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Probabilistic Analysis of Interdependent Infrastructures Subjected to Weather-Related Hazards

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Abstract

Simulation of the infrastructure performance using numerical models may significantly assist in developing strategies for improving its resilience to harmful effects of climate change. The paper presents a model for simulating the performance of interdependent infrastructure systems based on an extended network flow approach, i.e., infrastructure systems are considered as a network of nodes connected by directed edges. The model has been specifically developed to simulate the infrastructure performance at a community scale and has higher node resolution compared to typical models of infrastructure systems at the national level. The model is time dependent so that the infrastructure performance can be assessed at discrete points over a period of time. Parameters describing the performance of infrastructure assets (e.g., production and flow capacities, demands) can be treated as random variables or probabilities can be assigned to failures of the assets. The application of the model is illustrated by the probabilistic assessment of the performance of two interdependent infrastructure systems – electrical power and water, damaged by flooding.

Keywords: Infrastructure systems, interdependency, community-scale, network flow model, failure, probabilistic analysis.

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Introduction

Infrastructure of many countries is currently subject to ever increasing loads due to population growth and suffers from deterioration associated with ageing that makes it more vulnerable. It also faces new challenges due to climate change that may lead to an increase in the frequency and intensity of weather-related hazards, e.g., floods, storms, heatwaves. Such hazards can harm energy and water supply, transport, communication routes and other infrastructure systems (e.g., Pitt 2008).

Hence, there is a clear need for efficient strategies to improve the infrastructure resilience, both at national and local levels, that will ensure its continuous and reliable performance in the future. Simulation of the infrastructure performance, including the effects of its failures, may significantly assist in developing such strategies. Numerical models that can be used for such purpose need to meet certain requirements. First, the models should be able to take into account interdependencies between different infrastructure systems, i.e., when failure in one system may cause severe disruptions and failures in other systems (e.g., Rinaldi et al. 2001, Kröger 2008). For example, many infrastructure systems (e.g., water, gas, telecommunications) depend on an electricity system to function (e.g., Adachi and Ellingwood 2008, Winkler et al. 2011). Second, since conditions of infrastructure and demands on it may change rapidly during and after a hazard event it is also important to be able to simulate the infrastructure performance over a period of time, e.g., from the moment the infrastructure has been damaged by the hazard and until its full restoration. To carry out such analysis time-dependent (or dynamic) models are required (e.g., Trucco et al. 2012). In addition, there are major uncertainties associated with the prediction of hazard occurrence, its effects and damage induced to infrastructure by these effects (e.g., Barker and Haimes 2009). Under such circumstances the use of probabilistic models capable of explicitly accounting for such uncertainties is highly desirable (e.g., Hernandez-Fajardo and Dueñas-Osorio 2013).

Different approaches have been employed so far to examine various performance and vulnerability characteristics of interdependent infrastructure systems. Their extensive review has recently been presented by Quyang (2014), who broadly divided them into the following types: (1) empirical, (2) agent-based, (3) system dynamics-based, (4) economic theory-based, (5) network-based, and (6) other approaches. Other reviews and classifications of infrastructure models are also available (e.g., Pederson et al. 2006, Eusgeld et al. 2008).

The present paper concentrates on modelling the performance of interdependent infrastructures at a local (or community) scale. This means that smaller infrastructure systems compared to those at a national level need to be considered but higher model resolution (i.e., ability to take into account relatively small infrastructure components like emergency generators) is required. Network-based approaches seem to be very suitable for this purpose. These approaches are further subdivided into topology-based, which analyse the vulnerability of infrastructure systems based on their topologies, and flow-based, which consider the flow of commodities delivered by infrastructures (Quyang 2014). Since another modelling objective of this study is to quantitatively assess the amount of commodities/services, which can be supplied by infrastructures to costumers during and after a hazard event, a network flow approach is selected (e.g., Ahuja et al. 1993).

The paper starts with a brief overview of network flow models which have been proposed for modelling the performance of various infrastructure systems, including their interdependencies. After that a model developed in this study for simulation of the performance of interdependent infrastructure systems at a local scale is presented. The model is based on the concept of a multifunctional node (Svendsen and Wolthusen 2007), when a node can simultaneously perform production, consumption, transhipment and storage functions associated with different commodities and be a part of different infrastructure systems at the same time. This also provides capabilities to simulate local infrastructure features such as storage facilities and emergency generators. The infrastructure performance can be modelled over a period of time (e.g., from the moment it has been damaged by a hazard until its full restoration) at discrete time steps. In order to simulate the operation of damaged infrastructure a variable representing unmet demand (or shortage) is introduced along with the corresponding cost (or penalty) that is similar to a slack variable in the model of Lee et al. (2007). To take into account uncertainties associated with the infrastructure performance, including possible damage of the infrastructure assets by a hazard, parameters of the model (e.g., production and flow capacities, demands) can be treated as random variables or probabilities can be assigned to failures of nodes and edges. The application of the model is illustrated by an example that shows the probabilistic assessment of the performance of interdependent infrastructure systems (electrical power and water) damaged by a weather related hazard (flood).

