding to tariff  $\tau_k \in \Gamma$ , and  $i$  to  $S_l$ , corresponding to  $\tau_l \in \Gamma$  (where  $k \neq l$ ), then it must hold that  $\tau_k \prec^f \tau_l$ .*

*Proof.* Consider that the best group buying structure,  $\vec{S}^*$ , does not satisfy Lemma 3. This means that there are at least two customers  $i$  and  $j$  such that  $\frac{\sigma_i}{\mu_i} < \frac{\sigma_j}{\mu_j}$  and  $i \in S_l^*, j \in S_k^*$  where  $\tau_k \prec^f \tau_l$ . Towards a contradiction, consider another structure  $\vec{S}'$  that contains the same groups as  $\vec{S}^*$ , except that in coalitions  $S_k'$  and  $S_l'$  customers  $i$  and  $j$  are interchanged. Formally:  $S_m' = S_m^*$  for all  $m \neq l, k$ ,  $S_k' = S_k^* \setminus \{j\} \cup \{i\}$  and  $S_l' = S_l^* \setminus \{i\} \cup \{j\}$ . To compare the cost of  $\vec{S}^*$  and  $\vec{S}'$ , we only need to compare the cost of groups  $S_l^*, S_k^*$  with  $S_l', S_k'$  since the rest of groups are the same. We compare these groups based on their two cost components: the consumption term and the penalty term. Since all customers have the same  $\sigma$  and the number of customers in  $S_l'$  is the same as in  $S_l^*$  then the penalty term is the same among these groups. The same applies between groups  $S_k'$  and  $S_k^*$ . Regarding consumption terms, the difference between the cost of  $S_k^*$  and  $S_k'$  is  $(\mu_j \cdot p_k - \mu_i \cdot p_k)$  and the difference between the cost of  $S_l^*$  and  $S_l'$  is  $(\mu_i \cdot p_l - \mu_j \cdot p_l)$ . Adding both differences we obtain  $(p_l - p_k) \cdot (\mu_i - \mu_j)$ .

However  $p_l > p_k$  (because of the flatness relation) and  $\mu_i > \mu_j$  (because of the ratio inequality and  $\sigma_i = \sigma_j$ ), so the cost of  $\vec{S}'$  is less than  $\vec{S}^*$ , leading to a contradiction.  $\square$

Intuitively, Lemma 3 states that it is optimal to group customers which are poorer predictors (i.e. higher  $\frac{\sigma}{\mu}$ ) in either the

same or flatter tariffs than customers that are better predictors (i.e. lower  $\frac{\sigma}{\mu}$ ). However, the partition when grouping cannot be directly characterized by the inter-tariff thresholds (which only depend on the tariff themselves) as in the individual case because the coefficient of variation of a customer group depends on other customers that joined that group. Nevertheless, given that such an ordering relationship exists, the problem of deciding the optimal coalition structure is not a combinatorial one (as in the unrestricted case). Indeed, in Section VIII we propose an algorithm that is guaranteed to find such an optimal coalition structure in polynomial time.

#### B. Group Structure in Unrestricted Cases

Given the tractability results stated for the equal  $\sigma$ 's case, it is worth exploring if the same coefficient of variation ordering holds for the unrestricted case. Unfortunately, as we next show by means of a counterexample, such ordering restriction does not guarantee optimality in the general case.

**Example 4.** *Consider tariffs  $\tau_A, \tau_B, \tau_C$ , their inter-thresholds intervals ( $\lambda_{AB} = 0.4$  and  $\lambda_{BC} = 0.92$ ) and 10 customers as detailed in Fig. 1. Customers  $A_1 - A_9$  are small consumers, whose individual prediction error follows  $N(1, 1)$ , with  $\frac{\sigma}{\mu} = 1$ . Customer  $A_{10}$  is a much larger consumer with a prediction error following  $N(20, 15)$  and hence with a lower coefficient of variation ( $\frac{\sigma_{10}}{\mu_{10}} = 0.75$ ). When customers join tariffs on their own,  $A_1 - A_9$  will join the flattest tariff  $\tau_C$  (since  $\frac{\sigma_i}{\mu_i} > \lambda_{BC} \forall i = 1 \dots 9$ ), while  $A_{10}$  will join  $\tau_B$  (since  $\lambda_{AB} < \frac{\sigma_{10}}{\mu_{10}} < \lambda_{BC}$ ). Conversely, in the optimal group buying structure,  $A_1 - A_9$  will group together under the most predictive tariff  $\tau_A$  (since  $\frac{\sigma_{\{1:9\}}}{\mu_{\{1:9\}}} = \frac{\sqrt{9}}{9} < \lambda_{AB}$ ), while  $A_{10}$  remains alone under  $\tau_B$ . This is the optimal (cost minimizing) structure for this example: the grand coalition can not benefit from tariff  $\tau_A$  (as  $\frac{\sigma_{\{1:10\}}}{\mu_{\{1:10\}}} = 0.53 > \lambda_{AB}$ ), and it results in a higher total payment.*