Network flow models of infrastructure systems

In network flow models, an infrastructure system is considered as a network of nodes (or vertices) connected by directed edges (or arcs). In the traditional network flow approach the nodes usually represent physical infrastructure assets (e.g., electricity substations, water treatment works, water pumping stations, road intersections, hospitals, office and residential buildings) associated with supply, transhipment and demand of resources (e.g., water, wastewater, electricity), which are referred to as commodities. The edges model the flow/movement of commodities between the nodes and represent, e.g., power transmission and distribution lines, water and wastewater pipelines, roads. Mathematically, this means that infrastructure is presented as a directional graph (or digraph) $G = (V, E)$, where V is the set of nodes and E is the set of directed edges. In order to find the flows on the edges the problem is then formulated as a linear programming problem that aims to optimise the infrastructure performance by minimising its total operational cost, C (Ahuja et al. 1993)

$$\text{minimise} \quad C = \sum_{e \in E} c^{f,e} f_e \tag{1}$$

$$\text{subject to} \quad f^{i\leftarrow} - f^{i\rightarrow} = \kappa^i \quad \forall i \in V \tag{2}$$

$$\text{and} \quad 0 \le f_e \le f_e^{max} \quad \forall e \in V \tag{3}$$

where f_e and f_e^{max} are the flow rate of commodity on edge e and the maximum flow capacity of this edge, respectively, $c^{f,e}$ the cost associated with this flow, $f^{i\leftarrow}$ and $f^{i\rightarrow}$ are the in-flow and out-flow rates at node i (i.e., the sums of the flow rates over all edges transferring the commodity into and out of the node), respectively, and κ^i the demand/supply rate of the commodity at node i (κ^i is positive if it represents demand and negative otherwise). Eq. (1) represents the total cost of commodity flow within the network and in the context of linear programming is called the objective function. The flows, f_e, are variables to be determined from the solution, while $c^{f,e}$ and κ^i are known coefficients (or parameters). Eq. (2) is the balance equation for the commodity at node i, while Eq. (3) sets bounds on the variables; in the context of linear programming these equations are referred to as constraints.

It should be noted that models based on the traditional network flow approach only ensure continuity of the commodity flow at nodes, while physical laws governing the flow within an infrastructure system may not be fully satisfied. However, such models are still employed to analyse individual infrastructure systems, e.g., for optimisation/planning of power and water systems (Padiyar and Shanbhaq 1988, Manca et al. 2010). A major advantage of the network flow approach in the analysis of multiple interdependent infrastructure systems is that the same mathematical formulation can be used to describe the flow of commodities in different systems (e.g., power, water, gas, transport).

The traditional network flow approach has several limitations that preclude its direct use for analysis of interdependent infrastructure systems, in particular when they are damaged. For example, only a balanced network, in which the total supply of a commodity equals its total demand, can be analysed; there are no tools to model storage of commodities as well as interdependencies between infrastructure systems; and changes in the flow of commodities over time cannot be considered. A number of extensions of the traditional network flow approach have been proposed to overcome these limitations. To account for possible damage to infrastructure, when the total supply of a commodity is less than its total demand, Lee et al. (2007) introduced slack variables, which represented shortfall in meeting demands at nodes, and the corresponding weighting factors, which represented the costs associated with the shortfall. In addition, variables were also introduced to describe interdependence between individual infrastructure systems, which were modelled by separate networks so that the model was called the interdependent layer network (ILN) model. The node types included in the model were the same as in the traditional network flow approach, i.e., supply, demand and transhipment; edges were able to model both single and multicommodity flows. The model was time-invariant and in order to consider any change over time a new set of input data needed to be prepared and a new analysis to be run. The ILN model was mathematically formulated as a mixed-integer linear programming problem. The model implementation was demonstrated by guiding the restoration of services of three damaged interdependent infrastructure systems: electrical power, telecommunications and subway. Arboleda et al. (2009) then used the model to determine cost-effective strategies for restoring interdependent infrastructure systems (electrical power, water and transportation of medical supplies), which support a health care facility, after a natural hazard event. A similar model was later proposed by Shen (2013). To allow for the integration of the restoration planning and scheduling decisions the ILN model was later extended to a time-variant model; for individual infrastructure systems that was done by Nurre et al. (2012) and for interdependent infrastructure systems – by Cavdaroglu et al. (2013). This was achieved by using a multiperiod formulation of a network flow problem. In this formulation a considered time horizon is divided into a number of time periods and the network to be solved is a composition of multiple networks, each of them representing the network under consideration at one of these time periods.

A different network flow model for the analysis of interdependent infrastructures was presented by Quelhas et al. (2007). The model was developed to simulate the movement of bulk energy flows (electricity, natural gas and coal) in the US integrated energy system. The model comprises two types of nodes: source (or supply) nodes and transhipment nodes, which also represent demand. The production of natural gas and coal, storage facilities, energy conversion facilities (power plants) were represented by the edges outgoing from added dummy source nodes so that all restrictions on the production, storage and energy conversion were expressed through parameters of these edges. In particular, the so-called efficiency parameter, which related the flows at the beginning and at the end of an edge, was introduced. In order to account for the evolution of the system over time the multiperiod formulation, as in Cavdaroglu et al. (2013), was used. The model was later extended by Gil and McCalley (2011) to simulate large-scale disruptions in the system; in particular, to ensure a feasible solution when due to disruptions demand exceeded available supply a dummy supply node was added to the network. This node was connected to all demand nodes via edges having unlimited capacities. In order to prevent flows on these edges as long as supply met demand the costs associated with these flows were set higher compared to other edges.