Example 4 is of interest because it shows that the monotonic order of assigning customers to tariffs does not always hold:  $A_{10}$  has a lower coefficient of variation than  $A_1 - A_9$  individually, yet the optimal group buying structure still places her in a flatter (i.e. less predictive) tariff.

Note that, even in these unrestricted cases, the number of agents that are misclassified will be at most the number of inter-tariff thresholds. Hence counter-examples rely on one or a few customers having a very high share of the total consumption. Next, we provide an algorithm for the efficient computation of the optimal group structure with a large number of agents, and examine its performance experimentally.

### VIII. DYNAMIC PROGRAMMING SEARCH FOR OPTIMAL GROUP STRUCTURE

We present an algorithm that given a set of tariffs  $\Gamma$  computes the best group structure  $\{S_1^*, \dots, S_{|\Gamma|}^*\}$  such that  $\forall i, j \in N$  with  $\frac{\sigma_i}{\mu_i} < \frac{\sigma_j}{\mu_j}$   $i \in S_k, j \in S_l, k \neq l$  then  $\tau_k \prec^f \tau_l$ . The core of our approach takes its starting point in a Dynamic Programming (DP) algorithm formulated in [38] to solve combinatorial auctions with neighbouring assets. When applied to our problem, this algorithm computes the best group



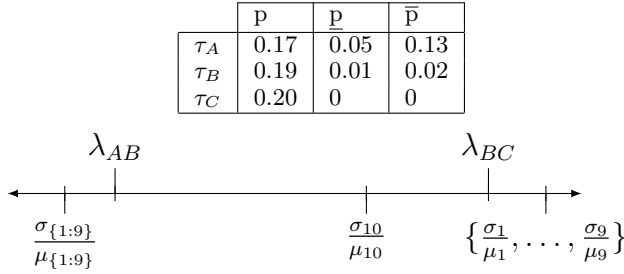


Fig. 1. Tariffs used in Example 4 (in  $\text{£/kWh}$ ) and the coefficient of variation intervals with  $\frac{\sigma}{\mu}$  for customers  $A_1 - A_{10}$  (taken individually) and the group formed by  $A_1 - A_9$ .

**Algorithm 1** Finding  $P^*$  in the  $\frac{\sigma}{\mu}$  interval with monotonic group tariff selection

**Require:**  $\{1, \dots, n\}$  (customers ordered by ascending coefficient of variation) and  $\Gamma$  (tariffs ordered by increasing flatness)

- 1: **for**  $i = 1$  to  $n$  **do**
- 2:  $C[i] \leftarrow c(\{1 : i\}, \Gamma)$ ;
- 3:  $T[i] \leftarrow k$   $\Gamma(k)$  is the optimal tariff for group  $\{1 : i\}$
- 4:  $P[i] \leftarrow 0$ ;
- 5: **if**  $i == n$  **then**
- 6:  $\bar{\tau} = |\Gamma| - 1$ ; /\*At least one tariff must remain\*/
- 7: **else**
- 8:  $\bar{\tau} = |\Gamma| - 2$ ; /\*At least two tariffs must remain\*/
- 9: **end if**
- 10: **for**  $j = 1$  to  $i - 1$  |  $T[j] \leq \bar{\tau}$  **do**
- 11:  $c' \leftarrow c(\{j + 1 : i\}, \Gamma) + C[j]$ ;
- 12:  $\tau' \leftarrow k$   $\Gamma(k)$  is the optimal tariff for group  $\{j + 1 : i\}$
- 13: **if**  $c' < C[i]$  and  $\tau > T[j]$  /\*Update\*/ **then**
- 14:  $(C[i], P[i], T[i]) = (c', j, \tau)$ ;
- 15: **end if**
- 16: **end for**
- 17: **end for**
- 18:  $P^*, T^* \leftarrow \{ \}; i \leftarrow n$ ;
- 19: **while**  $i > 0$  **do**
- 20:  $P^* \leftarrow P^* \cup \{ \{P[i] + 1 : i\} \}$
- 21:  $T^* \leftarrow T^* \cup T[i]$
- 22:  $i \leftarrow P[i]$ ;
- 23: **end while**
- 24: **return**  $(C[n], P^*, T^*)$