Svendsen and Wolthusen (2007) proposed a conceptual network flow model with multifunctional nodes, where the same node could act both as a consumer and a producer and also had storage facilities. The latter required explicit consideration of time since the amount of a stored commodity could change over time. This was achieved by considering the network states at discrete points in time. However, the model was not presented in enough detail and examples provided in the paper dealt mainly with topological analysis of infrastructure networks.

Extended network flow model

Model outline

The network flow model, which is described in the following, is an extension of the traditional network flow approach. The model is developed to simulate the performance of interdependent infrastructure systems at a local (community) scale under normal and extreme conditions (i.e., when infrastructure is damaged by weather related hazards). Like in the traditional approach infrastructure is presented as a digraph $G = (V, E)$. The nodes, V, represent infrastructure assets associated with production (including supply), consumption (including demand), transhipment and storage of commodities; e.g., electricity substations, water and wastewater treatment plants, water towers and pumping stations, which are mainly associated with production and transhipment of commodities, as well hospitals, shops, office and residential buildings, which are consumers of commodities. The edges, E, are similar to those in the traditional approach and each edge models the flow of a single commodity; e.g., power transmission and distribution lines, water and wastewater pipelines, etc.

The following extensions compared to the traditional approach have been made. First, when infrastructure is damaged a network representing the infrastructure may become unbalanced, i.e., the total demand of commodity, k, may exceed its available supply. To be able to model this, a variable, λ_k, representing unmet demand/shortage (i.e., the difference between actual demand and available amount of commodity) is introduced, along with the corresponding cost (or penalty), $c^{\lambda}{}_k$. Second, resilience of infrastructure systems can be increased through the use of local storage and production (e.g., emergency generators) facilities. To take these into account variables representing storage, s_k, and production, π_k (with the corresponding costs, $c^s{}_k$ and $c^{\pi}{}_k$, respectively) are added to the model. This means that a commodity may be available from supply (i.e., from sources outside of the considered local infrastructure), local production and storage. However, to produce commodity k another commodity (or commodities), n, may need to be consumed. To distinguish between traditional supply and internal production, the variable π_k is expressed as the sum of a variable $\pi_{k,k}$ representing traditional supply (i.e., supply from sources outside the boundaries of the local infrastructure, when commodities needed for the production of k are ignored) and $\pi_{n,k}$ representing the internal production (i.e., consumption of commodity n to produce k is taken into account). In a similar way, a variable κ_k representing consumption of commodity k is the sum of a variable $\kappa_{k,k}$, which denotes traditional demand, and $\kappa_{k,n}$ denoting consumption of k for production of commodity n. Third, the introduction of $\pi_{n,k}$ and $\kappa_{k,n}$ also enables to take into account interdependencies between different infrastructure systems as these variables are used to model the situations when the production of a commodity in one infrastructure system requires the consumption of a different commodity delivered by another system. Some of the definitions introduced above are illustrated in Figure 1.

Fourth, parameters of the infrastructure network under consideration (e.g., production capacities, consumer demands, flow capacities, amounts of stored commodities) may change with time. In order to take this into account the model is formulated as time-variant. This is done in the following way. The solution process starts at time t_0, which in principle may be any time depending on the purpose of the analysis. For example, the analysis can start just before a hazard strikes and the infrastructure network becomes damaged. After that, solutions are sought at discrete time steps Δt until, e.g., the infrastructure is restored. Since it is assumed that all variables and constants (except of the amount of stored commodities) remain unchanged within a time step, Δt may be different at different time steps. This time-variant formulation is similar to the one proposed by Svendsen & Wolthusen (2007).

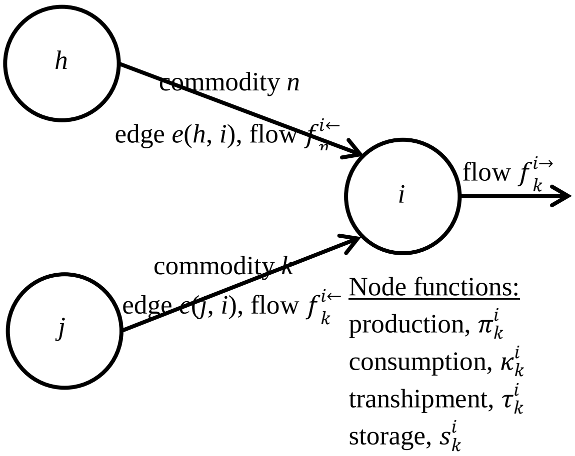

Figure 1. Basic elements of the extended network flow model.