buying structure when groups of consecutive consumers in the coefficient of variation interval are considered. Although this DP algorithm has a deterministic polynomial complexity<sup>3</sup>, it does not exploit the strictly monotonic tariff selection that characterizes our problem. To this end, we extend the algorithm to restrict search to structures in which groups are assigned to strictly increasing flatter tariffs. Thus, we develop an enhanced version of the DP algorithm that takes into account the specific structure of the problem. Specifically, our algorithm constrains search such that multiple groups cannot form around the same tariff (c.f. Lemma 2) and agents are assigned to groups in monotonic order of their flatness.

The pseudocode of the method is given in Algorithm 1. This algorithm requires, as an input, the set of customers ordered by ascending coefficient of variation and the set of available tariffs ordered by increasing flatness. Then, for each

customer  $i = 1 \dots n$ , the algorithm computes the subproblem corresponding to the best group-buying structure among  $i$  and other more predictable consumers (lines 1-17). The cost of such a subproblem is stored in  $C[i]$ , the first customer not included in the group of  $i$  is stored in  $P[i]$ , while the best tariff for such group is stored in  $T[i]$ . At the beginning of each iteration, the corresponding group-buying structure is initialized to one in which all customers join a single group (lines 2-4). Then, the algorithm computes the minimum number of tariffs that must not be selected to allow the formation of a new group (i.e. one in the case of the last customer and two for the rest). Then, for each  $j < i$  such that the subproblem  $j$  left insufficient tariffs ( $T[j] < \bar{\tau}$ ), the cost of such a structure is compared (and updated if it is the case) with the cost of the group  $\{j + 1 : i\}$  plus the cost of the best group structure among the first  $j$  customers,  $C[j]$  (lines 10-16). The structure is only updated if the tariff assigned to the last formed group,  $T[j]$ , is less flat than those assigned to the new group,  $\tau$  (line 13). By so doing, partitions that do not satisfy the flatness order among tariffs are discarded.

At the end of this process, the cost of the best group-buying structure explored during the execution of the algorithm is found in  $C[n]$ . Finally, the algorithm executes an iterative procedure (lines 18-23) to recover the groups that form such a structure ( $P^*$ ) and the set of tariffs to which they are assigned ( $T^*$ ). Although Algorithm 1 has the same worst-case quadratic computational complexity as the basic DP algorithm, in the practical instances (such as those considered in the experiments), the number of operations is significantly reduced. In particular, for the two tariffs case ( $|\Gamma| = 2$ ), Algorithm 1 reduces to linear complexity (i.e.  $\bar{\tau}$  is 0 for all iterations with exception of the last).

## IX. EXPERIMENTAL EVALUATION

Our experimental analysis makes use of a large dataset of around 3000 consumers on housing estates in the south of UK. For each customer, the dataset included her electricity consumption for every half hour during a three-month period - from January 2010 to March 2010. We take the sample mean over the daily consumption realizations of each customer as a point estimate for his/her  $\mu$  and the standard deviation of the data as an unbiased estimator for his/her  $\sigma$ .

The evaluation considers three tariffs (F, P and P<sup>+</sup>). Tariff F (*Flat*), corresponds to a flat tariff in which customers pay a fixed rate ( $\text{£}0.205$ ) per kW consumed. Tariff P (*Predictive*) reduces the baseline rate of tariff F at the cost of charging a penalty of  $\text{£}0.01/\text{£}0.03$  for each kW under/overconsumed respectively. Finally, tariff P<sup>+</sup> (*Highly Predictive*) offers the lowest baseline rate but severely penalizes any imbalance (with penalties of  $\text{£}0.17$  and  $\text{£}0.26$  for under/overconsumption). These figures were chosen such that baseline rates of these tariffs are similar to current supplier rates in the UK tariffs (see [2]). However, the penalty rate of tariff P<sup>+</sup> is set particularly high to favour the formation of multiple buying groups, given the focus on group structure generation of this paper.