Finally, there are significant uncertainties associated with the prediction of infrastructure damage caused by weather related hazards. Moreover, demands for commodities may also be uncertain. This can be taken into account by (i) assigning probabilities of failure to infrastructure assets represented by nodes and edges, and/or (ii) treating the demands, the production and/or flow capacities as random variables. When only the first approach is employed several (e.g., m) components (i.e., nodes and/or edges) of the infrastructure network under consideration may be in two states – of failure with probability $P_{f,j}$ and operational with probability ($1-P_{f,j}$), where $P_{f,j}$ denotes the probability of failure of the j-th component ($j = 1, \ldots, m$). It is worth to note that m may be significantly less than the total number of the network components since only a few of them may be vulnerable to a particular weather-related hazard. The whole infrastructure network then can be in 2^m states, each of which depends on the states of the components. The probabilities of occurrence of each of these infrastructure states can be easily calculated, e.g., the probability of the state when all the components are operational is $\prod_{j=1}^{m}(1-P_{f,j})$. When the number of the states is not too large (around 10^5 or less), each of the states can be analysed separately and results of the analyses can be aggregated in a probabilistic format (e.g., the probability distributions of commodity supplies available to customers at different points in time can be derived). The number of the states that need to be analysed can be reduced by neglecting the states with very low probabilities of occurrence. When a number of the infrastructure network parameters are represented by random variables a probabilistic analysis of the network can be carried out using Monte Carlo simulation. Since very efficient computational methods are available for solving linear programming problems the time required for the analysis of a medium-sized network (i.e., ranging between tens to hundreds of nodes that is typical for interdependent infrastructure systems at a local scale) should not be excessively long.

Node balance equations

In this section the relationships between various variables and constants representing the four functions of a node (see Figure 1) are described.

The balance equations at node *i* for commodity *k* are

$$f_k^{i\leftarrow} + \pi_k^i - \kappa_k^i - \tau_k^i - s_k^i = 0 \tag{4}$$

$$\tau_k^i - f_k^{i\rightarrow} = 0 \tag{5}$$

where $f_k^{i\leftarrow}$ and $f_k^{i\rightarrow}$ are the in-flow and out-flow rates, respectively, π_k^i the production rate, τ_k^i the transhipment rate, and s_k^i the rate of transferring the commodity to/out of storage. If no cost is associated with the transhipment of the commodity and there is no risk of damage of the transhipment function of node *i* then these two balance equations can be replaced by a single one

$$f_k^{i\leftarrow} + \pi_k^i - \kappa_k^i - f_k^{i\rightarrow} - s_k^i = 0 \tag{6}$$

It is worth to note that different commodities can be processed at a node. Hence, the balance equations should be satisfied at the node for each of these commodities. In-flow and out-flow rates, $f_k^{i\leftarrow}$ and $f_k^{i\rightarrow}$, are the sums of the flow rates over all edges transferring commodity *k* into and out of the node *i*, respectively.

The production rate of commodity *k* at node *i* is expressed as

$$\pi_k^i = \pi_{k,k}^i + \sum_{n\neq k} \pi_{n,k}^i \tag{7}$$

where $\pi_{k,k}^i$ is the supply rate, when the need in another commodity to produce commodity *k* is not taken into account (usually, at nodes that represent sources of supply of commodity *k* outside the boundaries of the local infrastructure network), $\pi_{n,k}^i$ the rate of production of commodity k that involves consumption of another commodity n. It is assumed that there is a linear relationship between the amounts of produced and consumed commodities so that

$$\pi_{n,k}^{i} = \alpha_{n,k}^{i} \kappa_{n,k}^{i} \tag{8}$$

where $\alpha_{n,k}{}^{i}$ is the coefficient relating the production rate of commodity k to the corresponding consumption rate, $\kappa_{n,k}{}^{i}$, of commodity n. Thus, Eq. (8) becomes

$$\pi_{k}^{i} = \pi_{k,k}^{i} + \sum_{n \neq k} \alpha_{n,k}^{i} \kappa_{n,k}^{i} \tag{9}$$

The assumption about a linear proportionality between produced and consumed commodities (Eq. (8)) is essential in order to formulate the model as a linear programming problem. This assumption is often correct only when additional conditions are met. For example, the ratio between the electrical power produced by a generator and the rate of the corresponding fuel consumption remains constant as long as the generator operates at a constant power, e.g., at its rated capacity. Similarly, the ratio between the flow of pumped water and the power consumed by the pump is constant when the total head and the pump speed do not change. If there is a clear nonlinearity in the relationship between the amounts of produced and consumed commodities in order to maintain a linear programming formulation this relationship can be approximated by a piecewise linear function.