We perform two sets of experiments, illustrating different features of the POU tariff model. First (in Section IX-A), we

<sup>3</sup>Quadratic computational requirements and linear computational requirements to the number of consumers [38].

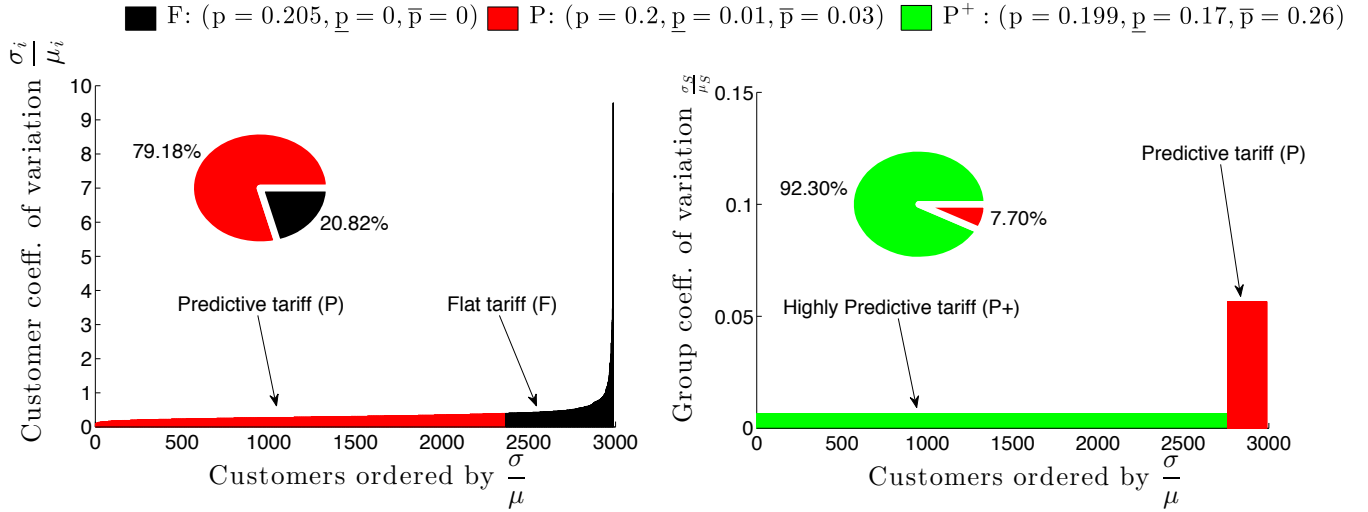


Fig. 2. Market segmentation when customers join tariffs individually (left); and through group-buying schemes (right). Tariffs are shown at the top.