The consumption rate of commodity k at node i is given by

$$\kappa_{k}^{i} = \kappa_{k,k}^{i} + \sum_{n \neq k} \kappa_{k,n}^{i} \tag{10}$$

where $\kappa_{k,k}{}^{i}$ is the rate of consumption of commodity k to satisfy demand of consumers, $\kappa_{k,n}{}^{i}$ the rate of consumption of commodity k to produce another commodity n. In order to take into account the possibility that not all demand is met when infrastructure is damaged $\kappa_{k,k}{}^{i}$ is presented as

$$\kappa_{k,k}^{i} = \kappa_{k}^{i,max} - \lambda_{k}^{i} \tag{11}$$

where $\kappa_{k}{}^{i,max}$ is the actual demand rate, i.e., the maximum/required rate of consumption of commodity k at node i at a given time and $\lambda_{k}{}^{i}$ the rate of unmet demand (or shortage). Eq. (10) can then be written as

$$\kappa_{k}^{i} = \kappa_{k}^{i,max} - \lambda_{k}^{i} + \sum_{n \neq k} \kappa_{k,n}^{i} \tag{12}$$

The introduction of storage as one of the node functions makes the model formulation time-variant. The amount of commodity *k* stored at node *i* at time *t* is denoted as $\omega_k^i(t)$

$$\omega_k^i(t) = \omega_k^i(t - \Delta t) + s_k^i \Delta t \tag{13}$$

where s_k^i is the rate of change in the amount of stored commodity and Δt the time step.

Taking into account Eqs. (9) and (12), Eq. (4) can be written as

$$f_k^{i\leftarrow} + \pi_{k,k}^i + \sum_{n\neq k} \alpha_{n,k}^i \kappa_{n,k}^i - \kappa_k^{i,max} + \lambda_k^i - \sum_{n\neq k} \kappa_{k,n}^i - \tau_k^i - s_k^i = 0 \tag{14}$$

Eq. (5) remains unchanged, while Eq. (6) can be updated in a similar way to Eq. (4).

Formulation of linear programming problem

As in the traditional network flow approach (see Eq. (1)), this extended model is formulated as a linear programming problem that minimises the cost of running of an infrastructure network. This cost, *C*, per unit of time within the time interval (t-Δt, t) can expressed as

$$C = \sum_{k=1}^{K} \left[\sum_{e\in E} c_k^{f,e} f_k^e + \sum_{i\in V} c_k^{\pi,i} \pi_k^i + \sum_{i\in V} c_k^{\tau,i} \tau_k^i + \sum_{i\in V} c_k^{\lambda,i} \lambda_k^i + \sum_{i\in V} c_k^{\omega i} \frac{1}{2} \left(\left(\omega_k^i(t) + \omega_k^i(t-\Delta t) \right) \right) \right] \tag{15}$$

where $c_k^{f,e}$, $c_k^{\pi,i}$, $c_k^{\tau,i}$, $c_k^{\lambda,i}$ and $c_k^{\omega,i}$ are the costs associated with flow, production, transhipment, shortage and storage of commodity *k* per unit of time, respectively, and *K* the total number of commodities. Note that the cost of storage (the last sum in Eq. (15)) is based on the average amount of stored commodity within the time interval (t-Δt, t). If to substitute Eqs. (9) and (13) into Eq. (15) the total cost becomes

$$C = \sum_{k=1}^{K} \left[\begin{array}{l} \sum_{e\in E} c_k^{f,e} f_k^e + \sum_{i\in V} c_k^{\pi,i} \left(\pi_{k,k}^i + \sum_{i\in V, n\neq k} \alpha_{n,k}^k \kappa_{n,k}^i \right) + \sum_{i\in V} c_k^{\tau,i} \tau_k^i + \sum_{i\in V} c_k^{\lambda,i} \lambda_k^i + \frac{1}{2} \sum_{i\in V} c_k^{\omega,i} s_k^i \Delta t \\ + \sum_{i\in V} c_k^{\omega,i} \omega_k^i(t-\Delta t) \end{array} \right] \tag{16}$$

where f_k^e, $\pi_{k,k}^i$, $\kappa_{n,k}^i$, τ_k^i, λ_k^i and s_k^i are the variables to be determined from the optimisation problem solution, while the rest of the parameters appearing in this equation are constants. This means, in particular, that the last sum in Eq. (16) is a constant and should be excluded from the objective function. In order to obtain the total cost, including the full cost of storage, this term can be added after solving the optimisation problem.

The optimization problem can then be formulated as minimise

$$C = \sum_{k=1}^{K}\left[\sum_{e\in E} c_k^{f,e} f_k^{e} + \sum_{i\in V} c_k^{\pi,i}\left(\pi_{k,k}^{i} + \sum_{i\in V, n\neq k}\alpha_{n,k}^{k}\kappa_{n,k}^{i}\right) + \sum_{i\in V} c_k^{\tau,i}\tau_k^{i} + \sum_{i\in V} c_k^{\lambda,i}\lambda_k^{i} + \frac{1}{2}\sum_{i\in V} c_k^{\omega,i} s_k^{i}\Delta t\right] \qquad (17)$$

subject to the following general constraints:

$$f_k^{i\leftarrow} + \pi_{k,k}^{i} + \sum_{n\neq k}\alpha_{n,k}^{i}\kappa_{n,k}^{i} + \lambda_k^{i} - \sum_{n\neq k}\kappa_{k,n}^{i} - \tau_k^{i} - s_k^{i} = \kappa_k^{i,max} \quad \forall i\in V \text{ and } \forall k\in K \qquad (18)$$