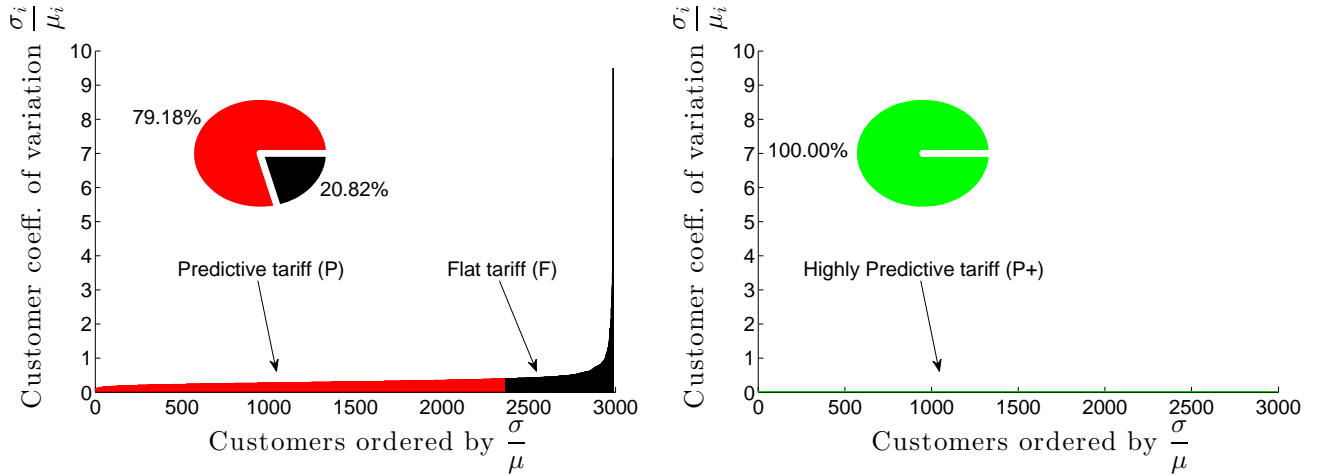


Fig. 3. Market segmentation, with same tariff parameters as in Fig. 2, except the  $P^+$  changed to  $P^+ : (0.199, 0.15, 0.2)$ .

study the optimal (i.e. minimising total payment) assignment of consumers to tariffs, and the resulting market segmentation, for both the case when consumers choose tariffs individually and through group buying. Second, in Section IX-B, we consider the allocation of the payments to the individual agents under a core-stable Shapley scheme, for a set of scenarios where consumer agents are divided into good, medium and poor predictors.

#### A. Optimal market segmentation of consumers into tariffs

In this section, given the data from 3000 consumers and our experimental set-up consisting of 3 tariffs (F, P and  $P^+$ ), we compute the optimal market segmentation of consumers around these tariffs. We first do this when consumer agents take an individually optimal decision about which tariff to join. Next, we do the same considering that consumers can join together under a tariff, through a group-buying scheme.

**Market segmentation with individual customer choice.** Fig. 2 (left) shows the coefficient of variation of each customer (ordered from the most to the least predictable), in the colour of its preferred tariff when joining individually. The figure also shows the share of customers that each tariff obtains as

a pie chart. Observe that the first 2366 customers with the lowest coefficient of variation prefer to join tariff P, whereas the remaining 622 customers prefer tariff F. Although a high percentage (i.e. 79.18%) of customers could benefit from a lower baseline price through tariff P, there remains a significant percentage (i.e. 20.82%) that are too unpredictable to benefit from this tariff, and choose the flat tariff F. Note that here tariff  $P^+$  gets no market share, as the risk of penalty imbalance under  $P^+$  remains too high for any individual customer. The total expected payment of the customers per day under a flat tariff is £8147.2, while the total payment with optimal market segmentation among the three POU tariffs with individual choice (without grouping) is £8097.

**Market segmentation with group buying.** We use the DP algorithm presented in the previous section to compute the most efficient group buying structure, under the constraint that customers with increasing coefficient of variation are assigned to flatter tariffs. Note the proposed DP algorithm has a worst-case quadratic performance in the number of customers, because of the additional structure of the problem it exploits, we found that in practice the number of operations is much less (around  $10^5$  or 2%) of the number of operations prescribed

by the quadratic bound (which is  $|N|^2/2 = 4.5 \cdot 10^6$ ). Fig. 2 (right) shows the coefficient of variation of the group joined by each customer (ordered from the most to the least predictable), again in the colour of the selected tariff. Observe that, by allowing grouping, the first 2758 customers, with the lowest coefficient of variation, prefer to group together under tariff  $P^+$  whereas the remaining 230 customers prefer to group under tariff  $P$ . The total expected payment of the customers per day under this market segmentation is £7952. Hence, there is a payment reduction for grouping this set of customers under such computed group-buying structure of £145 per day. It is worth noticing that the structure formed in the grouping case is completely different to the one from the individual choice case. Now, tariff  $F$  gets no market share, and the reduction of risk imbalance from grouping lets a high percentage of the customers (i.e. 92.30%) use the most predictive  $P^+$  tariff.

**Market segmentation after modifying tariff  $P^+$ .** Finally, we repeated the same experiments as above, by modifying tariff  $P^+$  (our most predictive tariff) to make it slightly less punitive of an imbalance (specifically, now  $\bar{p} = 0.15$  vs. 0.17 previously and  $\underline{p} = 0.2$  vs. 0.26). Experimental results for the new setting are shown in Fig. 3. With individual choice, results are the same (still no agent would choose  $P^+$  individually), but now in the group buying case, the grand coalition forms around the most predictive tariff  $P^+$ . Actually, in testing a number of realistic tariff configurations for this dataset, we found the grand coalition nearly always forms around the most predictive tariff in the case of group buying, due to the large reduction in prediction uncertainty achieved by 3000 consumers grouping together. However, as Fig. 2 shows this is not always the case, if the penalties for imbalance are set sufficiently high.

### B. Allocation of payments under a Shapley scheme

In this section, we consider how the total payment that each consumer agent makes for its bill varies, for a variety of scenarios. For all of these scenarios, we use the modified tariff  $P^+$  as defined above, (0.199, 0.15, 0.2), and two settings: one in which agents make predictions and payments under this tariff scheme individually, and one in which they join together as a group. For the case of group buying, the total payment is divided in a fair and core-stable way using the Shapley value solution concept<sup>4</sup>(defined in Section IV). Moreover, we use this section to explore a variety of realistic scenarios, in which good, medium and poor predicting consumers interact, and look at the savings they can get by grouping with consumers of other types.

To define such types, we order the customers from *good* predictors (with low standard deviation) to *poor* predictors (with high standard deviation). Then, Fig. 4(a) plots the expected daily payment of each customer when joining the  $P^+$  tariff on her own factored into two components: one corresponding to her consumption (in dark blue color) and another one corresponding to her penalty for imbalance (in light blue color). These correspond to the “consumption” and

“penalty” terms, as defined in Eq. 2 from Section III-B. Note that the penalty term is the only one that can be reduced by grouping and hence it is the focus of our analysis. These results show that while the total payment depends on both size and prediction error, the amount of penalty that each consumer pays when joining individually linearly increases with her standard deviation (i.e. the more predictable a customer consumption is, the lower the unit price that she pays for it). Given this, we cluster our consumer dataset into three types based on their standard deviation: *good* predictors ( $\sigma = 3.59$ ); *medium* predictors ( $\sigma = 8.87$ ), and *poor* predictors ( $\sigma = 17.9$ ). The characteristics of each customer type, as well as its frequency in the dataset (in parenthesis) are depicted in Fig. 4(a).

Then, Fig. 4(b) shows the expected payment of each customer type when joining individually and when grouped with the 3000 customers by types (i.e. using type frequencies as listed in Fig. 4(a)) under the Shapley allocation scheme. First, we observe that customers have incentives for grouping independently of their type (i.e. the expected penalty is significantly reduced for all types). Thus, the expected penalty under Shapley for a customer of the *good* type is reduced from £0.49 to £0.35 per day (i.e. a 28.5% reduction). Similarly, the expected penalty under Shapley for *medium* and *poor* customer types are reduced from £1.22 to £0.19 (i.e. 85% reduction) and from £2.25 to £0.70 (i.e. 69% reduction) per month respectively. Second, we observe that due to the fairness of the Shapley scheme, the penalty of the *poor* predictable type is higher than the penalty of the *medium* type which, in turn, is higher than those of the *good* type.

Now, whilst these results give us an idea of the incentives that each customer has to join our group-buying scheme in a real-world group sample, it also leads to the question of whether this result is affected by having relatively few *poor* predictable customers in the test set (i.e. 94% of customers are of the *good* predictable type). To study this, we run a second set of experiments in which we study how the incentives of customers vary with the inclusion of different proportions of agent types.

In more detail, Fig. 4(c) shows how the penalty in the Shapley value of a *good* and a *poor* predictor changes as the group is enlarged (up to 100 customers) with customers of the same type or with customers of the opposite type. Interestingly, it shows not only that a *good* predictor benefits from joining other *poor* predictors, but also that the incentive is greater than when joining with other *good* predictors. For example, compare the £0.27 daily penalty expected for a *good* predictor when joining a poor one, with the £0.35 per month penalty expected when joining another good predictor. A similar trend is observed for *poor* predicting customers: the reduction on their penalties when joining a group of *poor* predictors is greater than when joining a group of *good* predictors (£1.74 instead of £2.23 per month, for a group of 100 customers). Intuitively, this is because joining an agent with poor predictability provides a greater reduction in the joint prediction error.