$$\tau_k^{i} - f_k^{i\rightarrow} = 0 \quad \forall i\in V \text{ and } \forall k\in K \qquad (19)$$

$$\pi_{k,k}^{i} + \sum_{n\neq k}\alpha_{n,k}^{i}\kappa_{n,k}^{i} \le \pi_k^{i,max} \quad \forall i\in V \text{ and } \forall k\in K \qquad (20)$$

and variable bounds:

$$0 \le f_k^{e} \le f_k^{e,\max} \quad \forall e\in E \text{ and } \forall k\in K \qquad (21)$$

$$0 \le \pi_{k,k}^{i} \quad \forall i\in V \text{ and } \forall k\in K \qquad (22)$$

$$0 \le \kappa_{n,k}^{i} \quad \forall i\in V, \forall k\in K, \forall n\in K \text{ and } n\neq k \qquad (23)$$

$$0 \le \lambda_k^{i} \quad \forall i\in V \text{ and } \forall k\in K \qquad (24)$$

$$0 \le \tau_k^{i} \le \tau_k^{i,max} \quad \forall i\in V \text{ and } \forall k\in K \qquad (25)$$

$$-\frac{\omega_k^{i}}{\Delta t} \le s_k^{i} \le \frac{\omega_k^{i,\max} - \omega_k^{i}}{\Delta t} \quad \forall i\in V \text{ and } \forall k\in K \qquad (26)$$

where $f_k^{e,max}$ is maximum flow capacity on edge *e*, $\pi_k^{i,max}$ the maximum production capacity of commodity *k* at node *i*, $\tau_k^{i,max}$ the maximum transhipment capacity for commodity *k* at node *i* and $\omega_k^{i,max}$ the maximum storage capacity for commodity *k* at node *i*.

It is important to note that minimising the total cost separately at each time step does not give an optimal solution for the network performance over the whole time period under consideration. For example, when an infrastructure network is undamaged it is obvious that the optimal amount of a stored commodity is zero – if everything works properly there is no need for stored commodities because their storage incurs additional costs. However, when the network is damaged by a hazard the availability of commodities from storage may reduce the total cost (since there are additional high costs penalising for unmet demand). This problem can be resolved either by artificially setting negative costs for storage and assuming that the amount of a stored commodity equals the maximum capacity of the corresponding storage (under normal conditions this will prevent the use of stored commodities and adding more commodities to storage because of negative storage costs) or minimizing the total cost over all considered time steps, i.e., using the multiperiod formulation as in (Quelhas et al. 2007, Cavdaroglu et al. 2013). The second approach seems preferable. However, in order to use this approach the time required for restoring the infrastructure to its original undamaged condition needs to be known that may be difficult to predict. It should also be noted that the first approach does not allow to optimise the amount of stored commodities during a single analysis but this shortcoming can be overcome by running various scenarios with different storage capacities.

Probabilistic analysis of local infrastructure network

The application of the proposed model is illustrated by a hypothetical example, which examines the performance of two interdependent infrastructure systems (electrical power and water, see Figure 2) at a local level during a flood event. The power system includes four electricity substations - a primary substation and three secondary substations. The water system includes a water treatment plant, a water tower and a water pumping station, which consume electricity to pump water. Their maximum capacities are given in Table 1. It is assumed that 1 kWh is required to pump 7.5 m^3 of water. There are three nodes representing consumers: a hospital, a care home and surrounding residential area and a residential area; their daily demands for electricity and water are given in Table 2.

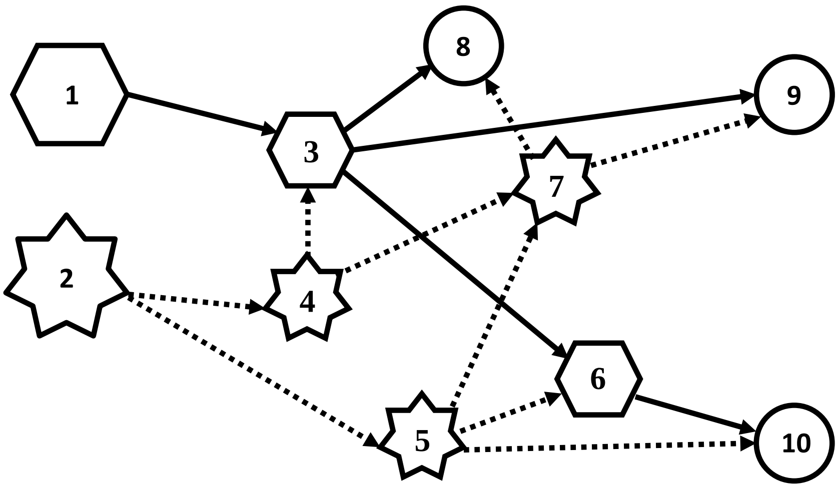

Figure 2. Local infrastructure network. Edges: solid lines - water pipelines, dashed lines – distribution power lines.

Node	Asset	Maximum capacity
	Electrical power system	
2	Primary substation	20 MWh/day
4,7	Secondary substation	5 MWh/day
5	Secondary substation	10 MWh/day
	Water system	
1	Water treatment plant	1000 m^3/day
3	Water tower	550 m^3
6	Water pumping station	200 m^3/day

Table 1. Description of infrastructure systems.

Node	Consumer	Daily demands	
		Electricity (MWh/day)	Water (m^3/day)
8	Hospital	1.3	35
9	Care home & residential area	3.1	75
10	Residential area	6.0	165

Table 2. Electricity and water demands by consumers.

Two options are considered: (i) without emergency generators; (ii) with emergency generators at the nodes 3, 6 and 8. Each generator has a power of 12 kW, a storage tank for 100 litres of fuel, and can produce 3 kWh per litre of consumed fuel (i.e., diesel).

It should be noted that in this example the costs of flow, storage, production and shortage do not represent actual costs and are assigned to ensure that the commodities are distributed between the consumers as intended. For example, in order to prevent the use of stored commodities (i.e., water and fuel), until there is no other way to meet demands, the costs of their storage are set negative. It is also assumed that among the consumers the hospital is the most important one and hence has the highest priority in receiving required commodities, followed by the care home and then the residential area. The costs of shortages of water and electricity for these consumers are set accordingly, i.e., the highest costs for the hospital, lower costs for the care home and the lowest ones for the residential area.

The performance of the infrastructure network during a 1-in-100-year flood, which usually represents the design event, is analysed. The three secondary electricity substations (nodes 4, 5 and 7) are located on a floodplain and may need to be shutdown depending on inundation depths. Flood modelling is not considered explicitly in this paper. It is assumed that a probabilistic flood map has been obtained for the plain using a hydraulic model and taking into account relevant sources of uncertainty. As a result, distribution functions, $f(d)$, of the inundation depth, d, at the locations of the electricity substations are available. Flood defences have been planned for the substations; taking these defences into consideration fragility curves, showing the probability of a substation flooding for a given inundation depth, $P_{f|d}(d)$, have been constructed. The probability of failure (i.e., flooding and subsequent shutdown), P_f, of an electricity substation can then be estimated as

$$P_f = \int_0^{\infty} P_{f|d}(d) f(d) dd \qquad (27)$$

For the three substations the following probabilities of failure in the case of the 100-year flood have been obtained: node 4 - 0.4, node 5 - 0.6 and node 7 - 0.2. Since the substations can only be in two states (operational and shutdown), the whole network can be in 8 (=2^3) states, whose probabilities are easily calculated analytically. The infrastructure network will function in a partially damaged condition until the secondary substations will return to operation, which may take several days. The aim of the example is to analyse what happens with the supply of electricity and water to the consumers when it takes up to six days to restore these substations. The analysis is carried out using a daily time step (i.e., Δt = 1 day). Each of the eight network states is analysed separately and results of the analyses are then aggregated using the probabilities of occurrence of the states in order to obtain the probability distributions of the amounts of electricity and water that can be supplied to the consumers. These amounts are expressed as a percentage of the corresponding demands.

Figures 3 and 4 present the results for the case without emergency generators. Figure 3 shows the probability distributions of the electricity supply to the consumers. For the hospital (node 8) and the care home with surrounding it residential area (node 9) only two events are possible – full (100%) supply with probability 0.61 and no (0%) supply with probability 0.39; for the residential area there are three possible events – 100% supply with probability 0.27, 0% supply with probability 0.60 and 93% supply (i.e., 93% of the daily electricity demand are supplied) with probability 0.13. As can be seen, disruptions in electricity supply may start on the first day and the results do not change over the considered

6 days. Disruptions in water supply (Figure 4) may occur on the first day only in the residential area (node 10) due to failure to supply electricity to the water pumping station (node 6). The water stored in the water tower ensures that this commodity is supplied in full during the first two days to the hospital (node 8) and the care home with surrounding it residential area (node 9). Disruptions in water supply to these consumers may start on the third day and the situation worsens on the sixth day.

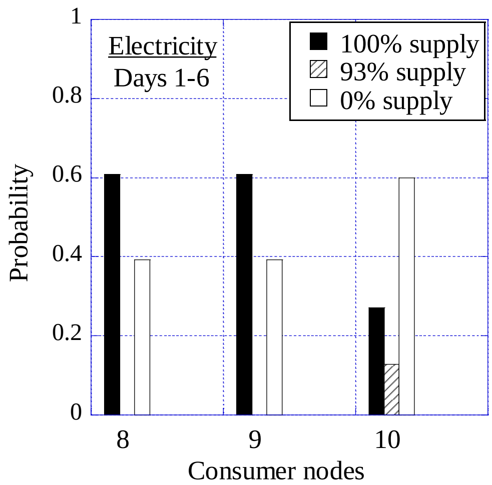

Figure 3. Probabilities of electricity supply to the consumers (without emergency generators).