<sup>4</sup>In general, computing the Shapley value exactly is exponential in the number of agents but, by restricting our setting to a number of agents types, this reduces the problem to one with a polynomial time complexity (c.f. [39]).

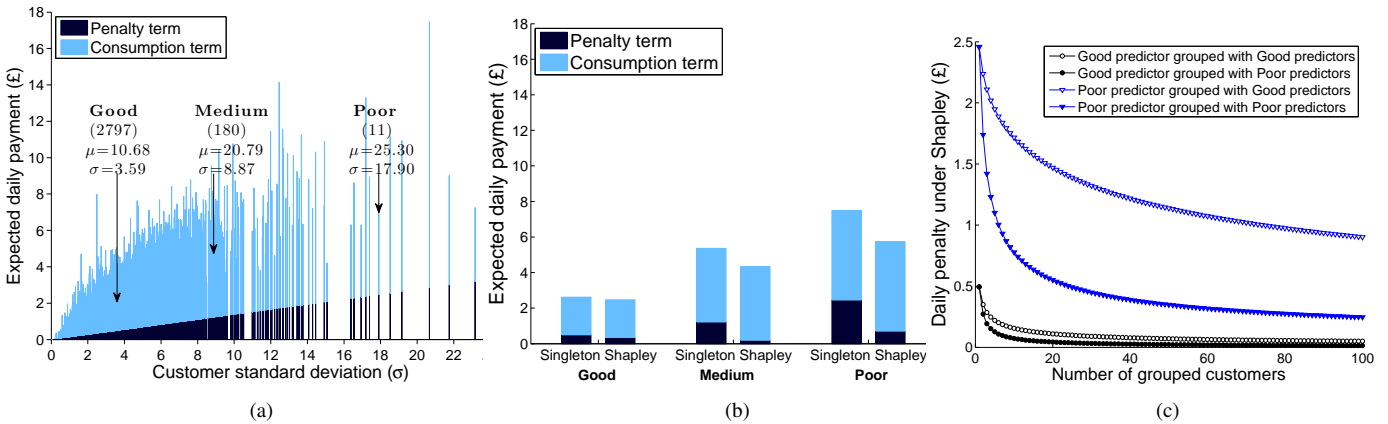


Fig. 4. Expected payments under  $P^+$  modified tariff for: (a) customers when joining individually ordered by predictability (increasing  $\sigma$ ); (b) customers types when joining individually and when grouping under Shapley; and (c) good/poor customers types when joining a group of all good/poor types (only the penalty term). Payments are split into the consumption term ( $\mu \cdot p$ ) and the remaining penalty term.

## X. CONCLUSIONS AND FUTURE WORK

This work provides a thorough analysis of market segmentation with multiple prediction-of-use tariffs. Besides giving a formal characterization of the problem, both for a single individual and group perspective, we provide a computationally tractable DP algorithm for determining efficient group buying structures and validate our approach on a large dataset of electricity consumers from the UK. Results demonstrate that the majority (i.e. 79.18%) of customers in the real-world dataset would benefit (i.e. lower their final electricity bill) from joining individually a more predictive POU tariff. Moreover, when considering group-buying schemes, this bill reduction gets larger and extends to all customers in the dataset (i.e. the group benefits from a further reduction of £145 per day w.r.t joining individually). Finally, distributing group benefit fairly using a Shapley allocation scheme results in individual reductions of the expected penalty, no matter the predictability of the customer (i.e. a customer with good predictability achieves a 28.5% penalty reduction whereas a customer with poor predictability achieves a 69% penalty reduction).

There are several directions we plan to explore in future work. From a practical perspective, we plan to explore the combination of POU with the more widely known time-of-use (TOU) tariffs [7]. For example, some suppliers may offer tariffs in which customers are only asked to predict their future consumption only for peak-time periods (i.e. using a POU structure for the peak times and a flat structure for the rest of the day). The advantage of this hybrid scheme over existing TOU pricing is that it will take into account the predictability rather than only the time at which energy is used. From a broader AI perspective [37], [40], this work can also be seen as opening the way for a wider range of techniques from coalitional game theory to be applied in the design of new group-buying schemes that allow customers to fully benefit from meaningfully managing their consumption as well as from their contribution to deliver energy efficient solutions.

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and algorithmic game theory techniques to building the next generation energy grids.



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