The corresponding results for the case, when the emergency generators are used, are not shown in graphical form. The main difference is that in this case there are no disruptions in water supply to all three consumers within the considered time period. Moreover, it has further been checked that with the relatively small amount of stored fuel the emergency generators at the water tower and the water pumping station are capable to ensure continuous supply of water to the consumers for up to 11 days when the electricity substations are not functioning. However, the emergency generator at the hospital has a relatively small effect on the situation with electricity supply – from the second day the results are exactly the same as in Figure 3. To have a more noticeable effect on the electricity supply to the hospital the amount of stored fuel for this generator needs to be increased.

It is worth to note that the model can be used to examine the performance of much more complicated infrastructure networks, with higher temporal resolution (e.g., hourly) and also enables to treat, if necessary, parameters of the network (e.g., demands, production and/or flow capacities) as continuous random variables.

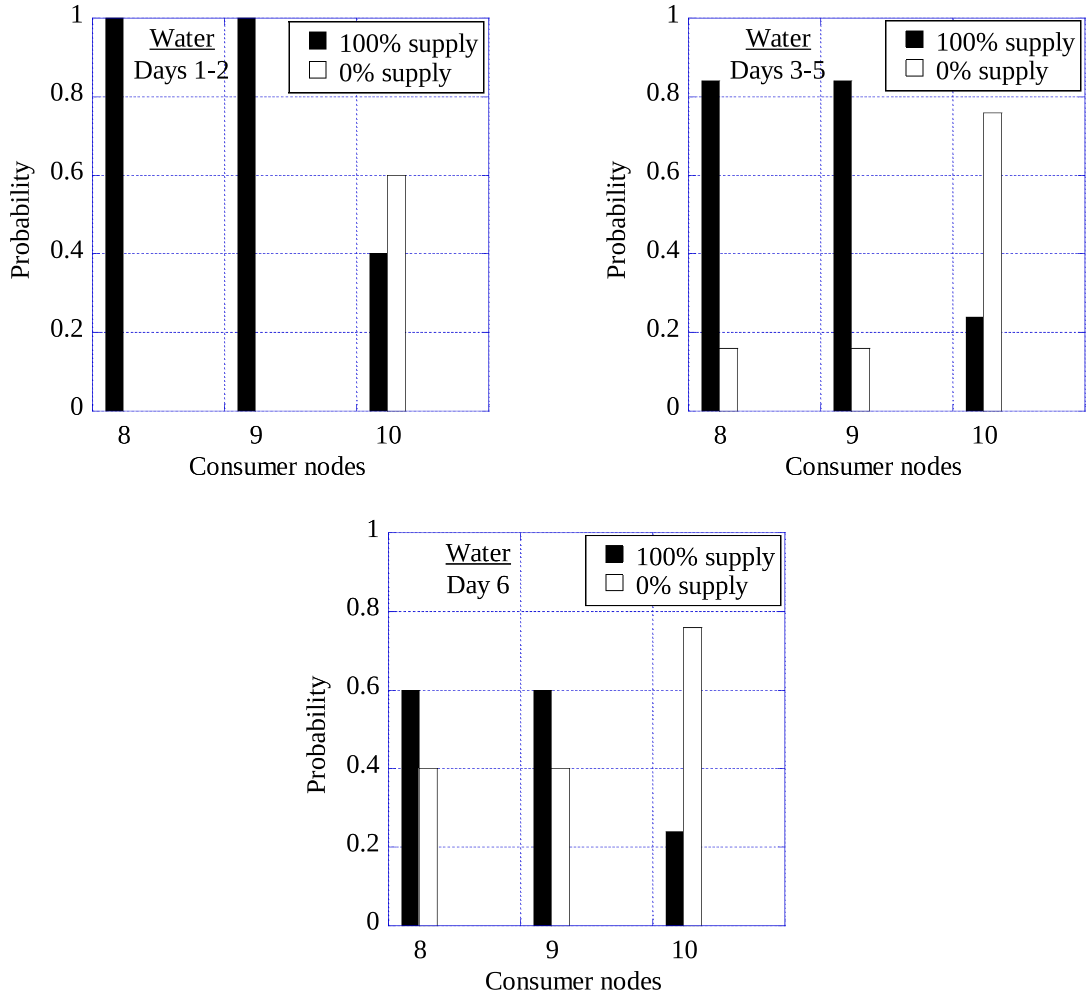

Figure 4. Probabilities of water supply to the consumers (without emergency generators).

Conclusions

A network flow model for simulation of the performance of interdependent infrastructures at a local scale has been presented in the paper. The model allows to conceive of a number of different infrastructure systems as a single network which ensures that their interdependencies are taken into account. A novel aspect of the model is a multifunctional node, which is capable to simulate production, consumption, transhipment and storage processes. The model also enables to carry out time-variant analysis and analyse networks with unbalanced supply and demand (the latter is essential to simulate the performance of damaged infrastructures). An example demonstrating the model application has been provided. Functioning of two small interdependent infrastructure systems – electrical power and water, during a flood event has been considered. It has been shown how uncertainties associated with damage/failure of the infrastructure assets can be taken into account. The example has also demonstrated how the model may be used to redistribute commodities between various consumers in accordance to the consumers' relative importance when the demand for these commodities exceeds their available supply.

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